Multi-hadron Operators with All-to-All Propagators

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Outline

- Introduction (motivation)
- All-to-all quark propagators
  - how it simplifies life ...
- Construction of interpolating operators and some effective masses/fit results
  - 3-quark operators
  - 2-quark operators
  - 4-quark operators
  - (5-quark operators ... eventually)
- Summary

Introduction

- Physically large lattices with light dynamical quarks are being generated by many collaborations
  ... but spectroscopy may not be so simple on these lattices

- As quark masses become lighter and volumes become larger, many single particle states start to mix with multi-particle states

- All-to-all quark propagators may become essential in studying these states

- Try the Noise-Dilution method (TrinLat) to construct explicit multi-particle operators

All-to-All Quark Propagators

Noise-Dilution Method

TrinLat (2005)

- Stochastic noise Z(4) noise source η (one for each quark)
- Dilute
$$\eta = \eta^{(0)} + \eta^{(1)} + \cdots \eta^{(N_{dil})}$$
- Invert Dirac operator to get solutions $\varphi^{(i)}$
Propagator $\sum_i \varphi^{(i)}(\vec{x}, t) \otimes \eta^{\dagger(i)}(\vec{x}_0, t_0)$
- Construct baryon and meson operators with $\varphi^{(i)}$ and $\eta^{(i)}$

Constructing Baryon Correlation Functions

(LHPC 2005)

$$C_{ij}^{(N)}(t) = c^{(i)}_{\mu\nu\tau}\,\overline{c}^{(j)}_{\overline{\mu\nu\tau}}$$

Group theory coefficients

$$\Big\{\ \widetilde{G}^{(uud)}_{(\mu|\overline{\mu})(\nu|\overline{\nu})(\tau|\overline{\tau})} + \widetilde{G}^{(uud)}_{(\tau|\overline{\nu})(\nu|\overline{\tau})(\mu|\overline{\mu})}$$
$$+\ \widetilde{G}^{(uud)}_{(\mu|\overline{\nu})(\nu|\overline{\mu})(\tau|\overline{\tau})} - \widetilde{G}^{(uud)}_{(\mu|\overline{\tau})(\nu|\overline{\nu})(\tau|\overline{\mu})}$$
$$-\ \widetilde{G}^{(uud)}_{(\mu|\overline{\nu})(\nu|\overline{\tau})(\tau|\overline{\mu})} - \widetilde{G}^{(uud)}_{(\nu|\overline{\nu})(\tau|\overline{\mu})(\mu|\overline{\tau})}$$
$$-\ \widetilde{G}^{(uud)}_{(\nu|\overline{\mu})(\tau|\overline{\nu})(\mu|\overline{\tau})} + \widetilde{G}^{(uud)}_{(\tau|\overline{\tau})(\nu|\overline{\nu})(\mu|\overline{\mu})}\ \Big\}$$

G's are the colour contracted 3-quark propagators

All-to-all simplifications

Source ($\bar{\mu}$'s) and sink (μ 's) indices could not be separated in the previous formula

All-to-all allows us to "separate" the source and sinks

$$C^{(N)}_{IJ}(t) = \sum_{\tilde{i}}^{N^A_{dil}} \sum_{\tilde{j}}^{N^B_{dil}} \sum_{\tilde{k}}^{N^C_{dil}} c^{(I)}_{\mu\nu\tau} B^{\tilde{i}\tilde{j}\tilde{k}}_{[ABC]\mu\nu\tau}(\vec{x}, t) \times$$

$$c^{(J)}_{\overline{\mu\nu\tau}} \Big\{ 2\overline{B}^{\tilde{i}\tilde{j}\tilde{k}}_{[ABC]\overline{\mu\nu\tau}} + 2\overline{B}^{\tilde{k}\tilde{j}\tilde{i}}_{[CBA]\overline{\mu\nu\tau}} - \overline{B}^{\tilde{k}\tilde{i}\tilde{j}}_{[ACB]\overline{\mu\nu\tau}} - \overline{B}^{\tilde{j}\tilde{i}\tilde{k}}_{[BAC]\overline{\mu\nu\tau}} - \overline{B}^{\tilde{k}\tilde{i}\tilde{j}}_{[ACB]\overline{\mu\nu\tau}} - \overline{B}^{\tilde{k}\tilde{i}\tilde{j}}_{[CAB]\overline{\mu\nu\tau}} - \overline{B}^{\tilde{j}\tilde{k}\tilde{i}}_{[BCA]\overline{\mu\nu\tau}} \Big\} (\vec{x}_0, t_0)$$

