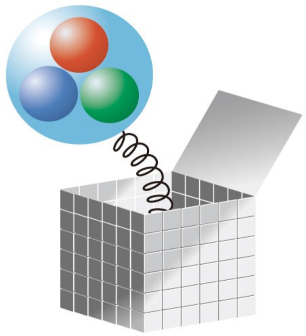
Light Meson spectrum with Nf=2+1 dynamical overlap fermions



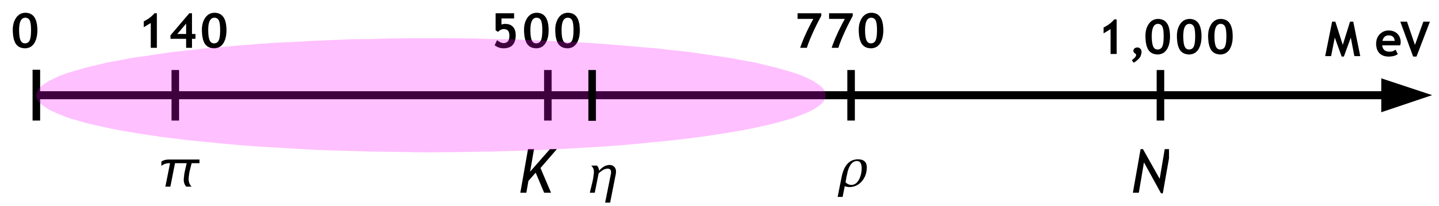
Jun Noaki (KEK)

for JLQCD-TWQCD Collaborations:

S.Aoki, T-W.Chiu, H.Fukaya, S.Hashimoto, T-H.Hsieh, T.Kaneko, H.Matsufuru, T.Onogi, E.Shintani and N.Yamada

Introduction

- QCD vs ChPT? (m_K OK?)



  - Lattice QCD (1st principle) ⟷ ChPT
  - Chiral properties of meson masses and decay consts.

- Numerical simulation with dynamical overlap fermions

  Neuberger, 1998

  - Exact chiral symmetry, No O(a) errors
  - Direct application of ChPT: no need of WChPT, SChPT, tmChPT...
  - Many technical challenges

    see S.Hashimoto's plenary, H.Matsufuru's poster

Plan

- Numerical simulation (Nf=2 and 2+1)
  - Setup parameters
  - Correlation functions with eidenmodes
- Analysis (Nf=2+1)
  - Finite size effects
  - Non-perturbative renormalization
- Chiral extrapolation
  - Results on the Nf=2 calculation
  - Preliminary results on the Nf=2+1
- Summary

Simulation setup

- Parameters

action: $S_{ov} + S_{Iwasaki} + S_{ex\text{-}Wilson}$ $(Q_{top} = 0)$

	$N_f = 2$	$N_f = 2+1$ (two ms's)
volume:	$16^3 \times 32$ $(1.9^3 \times 3.8\ \mathrm{fm}^4)$	$16^3 \times 48$ $(1.7^3 \times 5.2\ \mathrm{fm}^4)$
#config:	500 10,000 trajs./20	500 2,500 trajs./5
Cutoff: (r0=0.49 fm)	$1.67(2)(2)$ GeV	$1.83(1)$ GeV
quark mass:	6	5 x 2
mass range:	$290 - 750$ MeV	$315 - 810$ MeV $315 - 720$ MeV

With low-lying eigenmodes (Nf=2+1)

- Lowest 80 eigenpairs stored on disk

$$D_{ov} u_i = \lambda_i u_i$$

- Quark propagator   Giusti et al, 2003

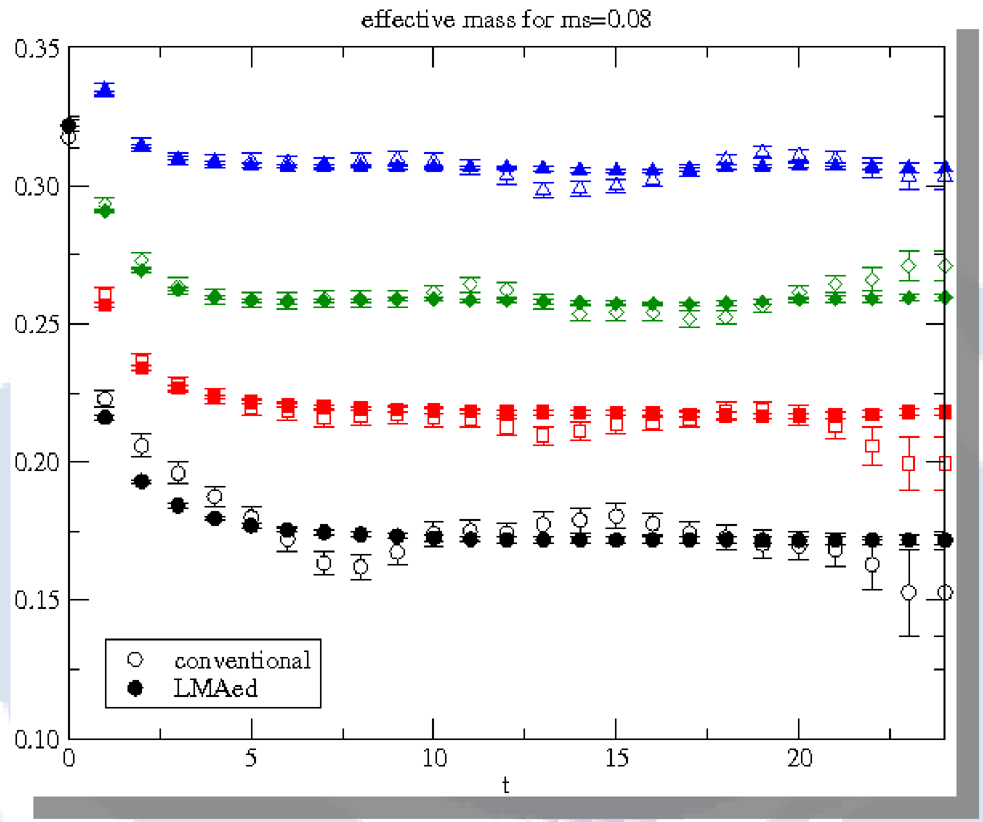
$$S_q(x,y) = \frac{\sum_{i=1}^{80} u_i(x) u_i(y)}{\lambda_i + m_q} + S_q^{\mathrm{High}}(x,y)$$

