Nucleon Generalized Form Factors with Domain Wall Fermions on an Asqtad Sea

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For the LHP Collboration

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Outline

- Physics Motivation
- Mixed Action Calculation
- Chiral Extrapolation
- Origin of the Nucleon Spin
- Comparison with Phenomenology
- Summary and Outlook

Physics Motivation

- Generalized parton distributions probe the light cone quark distribution $q(x, r_\perp)$ as a function of longitudinal momentum fraction x and transverse position $r_\perp$
- Specify total quark contribution to nucleon spin
- Reveal transverse structure of light cone wave function
- Synergy with experiment
  - Experiment measures convolutions of GPD's
  - Lattice measures moments of GPD's

Gauge Invariant Decomposition of Nucleon Spin

X. Ji PRL 78, 610 (1997)

$$
\begin{aligned}
J^i &= \frac{1}{2}\epsilon^{ijk}\int d^3x[T^{\alpha\nu}x^\mu - T^{\alpha\mu}x^\nu] = J^i_q + J^i_g \\
\vec{J}_q &= \int d^3x\, \psi^\dagger[\vec{\gamma}\gamma_5 + \vec{x}\times(-i\vec{D})]\psi \\
&= \frac{1}{2}[A_{20}(q^2=0) + B_{20}(q^2=0)] \\
\vec{J}_g &= \int d^3x[\vec{x}\times(\vec{E}\times\vec{B})] \\
&\neq \Delta g
\end{aligned}
$$

- A_{20} and B_{20} are generalized form factors defined below
- Cannot write J_g as sum of helicity and orbital contributions of local operators

Generalized form factors

$$\mathcal{O}_q^{\{\mu_1\mu_2\dots\mu_n\}} = \overline{\psi}_q\gamma^{\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}\psi_q$$

$$\bar{P} = \tfrac{1}{2}(P' + P)$$

$$\Delta = P' - P$$

$$t = \Delta^2$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\mu_1}|P\rangle &= \langle\!\langle\gamma^{\mu_1}\rangle\!\rangle A_{10}(t) \\
&+ \frac{i}{2m}\langle\!\langle\sigma^{\mu_1\alpha}\rangle\!\rangle\Delta_\alpha B_{10}(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\}}|P\rangle &= \bar{P}^{\{\mu_1}\langle\!\langle\gamma^{\mu_2\}}\rangle\!\rangle A_{20}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\langle\!\langle\sigma^{\mu_2\}\alpha}\rangle\!\rangle\Delta_\alpha B_{20}(t) \\
&+ \frac{1}{m}\Delta^{\{\mu_1}\Delta^{\mu_2\}}C_2(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\mu_3\}}|P\rangle &= \bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\!\langle\gamma^{\mu_3\}}\rangle\!\rangle A_{30}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\!\langle\sigma^{\mu_3\}\alpha}\rangle\!\rangle\Delta_\alpha B_{30}(t) \\
&+ \Delta^{\{\mu_1}\Delta^{\mu_2}\langle\!\langle\gamma^{\mu_3\}}\rangle\!\rangle A_{32}(t) \\
&+ \frac{i}{2m}\Delta^{\{\mu_1}\Delta^{\mu_2}\langle\!\langle\sigma^{\mu_3\}\alpha}\rangle\!\rangle\Delta_\alpha B_{32}(t),
\end{aligned}$$

Limits and Sum Rules

- Moments of parton distributions t →0

$$A_{n0} = \int dx x^{n-1} q(x)$$

- Electromagnetic form factors

$$A_{10} = F_1(t), \quad B_{10} = F_2(t)$$

- Total quark angular momentum

$$J_q = \tfrac{1}{2}[A(0)_{20} + B(0)_{20}]$$

- Momentum sum rule

$$1 = A_{20,q}(0) + A_{20,g}(0) = \langle x \rangle_q + \langle x \rangle_g$$

- Nucleon spin sum rule

$$\begin{aligned} \frac{1}{2} &= \frac{1}{2}\big(A_{20,q}(0) + A_{20,g}(0) + B_{20,q}(0) + B_{20,g}(0)\big) \\ &= \frac{1}{2}\Delta\Sigma_q + L_q + J_g \end{aligned}$$

Domain wall quarks on a staggered sea

- $\mathcal{O}(a^2)$ Tadpole improved staggered sea quarks (Asqtad)
  - Economical entre to chiral regime
  - MILC 2+1 flavor lattices with large L, small m_π publicly available
- Domain wall valence quarks
  - Chiral symmetry to within controlled m_{res}
  - Avoids operator mixing
  - $\mathcal{O}(a^2)$
  - Conserved 5-d axial current facilitates renormalization
- Mixed action ChPT   Chen, O'Connell, Walker-Loud, arXiv: 0706.00035
  - One-loop results have contiuum chiral behavior with low energy constants containing perturbative a-dependent corrections

Statistics for hadron structure

- Signal to noise degrades as pion mass decreases

$$\frac{\text{Signal}}{\text{Noise}} = \frac{\langle J(t)J(0)\rangle}{\frac{1}{\sqrt{N}}\sqrt{\langle |J(t)J(0)|^2\rangle - (\langle J(t)J(0)\rangle)^2}}$$

$$\sim \frac{Ae^{-M_N t}}{\frac{1}{\sqrt{N}}\sqrt{Be^{-3m_\pi t} - Ce^{-2M_N t}}}$$

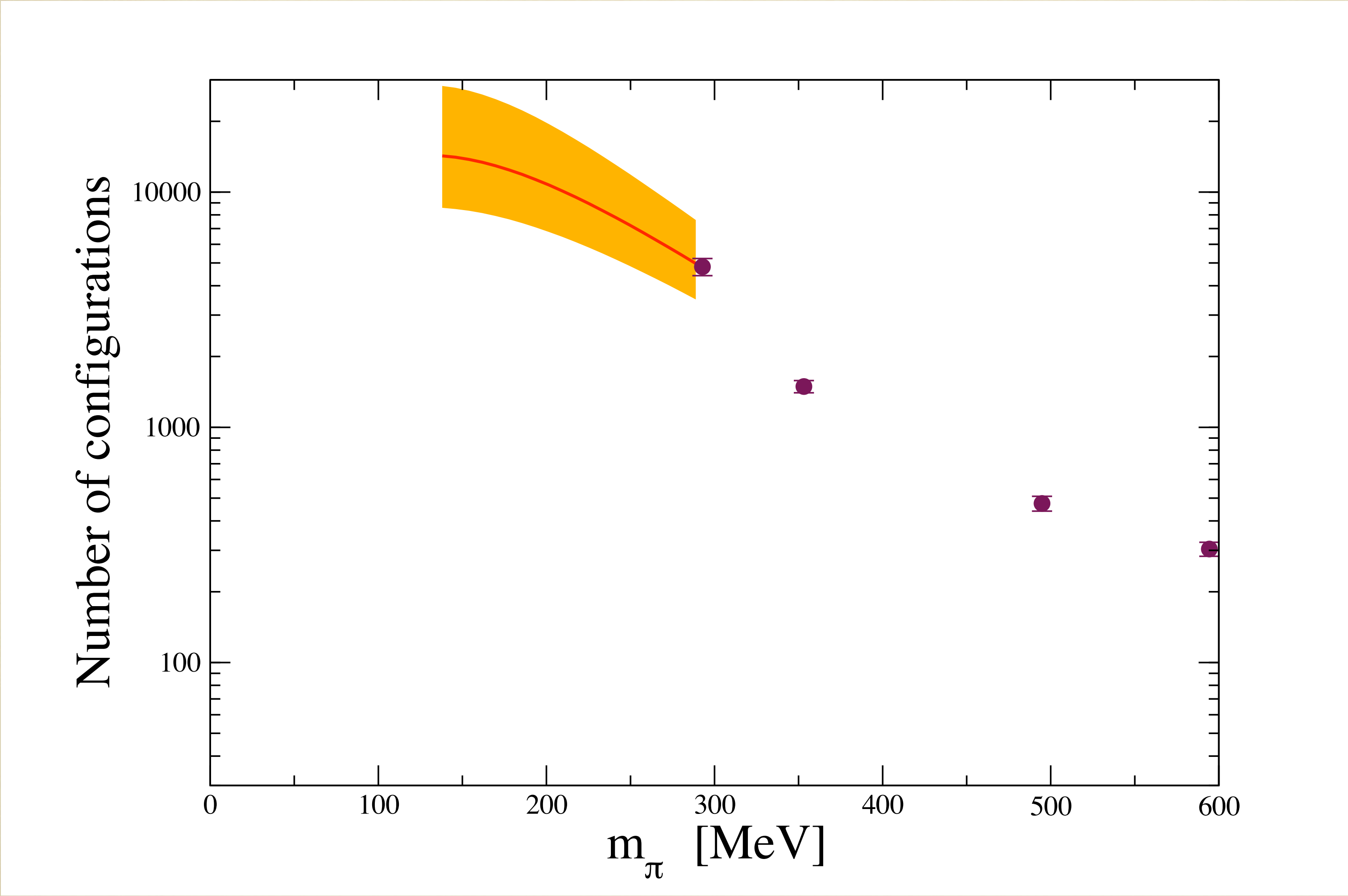
$$\sim \sqrt{N}De^{-(M_N - \frac{3}{2}m_\pi)}$$

- Due to different overlap of nucleon and 3 pions also have volume dependence: $\sqrt{V}$
- Kostas Orginos analyzed signal/noise correlation functions for mixed action data

Required Measurements

Measurements required for 3% accuracy at T=10

May need significantly more



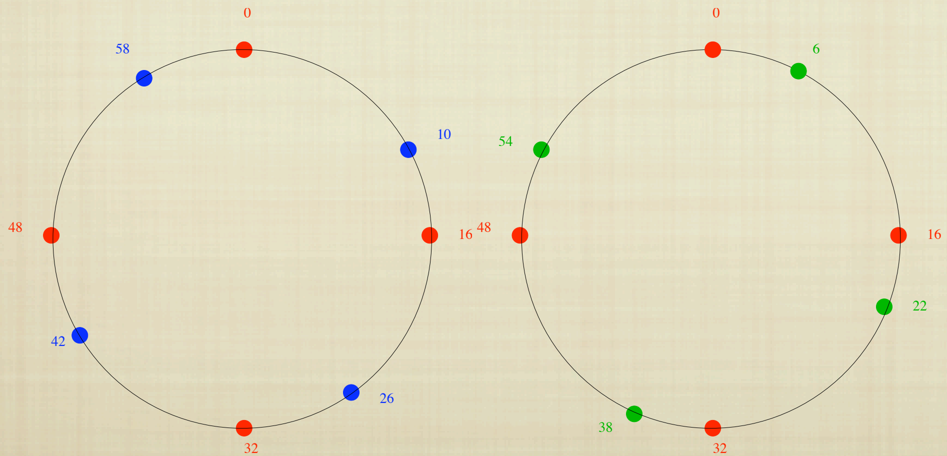
Numerical calculations

- MILC Asqtad configurations $N_F = 2+1$, $a = 0.125$ fm
- Domain wall valence quarks
  - $L_S = 16$, $M_5 = 1.7$
  - Valence quark mass tuned to Asqtad Goldstone pion mass.
  - Recent improvement: Factor 8 increase in # measurements

m_π	# configs	Vol	L (fm)	# measurements	
758	423	20^3	2.5	423	
688	348	20^3	2.5	348	
597	561	20^3	2.5	561	
495	477	20^3	2.5	477	
356	628	20^3	2.5	628	5024
353	274	28^3	3.5	274	2192
293	464	20^3	2.5		3712

Improvements in Measurements

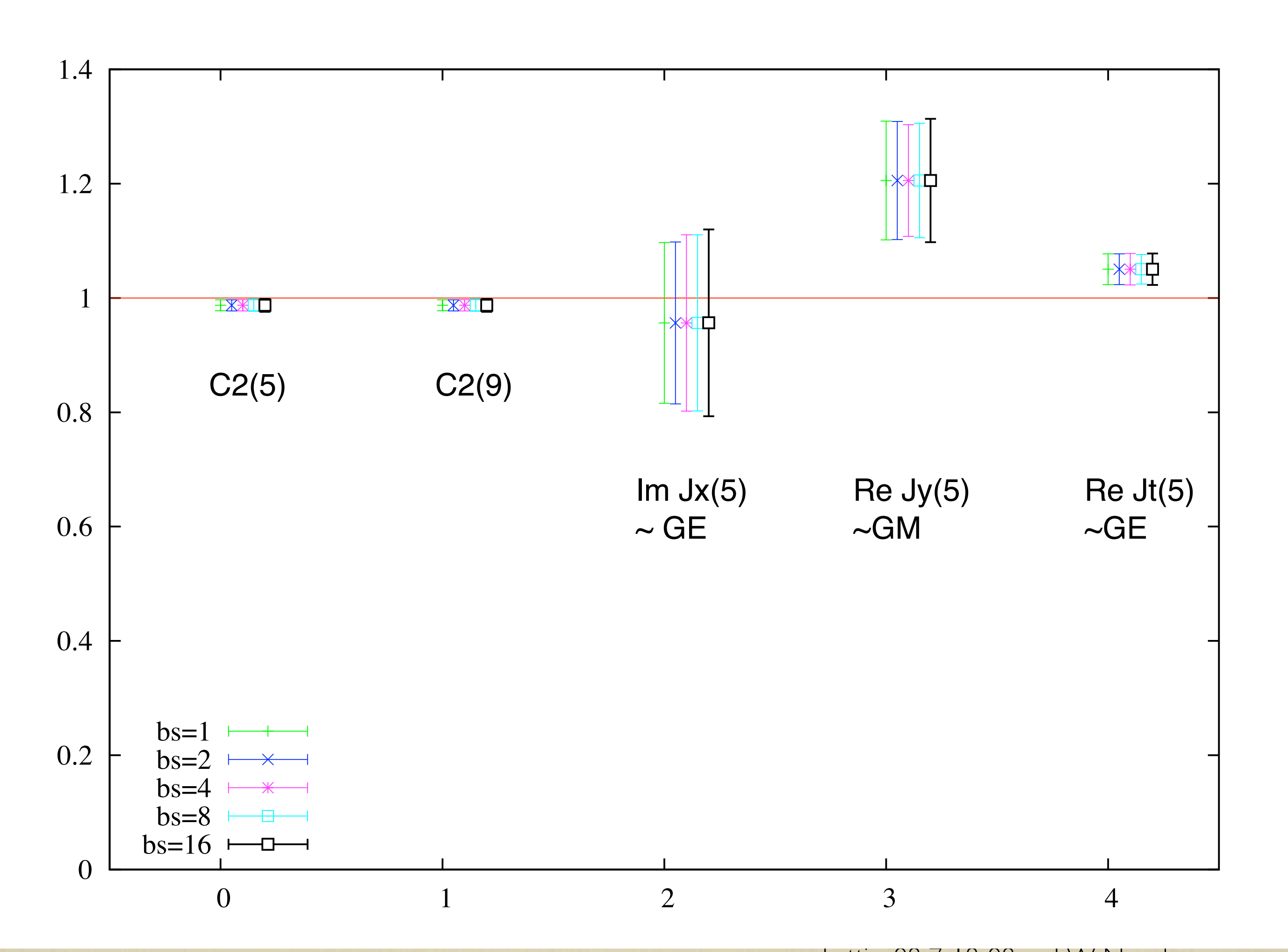
- 4 sets of forward propagators per configuration ● shifted spatially
- Coherent sequential propagators for 4 nucleon sinks ● and 4 antinucleon sinks ●
  - Save factor 4 in time
  - Gauge averaging cancels contributions from neighbors
- Shorter source sink separation
- Overall error reduction ~ factor 4



Statistical independence of measurements

Jackknife binning of correlation functions and matrix elements

Sergey Syritsyn



Perturbative renormalization

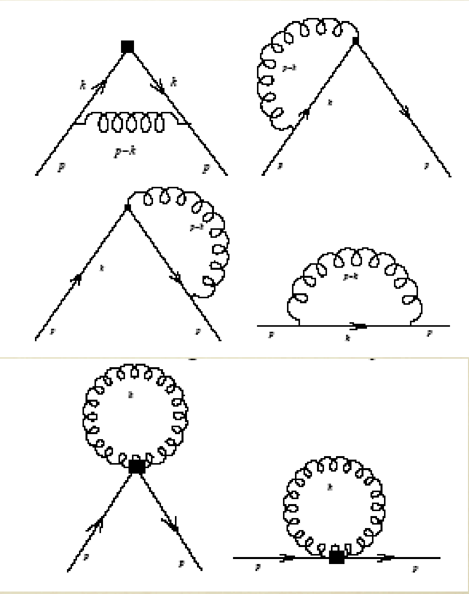
$$O_i^{\overline{MS}}(Q^2) = \sum_j \left(\delta_{ij} + \frac{g_0^2}{16\pi^2} \frac{N_c^2 - 1}{2N_c} \left(\gamma_{ij}^{\overline{MS}} \log(Q^2 a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}}) \right) \right) \cdot O_j^{LATT}(a^2)$$

HYP smeared domain wall fermions - B. Bistrovic

$$Z_{\mathcal{O}} = \frac{Z_{\mathcal{O}}^{pert}}{Z_A^{pert}} Z_A^{nonpert}$$

Evolve to $Q^2 = 4\ \text{GeV}^2$

operator	$H(4)$	HYP
$\bar{q}[\gamma_5]q$	$1_1^{\pm}$	0.981
$\bar{q}[\gamma_5]\gamma_\mu q$	$4_4^{\mp}$	0.976
$\bar{q}[\gamma_5]\sigma_{\mu\nu} q$	$6_1^{\mp}$	0.992
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}} q$	$6_3^{\pm}$	0.979
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}} q$	$3_1^{\pm}$	0.975
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}} q$	$8_1^{\mp}$	0.988
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}} q$	mixing	1.88×10^{-3}
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}} q$	$4_2^{\mp}$	0.987
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_\alpha D_{\beta\}} q$	$2_1^{\pm}$	0.993
$\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}} q$	$8_1^{\pm}$	0.994
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]} q$	$6_1^{\mp}$	0.982
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\{\nu]}D_{\alpha\}} q$	$8_1^{\pm}$	0.959



