Exotic phases of Finite Temperature $SU(N)$ gauge theories with massive fermions: F, Adj, A/S

Joyce Myers and Michael Ogilvie
jcmyers@wustl.edu
mco@wuphys.wustl.edu

Washington University in St. Louis

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Lattice 2008

Outline

Motivation: To understand the effects of massive fermions on the phase diagram of $SU(N)$ gauge theories for fermions in various representations.

- Phase diagram of a simple deformed Yang-Mills theory formulated on the lattice
- Phase diagram of a more complicated deformed Yang-Mills theory (compare with QCD(Adj))
- Phase diagram of $SU(N)$ gauge theories with massive fermions from the one-loop effective potential
  - Fermion representations: Fundamental (F), Antisymmetric (A), Symmetric (S), and Adjoint (Adj)
  - Boundary conditions: periodic (PBC), antiperiodic (ABC)
  - $N_c = 2$ through 9.
  - various N_f
- One-loop contribution to $\langle\bar{\psi}\psi\rangle_R$ $(R = F, A, S, Adj)$

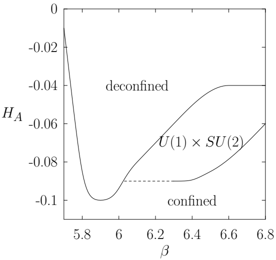
Lattice model

Last year we analyzed a simple deformed Yang-Mills theory on the lattice (Myers and Ogilvie 2008):

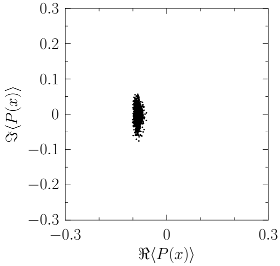
$$S_{lat,def} = S_W + \sum_{\mathbf{x}} V_{lat,def}\left[P(\mathbf{x})\right]$$

$$V_{lat,def}\left[P(\mathbf{x})\right] \equiv H_A \mathrm{Tr}_A P(\mathbf{x}) = H_A \left(\left| \mathrm{tr} P\left(\mathbf{x}\right) \right|^2 - 1 \right)$$

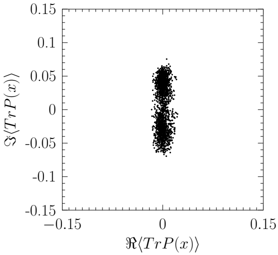
- Simulations in $SU(3)$ and $SU(4)$ revealed two interesting new phases.
- The simulations also showed that confined phase could be accessed perturbatively in $SU(3)$.



Phase diagram in $SU(3)$



$\langle P(x)\rangle$ in $SU(3)$, $\beta = 6.5$, $H_A = -0.055$



$\langle P(x)\rangle$ in $SU(4)$, $\beta = 11$, $H_A = -0.12$

Deformed Yang-Mills theory

To keep the confined phase accessible for $N > 3$ additional terms were required in the deformation potential (Ogilvie et al 2007, Unsal and Yaffe 2008):

$$V_{def}(P) \equiv \frac{1}{\beta} \sum_{n=1}^{\lfloor N/2 \rfloor} a_n \left|\mathrm{tr}\left(P^n\right)\right|^2 = \frac{1}{\beta} \sum_{n=1}^{\lfloor N/2 \rfloor} a_n \sum_{i,j=1}^{N} \cos\left[n\left(v_i - v_j\right)\right]$$

where $\lfloor N/2 \rfloor$ is the integer part of $N/2$.
Including the boson contribution from pure Yang-Mills theory

$$V_{model}(P) = \frac{1}{\beta^4}\left[\frac{1}{24\pi^2} \sum_{i,j=1}^{N} [v_i - v_j]^2 \left(2\pi - [v_i - v_j]\right)^2 - \frac{\pi^2}{45}\left(N^2 - 1\right)\right]$$

$$+ \frac{1}{\beta} \sum_{n=1}^{\lfloor N/2 \rfloor} a_n \sum_{i,j=1}^{N} \cos\left[n\left(v_i - v_j\right)\right]$$

- We minimize this potential to determine the phase diagram for a range of values of the a_n