- Meson correlator

DeGrand & Schaefer, 2004;
Giusti et al., 2004

$$C(t) = C^{HH}(t) + C^{LH}(t) + C^{HL}(t) + C^{LL}(t)$$

Average over source locations (time slices)



Finite size effects

- $m_\pi L \simeq 2.7$ for the lightest mass
  - Standard FSE
  - Fixed topology effect (Q=0) Brower et al, 2003
- Correction with ChPT ⬅ exact chiral symm.
  - Standard FSE: resummed Luscher's formulae Colangelo et al, 2005
  - Fixed topology effect (NLO ChPT):

eg. Nf=2 ChPT

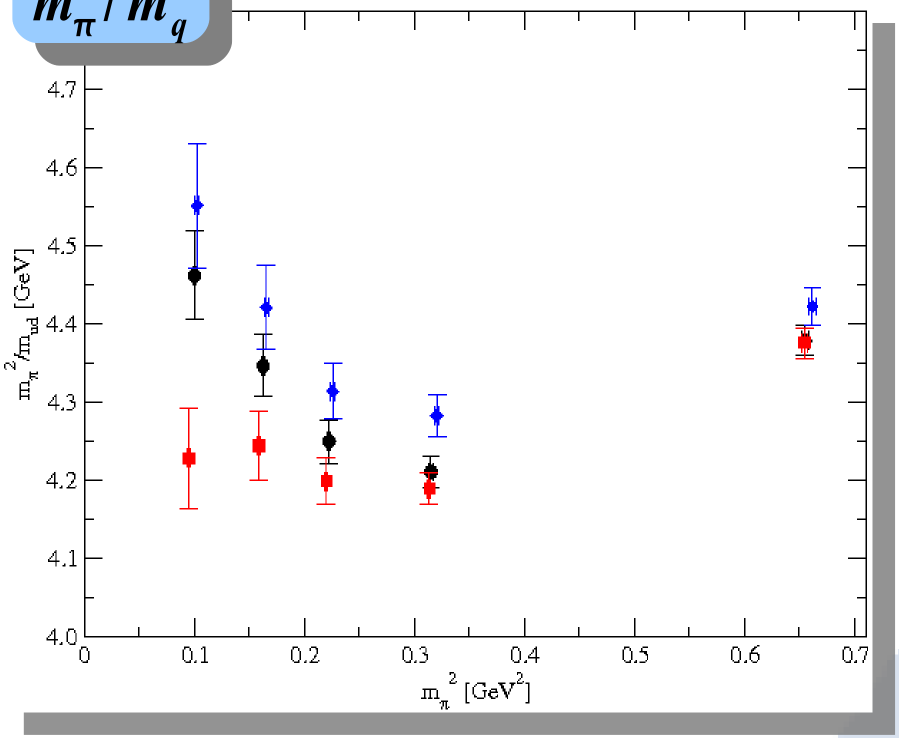
$$\frac{m_{\pi,Q=0}}{m_\pi(\theta=0)} = 1 - \frac{1}{16 V \chi_{\mathrm{top}}}\left[1 + \xi\left(1 + \ln\left(m_\pi/\Lambda_3\right)^2\right)\right]$$

$$\frac{f_{\pi,Q=0}}{f_\pi(\theta=0)} = 1 + \frac{1}{4 V \chi_{\mathrm{top}}}\xi\left(1 + \ln\left(m_\pi/\Lambda_4\right)^2\right)$$