Overdetermined system for form factors

Calculate ratio

$$R_{\mathcal{O}}(\tau, P', P) = \frac{C^{3\mathrm{pt}}_{\mathcal{O}}(\tau, P', P)}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}, P')} \left[\frac{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}} - \tau + \tau_{\mathrm{src}}, P)\, C^{2\mathrm{pt}}(\tau, P')\, C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}, P')}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}} - \tau + \tau_{\mathrm{src}}, P')\, C^{2\mathrm{pt}}(\tau, P)\, C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}, P)} \right]^{1/2}$$

Schematic form

$$\begin{aligned}
\langle \mathcal{O}_i^{cont} \rangle &= \sum_j a_{ij} \mathcal{F}_j \\
\langle \mathcal{O}_i^{cont} \rangle &= \sqrt{E'E} \sum_j Z_{ij} \overline{R}_j \\
\overline{R}_i &= \frac{1}{\sqrt{E'E}} \sum_{jk} Z^{-1}_{ij} a_{jk} \mathcal{F}_k \\
&\equiv \sum_j a'_{ij} \mathcal{F}_j \,.
\end{aligned}$$

Chiral extrapolation of GPD's

Haegler et al, LHPC, Phys Rev D77, 094502 (2008)

- Fundamental problem - large pion masses
- Covariant Baryon Chiral Perturbation theory gives consistent fit to matrix elements of twist-2 operators for wide range of masses
  (Dorati, Gail, Hemmert, Nucl Phys A798, 96 (2008)
- HBChPT expands in $\epsilon = \left\{ \frac{m_\pi}{\Lambda_\chi}, \frac{p}{\Lambda_\chi}, \frac{m_\pi}{M_N^0}, \frac{p}{M_N^0} \right\}$

  $\Lambda_\chi = 4\pi f_\pi \sim 1.17\,\mathrm{GeV}, \quad M_N^0 \sim 890\,\mathrm{MeV}$
- CBChPT resums all orders of $\left(\frac{1}{M_N^0}\right)^m$

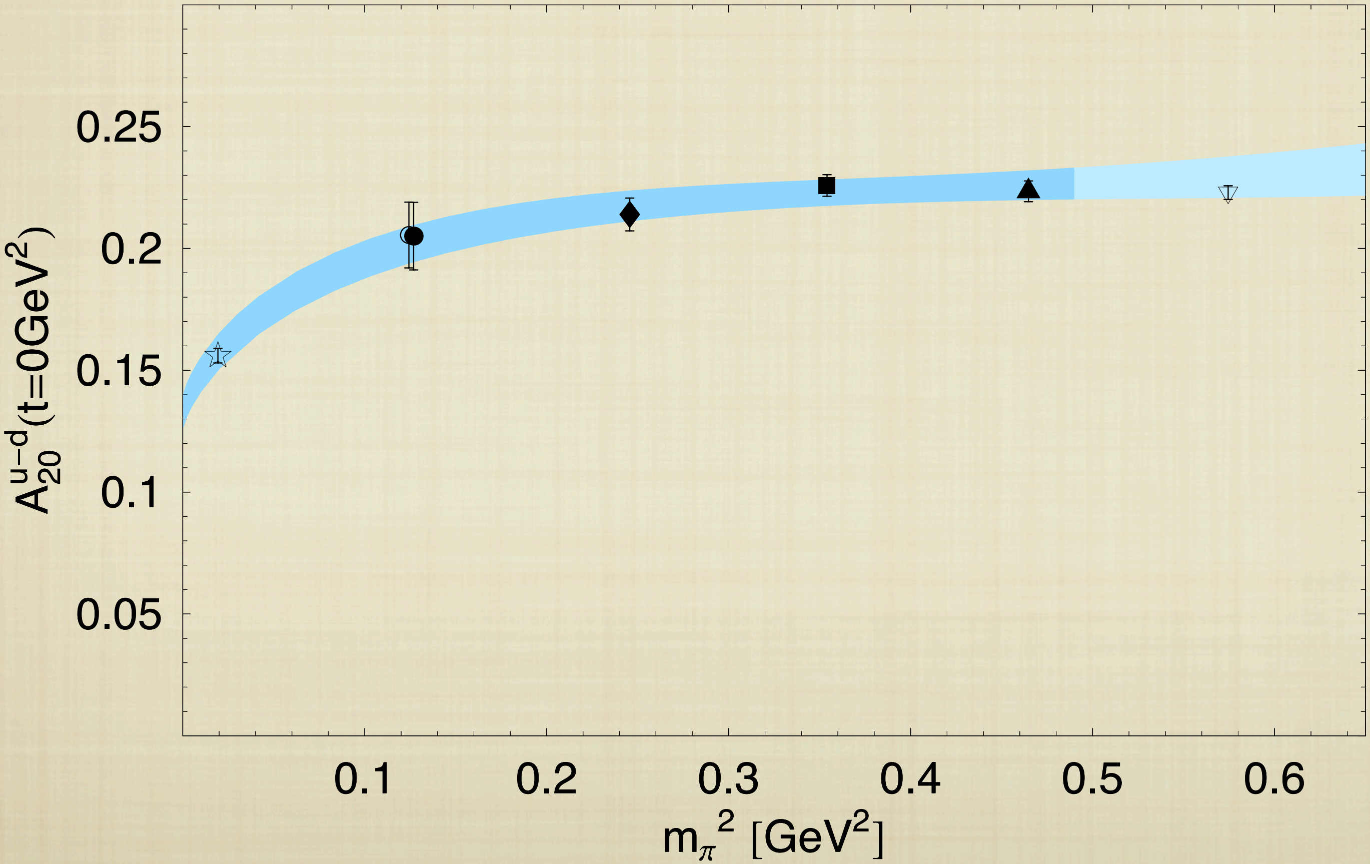
Chiral extrapolation of $\langle x \rangle_q^{u-d} = A_{20}^{u-d}(t=0)$

Chiral extrapolation O(p^2) CBChPT (Dorati, et al, NP A798, 96 (2008))

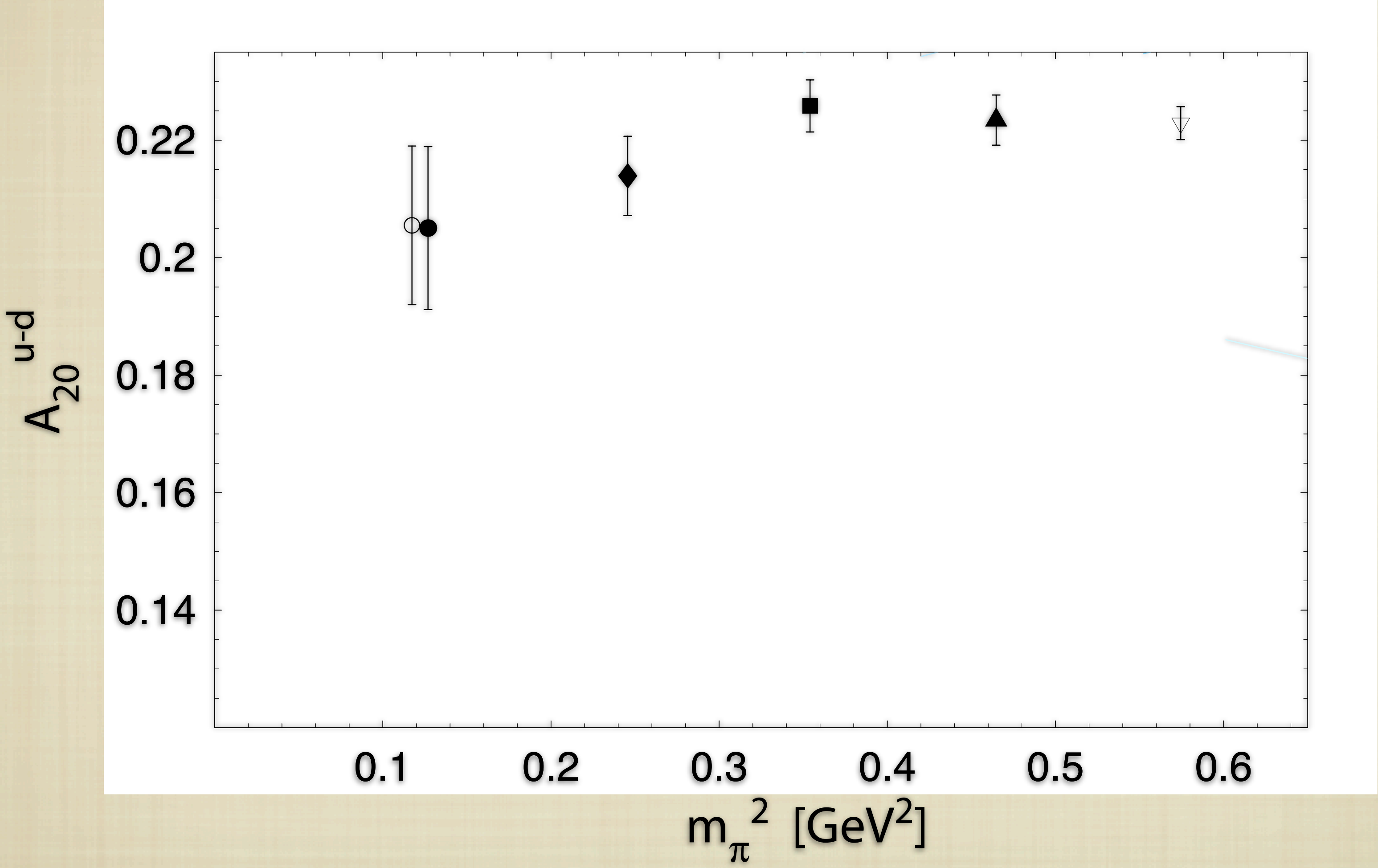
Global fit to A, B, C with 9 fit parameters

$$A_{20}^{u-d}(t, m_\pi) = A_{20}^{0,u-d}\left(f_A(m_\pi) + \frac{g_A^2}{192\pi^2 f_\pi^2} h_A(t, m_\pi)\right) + \widetilde{A}_{20}^{0,u-d} j_A(m_\pi) + A_{20}^{m_\pi,u-d} m_\pi^2 + A_{20}^t t$$

$$\sim a\left(1 - \frac{3g_A^2 + 1}{4\pi f_\pi^2} m_\pi^2 \ln m_\pi^2\right) + b m_\pi^2 \cdots$$



Chiral extrapolation of <x>



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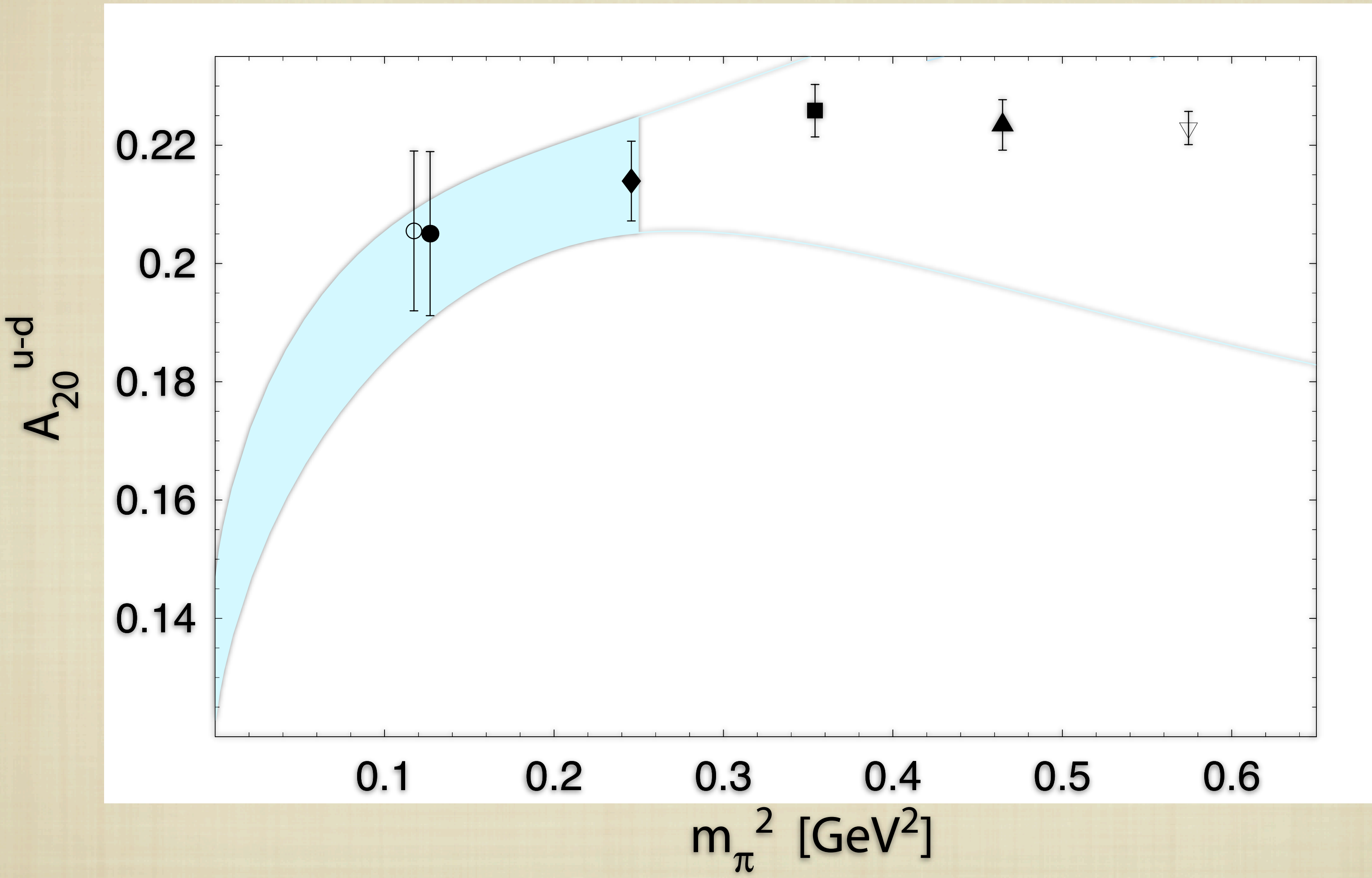
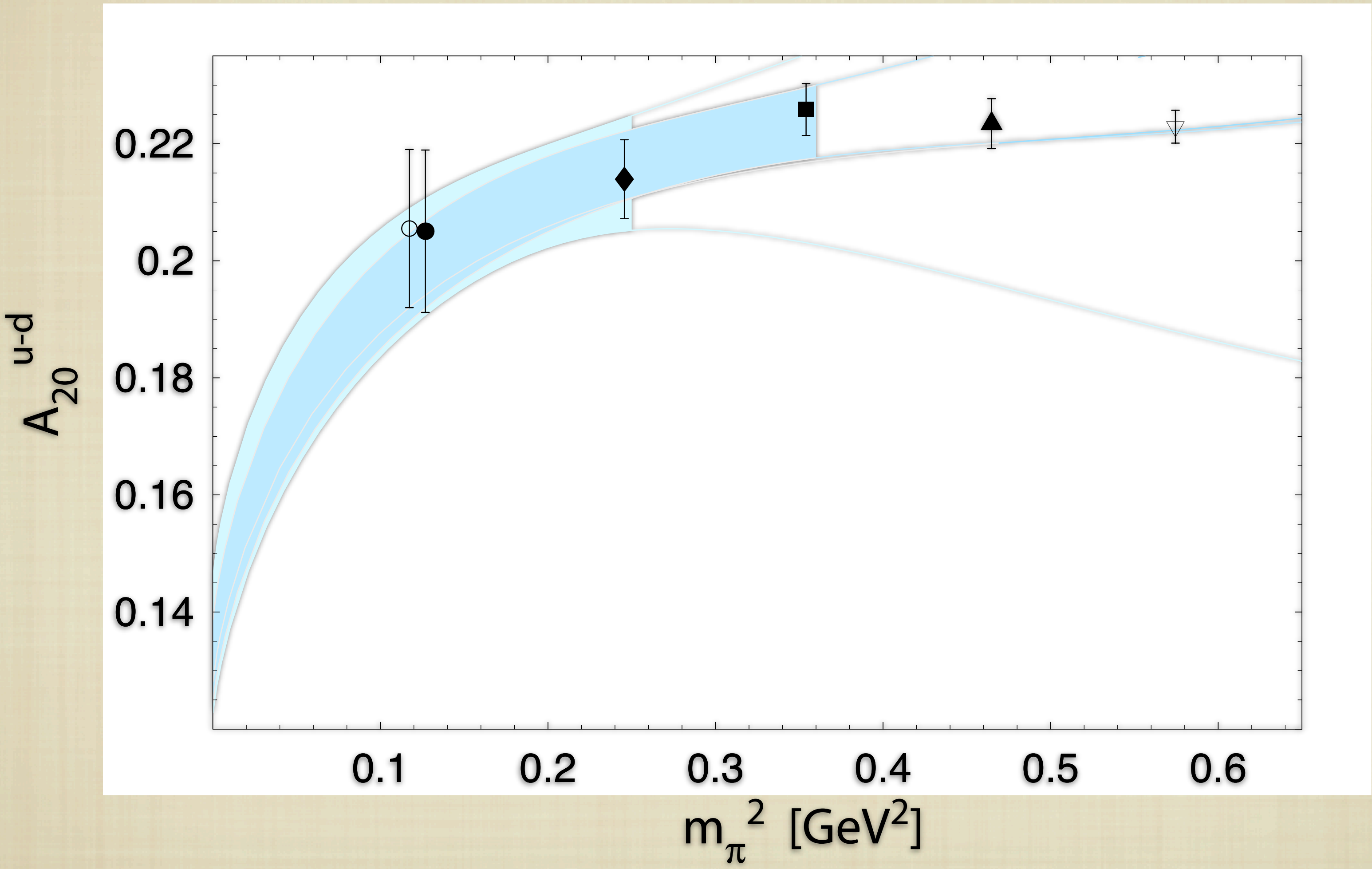
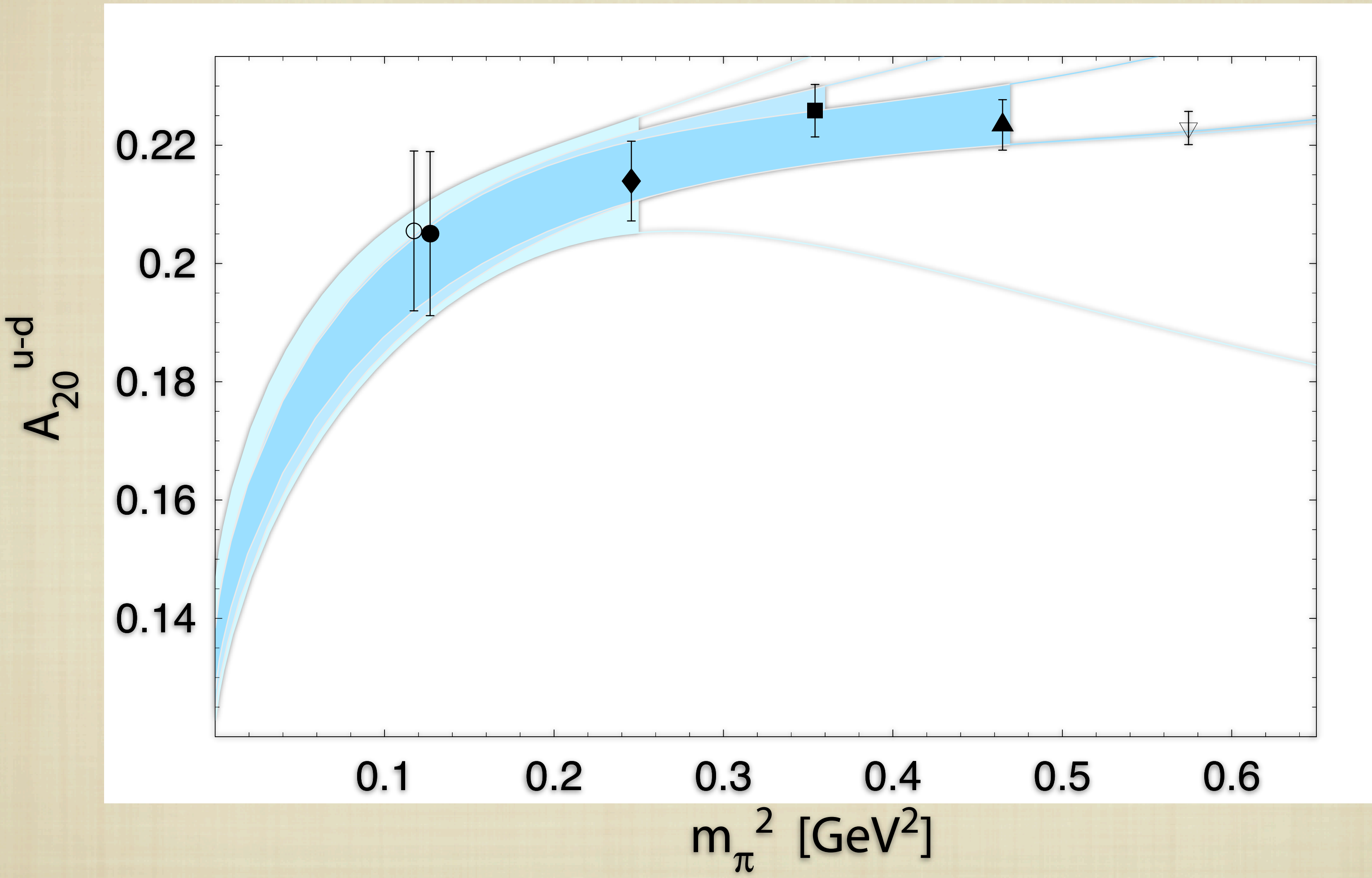