One-loop effective potential

The one-loop effective potential for N_f Majorana fermions ($N_{f,Dirac} = \frac{1}{2}N_f$) of mass m in a background Polyakov loop $P = diag\{e^{iv_1}, e^{iv_2}, ..., e^{iv_N}\}$ gauge field is (Meisinger and Ogilvie 2001):

$$V_{eff}(P,m) \equiv -\frac{1}{\beta V_3}\ln Z(P,m)$$
$$=\frac{1}{\beta V_3}\left[-N_f \ln\det\left(-D_R^2(P)+m^2\right)+\ln\det\left(-D_{adj}^2(P)\right)\right]$$

$$=\frac{m^2 N_f}{\pi^2\beta^2}\sum_{n=1}^{\infty}\frac{(\pm 1)^n}{n^2}\mathrm{Re}\left[\mathrm{Tr}_R\left(P^n\right)\right]K_2\left(n\beta m\right)$$
$$+\frac{1}{\beta^4}\left[\frac{1}{24\pi^2}\sum_{i,j=1}^{N}[v_i - v_j]^2\left(2\pi - [v_i - v_j]\right)^2 - \frac{\pi^2}{45}\left(N^2-1\right)\right]$$

where we have $(+1)^n$ for periodic boundary conditions (PBC) and $(-1)^n$ for antiperiodic boundary conditions (ABC) applied to fermions.

Chiral Condensate:

$$\langle\bar{\psi}\psi\rangle_{1-loop}(m) = -\lim_{V_4\to\infty}\frac{1}{V_4 N_f}\frac{\partial}{\partial m}\ln Z(m) = \frac{1}{N_f}\frac{\partial}{\partial m}V_{\mathrm{eff}}(P,m)$$

Possible phases of QCD for PBC and ABC

- ABC
  - confined phase

$$\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$$

  - deconfined phase

$$\mathbf{v} = \{0,0,0\}, \{\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}\}, \{\frac{4\pi}{3}, \frac{4\pi}{3}, \frac{4\pi}{3}\}$$

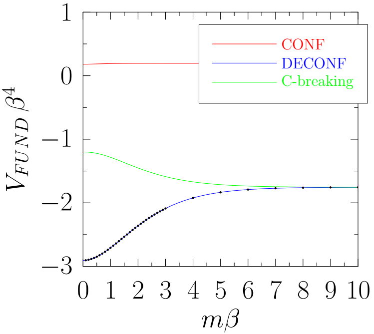
- PBC
  - confined phase
  - deconfined phase
  - $\mathscr{C}$-breaking phase (P is not invariant under $P \to P^*$.)

$$\mathbf{v} = \{\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}\}$$

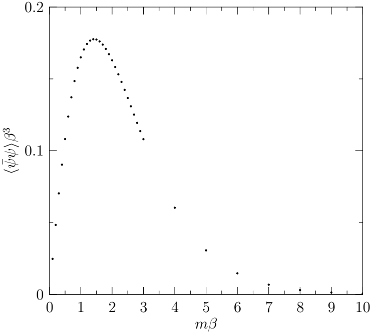
Note: In QCD(F) with PBC on fermions, $\mathscr{C}$-symmetry is only broken for N odd. For N even, $Tr_F P$ is magnetized along the negative real axis ($\mathbf{v} = \{\pi, \pi, ...\}$).

V_{EFF} and $\langle \bar{\psi}\psi \rangle$ in perturbative QCD

- We calculate V_{eff} for fermions in the fundamental representation to which ABC are applied.



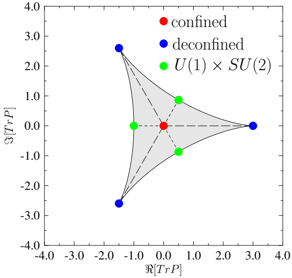
$V_{F,-}$, $Nc = 3$, $Nf = 2$ (1 Dirac flavour)



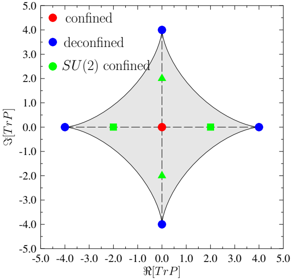
$\langle \bar{\psi}\psi \rangle_{F,1-loop}$ for $N_c = 3$

- Only the deconfined phase is accessible in the perturbative limit.
- The fermion contribution to V_{eff} vanishes as $m\beta \to \infty$.
- The inflection point in V_{EFF} at $m\beta \approx 1.4$ implies a large one-loop contribution to $\langle \bar{\psi}\psi \rangle$.