Three-Quark Colour-Singlet Operators (baryons)

$$\mathrm{B}^{i,j,k}_{\mu\nu\tau\ [012]}(\vec{x},t) = \epsilon_{abc}\psi^{(i)a}_{\mu\ [0]}(\vec{x},t)\psi^{(j)b}_{\nu\ [1]}(\vec{x},t)\psi^{(k)c}_{\tau\ [2]}(\vec{x},t)$$

$$\overline{\mathrm{B}}^{i,j,k}_{\overline{\mu\nu\tau}\ [012]}(\vec{x},t) = \epsilon_{abc}\eta^{(i)a\dagger}_{\overline{\mu}\ [0]}(\vec{x},t)\eta^{(j)b\dagger}_{\overline{\nu}\ [1]}(\vec{x},t)\eta^{(k)c\dagger}_{\overline{\tau}\ [2]}(\vec{x},t)$$

Note that the quarks may be displaced ...

$$U_y(\vec{x},t)U_y(\vec{x}+a\hat{e}_y,t)U_y(\vec{x}+2a\hat{e}_y,t)\psi(\vec{x}+3a\hat{e}_y,t)$$

where the $U_i(\vec{x},t)$'s are the gauge field

Store some of the momentum projected operators

$$B^{i,j,k}_{\mu\nu\tau\ [012]}(\vec{p},t) = \sum_{\vec{x}} e^{-\vec{p}\cdot\vec{x}} B^{i,j,k}_{\mu\nu\tau\ [012]}(\vec{x},t)$$

Baryon Correlators

Preliminary results presented in Regensburg (Latt'07)

Noise level ~ point-to-all propagators

with Time-Spin dilution

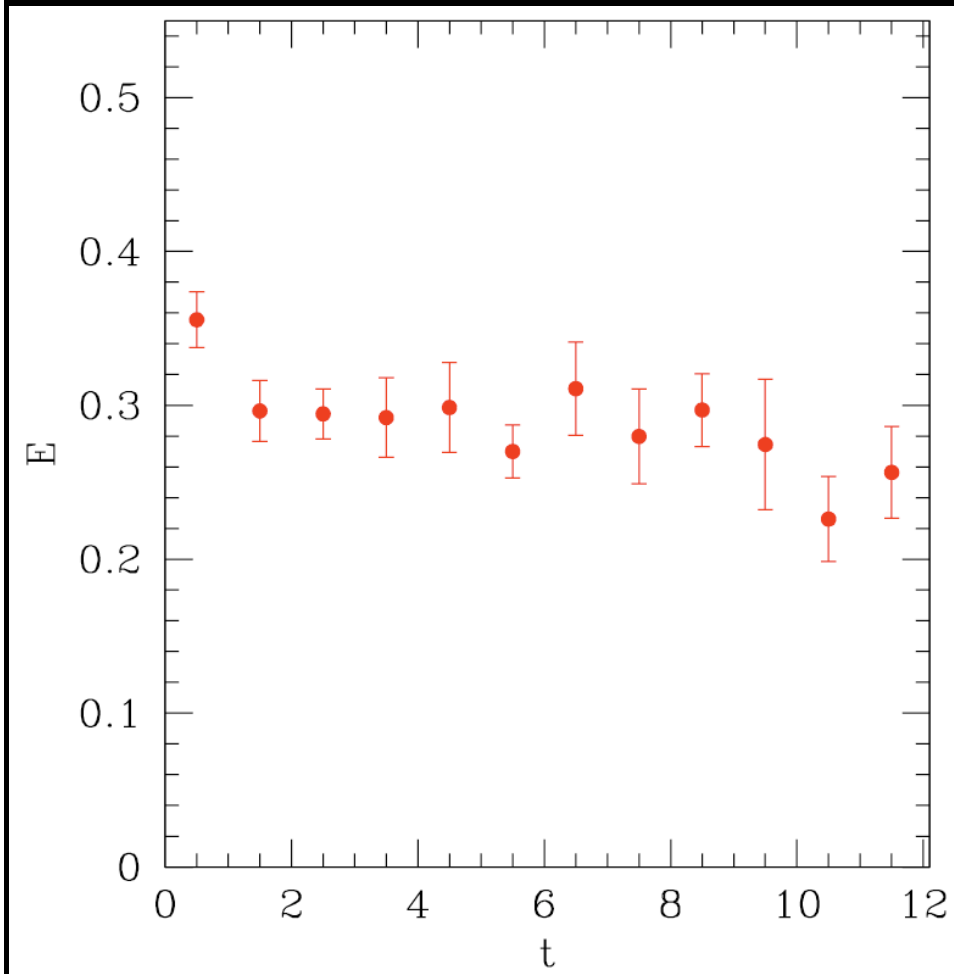
The dilution method does **not** ruin the diagonalization procedure (which is needed for excited states)