Aoki et al, 2007; JLQCD-TWQCD, 2007; see also Chiu's talk

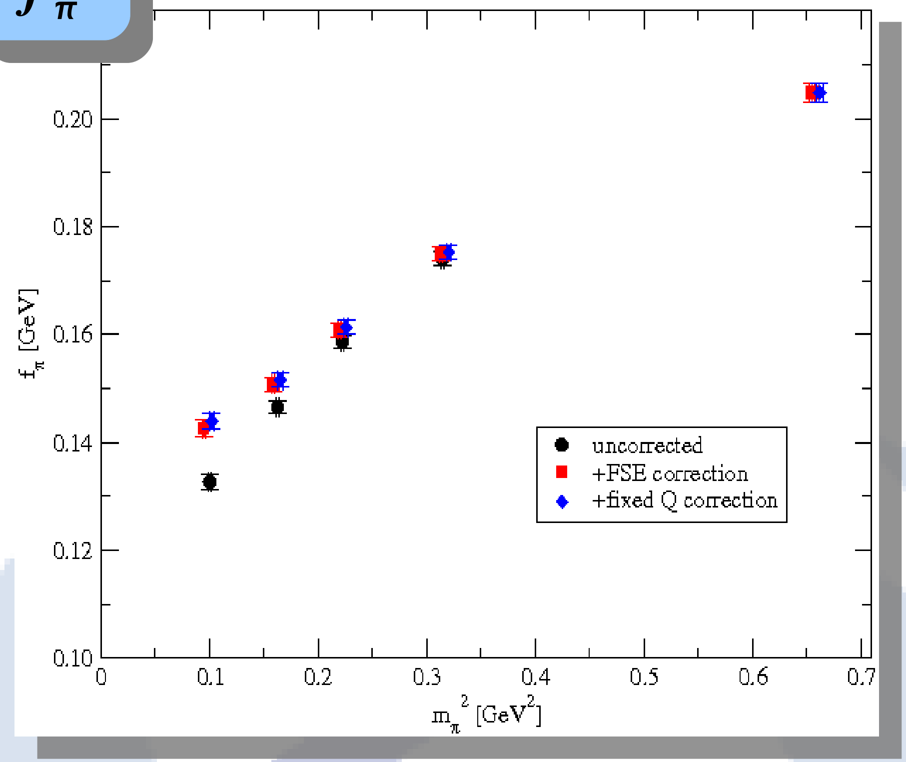
Actual Corrections (Nf=2+1)

m_π^2 / m_q



- Two corrections almost cancel. +2% at most.

f_π



- Fixed Q correction is tiny. +8% at most.

Quark mass renormalization

- RI/MOM condition   Martinelli et al., 1995

In Landau gauge:
$$Z_q^{-1} Z_\Gamma \Lambda_\Gamma(p) = 1$$

- Vertex functions   (WTI+OPE)

$$\Lambda_P(p) = A \cdot \frac{\langle \bar\psi \psi \rangle}{m_q} + Z_q Z_m + B_P \cdot m_q^2$$
$$\Lambda_S(p) = A \cdot \frac{\partial}{\partial m_q} \langle \bar\psi \psi \rangle + Z_q Z_m + B_S \cdot m_q^2$$

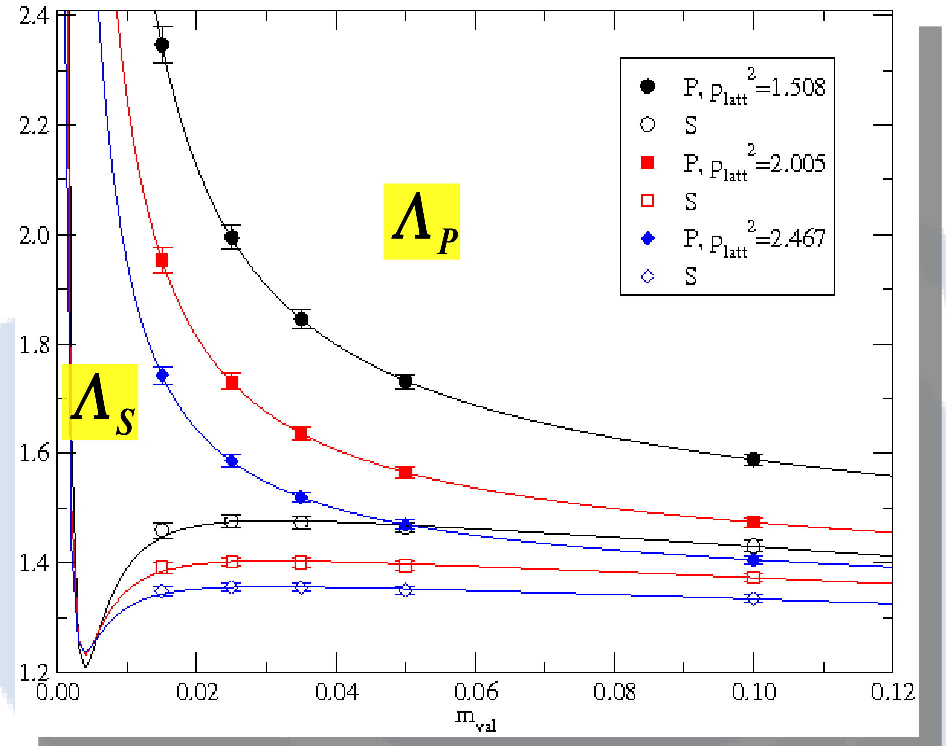
- Control of m_q dependence

with real data:
$$\langle \bar\psi \psi \rangle = \left\langle \frac{1}{V} \sum_{i=1}^{80} \frac{2 m_q}{m_q^2 + \lambda_i^2} \right\rangle$$

- 3-loop matching at chiral limit

$$Z_m^{\overline{MS}}(2\,\mathrm{GeV}) = 0.815(8)(3)$$

ms=0.100,mud=0.035,
simul fit for same color



Plan

- Numerical simulation (Nf=2 and 2+1)
  - Setup parameters
  - Correlation functions with eidenmodes
- Analysis (Nf=2+1)
  - Finite size effects
  - Non-perturbative renormalization
- Chiral extrapolation
  - Results on the Nf=2 calculation
  - Preliminary results on the Nf=2+1
- Summary