Phases of adjoint QCD: $N_c = 3$, $N_c = 4$, PBC, $N_f > 1$



$N_c = 3$



$N_c = 4$

confined: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

$U(1) \times SU(2)$: $\mathbf{v} = \{0, \pi, \pi\}$

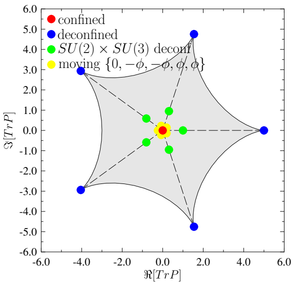
deconfined: $\mathbf{v} = \{0, 0, 0\}$

confined: $\mathbf{v} = \{0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}\}$

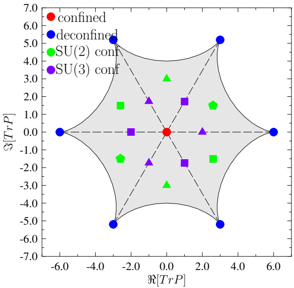
$SU(2)$ conf: $\mathbf{v} = \{-\frac{\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0\}$

Phases of adjoint QCD: $N_c = 5$, $N_c = 6$, PBC, $N_f > 1$



$N_c = 5$



$N_c = 6$

confined: $\mathbf{v} = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

inconst: $\mathbf{v} = \{0, -\phi, -\phi, \phi, \phi\}$

$SU(2) \times SU(3)$ dec: $\mathbf{v} = \{\pi, \pi, 0, 0, 0\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0, 0\}$

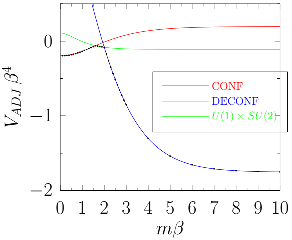
confined: $\mathbf{v} = \{\frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}\}$

$SU(3)$ conf: $\mathbf{v} = \{0, \frac{2\pi}{3}, -\frac{2\pi}{3}, 0, \frac{2\pi}{3}, -\frac{2\pi}{3}\}$

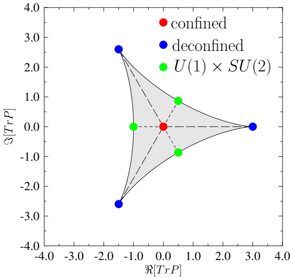
$SU(2)$ conf: $\mathbf{v} = \{-\frac{\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0, 0, 0\}$

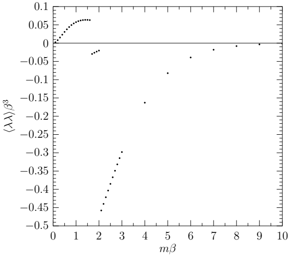
$SU(3)$ Adjoint QCD (PBC) $N_f = 2$



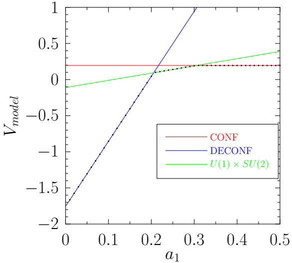
V_{ADJ}, $Nc = 3$, $Nf = 2$



V_{ADJ}, $Nc = 3$ phases



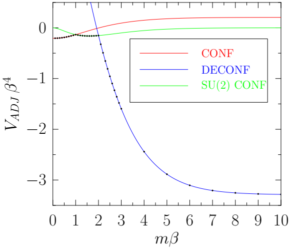
$\langle\lambda\lambda\rangle_{ADJ}$



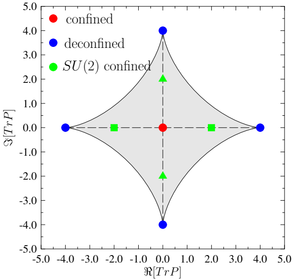
V_{model}, deformed YM, $Nc = 3$

- The data points (black dots) were found by minimizing V_{eff} with respect to the Polyakov loop eigenvalues v_i.
- The confined phase is accessible perturbatively for $m\beta \le 1.6$.
- There is a dramatic jump in $\langle\bar{\psi}\psi\rangle$ corresponding to the deconfinement transition
- The model has the same phases as QCD(Adj)