The big advantage is that we can construct multi-hadron correlation functions

Detailed dilution study with higher statistics will be presented by J. Bulava on Thursday

Nucleon Effective Mass

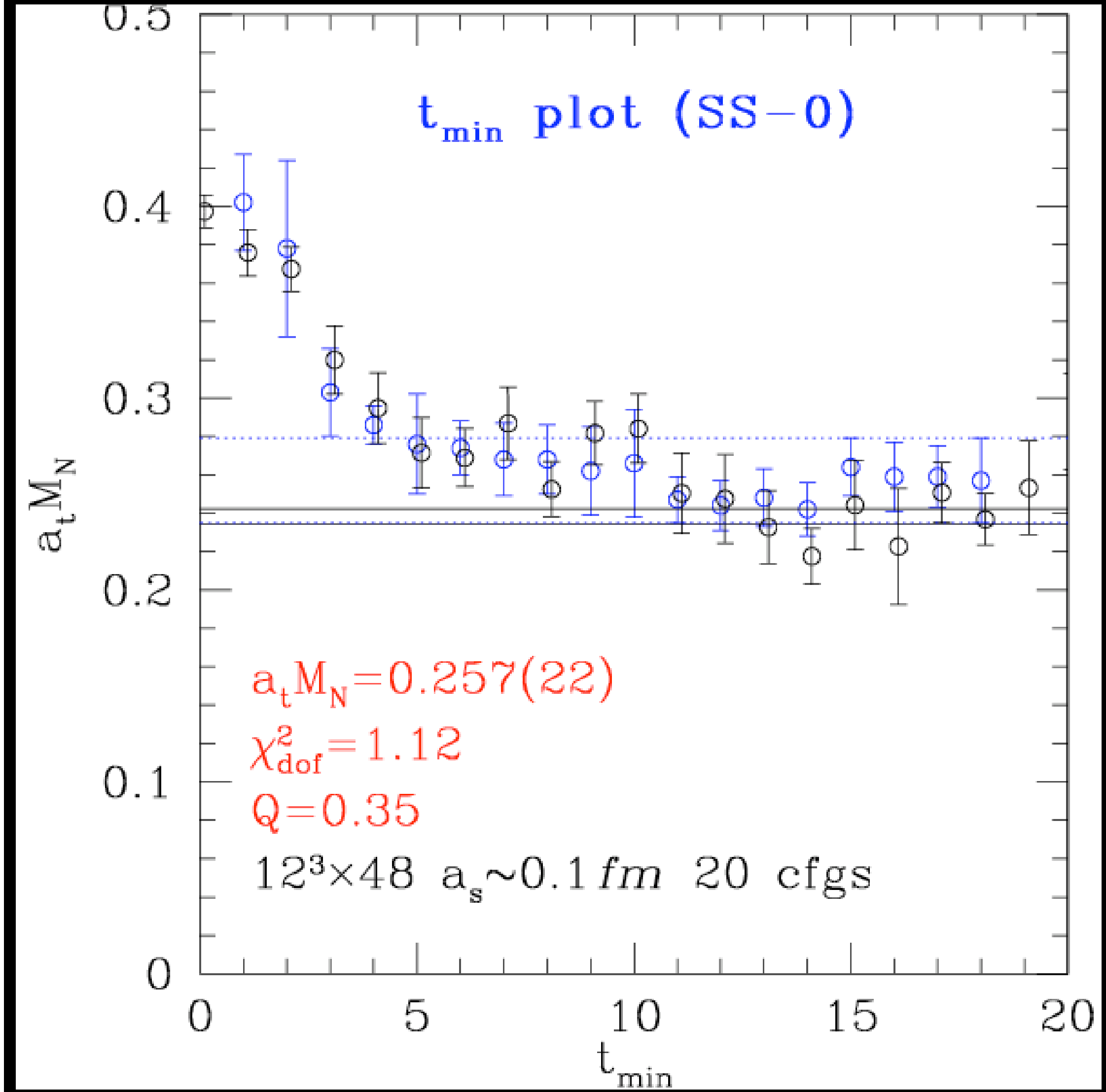
Ground State Nucleon



- $12^3 \times 48$ Lattice
- M(PS)~700 MeV
- 20 configurations
- Optimized operator
- Time+Spin Dilution

... looking pretty good for 3-quark states ...