Nf=2 NLO ChPT

- Chiral expansion

$$m_\pi^2/m_q = 2B\left(1+x\ln x\right)+c_3 x+O\left(x^2\right)$$
$$f_\pi = f\left(1-2x\ln x\right)+c_4 x+O\left(x^2\right)$$

where $x = \dfrac{2B\,m_q}{(4\pi f)^2}$

- Equivalent expansion parameters

in the valid region of NLO,

$$x \Leftrightarrow \hat{x} = \left(\frac{m_\pi}{4\pi f}\right)^2 \quad \text{or} \quad \xi = \left(\frac{m_\pi}{4\pi f_\pi}\right)^2$$

Effectively resum higher order effects

-Fits as a func of m_q, m_pi^2 and xi

-Simultaneous fit and independent fits

-Comparison through chi^2 is inadequate.

- Three fit curves on one figure

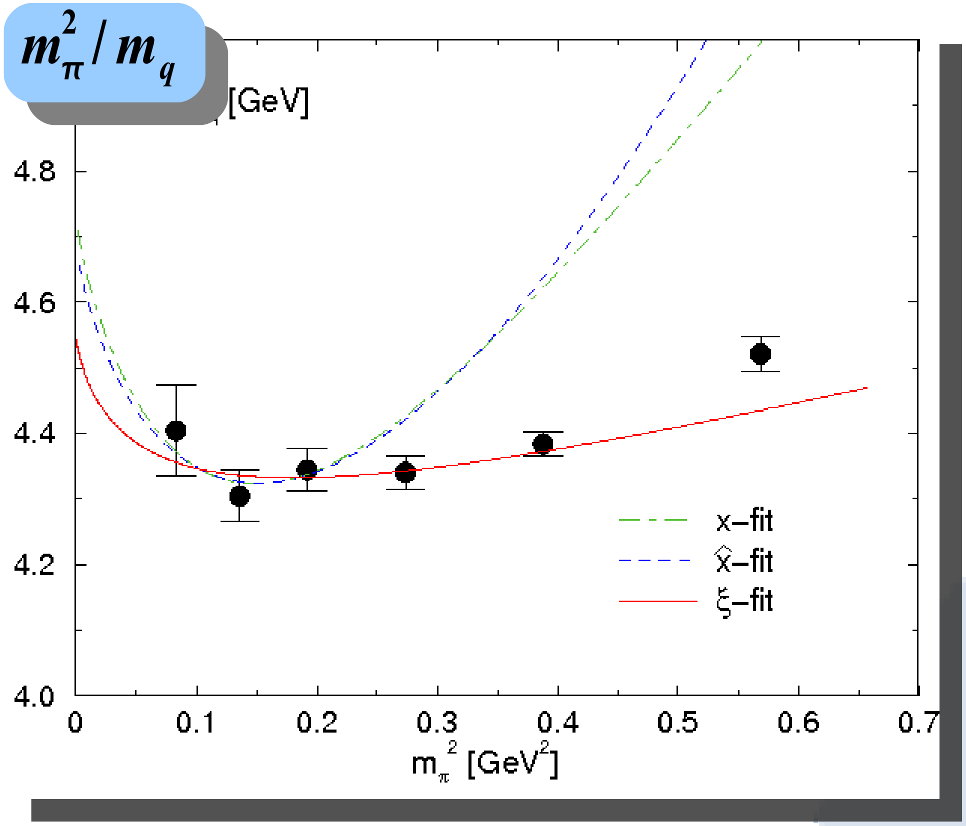
rescale of horizontal axis

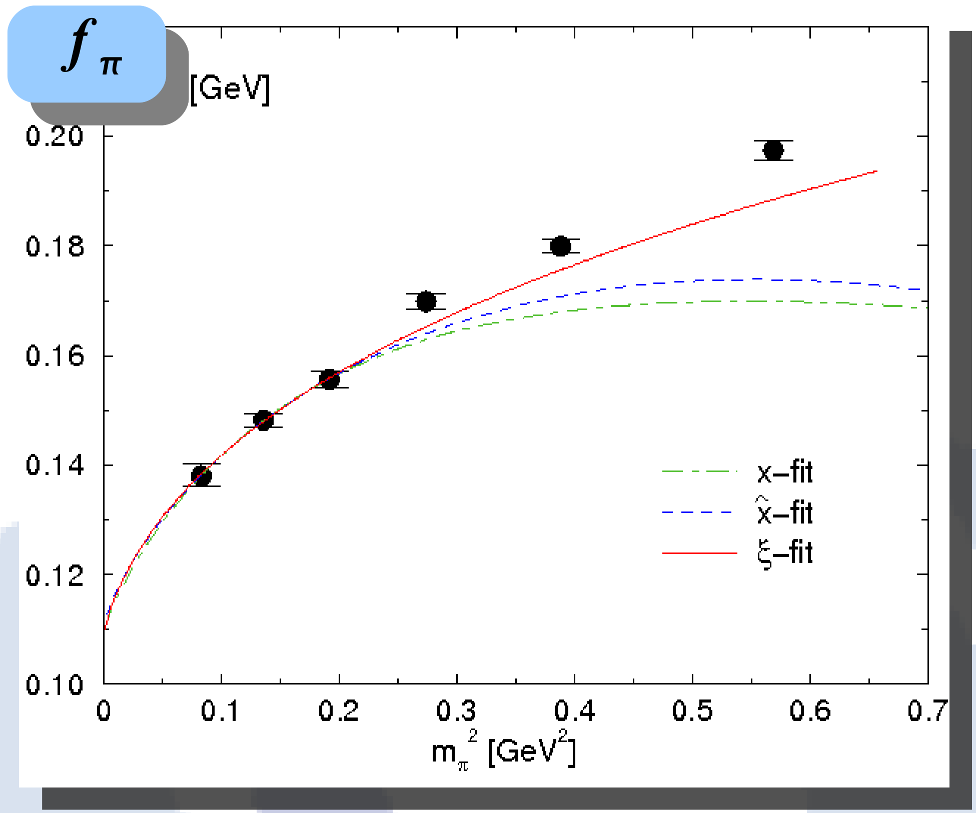
$$m_\pi^2 = \left(m_\pi^2/m_q\right)_{\text{curve}}\times m_q \;, \qquad m_\pi^2 = \left(4\pi f_\pi\right)^2_{\text{curve}}\times \xi$$