$SU(4)$ Adjoint QCD (PBC) $N_f = 2$

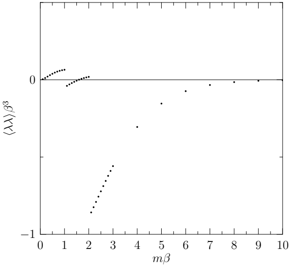


V_{ADJ}, $Nc = 4$, $Nf = 2$

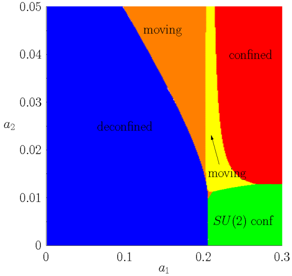


V_{ADJ}, $Nc = 4$ phases

- The confined phase is accessible perturbatively for $m\beta \leq 1.0$.



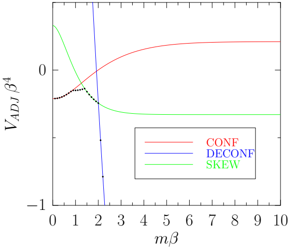
$\langle\lambda\lambda\rangle_{ADJ}$



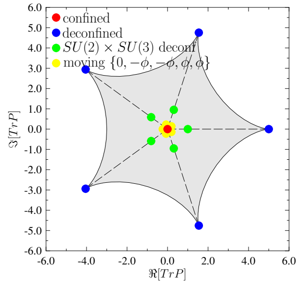
V_{model}, deformed YM, $Nc = 4$

- The model has the same phases as QCD(Adj), and more, but the additional phases can be circumnavigated.

$SU(5)$ Adjoint QCD (PBC) $N_f = 2$

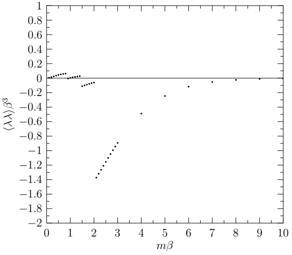


V_{ADJ}, $Nc = 5$, $Nf = 2$

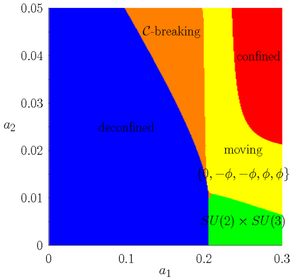


V_{ADJ}, $Nc = 5$ phases

- The confined phase is accessible perturbatively for $m\beta \leq 0.8$.
- A moving phase is found between the confined and $SU(2) \times SU(3)$-dec phases.



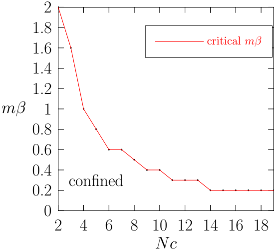
$\langle\lambda\lambda\rangle_{ADJ}$



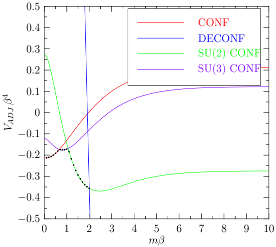
V_{model}, deformed YM, $Nc = 5$

- The model includes the phases of QCD(Adj).
- The (non-$\mathscr{C}$-breaking) moving phase of the model is the same as that of QCD(Adj).

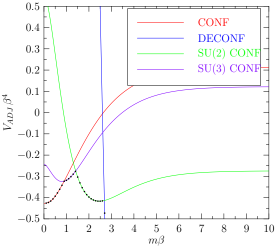
Accessibility of the confined phase as $N \to \infty$, or as N_f is increased



Range of $m\beta$ for which the confined phase is accessible in QCD(Adj) with $N_f = 2$



V_{eff} for $N_c = 6$, $N_f = 2$



V_{eff} for $N_c = 6$, $N_f = 3$

- As $N \to \infty$ the maximum $m\beta$ for which the confined phase is accessible, $(m\beta)_{crit}$, decreases.
- However, as N_f increases, $(m\beta)_{crit}$ increases (we must have $N_f \leq 5$ Majorana flavours to preserve asymptotic freedom).