Nucleon Fits



20 configs

Single-Site op only

Time+Spin-dilution

black solid line from conventional point propagators (200 cfgs + backward averaging)

* fitting method is being revised

Two-Quark Colour-Singlet Operators (mesons)

$$M^{i,j}_{[01]}(\vec{x},t) = \eta^{\dagger(i)a}_{\mu[0]}\Gamma_{\mu\nu}\psi^{(j)a}_{\nu[1]}$$

$$M^{i,j}_{[10]}(\vec{x},t) = \eta^{\dagger(i)b}_{\mu[1]}\Gamma_{\mu\nu}\psi^{(j)b}_{\nu[0]}$$

Meson Correlation Function

$$C(t,t_0;\vec{p}) = M^{i,j}_{[10]}(\vec{p},t)M^{j,i}_{[01]}(\vec{p},t_0)$$

(

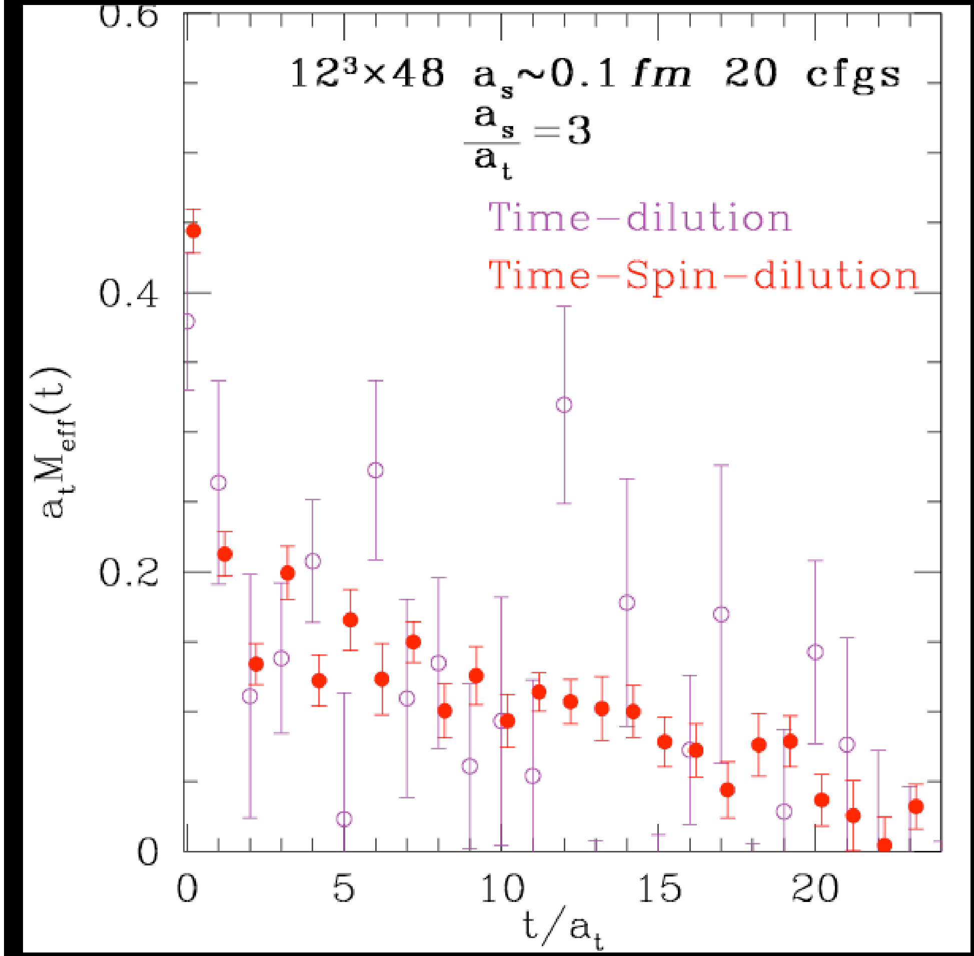
One can also use γ_5 hermiticity to write

$$M^{i,j}_{[10]}(\vec{x},t) = \eta^{\dagger(i)b}_{\mu[1]}\Gamma_{\mu\nu}\gamma_5\eta^{(j)b}_{\nu[0]}$$

$$M^{i,j}_{[10]}(\vec{x},t) = \psi^{\dagger(i)b}_{\mu[1]}\gamma_5\Gamma_{\mu\nu}\psi^{(j)b}_{\nu[0]} \quad \text{etc}$$

