# *Stochastic All-to-All Propagators for Baryon Correlators*

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# Outline

# Motivation

LHPC Spectrum effort:

- Goal: extract a large number of low-lying excited hadron states
- Requires a large variational basis of operators
- Multi-particle and non-zero momentum operators are required to identify multi-hadron states in the spectrum
- Recall talks from E. Engelson, N. Mathur, R. Edwards, C. Morningstar, M. Peardon, J. Foley, J. Juge.

# Stochastic All-to-all Propagators

Estimate all elements of the quark propagator, $M^{-1}_{(\alpha a|\beta b)}(\mathbf{x}, t|\mathbf{x}_0, t_0)$:

- Generate $N_r$ random $(Z_4)$ sources: $\eta^{(r)}_{\alpha a}(\mathbf{x}, t)$
- Solve

$$M_{(\alpha a|\beta b)}(\mathbf{x}, t|\mathbf{x}', t')\ \phi^{(r)}_{\beta b}(\mathbf{x}', t') = \eta^{(r)}_{\alpha a}(\mathbf{x}, t)$$

  for the $\phi^{(r)}_{\alpha a}(\mathbf{x}, t)$
- The quark propagator is given by

$$M^{-1}_{(\alpha a|\beta b)}(\mathbf{x}, t|\mathbf{x}_0, t_0) = E\left[\phi_{\alpha a}(\mathbf{x}, t)\ \eta^{*}_{\beta b}(\mathbf{x}_0, t_0)\right]$$

# The Dilution Method [1]



- Instead of adding more noise sources, Dilute a single noise source:

$$\eta^{(r)}_{\alpha a}(\mathbf{x},t) = \sum_{d=1}^{N_d} P^{[d]}_{(\alpha a|\beta b)}(\mathbf{x},t|\mathbf{x}',t')\,\eta^{(r)}_{\beta b}(\mathbf{x}',t')$$
$$= \sum_{d=1}^{N_d} \eta^{(r)[d]}_{\alpha a}(\mathbf{x},t)$$

- Solve

$$M_{(\alpha a|\beta b)}(\mathbf{x},t|\mathbf{x}',t')\,\phi^{(r)[d]}_{\beta b}(\mathbf{x}',t') = \eta^{(r)[d]}_{\alpha a}(\mathbf{x},t)$$

- Examples of dilution schemes:
  - Time
  - Time + spin + color
  - Time + spatial even-odd

# Implementation for Baryon Two-point Functions



Given the $\phi^{(r)[d]}_{\alpha a}(\mathbf{x},t)$ and $\eta^{(r)[d]}_{\alpha a}(\mathbf{x},t)$,

- Form source and sink baryon operators:

$$\Gamma^{(r)[d_A d_B d_C]}_{\ell}(t) = c^{(\ell)}_{\alpha\beta\gamma;ijk} \sum_{\mathbf{x}} \epsilon_{abc}\ \widetilde{\phi}^{(r)[d_A]}_{\alpha ai}(\mathbf{x},t)\ \widetilde{\phi}^{(r)[d_B]}_{\beta bj}(\mathbf{x},t) \times \widetilde{\phi}^{(r)[d_C]}_{\gamma ck}(\mathbf{x},t)$$

$$\Omega^{(r)[d_A d_B d_C]}_{\ell}(t) = c^{(\ell)}_{\alpha\beta\gamma;ijk} \sum_{\mathbf{x}} \epsilon_{abc}\ \widetilde{\eta}^{(r)[d_A]}_{\alpha ai}(\mathbf{x},t)\ \widetilde{\eta}^{(r)[d_B]}_{\beta bj}(\mathbf{x},t) \times \widetilde{\eta}^{(r)[d_C]}_{\gamma ck}(\mathbf{x},t)$$

