A Variational Study of the Nucleon Wavefunction

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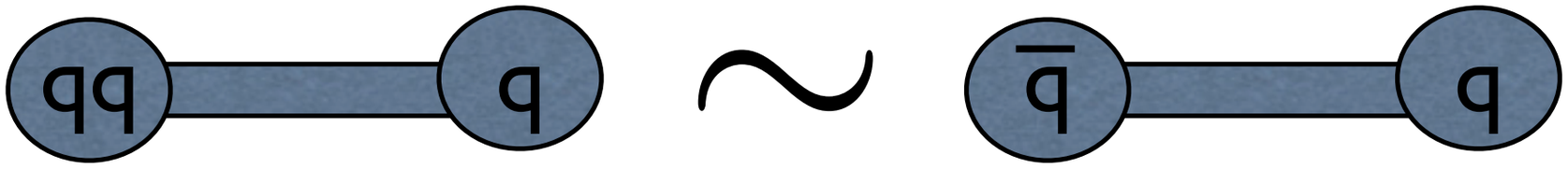
work done under the supervision of John Negele

outline

- physical motivation
- variational method
- exploratory calculation
  - parameters explored
  - results
- summary

Physical Motivation

- spatial nucleon wavefunction (quarks)
- gluons
- diquarks
  - "good" (scalar) diquarks $(\mathrm{u}\ \mathrm{C}\gamma_5\ \mathrm{d})$
  - "bad" (vector) diquarks $(\mathrm{u}\ \mathrm{C}\gamma_\mu\ \mathrm{d})$
  - motivated by one-gluon exchange
- flux tube model (c.f. Regge trajectories)



The Method

For a trial source

$$|\,\mathrm{J}\,\rangle = \mathrm{J}\,|\,\Omega\,\rangle$$

we calculate the (momentum-projected) two-point function

$$\langle\,\bar{\mathrm{J}}(t)\,|\,\mathrm{J}(0)\,\rangle \;\; = \;\; \sum_n \left|\,\langle\,\bar{\mathrm{J}}(0)\,|n\rangle\,\right|^2 \, e^{-E_n t}$$

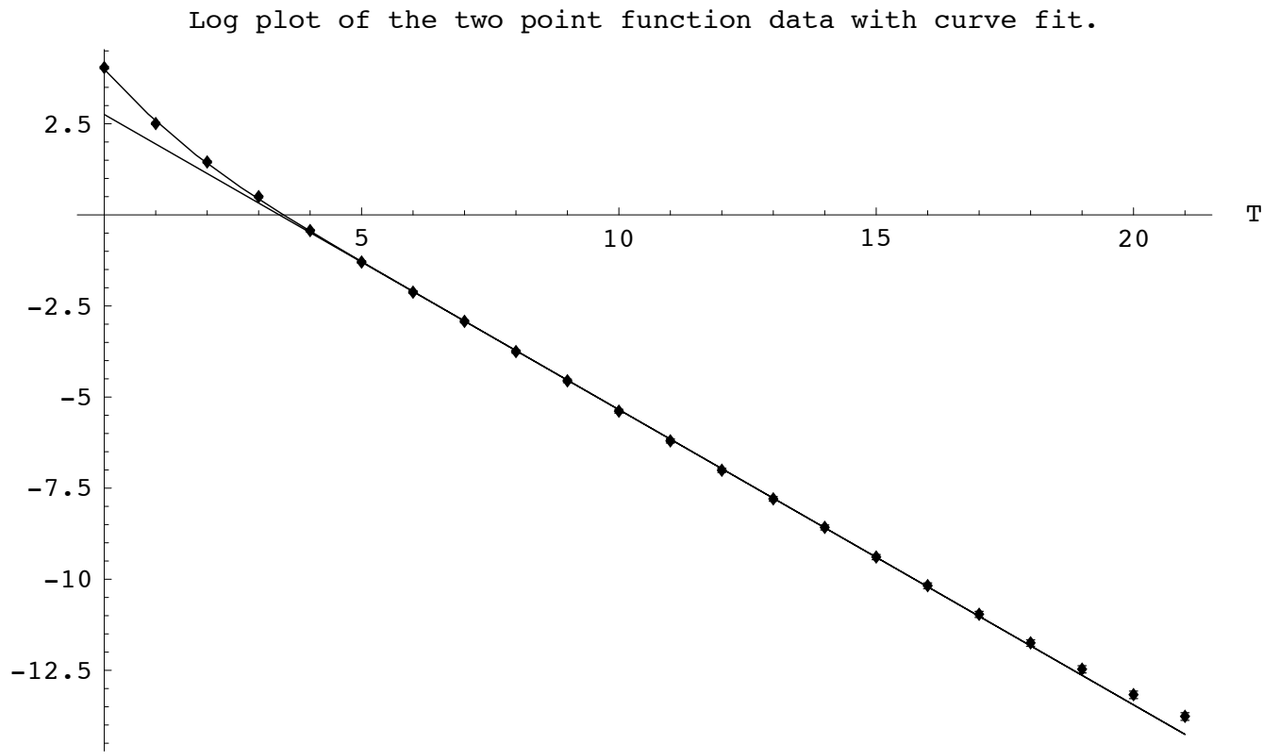
For large *t*, the ground state dominates the sum...

... so we fit the two-point function to a sum of exponentials:

$$C(t) \equiv \langle\, \bar{\mathrm{J}}(t)\,|\,\mathrm{J}(0)\,\rangle \quad \doteq \quad A_0\, e^{-m_0 t} + A_1\, e^{-m_1 t}$$

and estimate the (normalized) ground-state overlap for our trial source:

$$\frac{\left|\langle \bar{\mathrm{J}}(0)|0\rangle\right|^2}{\sum_n \left|\langle \bar{\mathrm{J}}(0)|n\rangle\right|^2} = \frac{A_0}{C(0)}$$



computational side note:

propagator correction at t = t_{source}

Using the "naive" Wilson propagator in the calculation of the two-point function is incorrect at t_{source}! This can be seen by examining the transfer matrix formalism:

$$\langle \hat{O}_1 \hat{O}_2 \rangle = \frac{\int DU D\bar{q} Dq \, e^{-S(U,\bar{q},q)} O_1 O_2}{\int DU D\bar{q} Dq \, e^{-S(U,\bar{q},q)}} = \frac{\langle 0 | N\{\hat{T} \ldots \hat{T} \hat{O}_1 \hat{T} \ldots \hat{T} \hat{O}_2 \hat{T} \ldots \hat{T}\} | 0 \rangle}{\langle 0 | \hat{T} \ldots \hat{T} | 0 \rangle}$$

We want to find T (and appropriate coordinate transform) such that we recover the action exponential in the path integral.
Writing out the Wilson action, one finds that it is necessary to define the following normal ordering convention for the fermion field operators:

$$\begin{aligned} \text{upper:} \qquad N\{q^U \bar{q}^U\} &= q^U \bar{q}^U \\ \text{lower:} \qquad N\{q^L \bar{q}^L\} &= \bar{q}^L q^L \end{aligned}$$