)

Pion Effective Mass

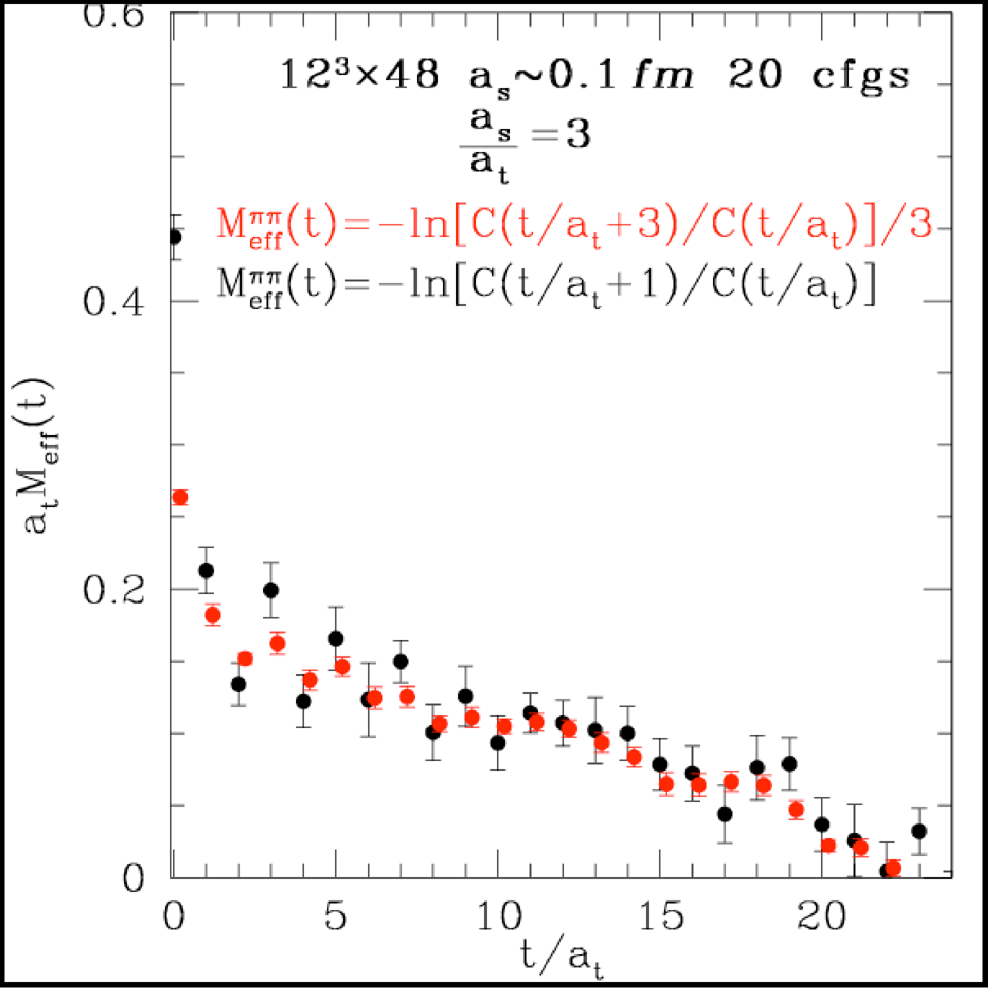


Ground State Pion

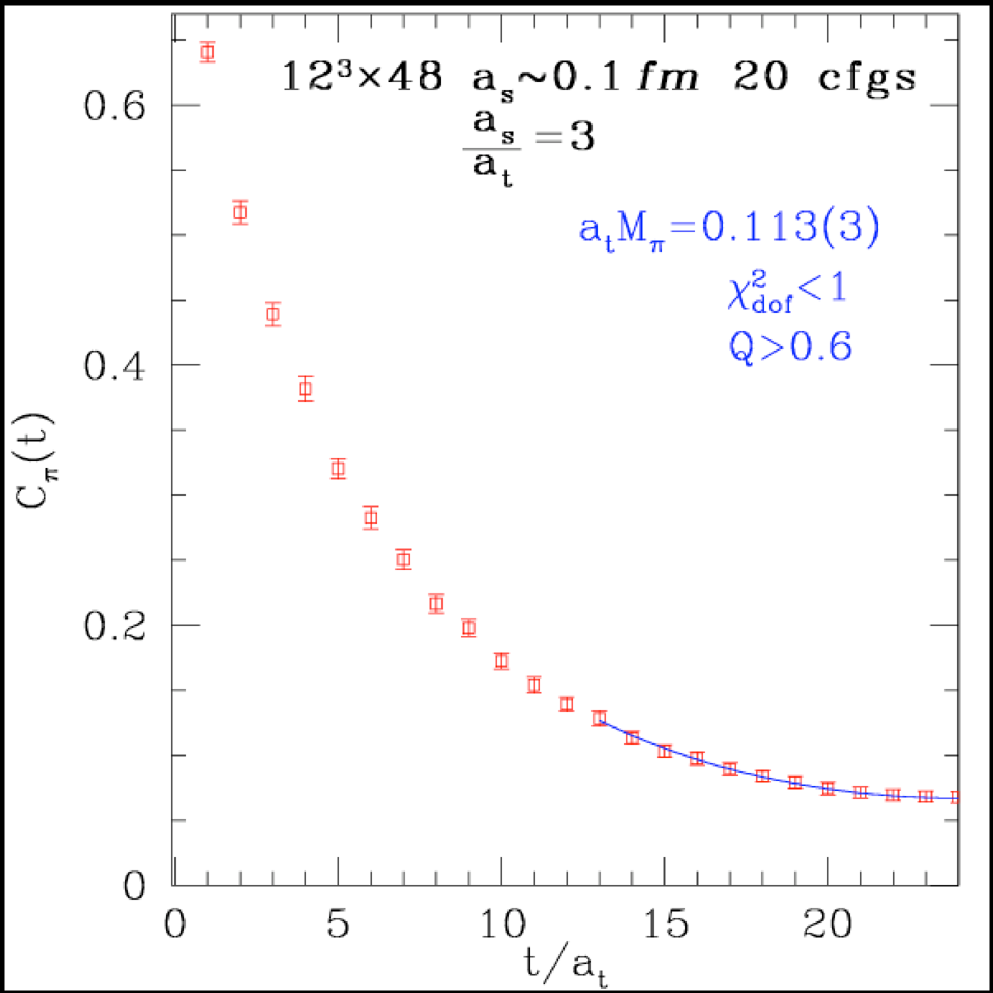
- $12^3 \times 48$ Lattice
- M(PS)~700 MeV
- 20 configurations
- simplest γ_5 operator
- Time Dilution and Time+Spin Dilution

“Large” effective mass errors are an artifact ...

Noise on different timeslices are independent; this causes the local definition of effective masses to be noisy