- Combine them to form two-point functions

# Advantages of the Dilution Method



- Expected to approach exact all-to-all *faster than* $1/\sqrt{N_d}$ as $N_d \rightarrow N_t \times N_s \times N_c \times V$
- *Complete factorization* of source and sink in correlation functions
  - Great for a large variational basis
  - Use the same operators to make multi-hadrons
- All elements of quark propagator are calculated
  - Non-zero momentum projections require spatial sum at source
  - Disconnected Diagrams

# Dilution Scheme Tests

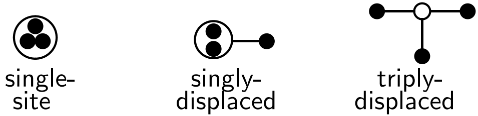


- 100 quenched gauge configurations with: $L_s = 12$, $L_t = 48$, $a_s \approx .1$ fm, $\beta = 6.1$, $m_\pi \approx 700$ MeV
- Choose a few relevant observables for comparison of dilution schemes, point-to-all
- Question: Assuming time dilution, is it better to add more noise sources or more dilution projectors?

# Basis For Comparison

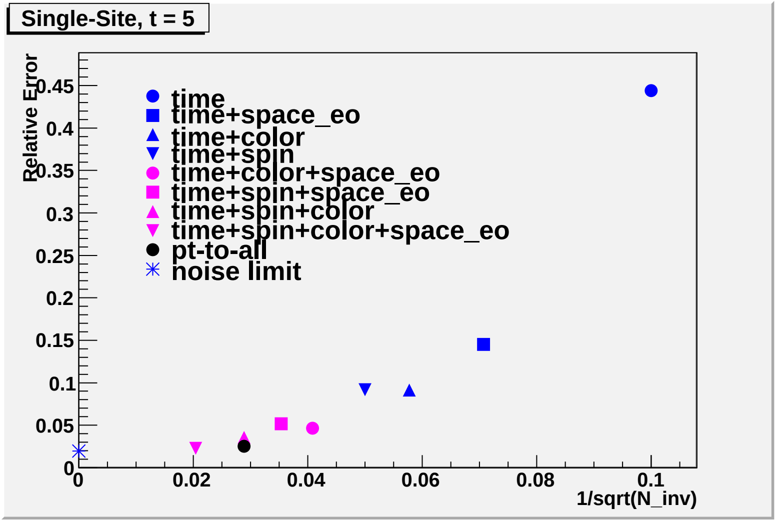


- Examined Single-Site, Singly-Displaced, and Triply-Displaced baryon operators



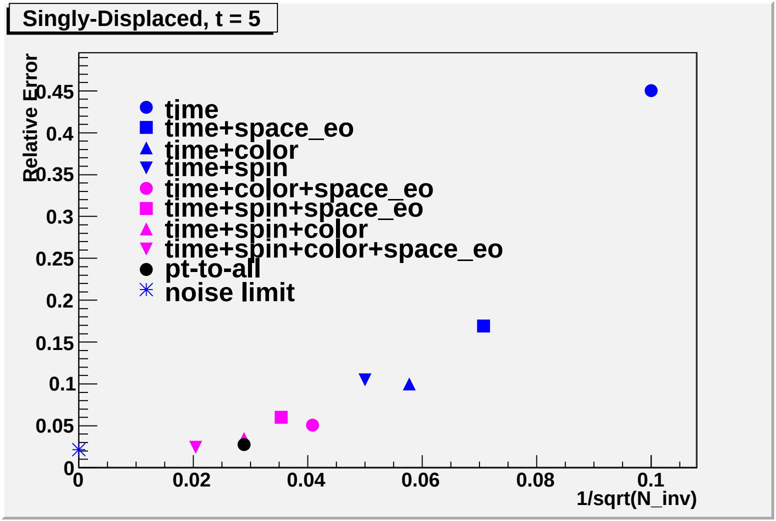

- Diagonal correlators evaluated at several time separations is the measure of choice