Direct comparison is possible.

Fit curves (Nf=2)

- Using the lightest 3 data points,

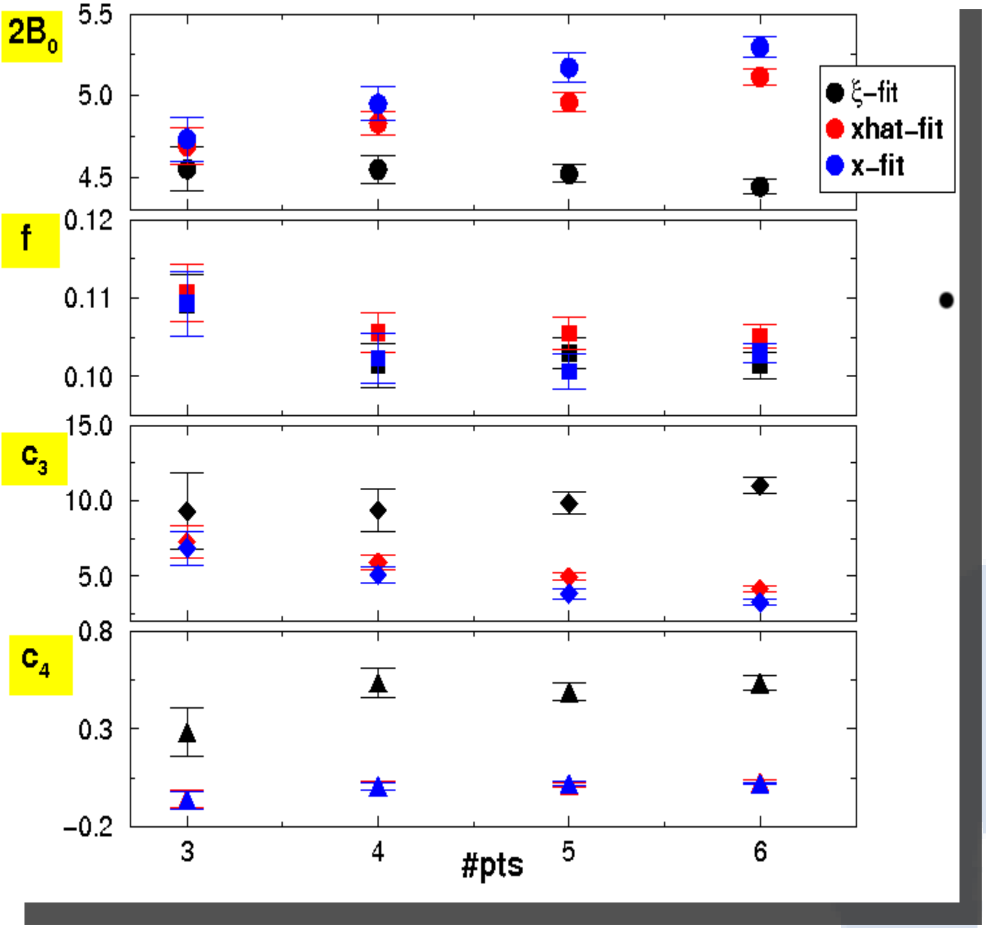




- NLO is OK for the lightest three data.
- Xi-fit describes the data beyond the fitted region.

Validity of NLO?

- Fit parameters for different mass ranges



- Convergence at the 3rd pt
- Threshold is between 3,4th points ⟷ ~450MeV

- Two options:
  - Analyse data below 450 MeV instead of full info from lattice. Statistical error is larger.
  - Use heavier mass points by including NNLO terms. Only the xi-fit is useful.