... where upper (lower) refers to the upper (lower) half spinor in the Dirac basis.
The operators anticommute ***except*** when on the same timeslice, in which case we pick up a correction term from the anticommutator:

$$P_{correct}(x,y) \quad = \quad P_{naive}(x,y) - \frac{(1-\gamma_0)}{2} B^{-1}(\vec{x},\vec{y}) \, \delta_{x_0,y_0}$$

where

$$B(\vec{x},\vec{y}) \quad = \quad \mathbb{I} - \kappa \sum_{j=1}^{3} \left[U_j^\dagger(y) \delta_{x-y-\hat{\jmath}} + U_j(x) \delta_{x-y+\hat{\jmath}} \right]$$

Exploratory Calculation:

- Quenched Wilson, 16^3x32 lattice

$$\beta = 6.0,\ \kappa = 0.1530\ \ (m_\pi \approx 900\ \mathrm{MeV})$$

- Why not use domain wall fermions?
  - nonlocal transfer matrix
  - oscillating terms (SS & JN, hep-lat: 0710.0425)

Exploratory Calculation:

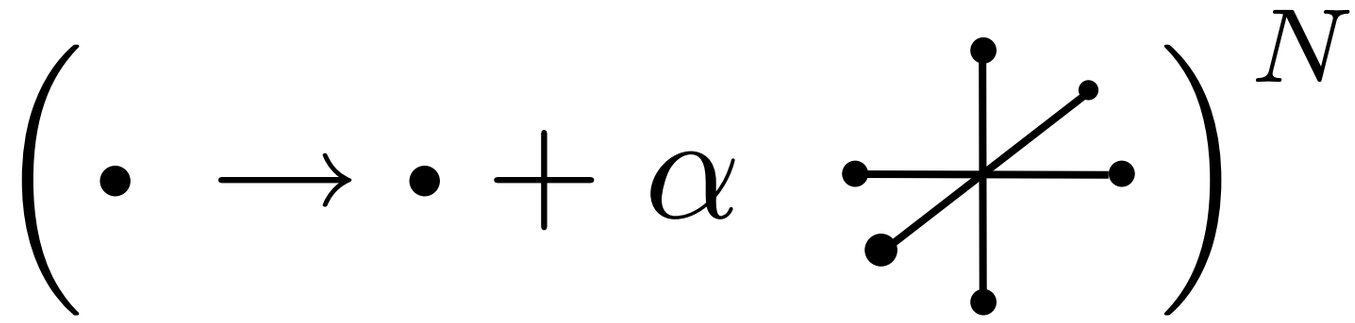
- Source interpolating field motivated by diquark model of nucleon:

$$\mathrm{J} = (\ \mathrm{U}\ \mathrm{C}\gamma_5\ \mathrm{D}\ )\ \mathrm{U}$$

- Variational parameters:
  - spatial extent of quark sources (quark smearing)
  - number of dirac components (2 vs. 4)
  - gluon wavefunction (gauge field smearing)
  - relative size of quark, diquark
  - relative position of quark, diquark

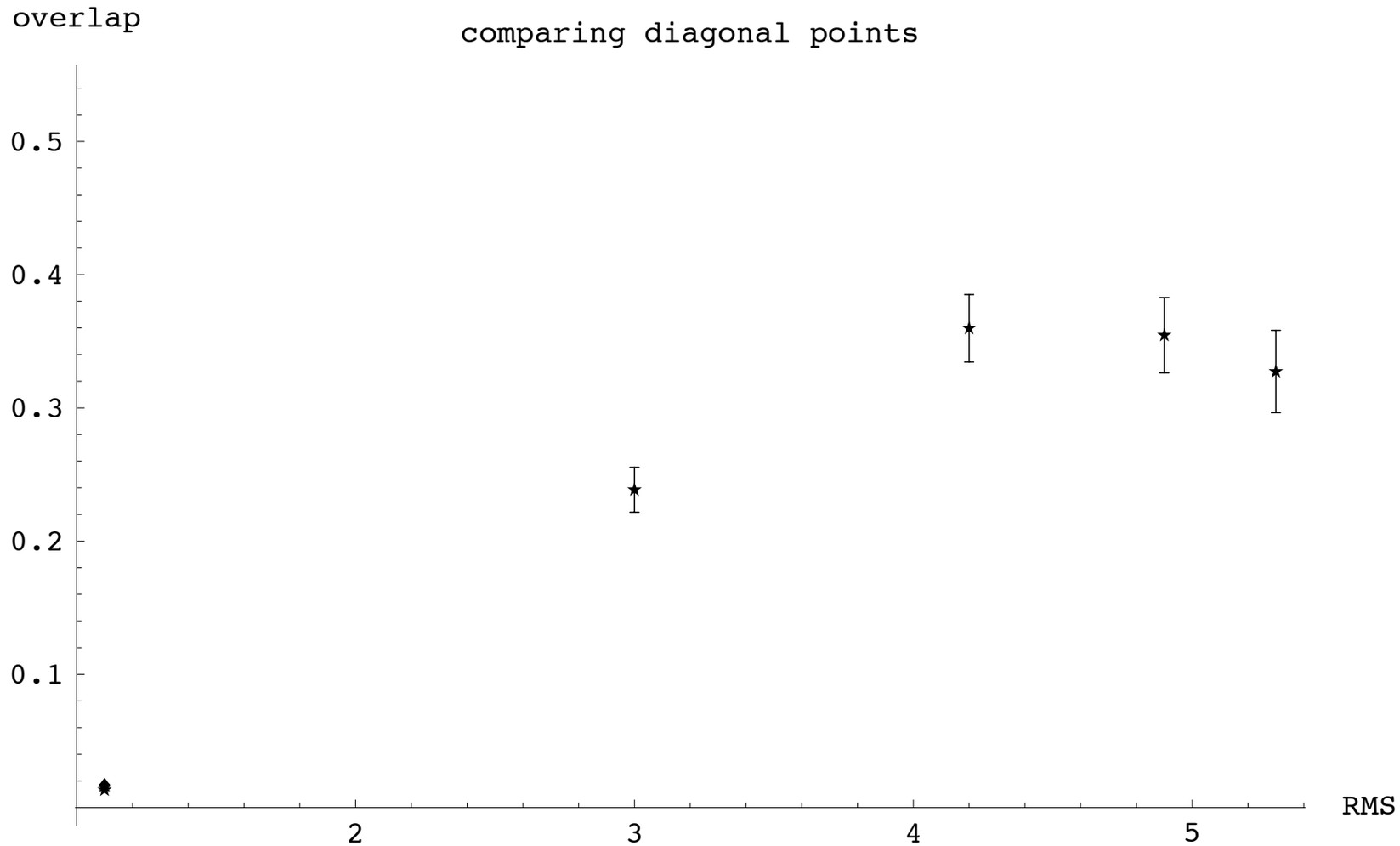
Quark Smearing

- Gauge-invariant smearing:

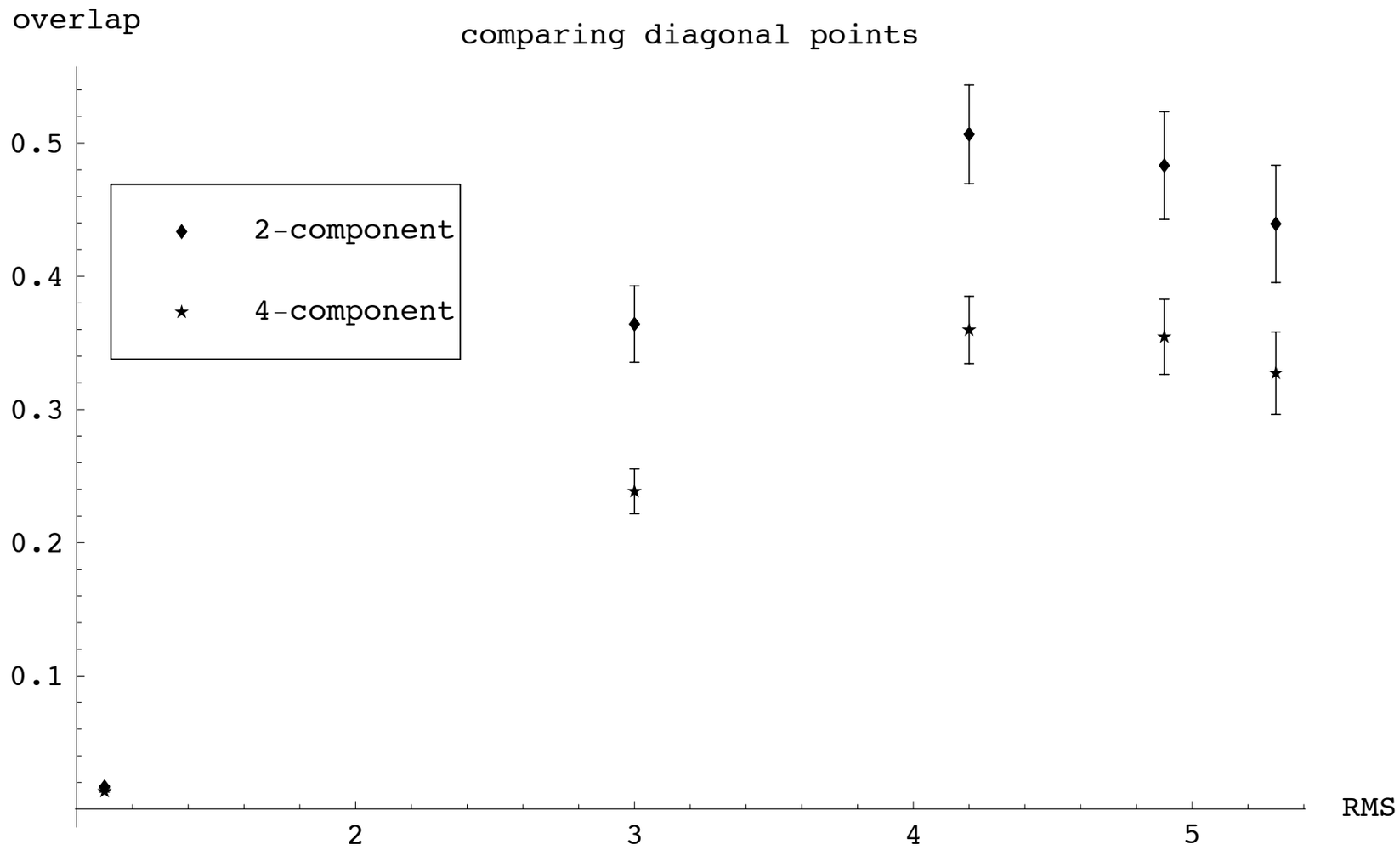


- Fix $\alpha = 3.0,$ vary N

- four-component spinors: variation with quark radius



- two-component versus four-component spinors



- using only two components: $0.35 \rightarrow 0.50$!

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Apparently, we've been using a very poor trial wavefunction in the lower components. For any dirac spinor:

$$u = \begin{pmatrix} u^U \\ u^L \end{pmatrix}$$

the dirac equation implies:

$$u^L = \frac{1}{E+m}(\mathbf{p}\cdot\sigma)u^U$$

So, if we put the upper components into something like an s-wave state, the lower components ought to be in a p-wave. But we've been setting: $u^L \approx u^U$, which is clearly a bad approximation to the true wavefunction. It's better to set $u^L = 0$ than to introduce so many excited state contaminants.

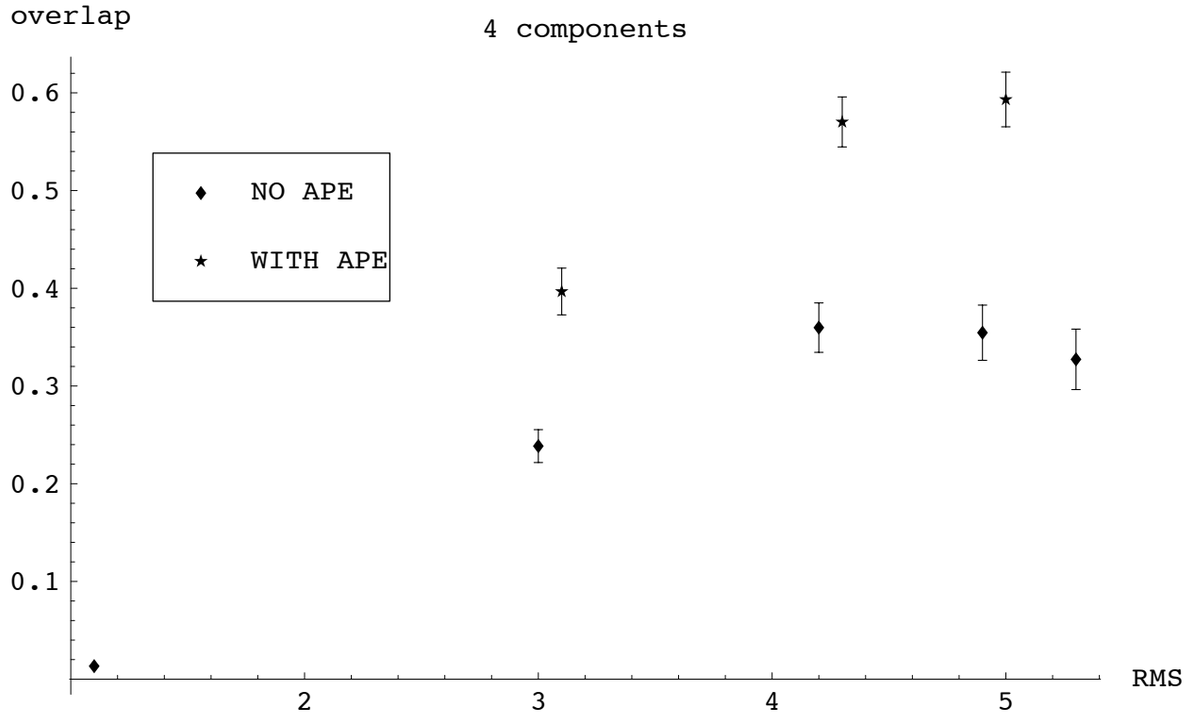
Add Gauge Smearing:

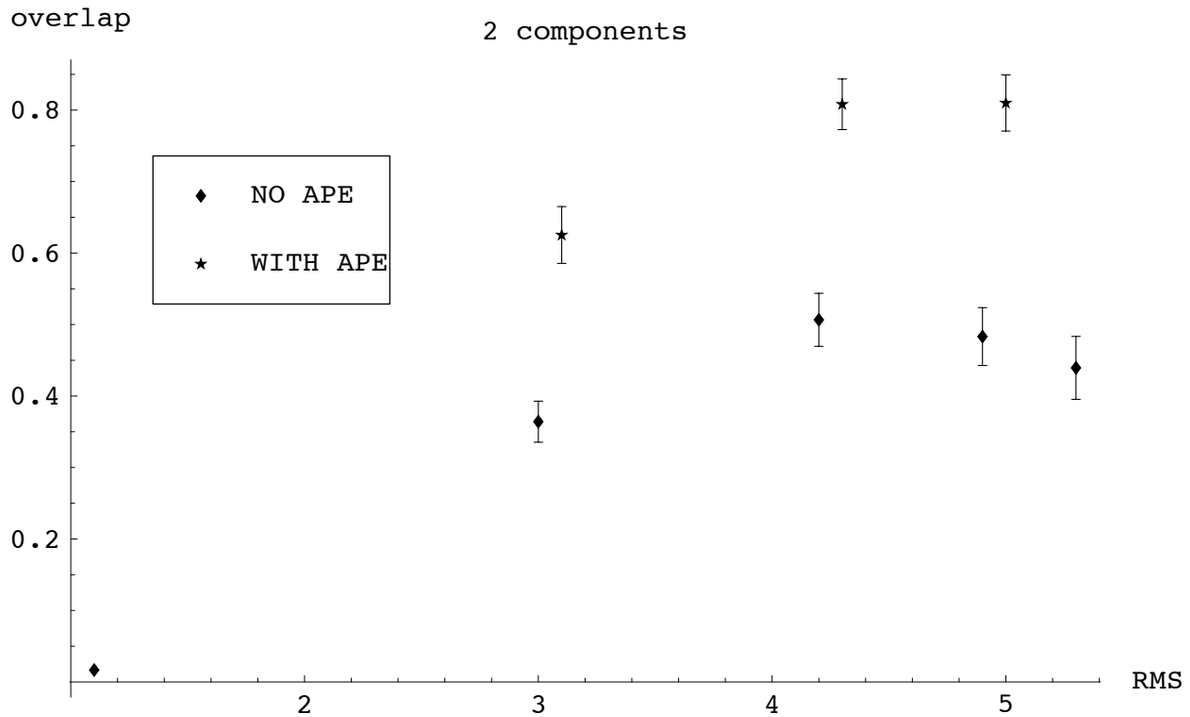
- Smear gauge links used in constructing the source:

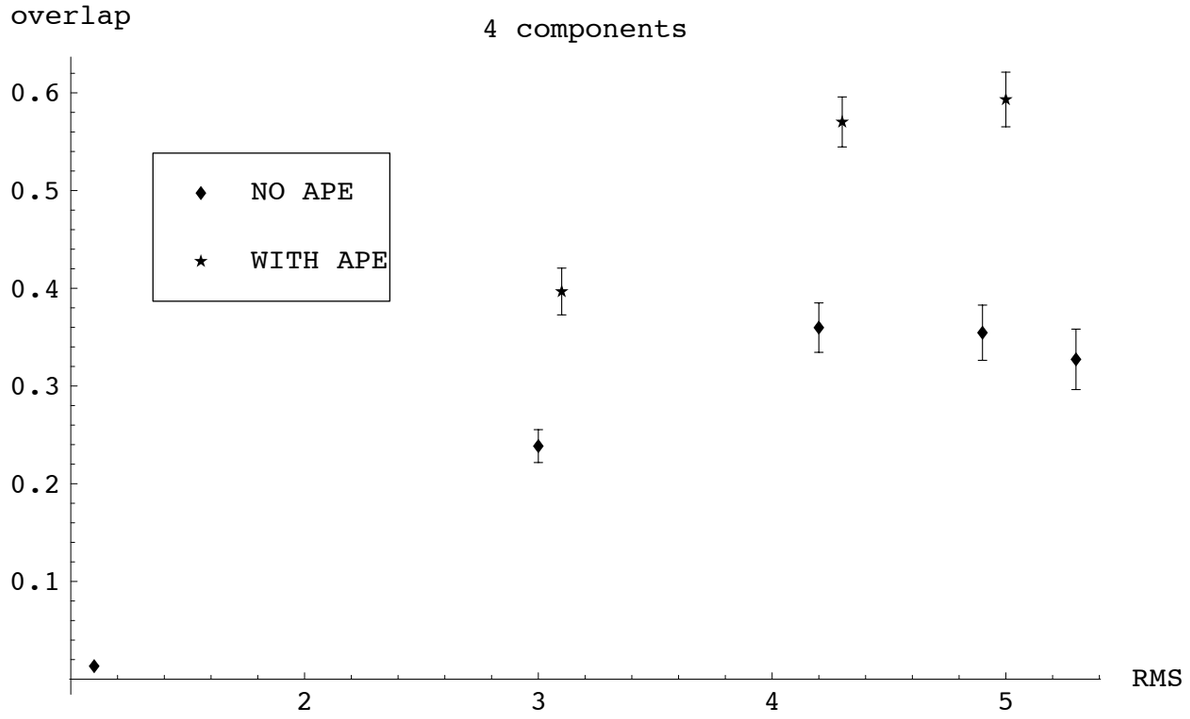
$$\left(\quad \Big| \rightarrow (1-c) \Big| \; + c \sum \square \; \right)^N$$