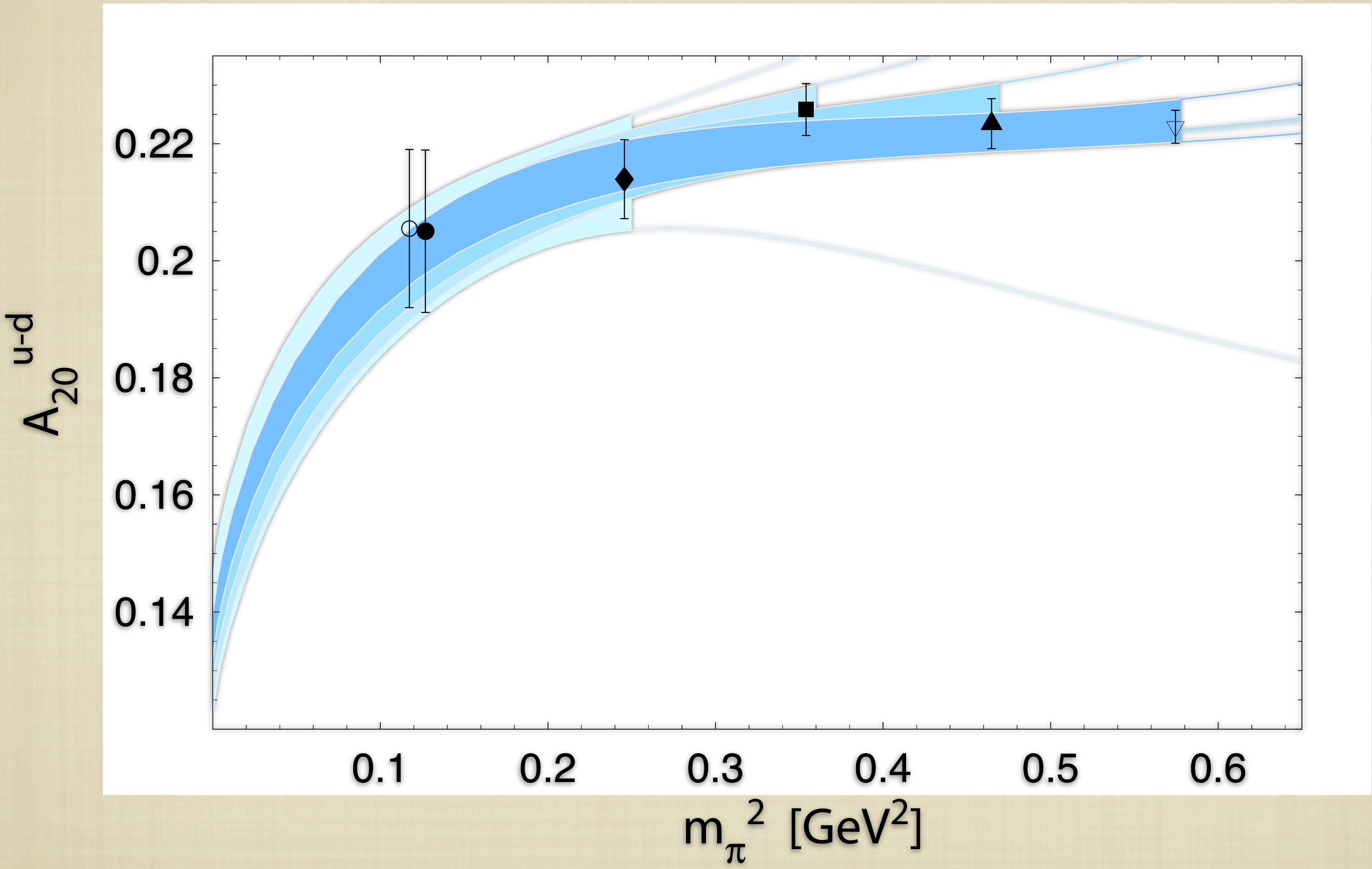
Chiral extrapolation of <x>



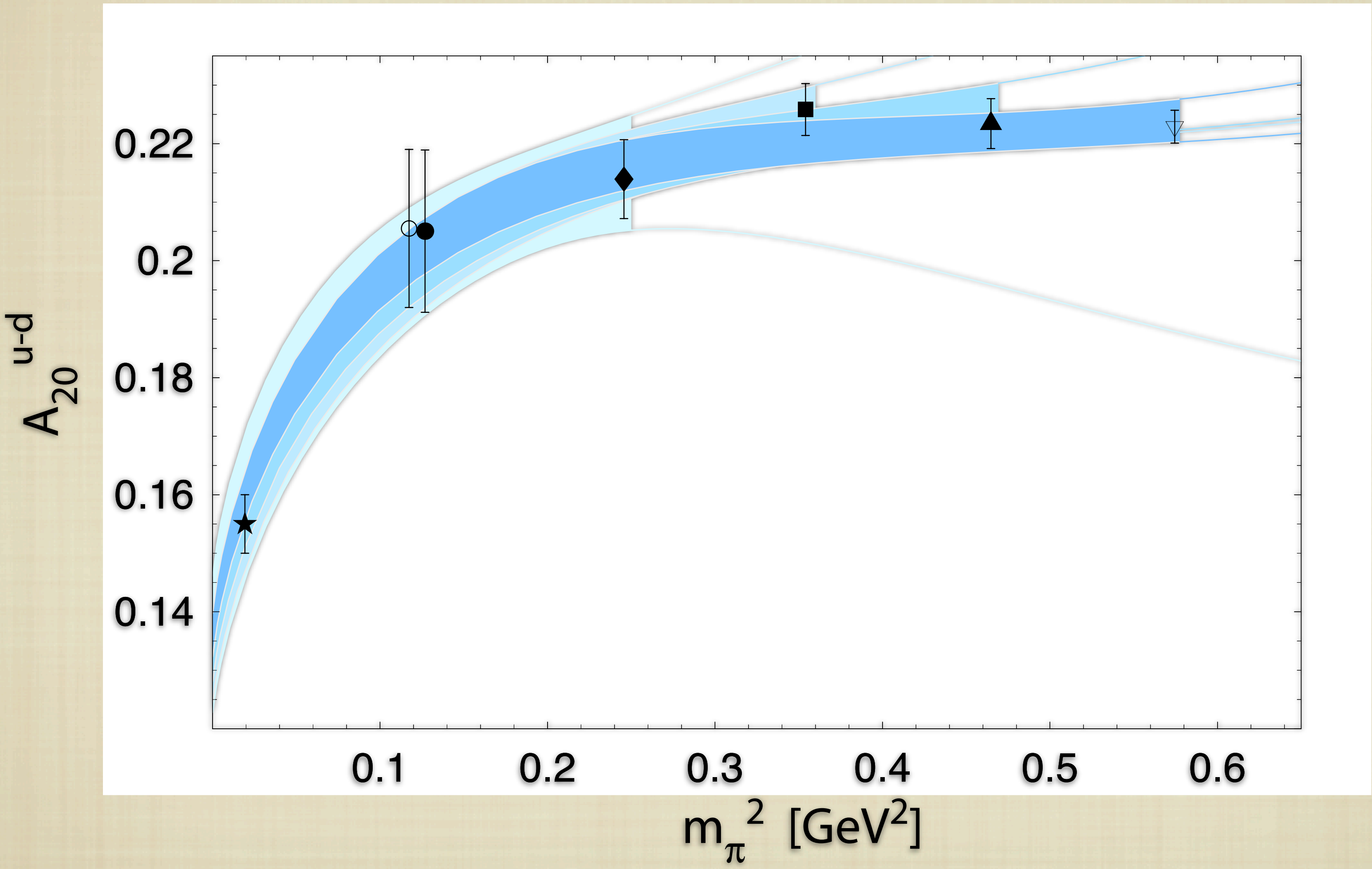
Chiral extrapolation of <x>



Chiral extrapolation of <x>



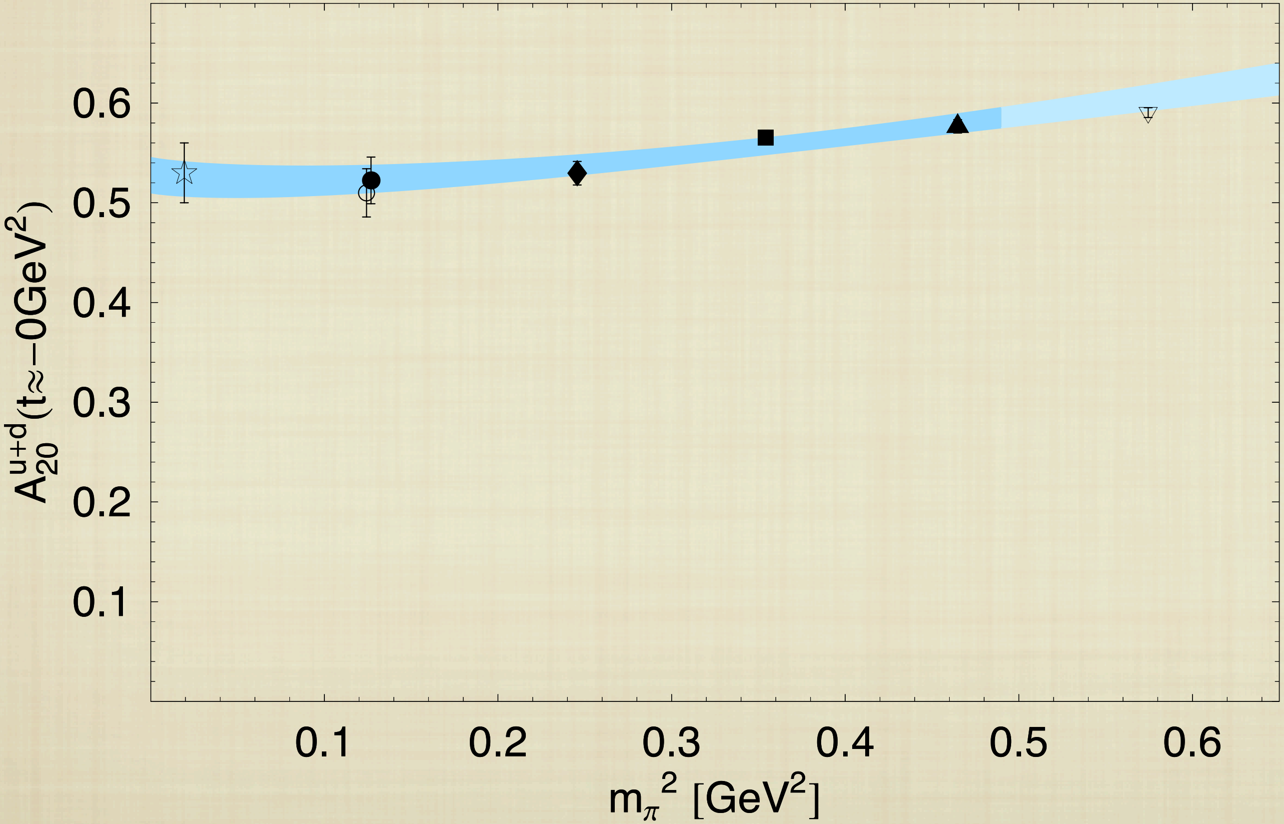
Chiral extrapolation of <x>



Chiral extrapolation of $\langle x \rangle_q^{u+d} = A_{20}^{u+d}(t=0)$

Chiral extrapolation O(p²) CBChPT (Dorati, Hemmert, et. al.)

Note: connected diagrams only

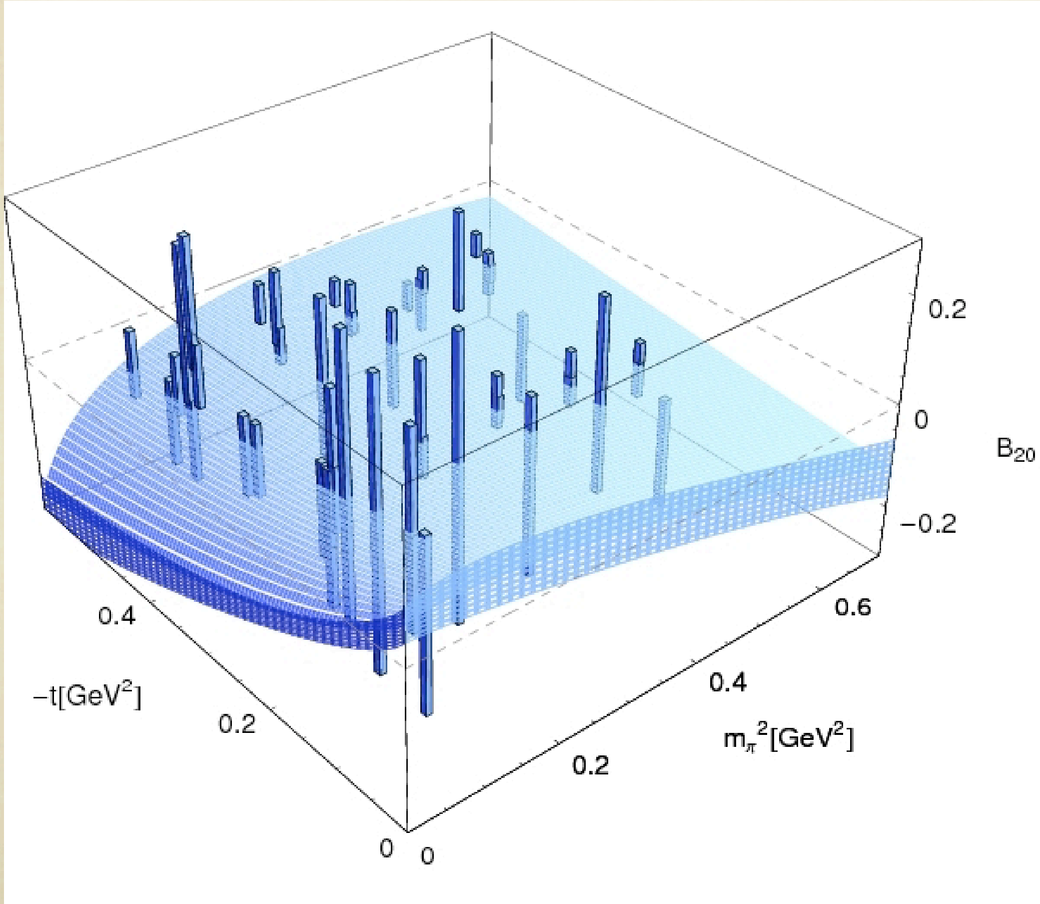


Chiral Extrapolation of $B_{20}^{u+d}(t, m_\pi)$

Chiral extrapolation O(p^2) CBChPT +O(p^3) corrections

Note: connected diagrams only (Dorati, et. al.)

$$B_{20}^{u+d}(t,m_\pi) = A_{20}^{0,u+d} h_B^{u+d}(t,m_\pi) + \Delta B_{20}^{t,u+d}(t,m_\pi) + \frac{m_N(m_\pi)}{m_N}\left\{B_{20}^{0,u+d} + + \delta_B^t\, t + \delta_B^{m_\pi}\, m_\pi^2\right\}\ldots$$