# Results

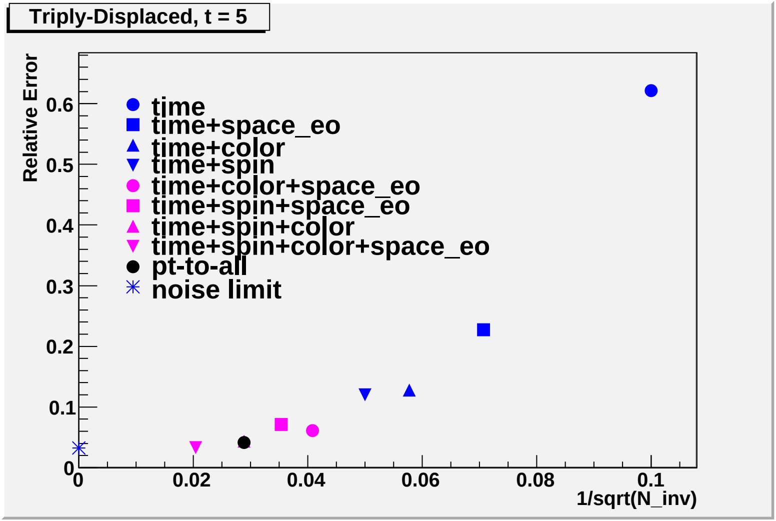

Single-Site, t = 5
Relative Error
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
time
time+space_eo
time+color
time+spin
time+color+space_eo
time+spin+space_eo
time+spin+color
time+spin+color+space_eo
pt-to-all
noise limit
0
0.02
0.04
0.06
0.08
0.1
1/sqrt(N_inv)

# Results




Singly-Displaced, t = 5
Relative Error
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.02
0.04
0.06
0.08
0.1
1/sqrt(N_inv)
time
time+space_eo
time+color
time+spin
time+color+space_eo
time+spin+space_eo
time+spin+color
time+spin+color+space_eo
pt-to-all
noise limit

# Results

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Motivation and Background

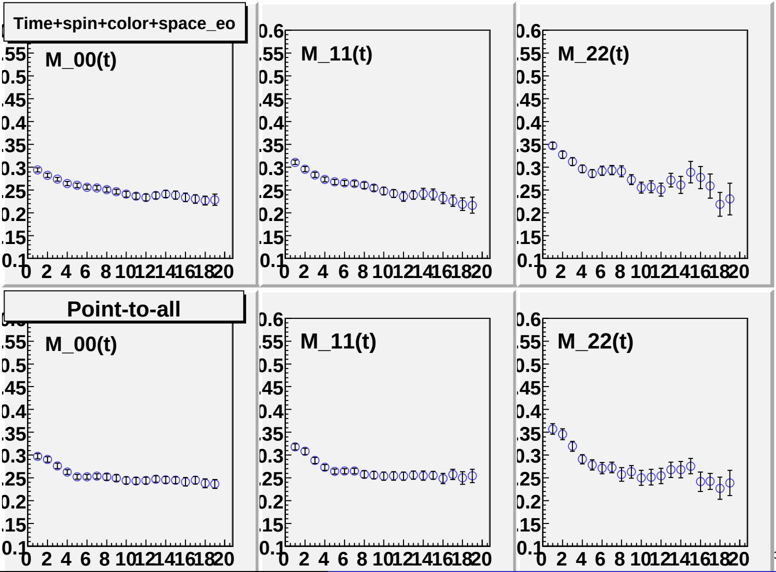
Methods

Dilution Scheme Tests

**Results and Conclusions**

# Results - Effective Masses





# Conclusions



- Adding more dilution projectors beats adding more noise sources, up to a point
- time + spin + color dilution is roughly equivalent to point-to-all method
- time + spin + color + spatial-even-odd dilution is consistent with the gauge noise
- Currently working on a alternative method that give exact all-to-all for less. Stay Tuned!

J. Foley, et al., Comput. Phys. Commun. **172**, 145 (2005).

S. Basak, et al., Phys. Rev. D **72**, 94506 (2005).

A. Lichtl, P.H.D. Thesis [hep-lat/0609019].