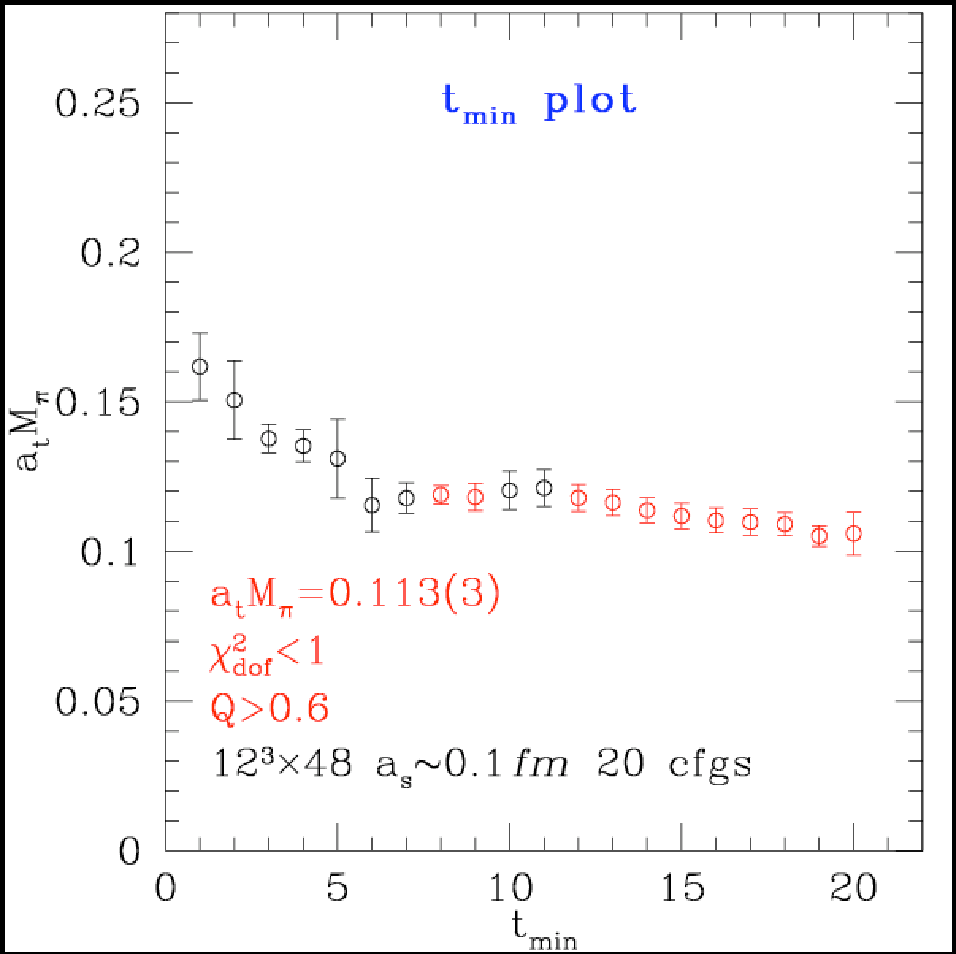
Single Pion Correlator



Ground State Pion

- 12^3 x 48 Lattice
- M(PS)~700 MeV
- 20 configurations
- Time+Spin Dilution
- simplest γ_5 operator
- same smearing as nucleon

Single Pion Correlator Fits



Ground State Pion

- $12^3 \times 48$ Lattice
- M(PS)~700 MeV
- 20 configurations
- Time+Spin Dilution
- simplest γ_5 operator
- same smearing as nucleon

Fits are very stable

Two-Pion Correlation Function

Simplest multi-hadron state

I=2 $\pi\pi$

$$C_{\pi\pi}(t, t_0)$$

$$= \langle \mathcal{O}_\pi(t)\mathcal{O}_\pi(t)\mathcal{O}_\pi^\dagger(t_0)\mathcal{O}_\pi^\dagger(t_0)\rangle$$

$$= \langle \bar{\psi}\gamma_5\psi(\vec{x}_1, t)\bar{\psi}\gamma_5\psi(\vec{x}_2, t)\bar{\psi}\gamma_5\psi(\vec{x}_3, t_0)\bar{\psi}\gamma_5\psi(\vec{x}_4, t_0)\rangle$$

$$= \text{glue-exchange (D) - quark exchange (C)}$$

$$= A\left[e^{-E_{\pi\pi}t} + e^{-E_{\pi\pi}(L_t - t)}\right] + Be^{-M_\pi L_t}$$

All-to-all construction

$$C_{\pi\pi}(t,t_0) = \langle \mathcal{O}_\pi(t)\mathcal{O}_\pi(t)\mathcal{O}^\dagger_\pi(t_0)\mathcal{O}^\dagger_\pi(t_0)\rangle$$

$$\propto \left(M^{i,j}_{[10]}(\vec{p},t)M^{j,i}_{[01]}(\vec{p},t_0)\right)\left(M^{k,l}_{[32]}(\vec{p},t)M^{l,k}_{[23]}(\vec{p},t_0)\right)$$

$$-\left(M^{i,j}_{[01]}(\vec{p},t)M^{j,k}_{[12]}(\vec{p},t_0)M^{k,l}_{[23]}(\vec{p},t)M^{l,i}_{[30]}(\vec{p},t_0)\right)$$

where

$$M^{i,j}_{[AB]}(\vec{p},t) = \sum e^{-i\vec{p}\cdot\vec{x}}\eta^{\dagger(i)b}_{\mu[A]}\gamma_{5\mu\nu}\psi^{(j)b}_{\nu[B]}(\vec{x},t)$$

but recall that these are the matrices that were needed to make the pion correlation function