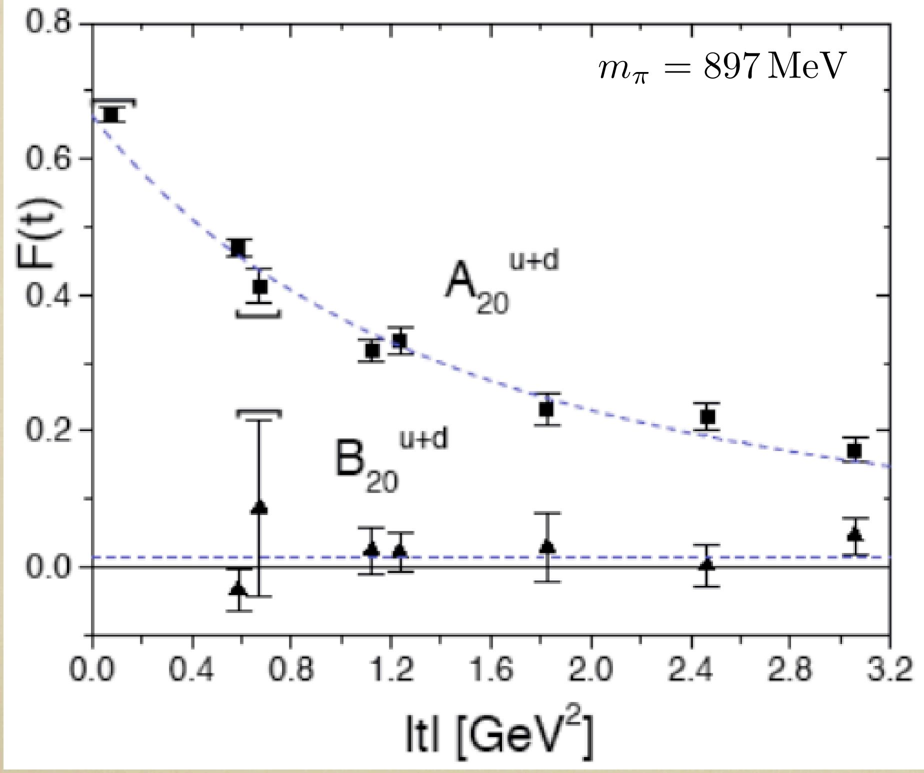
Quark contributions to proton spin

Spin inventory for heavy quarks

Quark spin contribution $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1 \rangle_{\Delta u + \Delta d} \quad \sim \frac{1}{2}0.682(18)$

Total quark contribution (spin plus orbital)

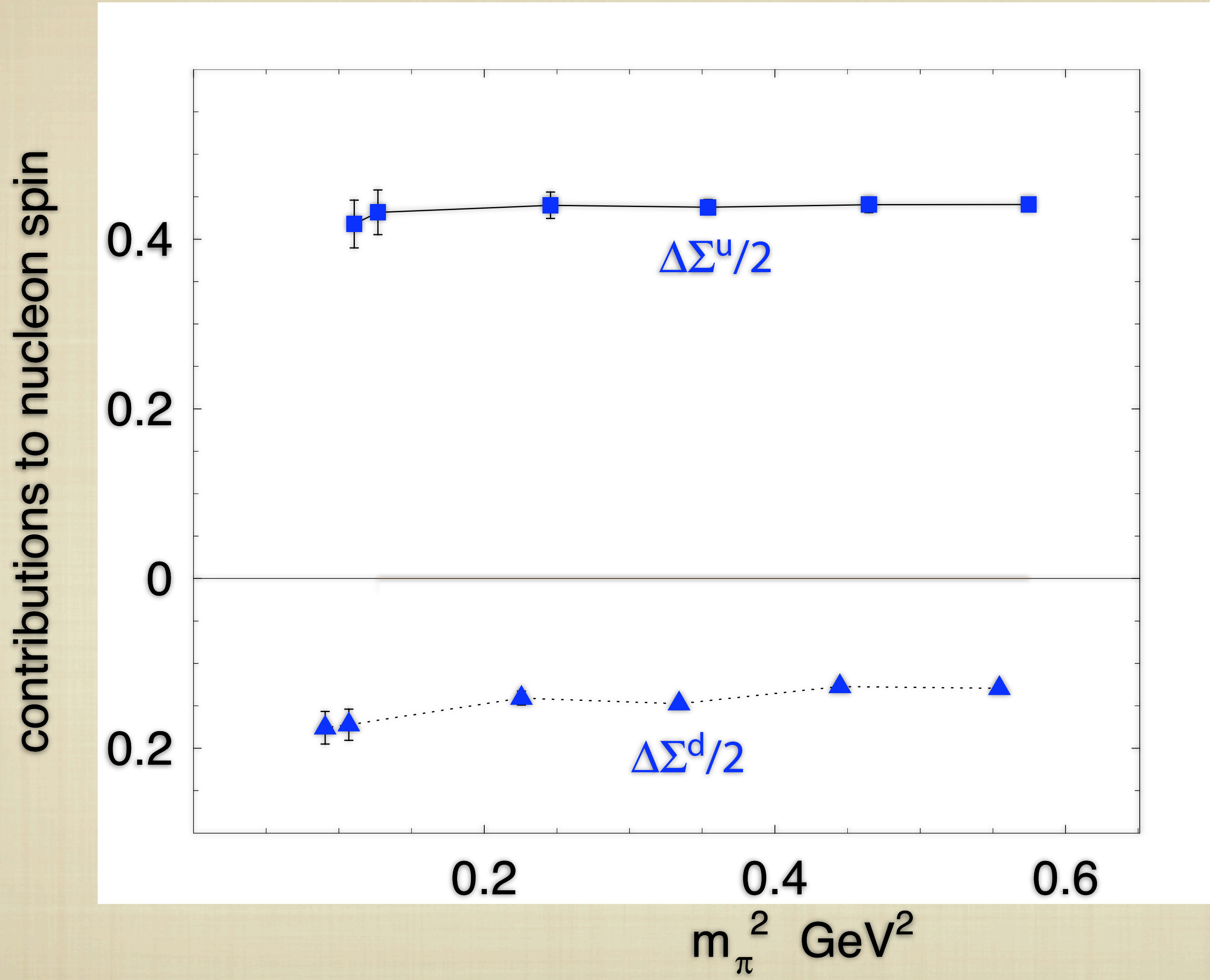
$$J_q = \frac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)] = \frac{1}{2}[\langle x \rangle_{u+d} + B_{20}^{u+d}(0)] \quad \sim \frac{1}{2}0.675(7)$$