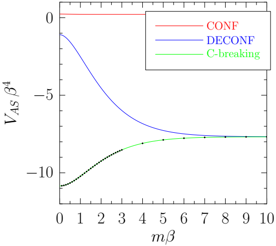
Orientifold Planar Equivalence

The story:

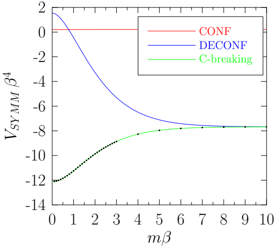
- Armoni, Shifman, and Veneziano(2003 - 2004) prove non-perturbatively the equivalence of the bosonic sectors of QCD(Adj) with N_f Majorana fermions and QCD(AS/S) with N_f Dirac fermions, in the planar limit.
- Unsal and Yaffe (2006) show that on $S^1 \times \mathbb{R}^3$ $\mathscr{C}$-symmetry is broken in QCD(A/S) when PBC are applied to fermions.
- DeGrand and Hoffman (2007), Lucini et al (2007) showed using lattice simulations that the $\mathscr{C}$-breaking is lifted as S^1 is decompactified
- Lucini et al. (2008) non-perturbatively prove orientifold equivalence in the quenched approximation (in the absence of $\mathscr{C}$-breaking) using lattice simulations and calculate the quark condensate in QCD(A/S/Adj)

We compare (to 1-loop order) the phase diagrams of $QCD(A), QCD(S)$ with $N_f = 2$ (1 Dirac flavour), to $QCD(Adj)$ with $N_f = 1$ (Majorana flavour), for massive fermions with PBC.

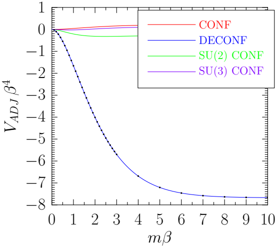
$SU(6)$ QCD(A) (left), (S) (middle), and (Adj) (right) for PBC on fermions



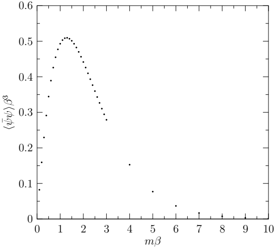
$V_{A,+},\ Nc = 6,\ Nf = 2$



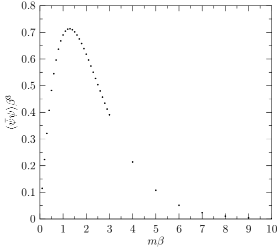
$V_{S,+},\ Nc = 6,\ Nf = 2$



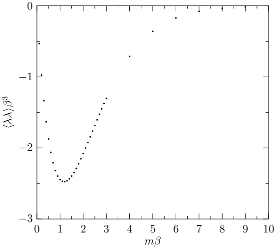
$V_{Adj,+},\ Nc = 6,\ Nf = 1$



$\langle\bar\psi\psi\rangle_A$ for $N_c = 6$



$\langle\bar\psi\psi\rangle_S$ for $N_c = 6$



$\langle\lambda\lambda\rangle_{Adj}$ for $N_c = 6$

The C breaking phase of QCD(AS/S)

- The $\mathscr{C}$-breaking phase is favoured in the case where PBC are applied to fermions in the A and S representations (When ABC are used the deconfined phase is favoured).
- For example, when $N_c = 6$ the $\mathscr{C}$-breaking phase has the Polyakov loop eigenvalues

$$\mathbf{v} = \{\frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}\}$$

- P is clearly not invariant under $P \to P^*$.

Conclusions

One-loop PT:

- In QCD(Adj) for $N_f \geq 2$ there are several exotic phases occuring between the confined and deconfined phases
- In QCD(Adj) for $N_f \geq 2$, as N increases, $(m\beta)_{crit}$, below which the confined phase is accessible, decreases.
- In QCD(Adj) for $N_f \geq 2$, as N_f is increased, the confined phase is accessible for a larger $(m\beta)_{crit}$.
- In $QCD(A/S)$ with PBC for fermions the $\mathscr{C}$-breaking phase is favoured for all $m\beta$.
- For all representations there is a clear one-loop contribution to $\langle\bar{\psi}\psi\rangle$ for small $m\beta$.
  - In QCD(AS), QCD(S), QCD(F) there is an inflection point in V_{eff} at which $\langle\bar{\psi}\psi\rangle \neq 0$, in the deconfined phase.
  - In QCD(Adj) for $N_f \geq 2$ (with PBC on fermions) the chiral condensate peaks at the transition to the deconfined phase
- In QCD(Adj) for $N_f = 1$ the deconfined phase is favoured for all $m\beta$.

The deformed Yang-Mills theory finds all the phases of QCD(Adj), and the a_n can be slowly varied to go through the phases in the same order.