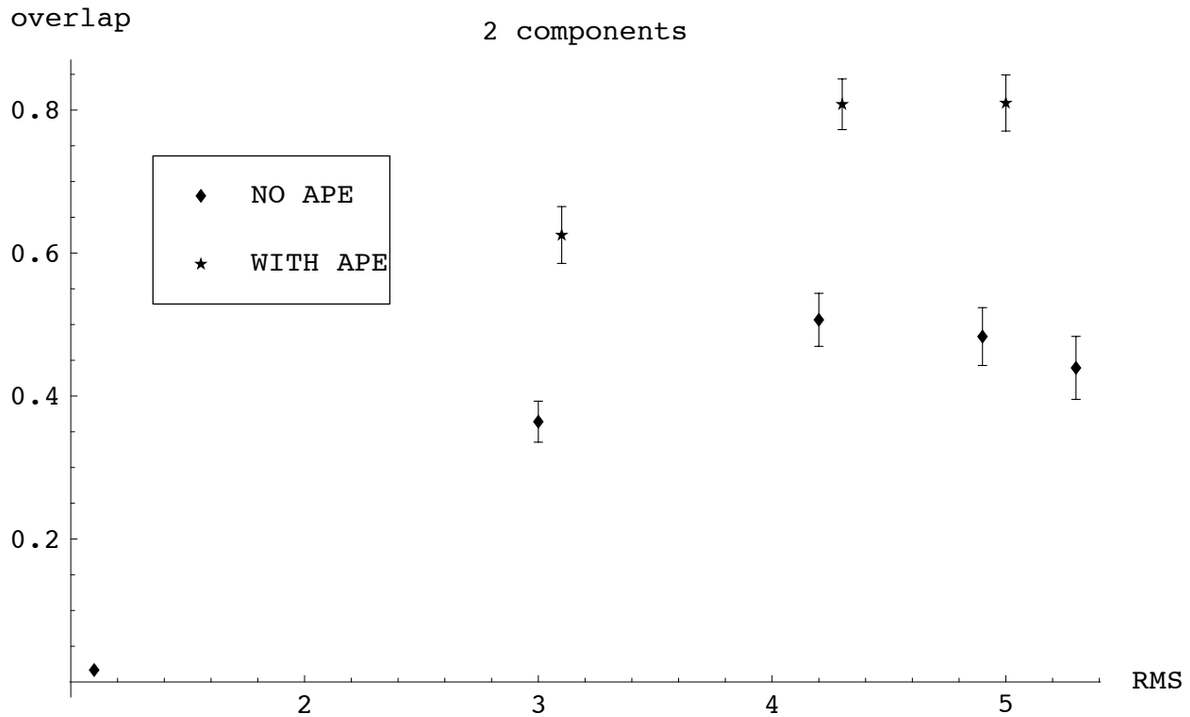
- Parameters used:

$$c = 0.26, \quad N = 25$$


overlap
4 components
0.6
0.5
0.4
0.3
0.2
0.1
NO APE
WITH APE
2
3
4
5
RMS


overlap
2 components
0.8
0.6
0.4
0.2
NO APE
WITH APE
2
3
4
5
RMS





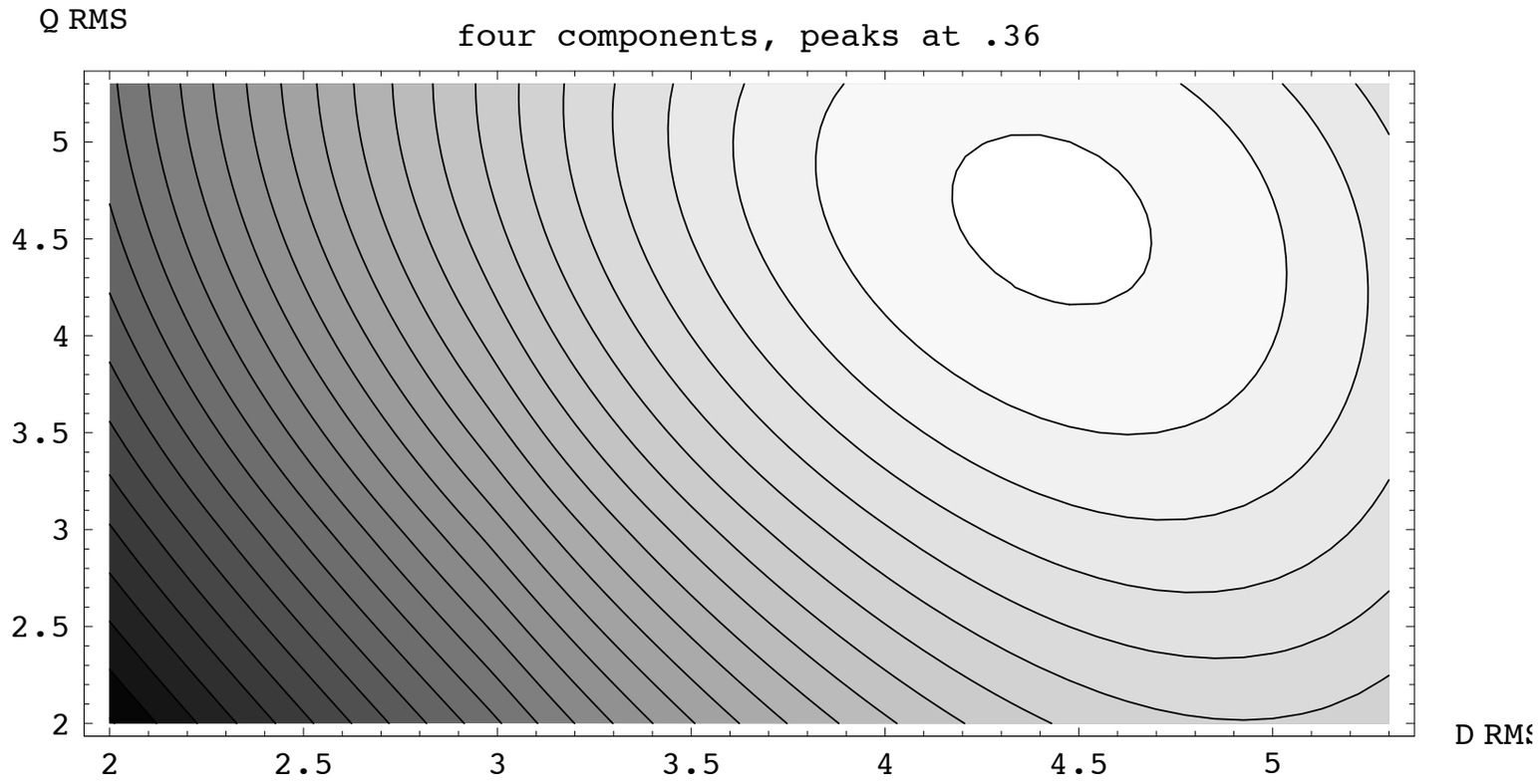
- Improves overlap: $0.50 \rightarrow 0.80$