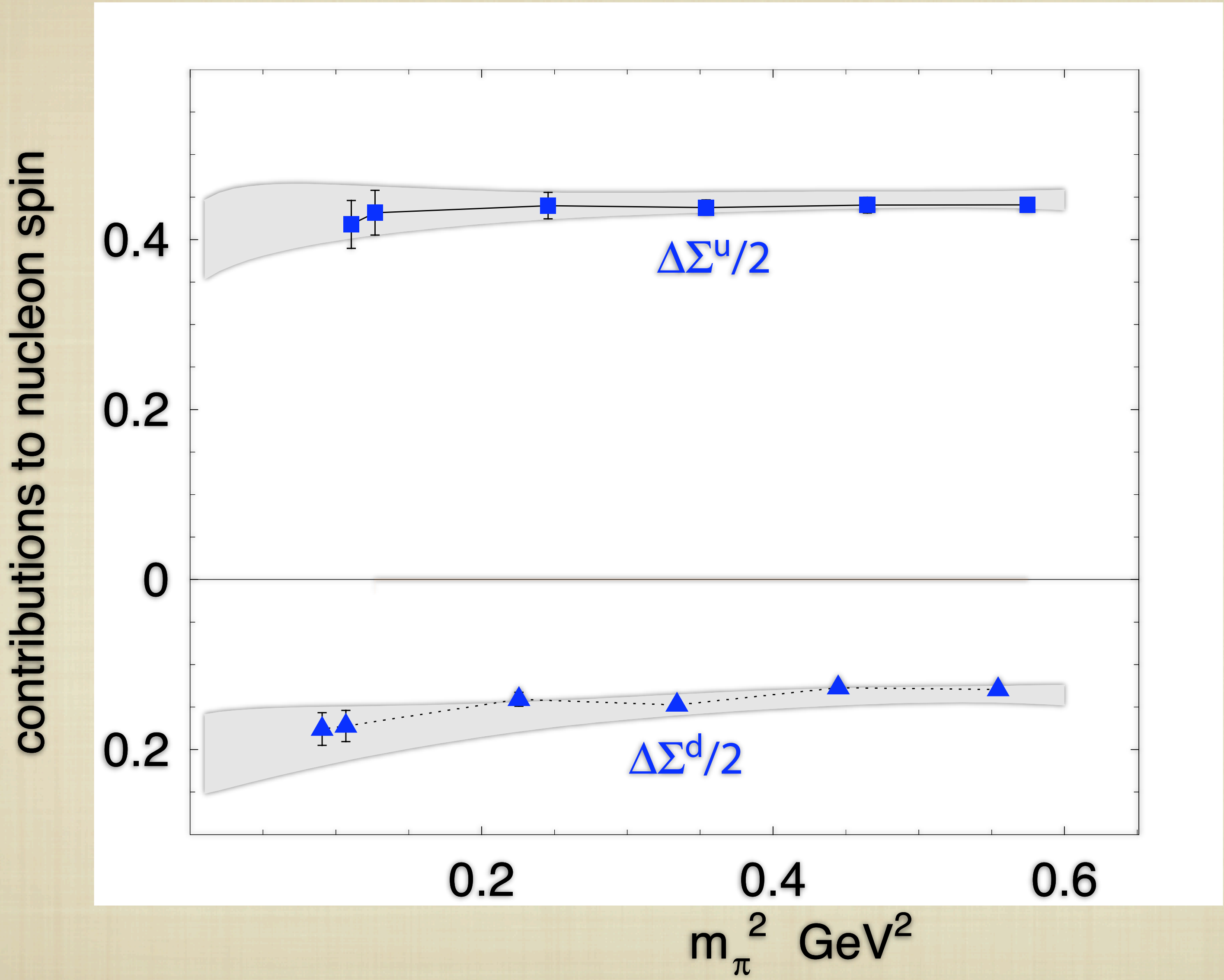
Spin Inventory

68% quark spin
0% quark orbital
32% gluons

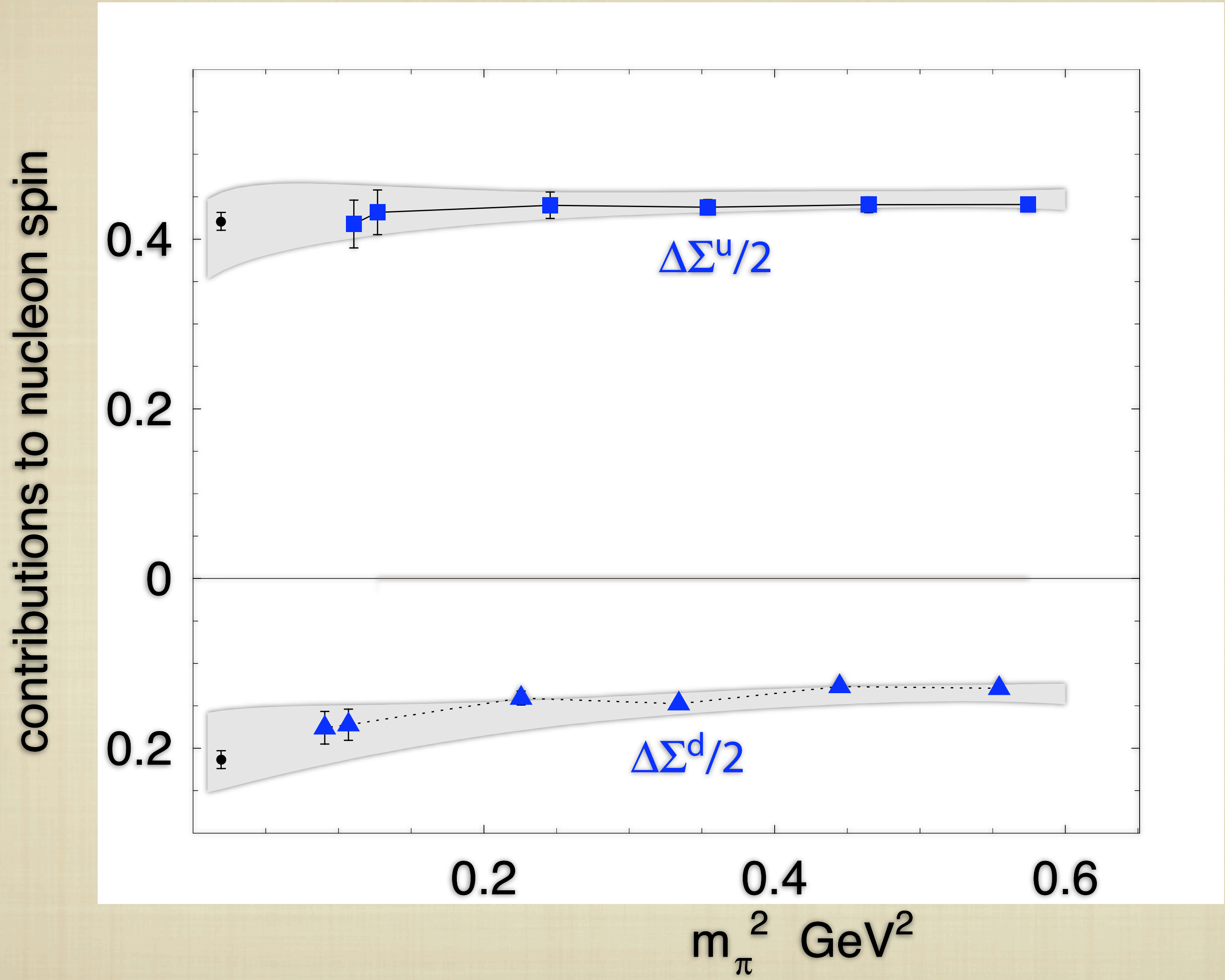
Quark contributions to the proton spin



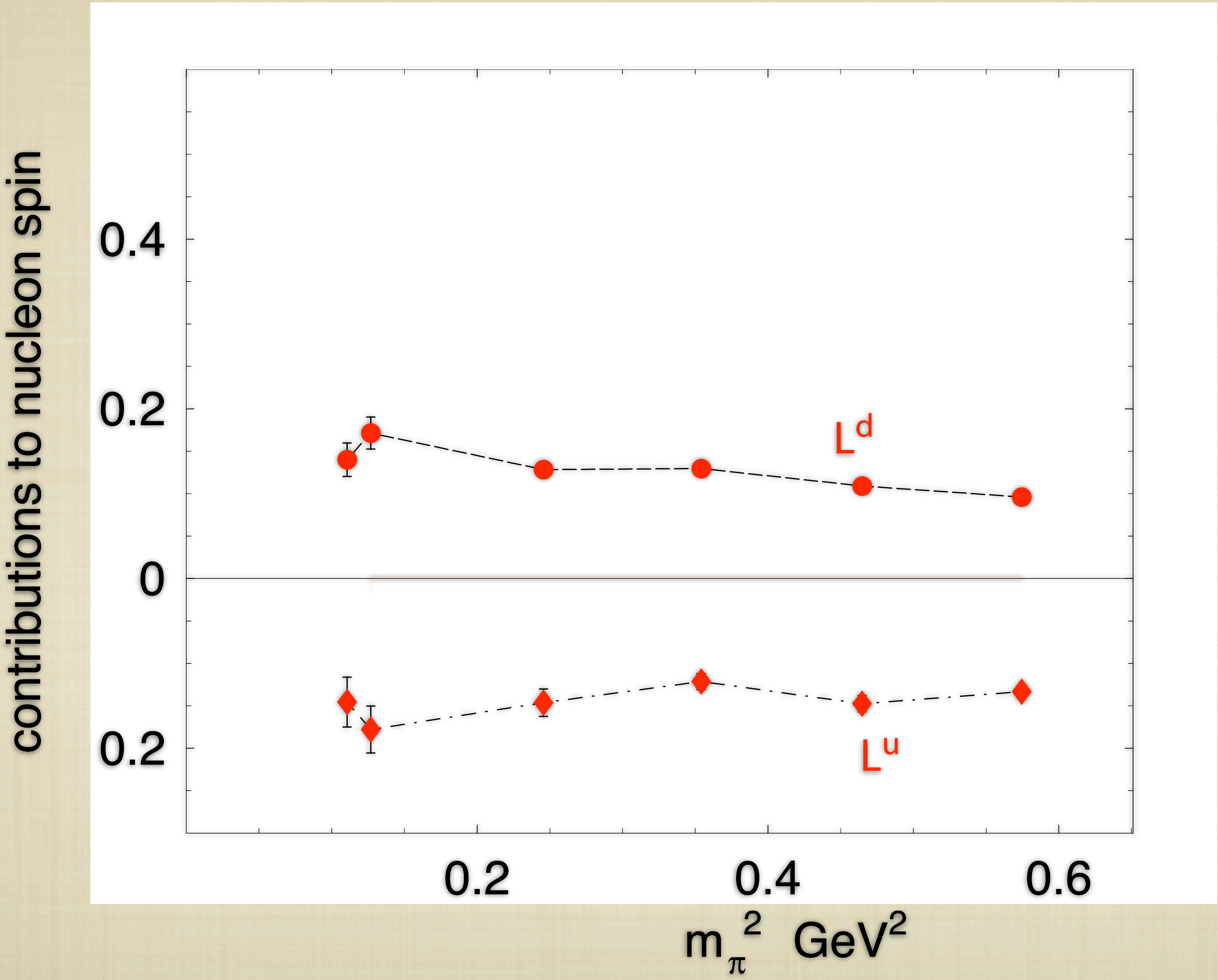
Quark contributions to the proton spin



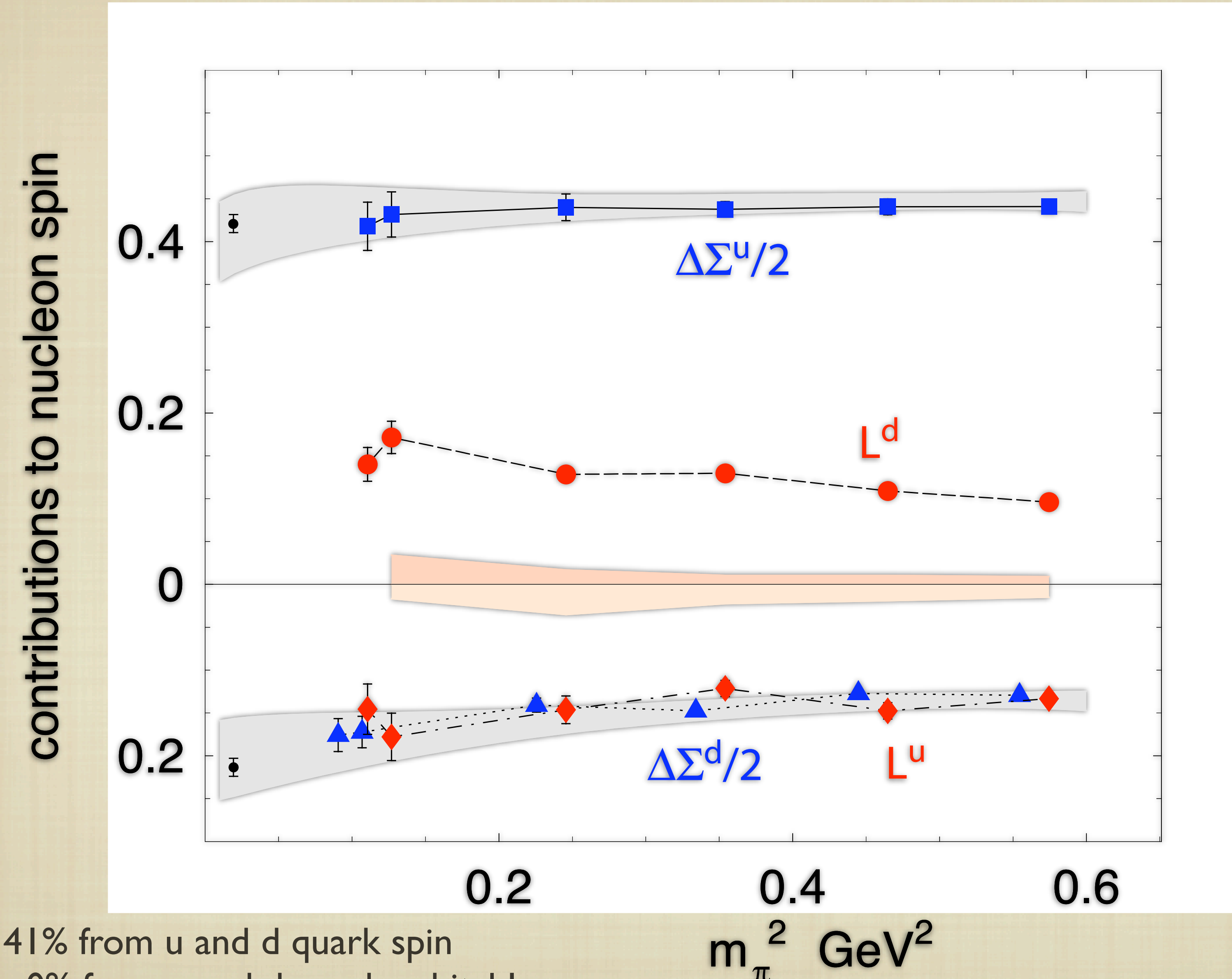
Quark contributions to the proton spin



Quark contributions to the proton spin



Quark contributions to the proton spin



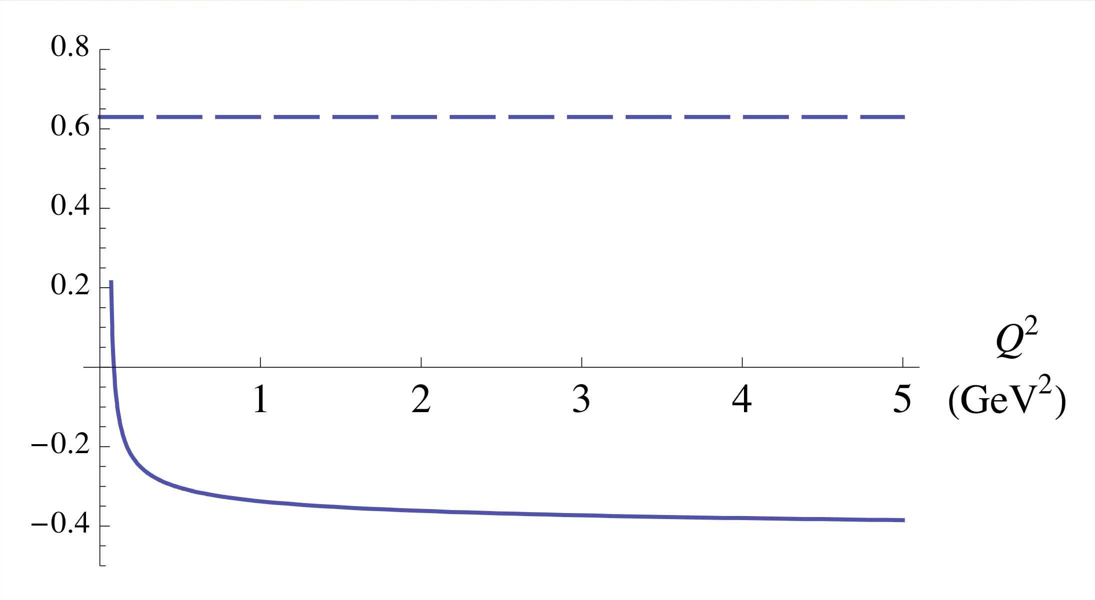
41% from u and d quark spin
~0% from u and d quark orbital L
remainder from glue

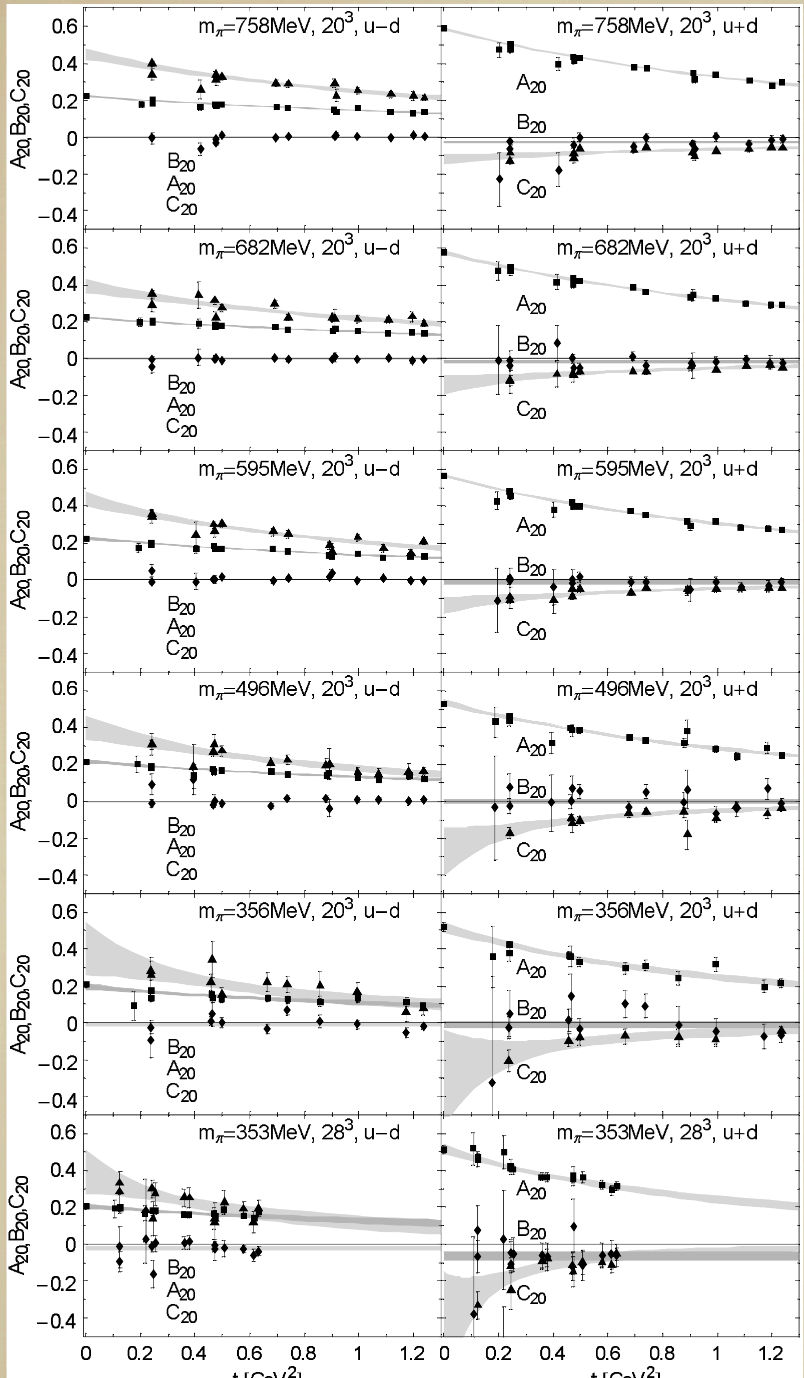
Evolution of nonsinglet angular momentum

Nonsinglet J has simple evolution A. W. Thomas arXiv:0803.2775 [hep-ph]

Spin conserved, so large change in L

$$L^{u-d}(t) + \frac{\Delta\Sigma^{u-d}}{2} = \left(\frac{t}{t_0}\right)^{-\frac{32}{81}} \left(L^{u-d}(t_0) + \frac{\Delta\Sigma^{u-d}}{2}\right) \qquad t = \ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)$$





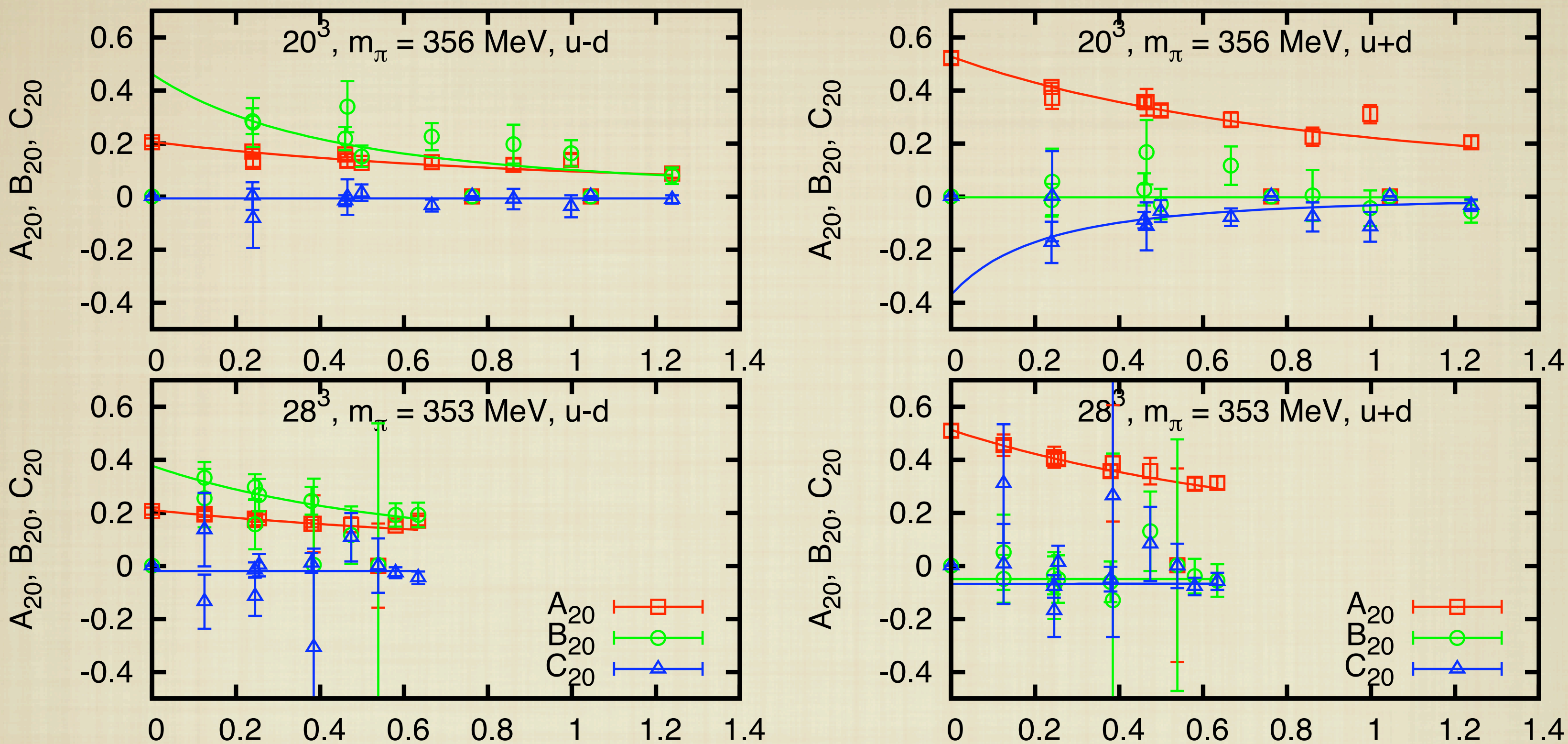
First x moments:

$$A_{20}, B_{20}, C_{20}$$

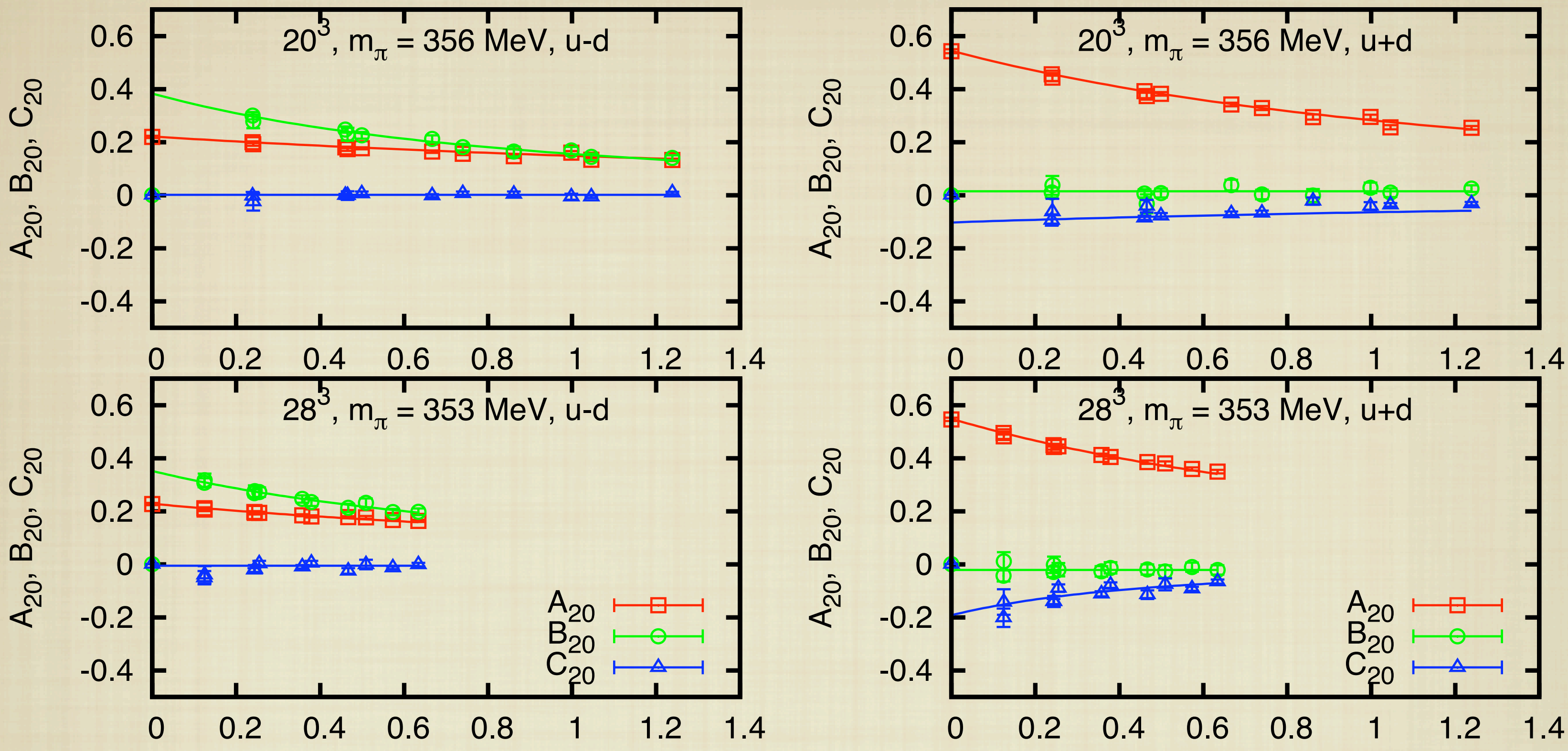
Consistent with large N behavior [Goeke et. al.]

$$|A_{20}^{u+d}| > |A_{20}^{u-d}|$$
$$|B_{20}^{u-d}| > |B_{20}^{u+d}|$$
$$|C_{20}^{u+d}| > |C_{20}^{u-d}|$$