Extension to NNLO (Nf=2)

- NLO

$$\frac{m_\pi^2}{m_q} = 2B\left(1+\xi\ln\xi\right)+c_3\xi$$

$$f_\pi = f\left(1-2\xi\ln\xi\right)+c_4\xi$$

- NNLO

$$\frac{m_\pi^2}{m_q} = 2B\left[1+\xi\ln\xi+\frac{7}{2}(\xi\ln\xi)^2+\left(\frac{2c_4}{f}-\frac{4}{3}(\tilde{L}+16)\right)\xi^2\ln\xi\right]+c_3(\xi-9\xi^2\ln\xi)+K_1\xi^2$$

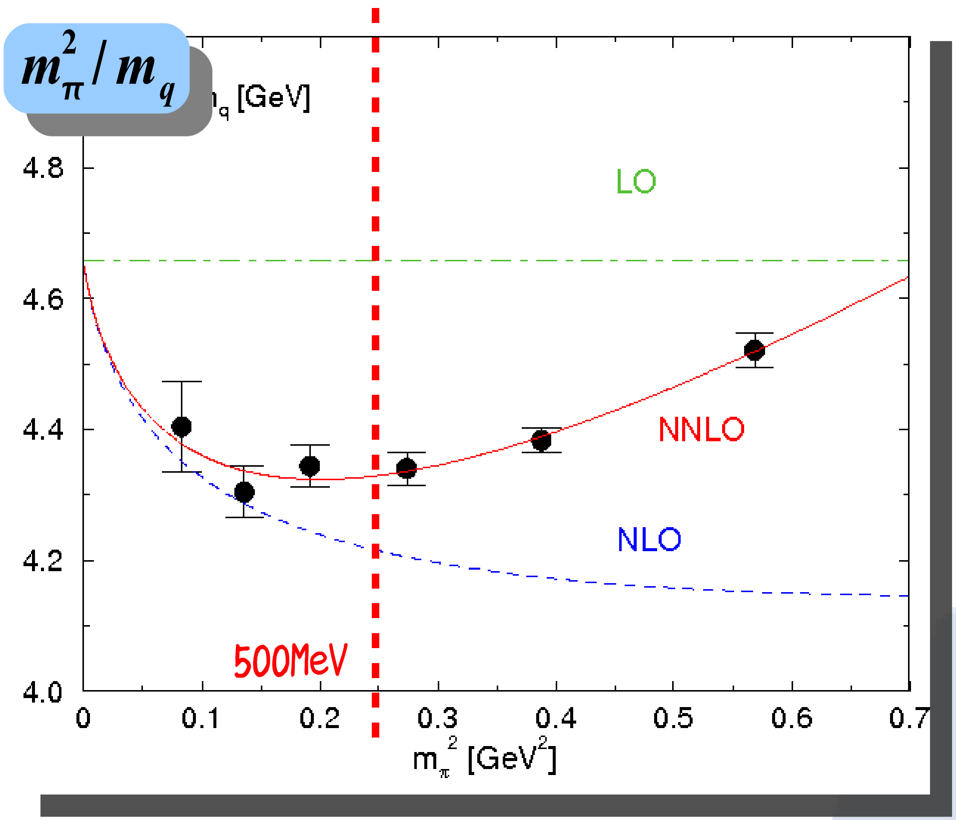
$$f_\pi = f\left[1-2\xi\ln\xi+5(\xi\ln\xi)^2-\frac{3}{2}\left(\tilde{L}+\frac{53}{2}\right)\xi^2\ln\xi\right]+c_4(\xi-10\xi^2\ln\xi)+K_2\xi^2$$

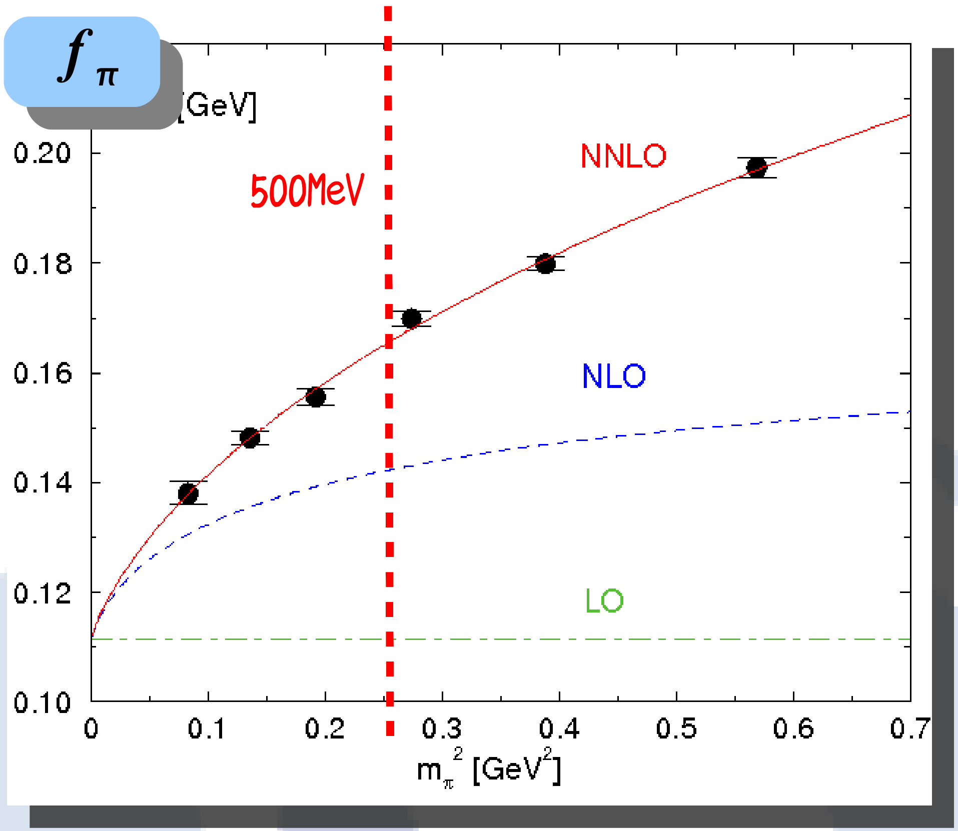
input: $\tilde{L} = 7\ln\left(\frac{\Lambda_1}{4\pi f}\right)^2+8\ln\left(\frac{\Lambda_2}{4\pi f}\right)^2$ from phenomenology

- Large shift of c3 and c4.
- Simultaneous fit is necessary for NNLO.