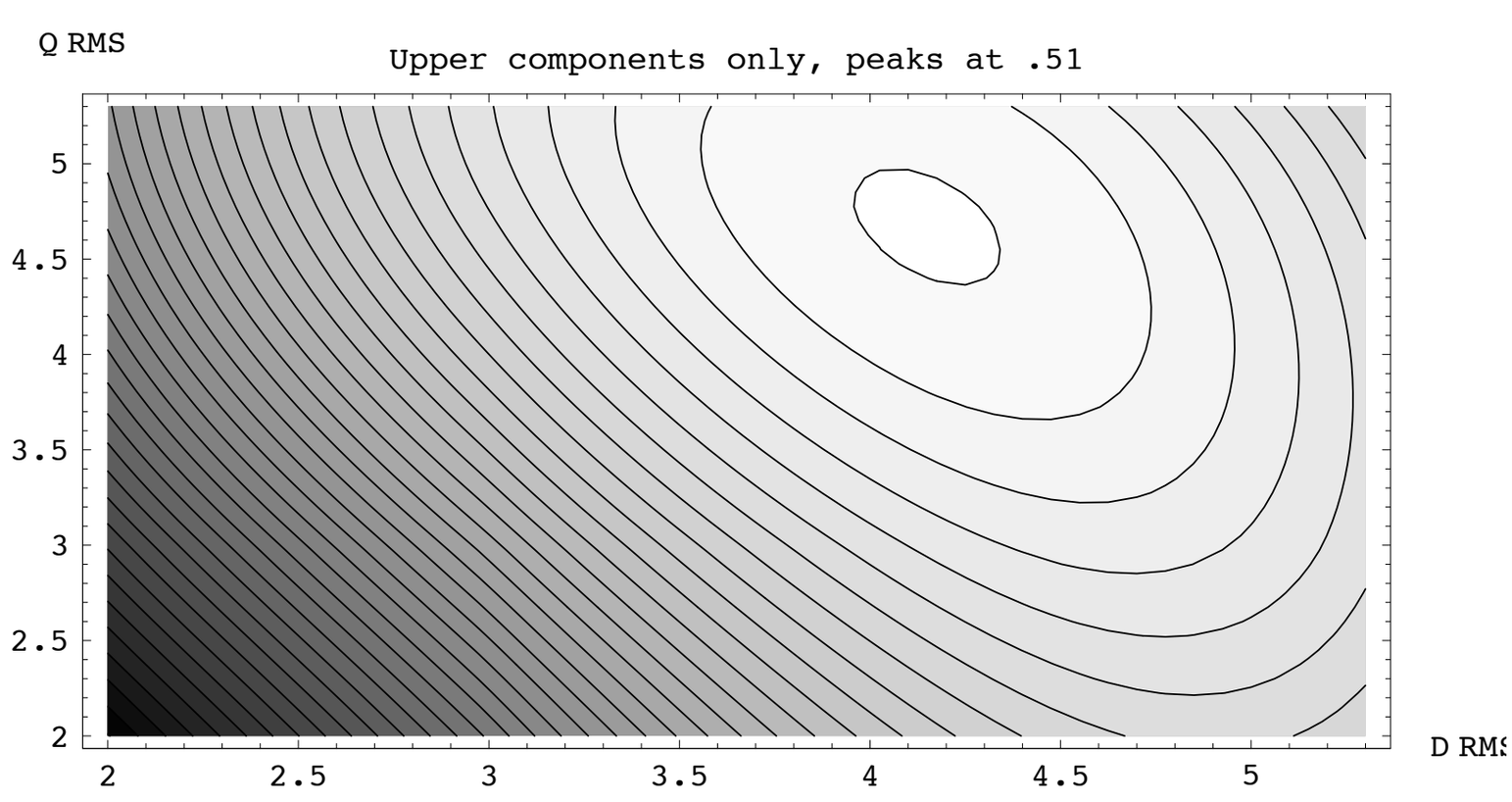
Change Relative Size of Quark, Diquark:

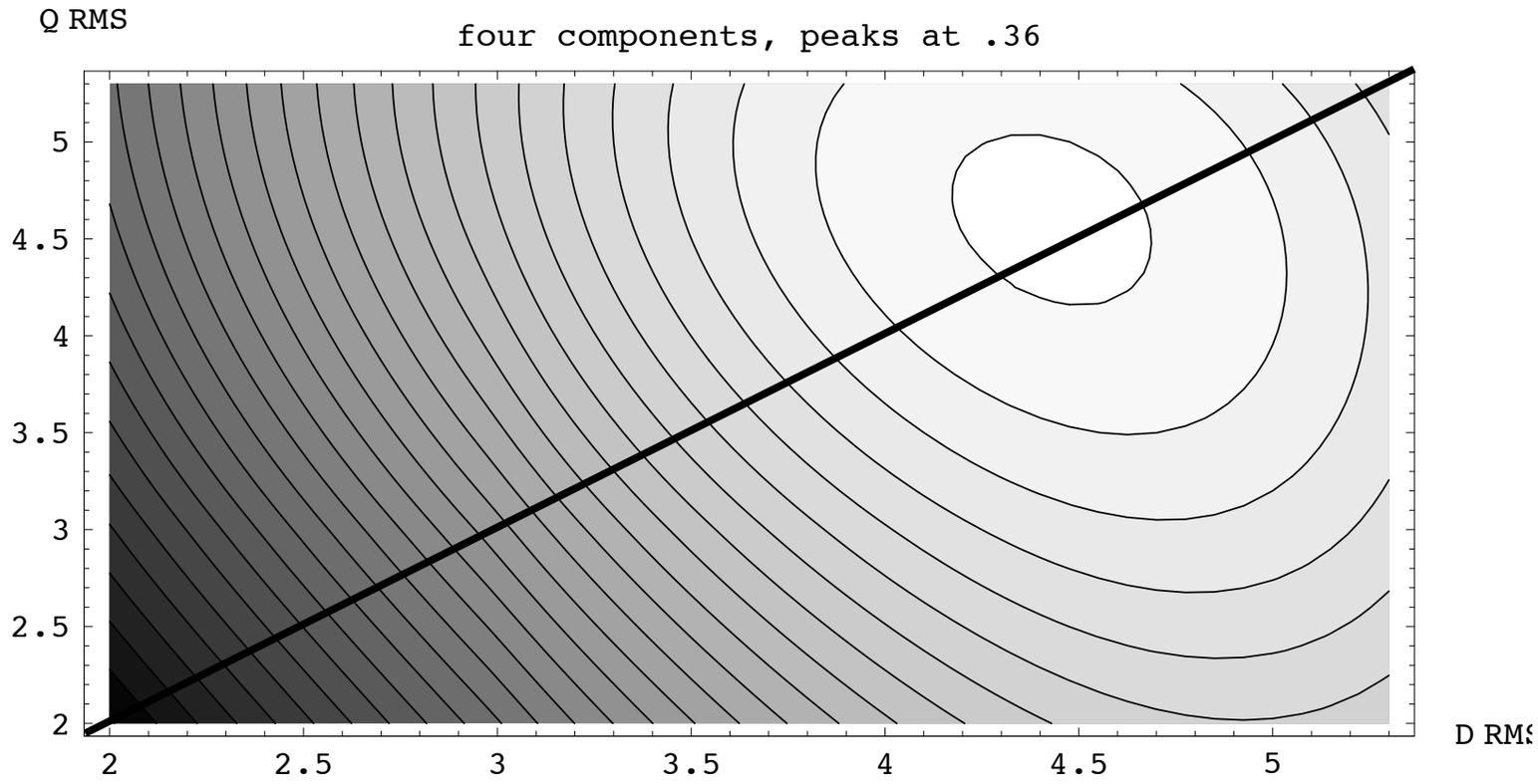
- Perhaps quarks in diquark feel a different potential than single quark does....
- Use trial source:

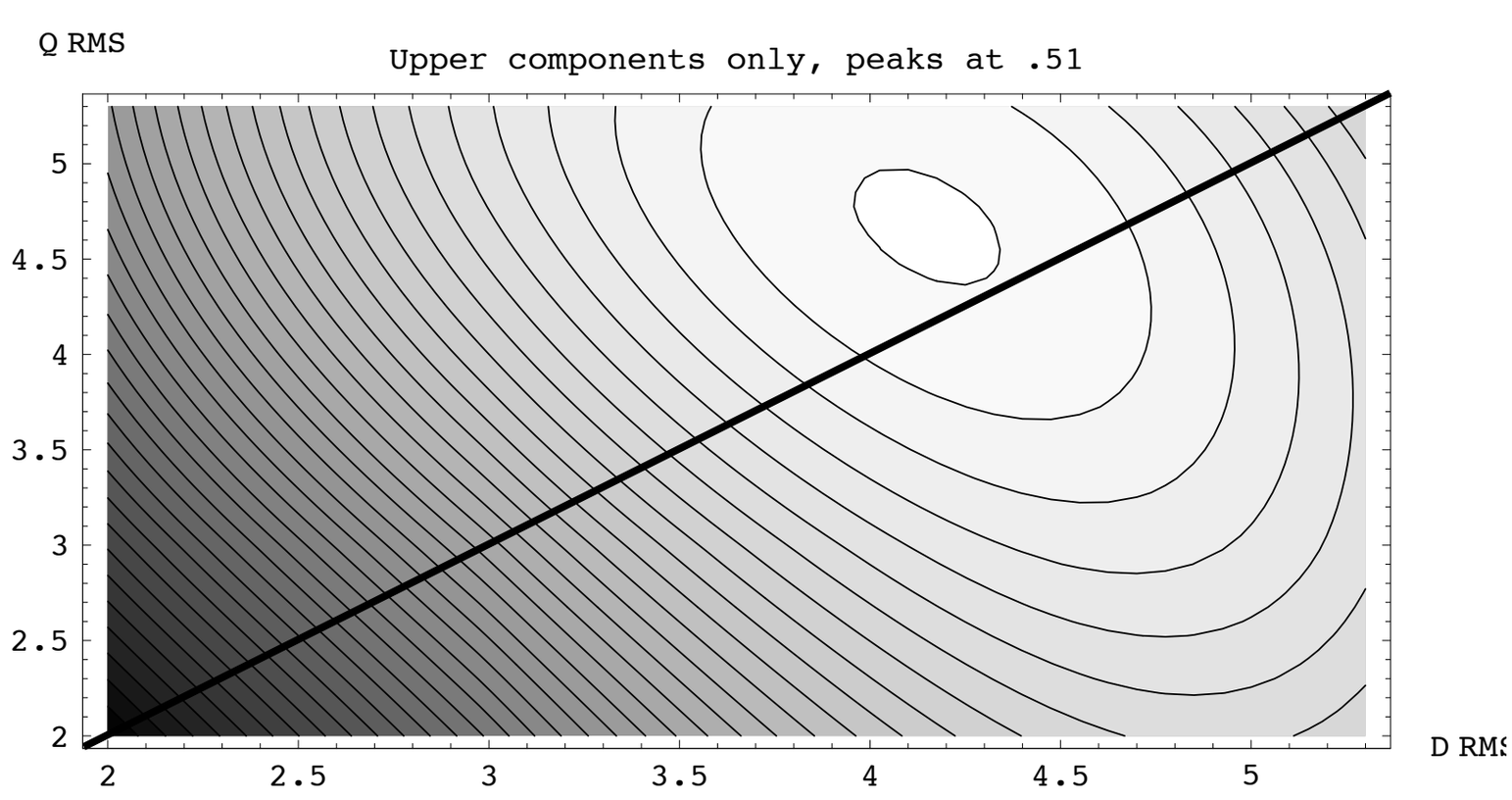
$$\mathrm{J} = (\ \mathrm{U}_{s1}\ \mathrm{C}\gamma_5\ \mathrm{D}_{s1}\ )\ \mathrm{U}_{s2}$$

- Want to find maximum overlap in two-dimensional parameter space


Q RMS
four components, peaks at .36
5
4.5
4
3.5
3
2.5
2
2
2.5
3
3.5
4
4.5
5
D RMS


Q RMS
Upper components only, peaks at .51
5
4.5
4
3.5
3
2.5
2
2
2.5
3
3.5
4
4.5
5
D RMS


Q RMS
four components, peaks at .36
5
4.5
4
3.5
3
2.5
2
2
2.5
3
3.5
4
4.5
5
D RMS


Q RMS
Upper components only, peaks at .51
5
4.5
4
3.5
3
2.5
2
2
2.5
3
3.5
4
4.5
5
D RMS

Try displacing quark:

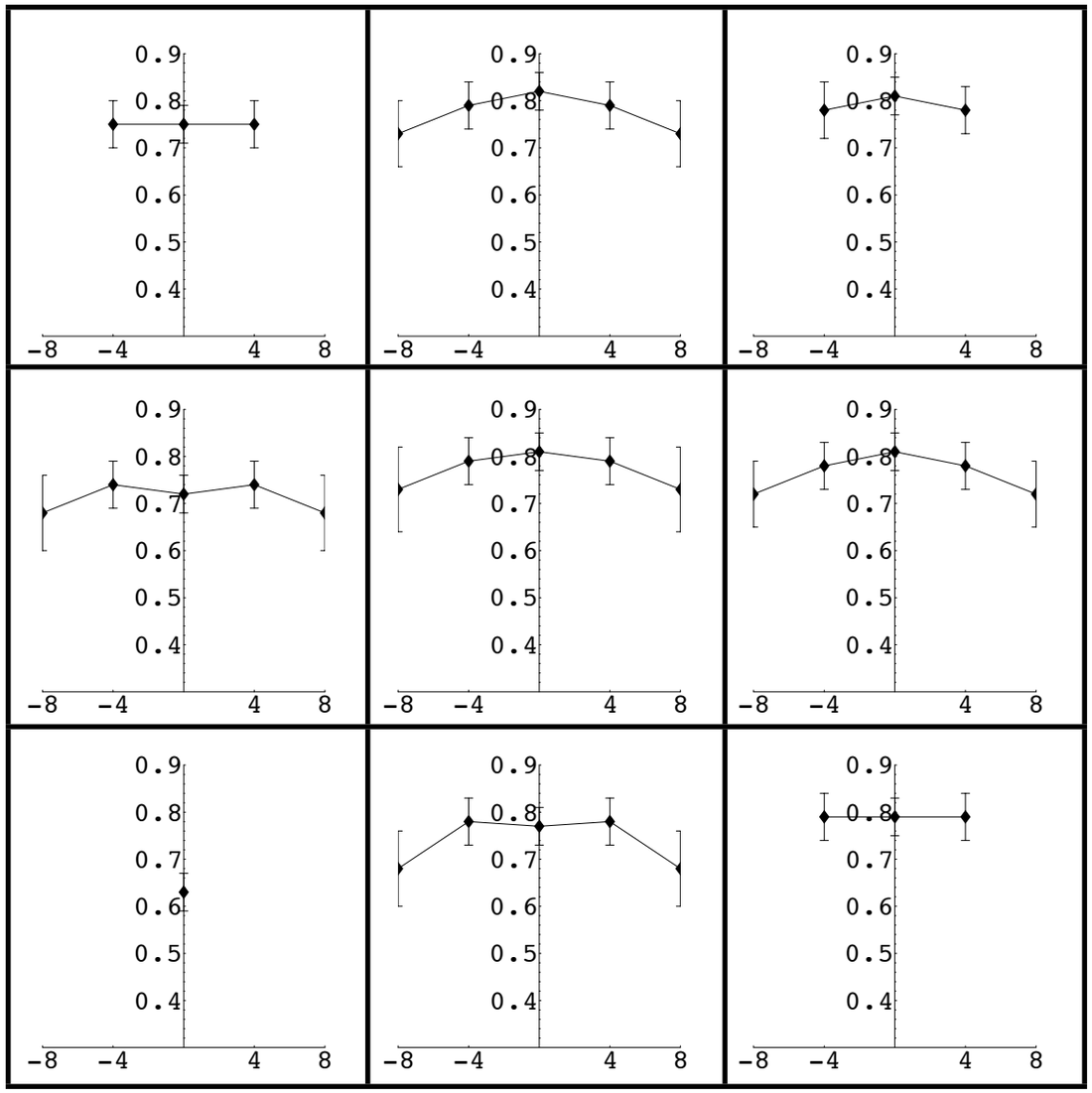
- Motivated by flux tube model
- Use a source with quark, diquark separated:

$$\mathrm{J} = (\ \mathrm{U}_{s1}(x)\ \mathrm{C}\gamma_5\ \mathrm{D}_{s1}(x)\ )\ \mathrm{U}_{s2}(y)$$

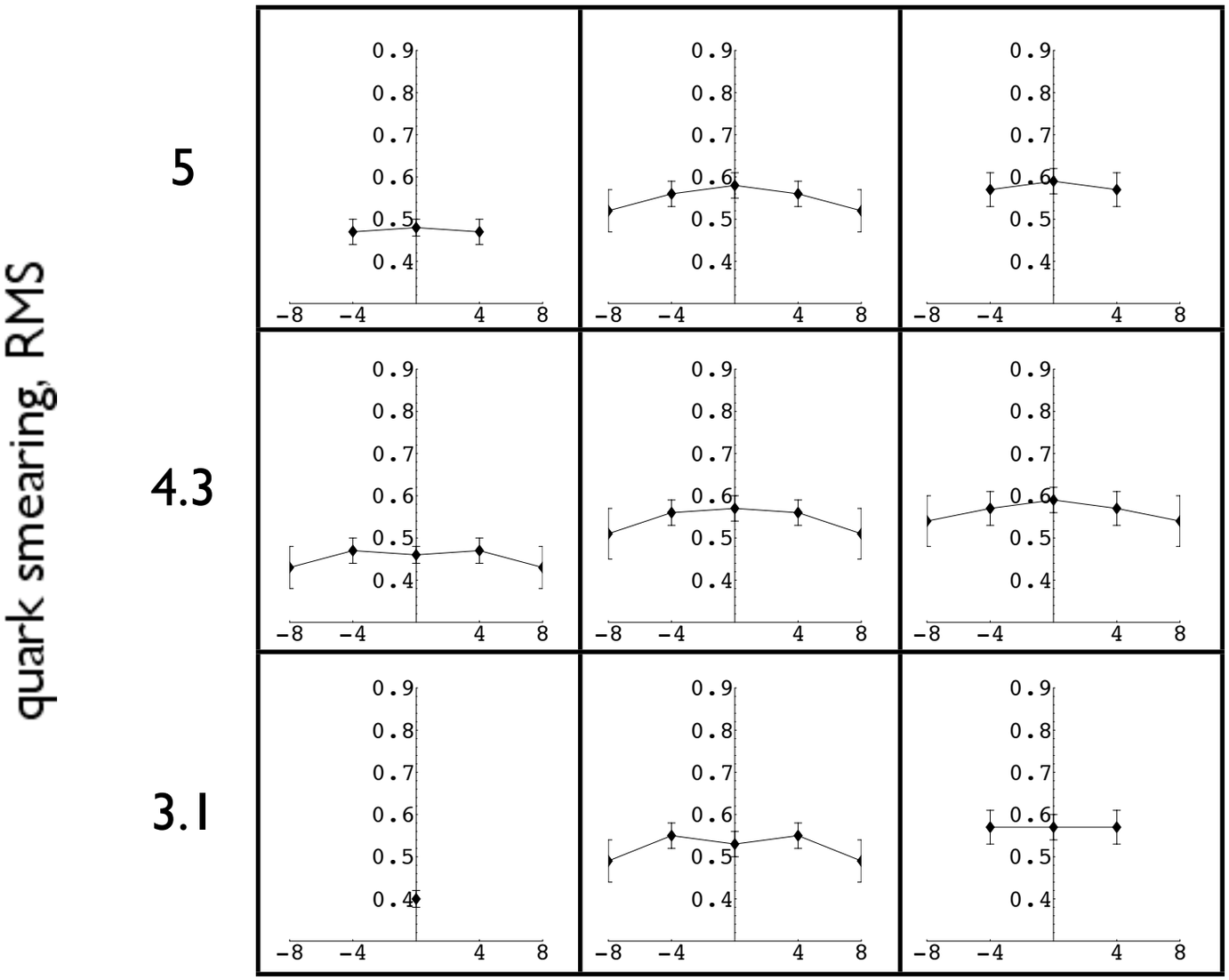
- Symmetrize sink:

$$C(t) = \sum_{\vec{r}} \sum_{\hat{j}} \langle\ (\bar{u}(\vec{r},t) C\gamma_5 \bar{d}(\vec{r},t))\bar{u}(\vec{r}+\ell\hat{j},t)\ (u(0,0)C\gamma_5 d(0,0))u(\ell\hat{x},0)\ \rangle$$

upper components only



four component spinors



3.1 4.3 5

diquark smearing, RMS

Summary

- What can we say about the optimal nucleon source?
  - Point vs. extended:
  - 2- vs. 4-component:
  - Smearing gauge field:
  - Relative size of quark, diquark?
  - Relative location of quark, diquark?

Summary

- What can we say about the optimal nucleon source?
  - Point vs. extended: $\sim 0.01\% \rightarrow 35\%$
  - 2- vs. 4-component:
  - Smearing gauge field:
  - Relative size of quark, diquark?
  - Relative location of quark, diquark?

Summary

- What can we say about the optimal nucleon source?
  - Point vs. extended: $\sim 0.01\% \rightarrow 35\%$
  - 2- vs. 4-component: $35\% \rightarrow 50\%$
  - Smearing gauge field:
  - Relative size of quark, diquark?
  - Relative location of quark, diquark?

Summary

- What can we say about the optimal nucleon source?
  - Point vs. extended: $\sim 0.01\% \rightarrow 35\%$
  - 2- vs. 4-component: $35\% \rightarrow 50\%$
  - Smearing gauge field: $50\% \rightarrow 80\%$
  - Relative size of quark, diquark?
  - Relative location of quark, diquark?