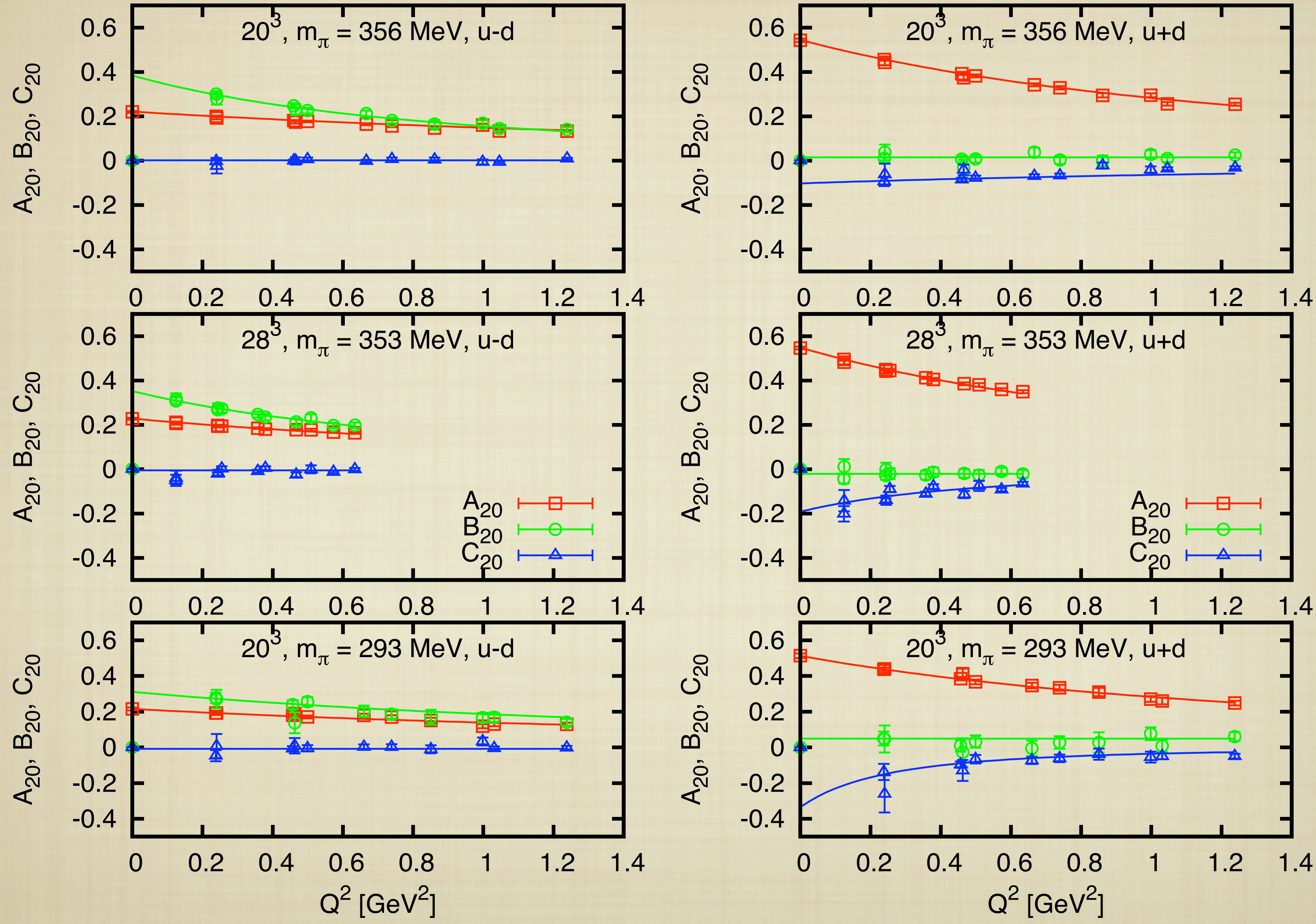
A_{20}, B_{20}, C_{20} Original Data



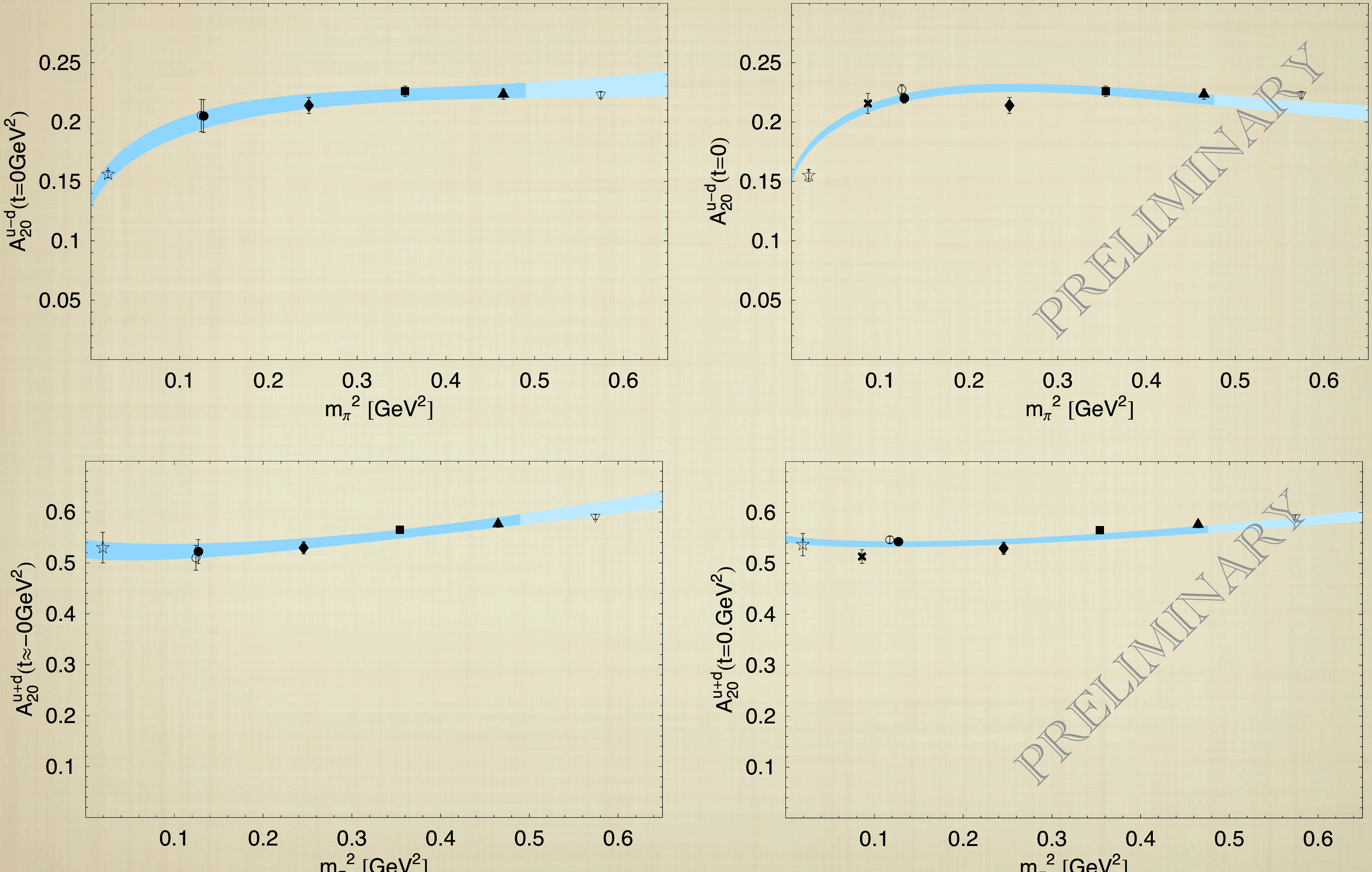
A_{20}, B_{20}, C_{20} New Data



A_{20}, B_{20}, C_{20} New Data



High statistics data for low masses



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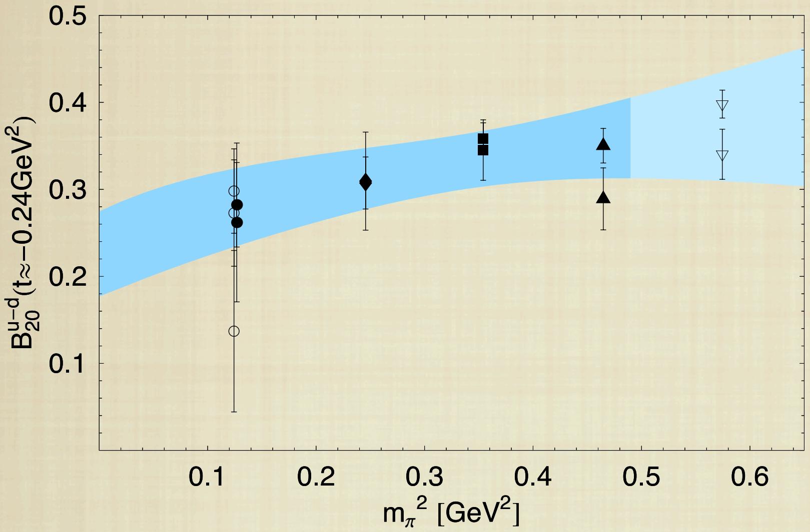
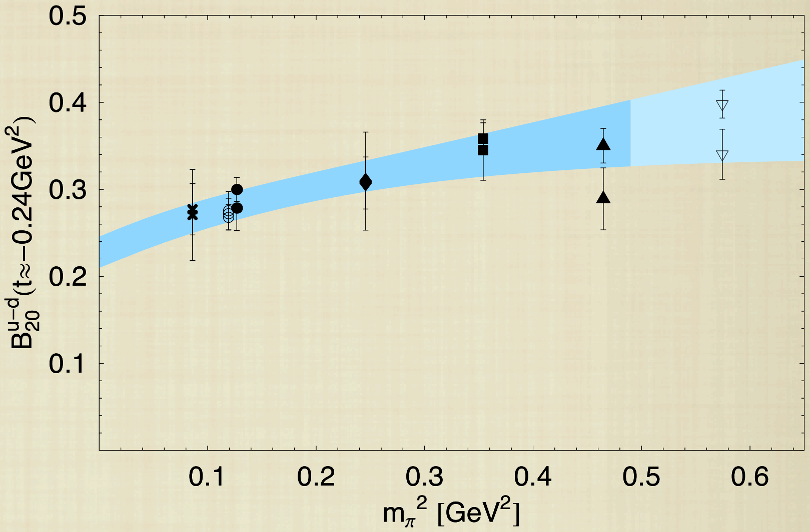
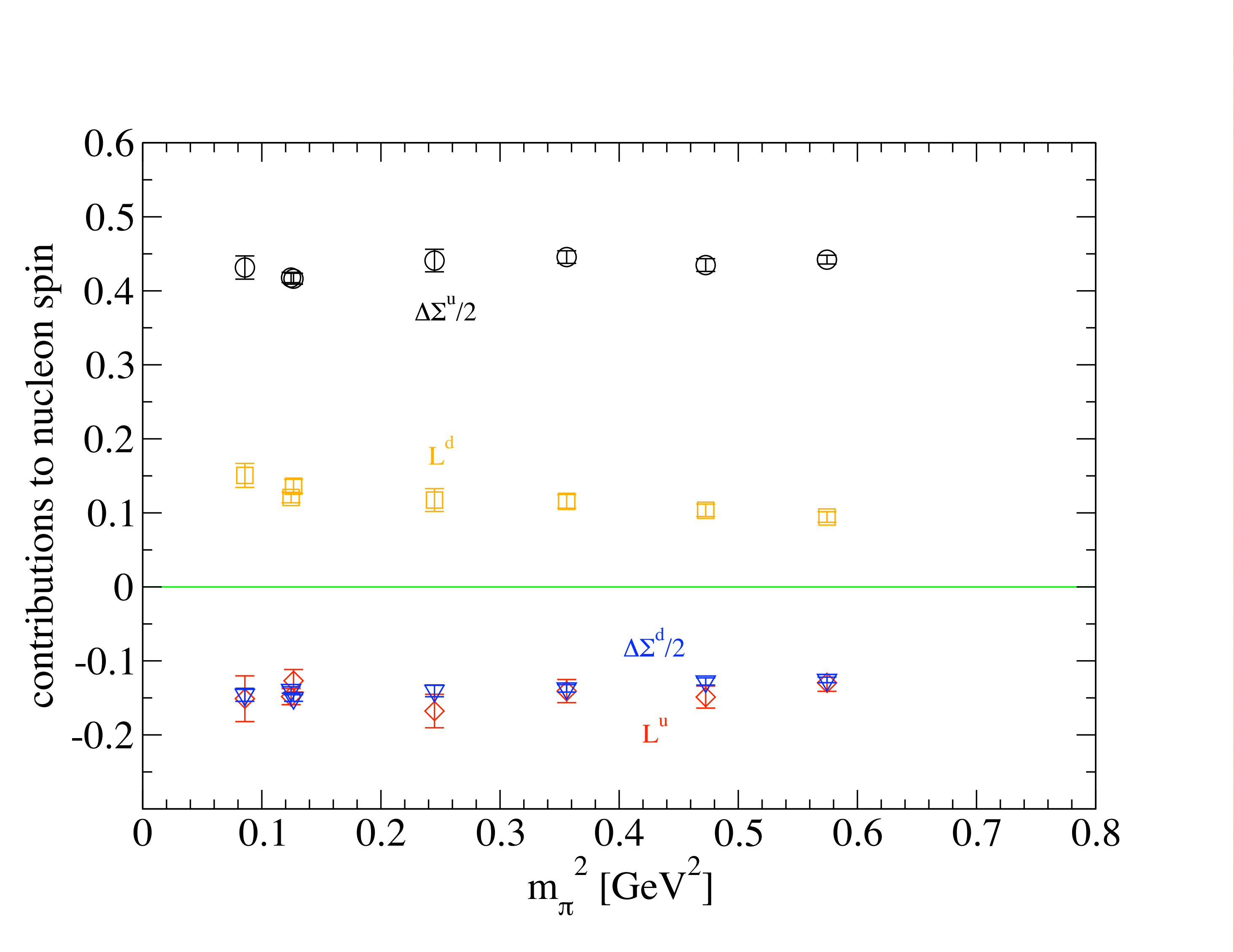
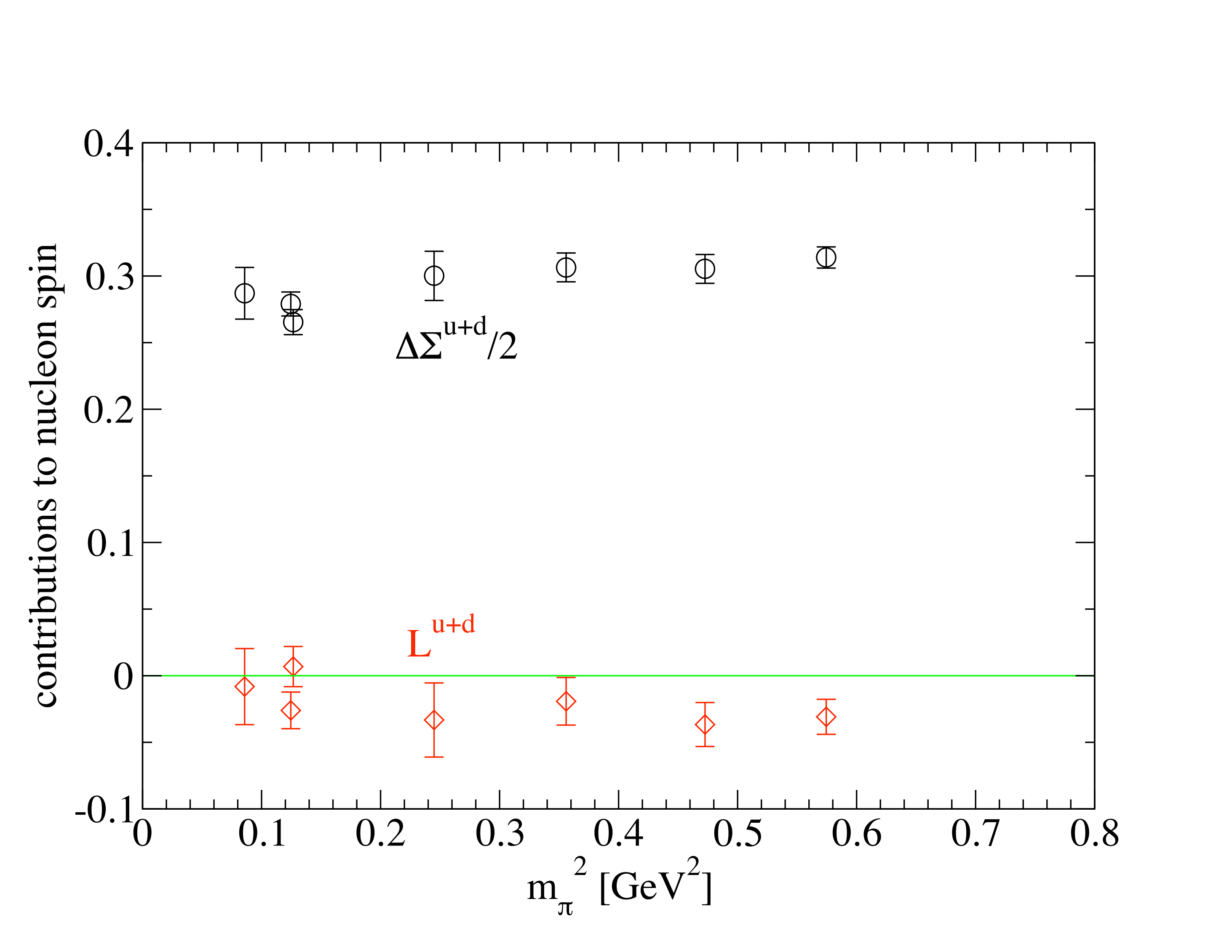