Convergence of ChPT (Nf=2)

- Using all data points, chi^2/dof = 1.42

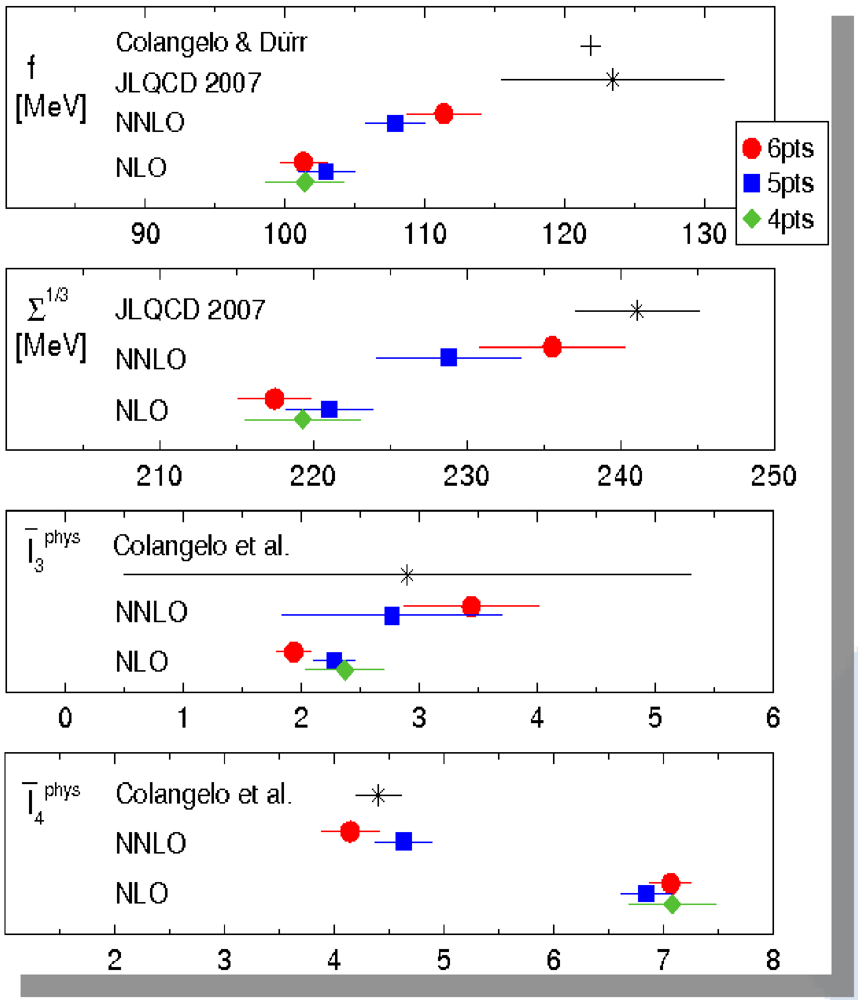




@500MeV, $\dfrac{|\mathrm{NNLO-NLO}|}{|\mathrm{NLO-LO}|}$ = 0.3 and 0.7

Low energy parameters (Nf=2)

- NLO vs NNLO



- NLO for >500 MeV is indeed problematic.
- LECs

$$f = 111.4(2.7)\left(^{+0.0}_{-3.5}\right)\left(^{+6.0}_{-0.0}\right)\,\mathrm{MeV}$$

$$\Sigma^{\overline{MS},2\mathrm{GeV}} = [235.6(4.9)\left(^{+0.0}_{-6.7}\right)\left(^{+12.7}_{-0.0}\right)\mathrm{MeV}]^3$$

(stat.)(6-5pts)(latt. scale)

$$\bar{l}_3^{phys} = 3.44(57)\left(^{+0.0}_{-68}\right)\left(^{+32}_{-0}\right)$$

$$\bar{l}_4^{phys} = 4.14(25)\left(^{+49}_{-0}\right)\left(^{+32}_{-0}\right)$$

(stat.)(6-5pts)(renorm. points)