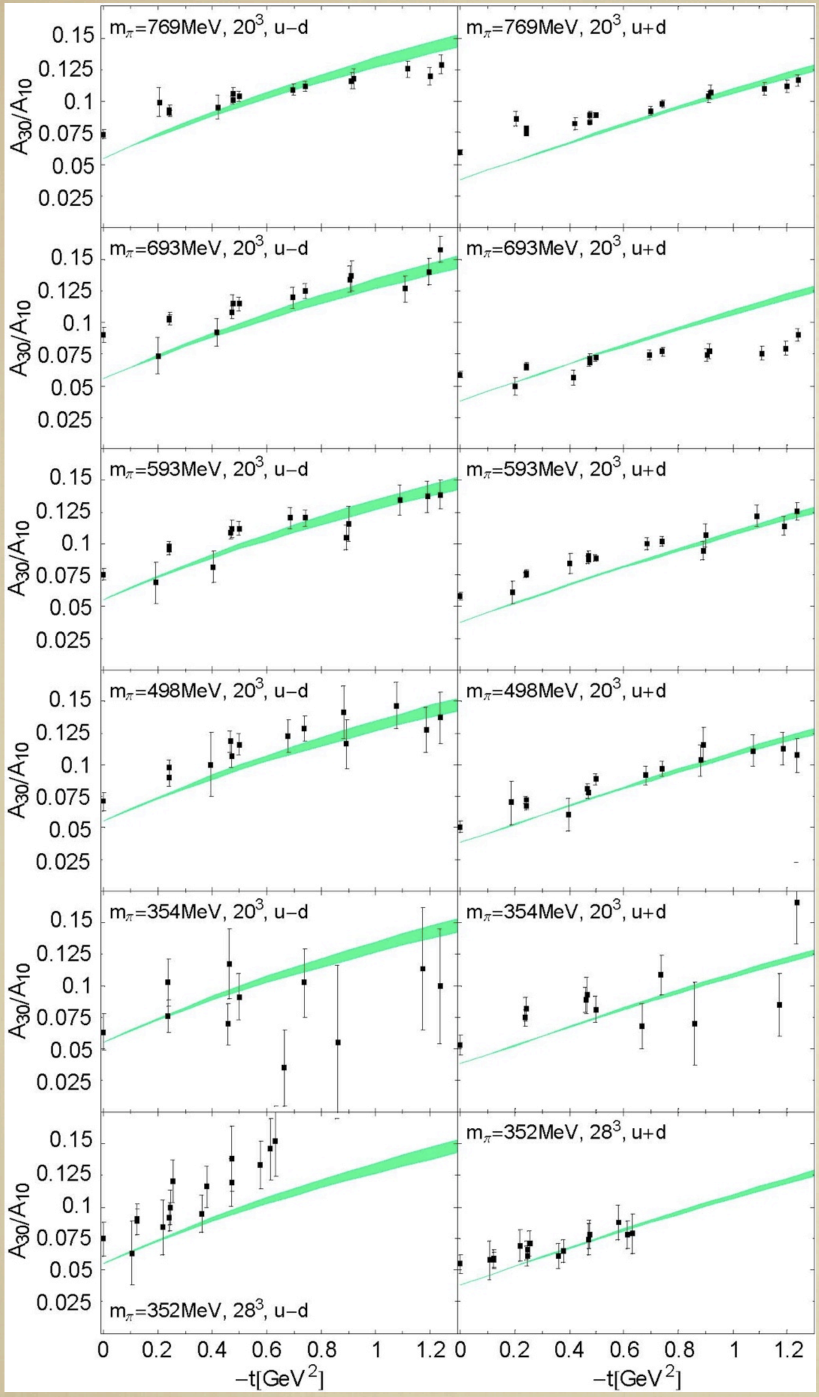


Quark contributions to the proton spin



Quark contributions to the proton spin

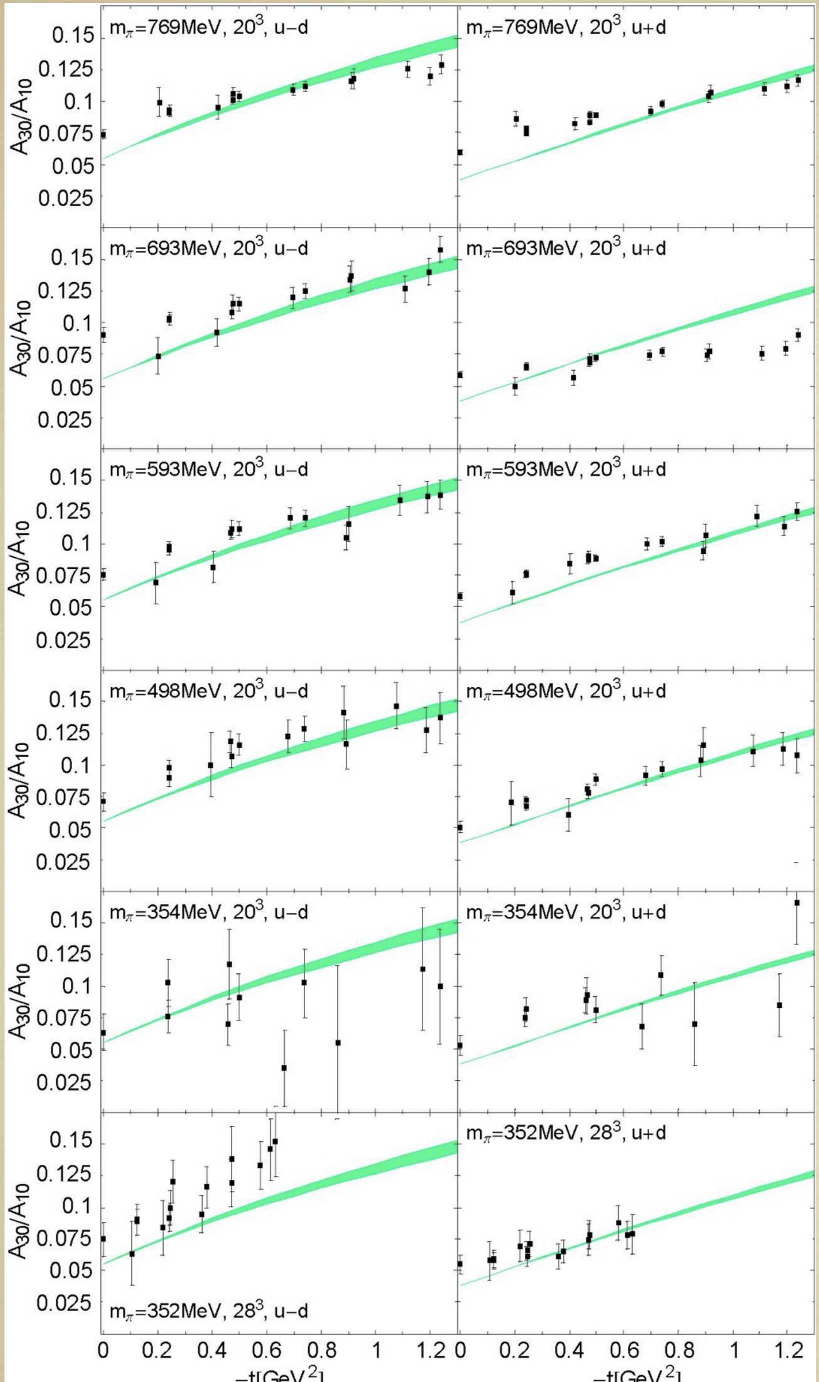




Comparison with Phenomenology

Ratios A_{30} / A_{10}

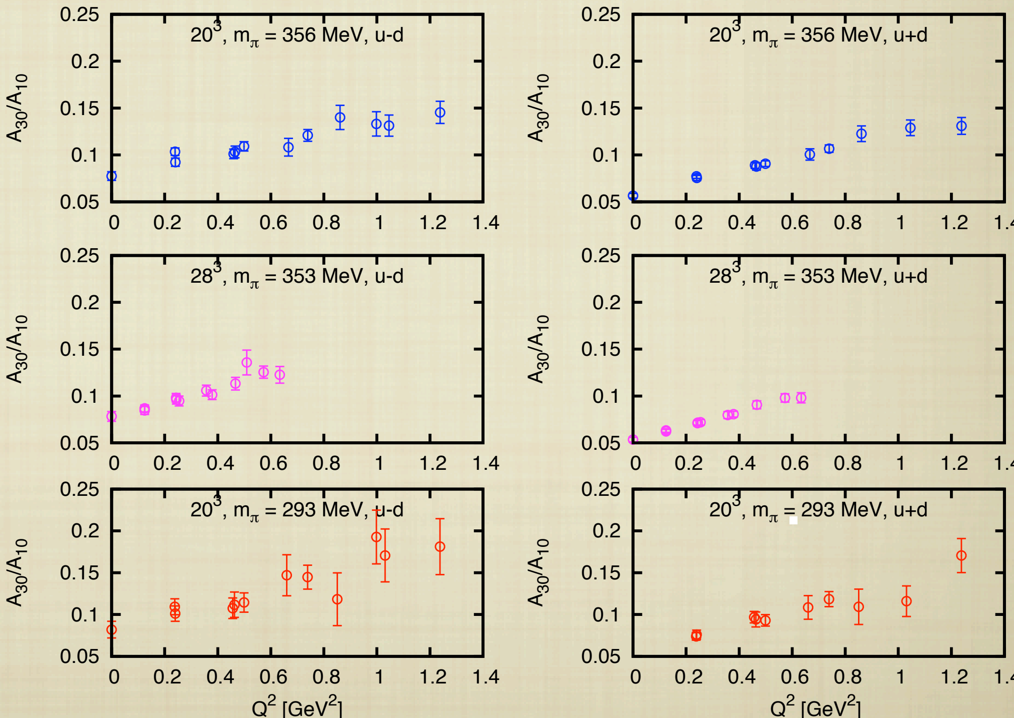
GPD parameterization:
Nucleon form factors,
CTEQ parton distributions,
Regge behavior,
Ansatz
Diehl, Feldmann, Jakob, Kroll EPJC 2005



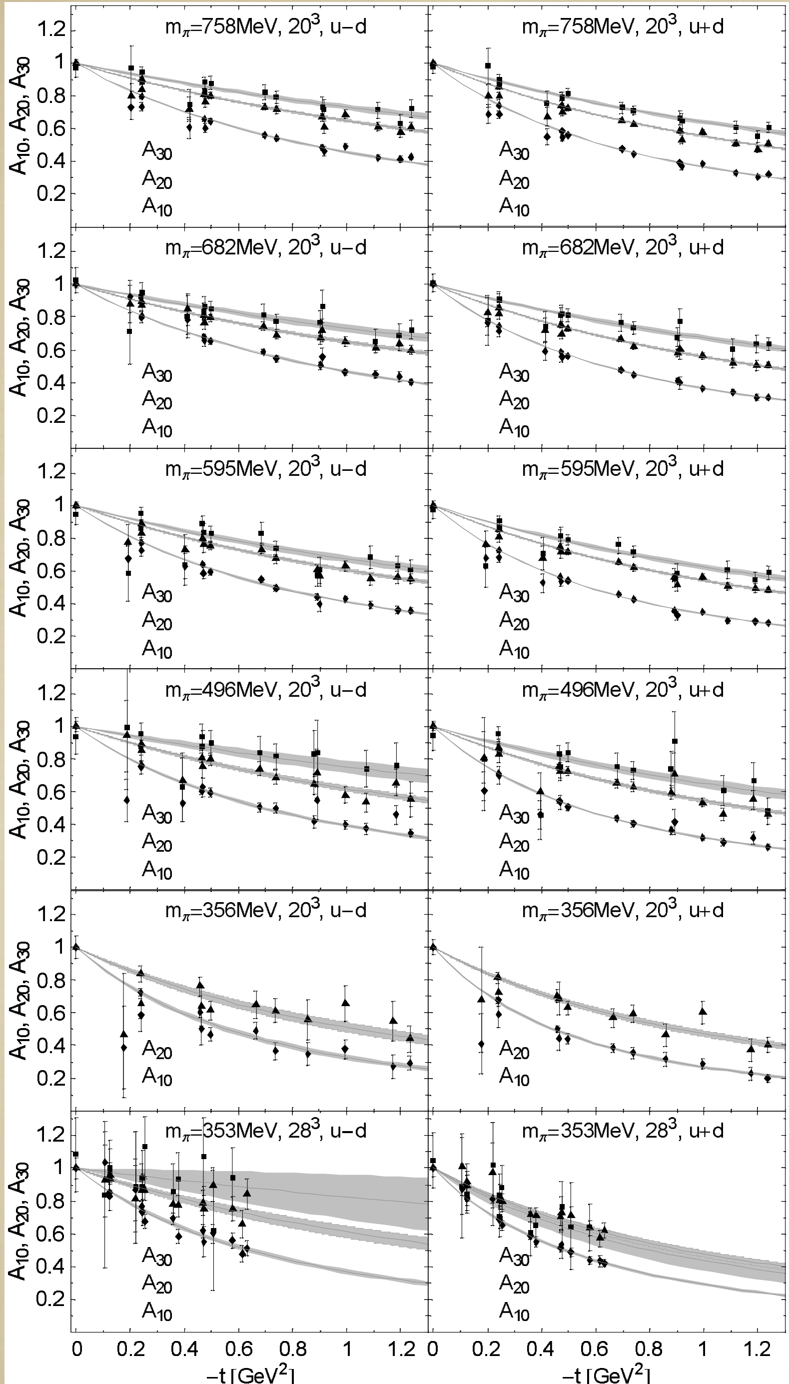
Comparison with Phenomenology

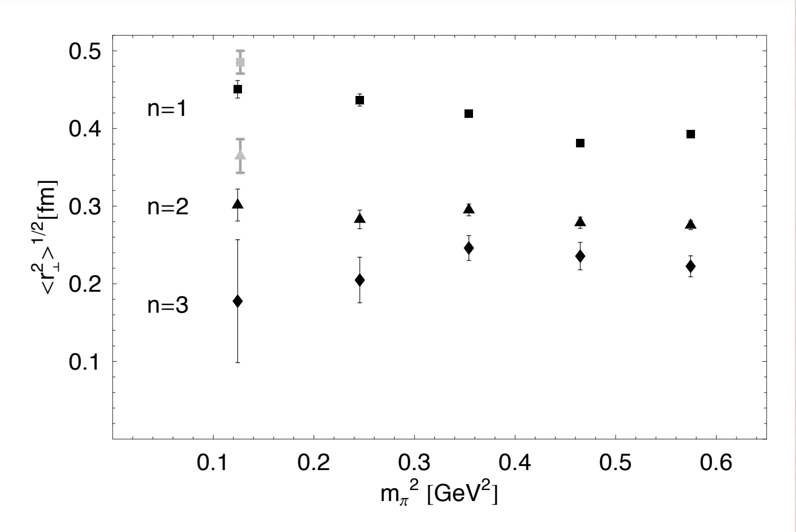
Ratios A_{30} / A_{10}

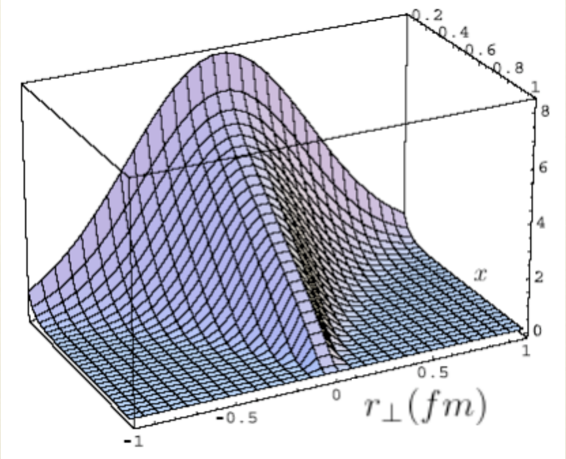
New Data

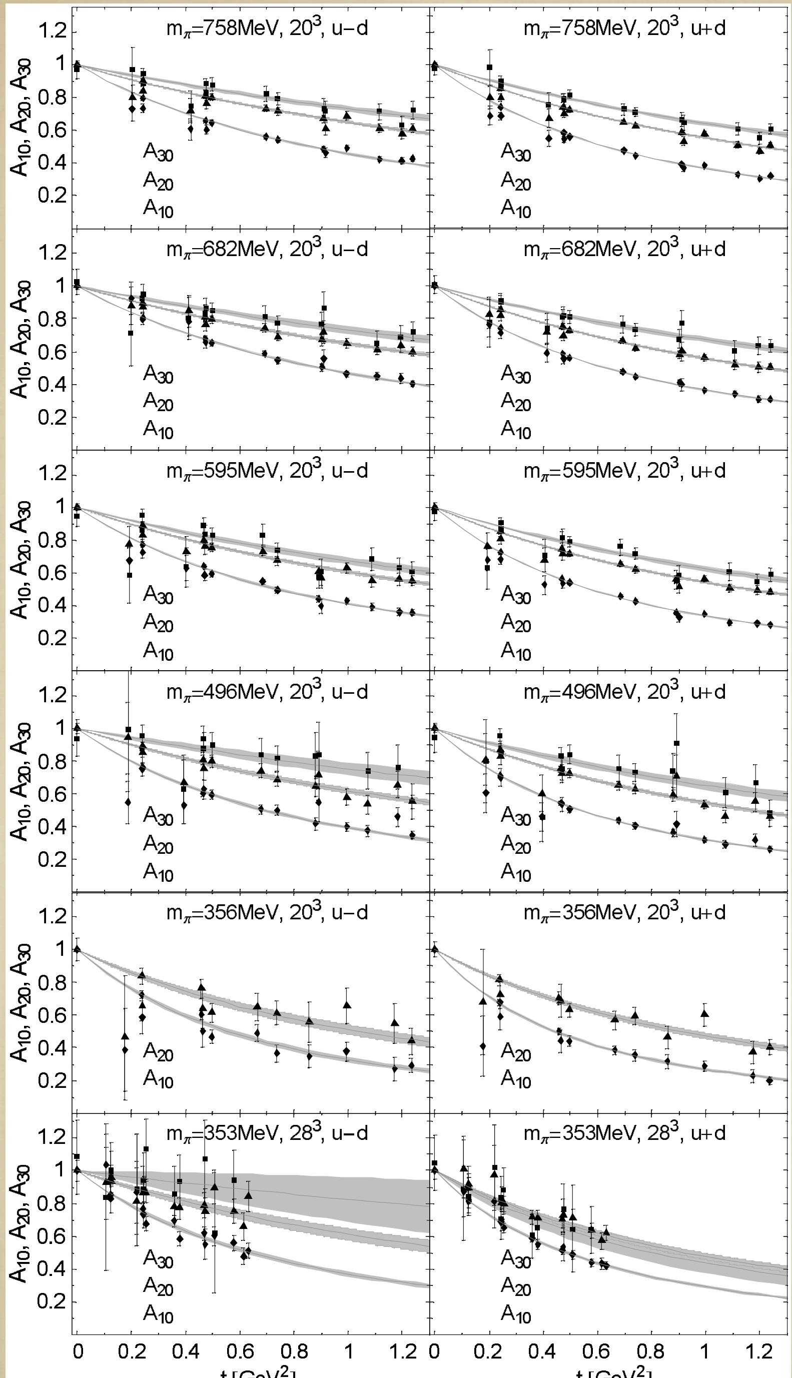


Generalized form factors A_{10}, A_{20}, A_{30}





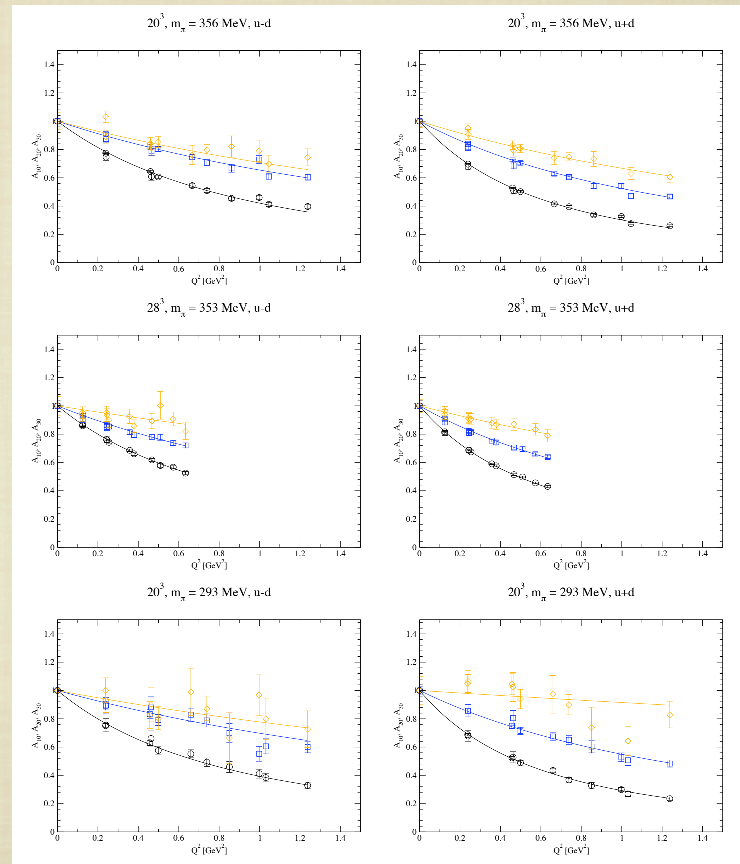




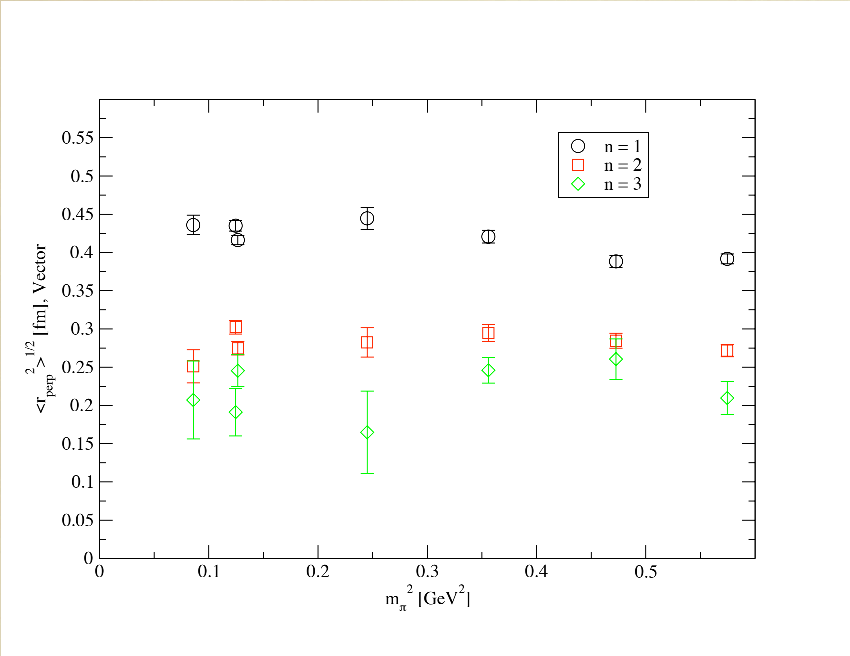
Generalized form factors

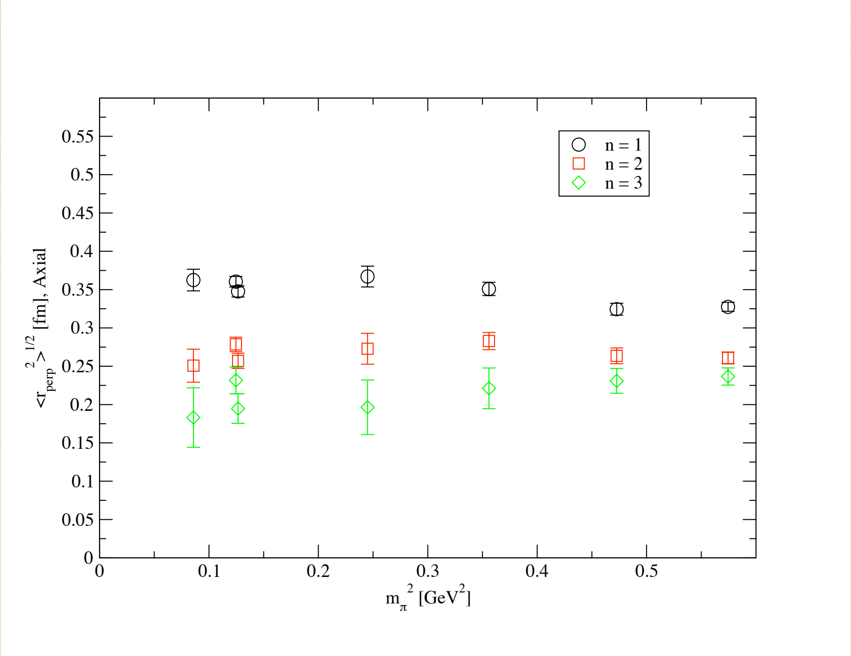
A_{10}, A_{20}, A_{30}

New Data



2-d rms Radii for $A_{n0}, \tilde{A}_{n0}$





Summary

- Mixed action calculations of generalized form factors:
  - Quark orbital angular momentum has unintuitive sign and contributes negligibly to total nucleon spin
  - Constraints on GPD's complementary to experiment
  - Measure transverse size
- CBChPT describes behavior to surprisingly high pion masses
- Improved statistics for light mass ensembles by factor 4:
  - understand systematics and improve statistics of higher masses

Outlook

- Dynamical DW calculations with RBC and UKQCD
  Mixed action results compare well with DW calculations
  see Sergey Syritsyn's talk - 5:40

- Flavor singlet sector

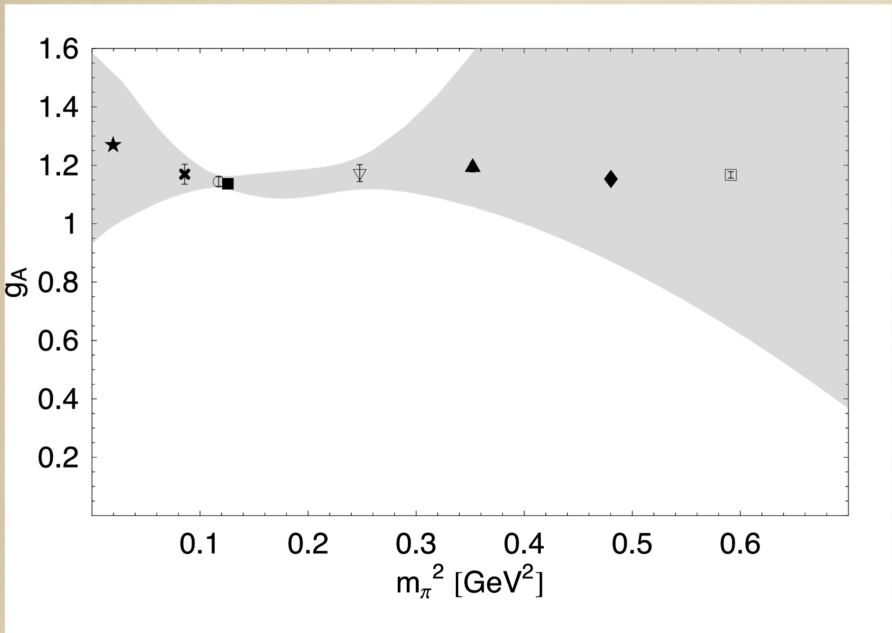
  - Calculate $A_{20}^{(g)}(0) + B_{20}^{(g)}(0)$ from $\langle P|T_{\mu\nu}^{(g)}|P'\rangle$
    using improved gluon operators

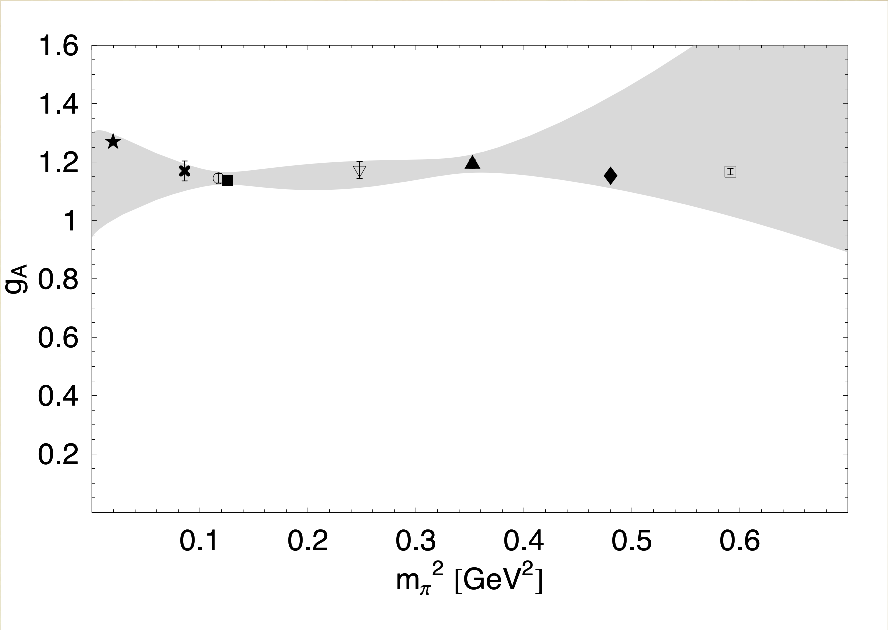
  - Quark contributions from disconnected diagrams

  - Calculate renormalization and mixing coefficients Z_{ij}

Backup slides

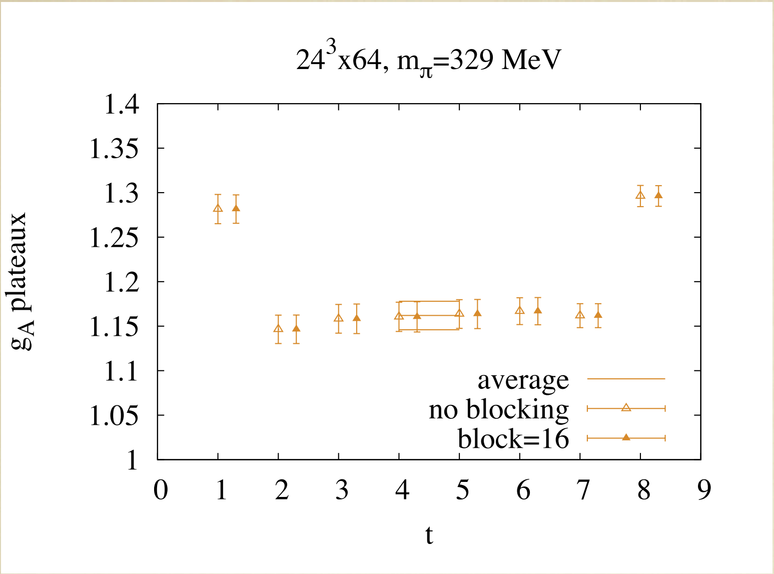
Chiral extrapolation of g_A

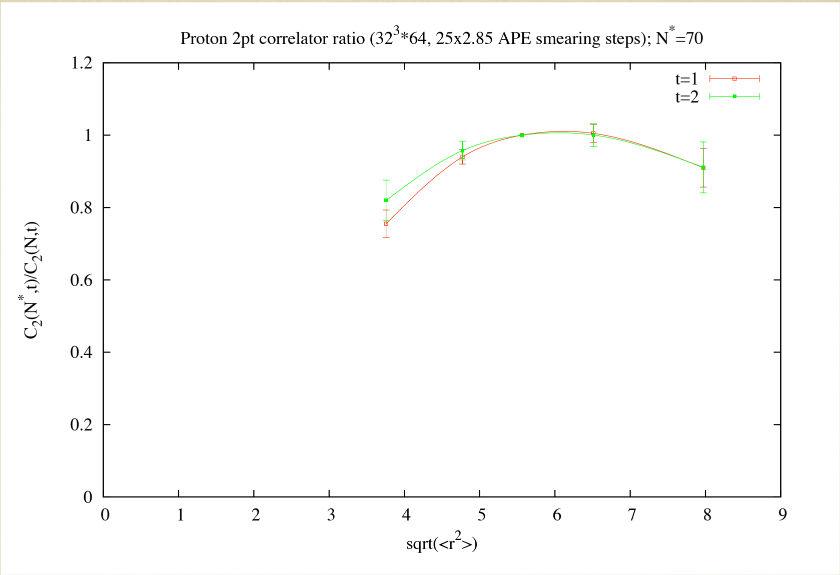




Masses below 500 MeV consistent with extrapolation to experimental point, but higher masses are not.

Plateaus for g_A





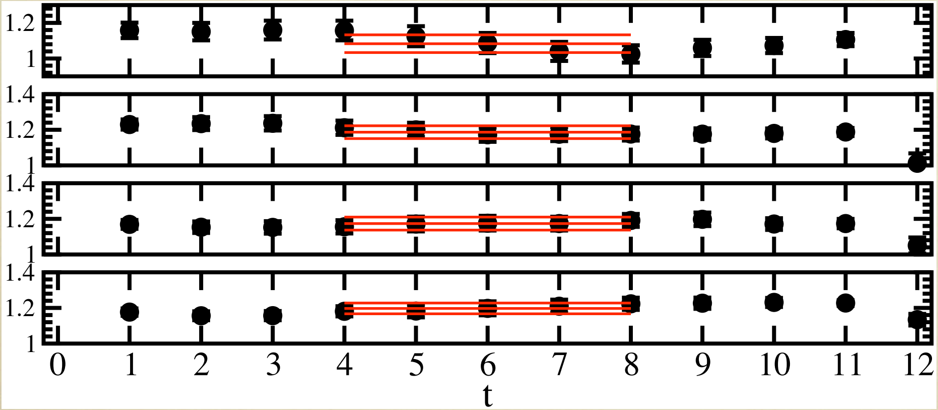
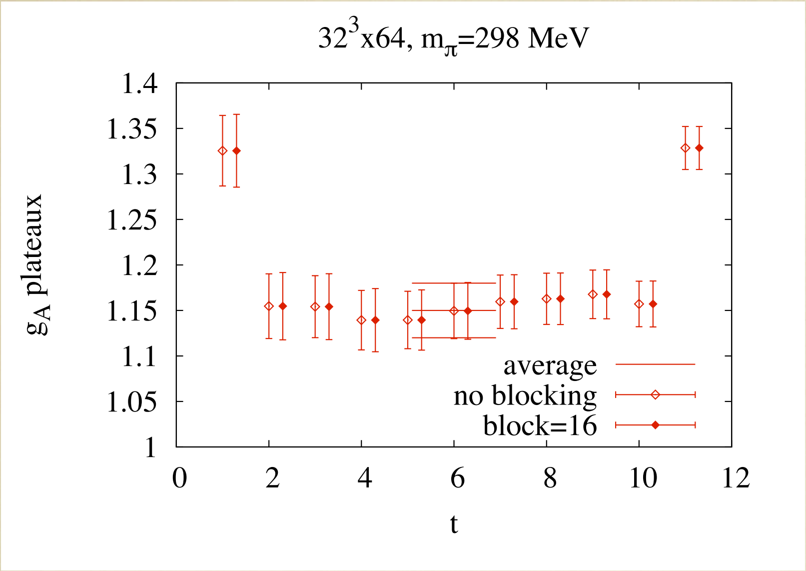


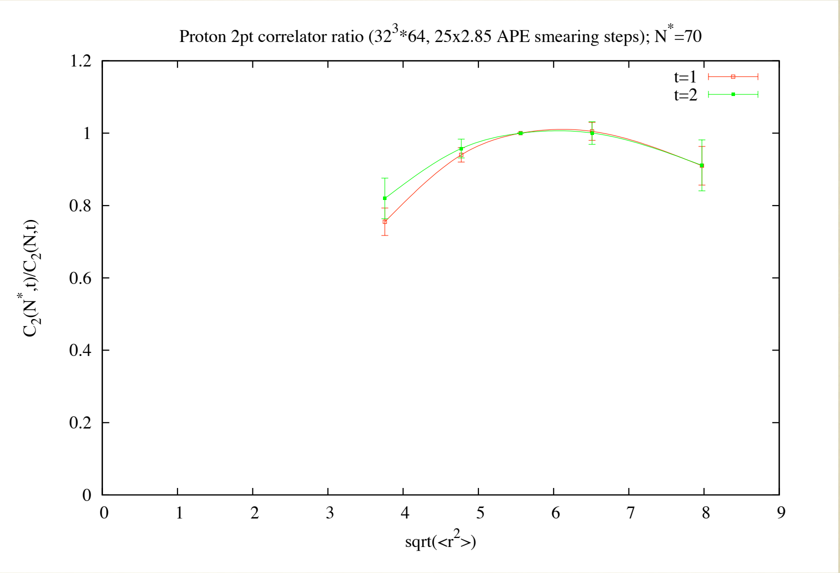
FIG. 1: Plateaus of g_A. $V = (2.7\ \mathrm{fm})^3$ and $m_f = 0.005$, 0.01, 0.02, and 0.03, from top to bottom.

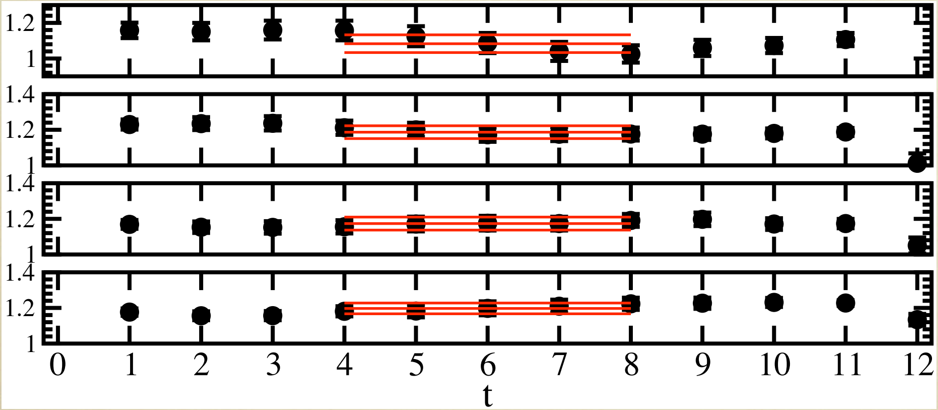
331 MeV

arXiv 0801.4016

Plateaus for gA







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