- Physical quantities

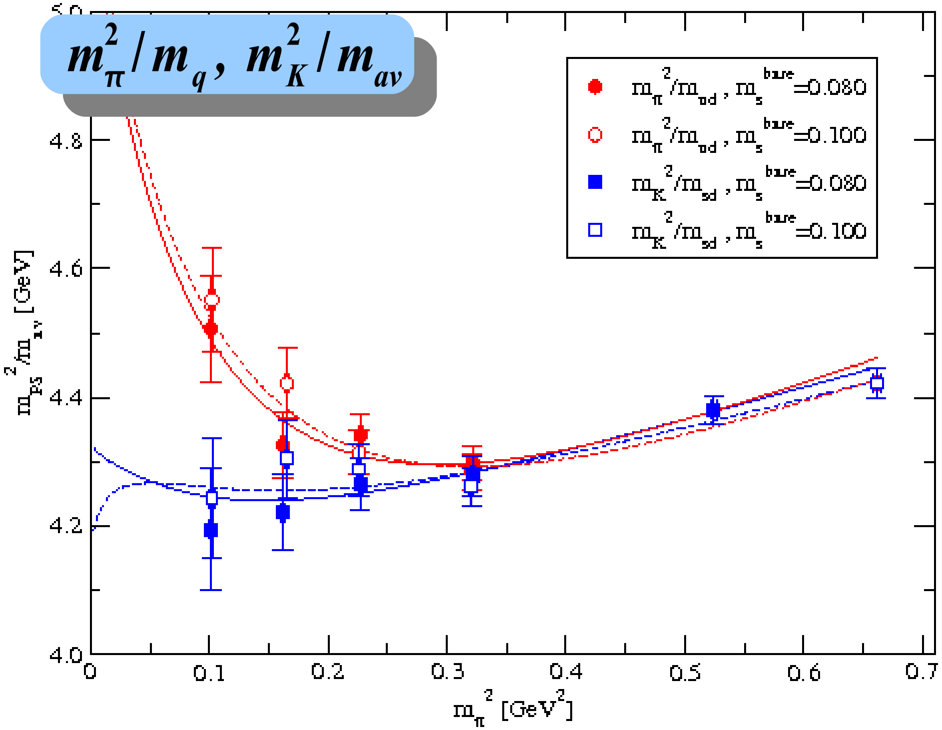
$$m_{ud}^{\overline{MS},2\mathrm{GeV}} = 4.44(15)\left(^{+9}_{-0}\right)\left(^{+0}_{-23}\right)\mathrm{MeV}$$

$$f_\pi = 119.3(2.4)\left(^{+0.0}_{-2.8}\right)\left(^{+6.4}_{-0.0}\right)\mathrm{MeV}$$

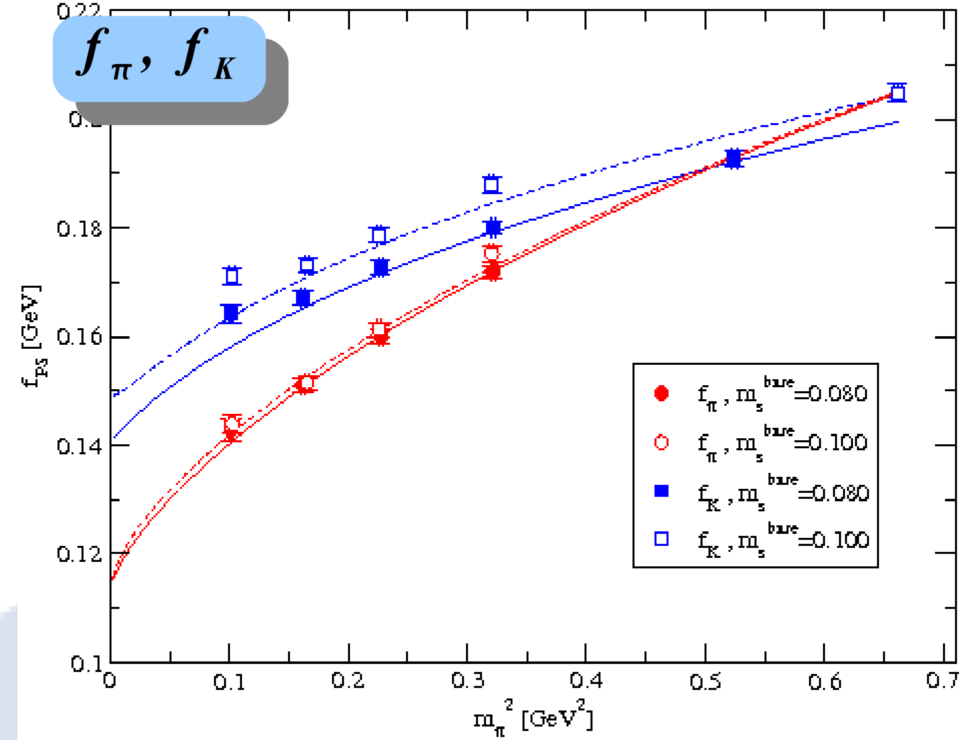
Preliminary result for Nf=2+1

- Simultaneous fit to the full NNLO expressions Amoros et al. 1999

$m_\pi^2/m_q,\ m_K^2/m_{av}$



$f_\pi,\ f_K$



- 16 params for 20 data points with input L_1,2,3,7. Using (ξ_π, ξ_{η_s})
- chi^2/dof = 9.7
- Preliminary result:

$m_{ud}^{\overline{MS},2GeV} = 3.76(45)$ MeV, $m_s^{\overline{MS},2GeV} = 116(12)$ MeV,

$f_K/f_\pi = 1.201(30)$ study of LECs is ongoing.

Summary

- Nf=2 and 2+1 dynamical overlap fermions
  - Exact chiral symm. No need of XChPT.
  - Improvements of the data with eigenmodes.
  - FSE corrections using ChPT calculations. (shevere with current resource.)
  - Non-perturbative renormalization for quark mass.
- Nf=2 ChPT is tested
  - Xi-expansion shows better convergence behavior.
  - ~450 MeV is the upper limit of NLO ChPT (Kaon is out).
  - NNLO analysis needed beyond this scale.
  - 1/a is a source of large systematic error
- Extension to Nf=2+1
  - ChPT test is to be completed on a 16^3x48 lattice.
  - Generation on a 24^3x48 lattice has started.