So all that we really need are the matrices

$$M^{i,j}_{[01]}(t) \quad M^{i,j}_{[10]}(t) \quad M^{i,j}_{[12]}(t)$$

$$M^{i,j}_{[30]}(t) \quad M^{i,j}_{[32]}(t) \quad M^{i,j}_{[23]}(t)$$

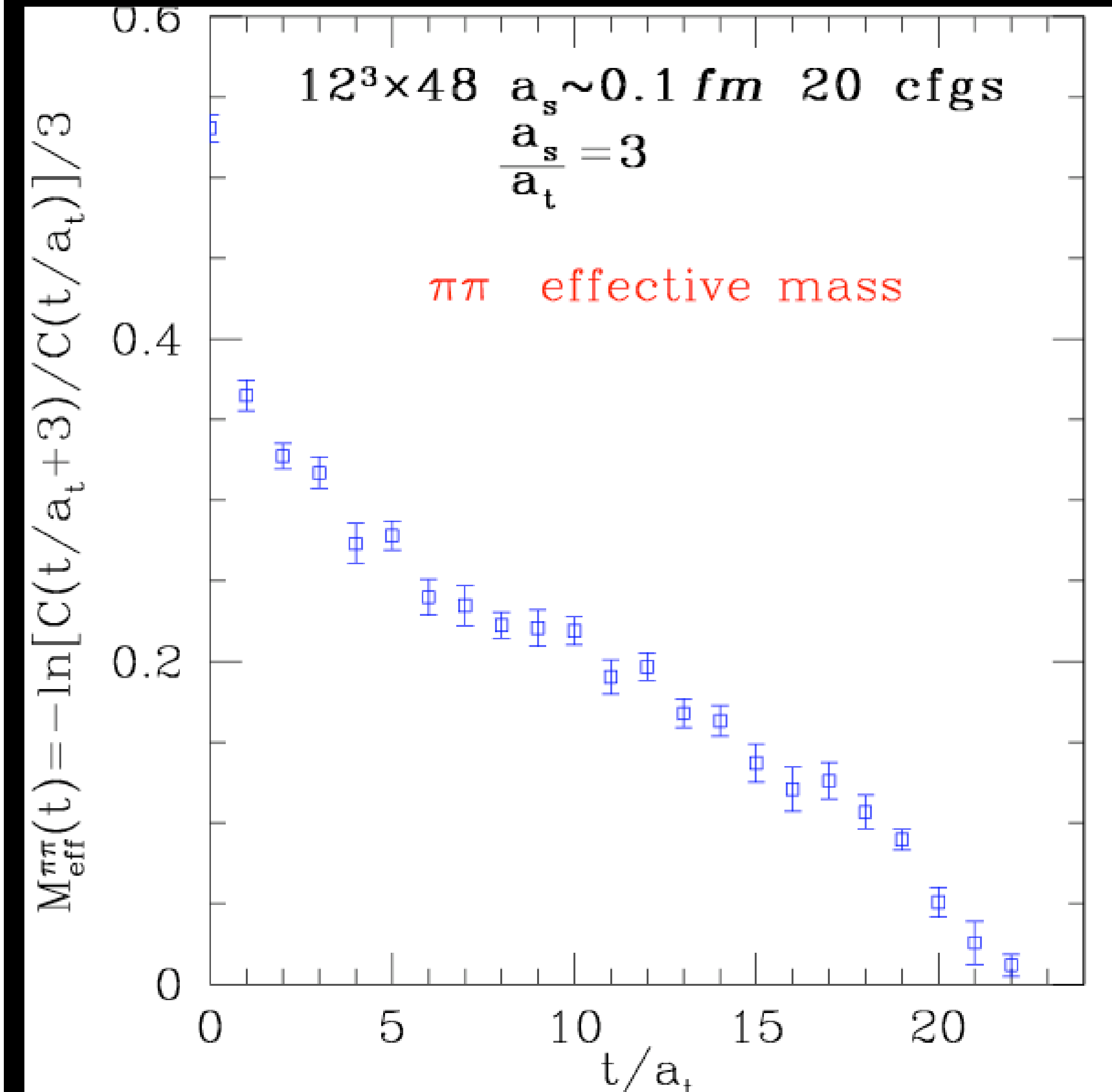
to make the two-pion correlation function

Size of Matrix

$$(N_{\rm dil} \times N_{\rm dil}) \times N_t$$

sizes of these matrices are small ...
(no spatial indices)

Two-Pion Effective Mass



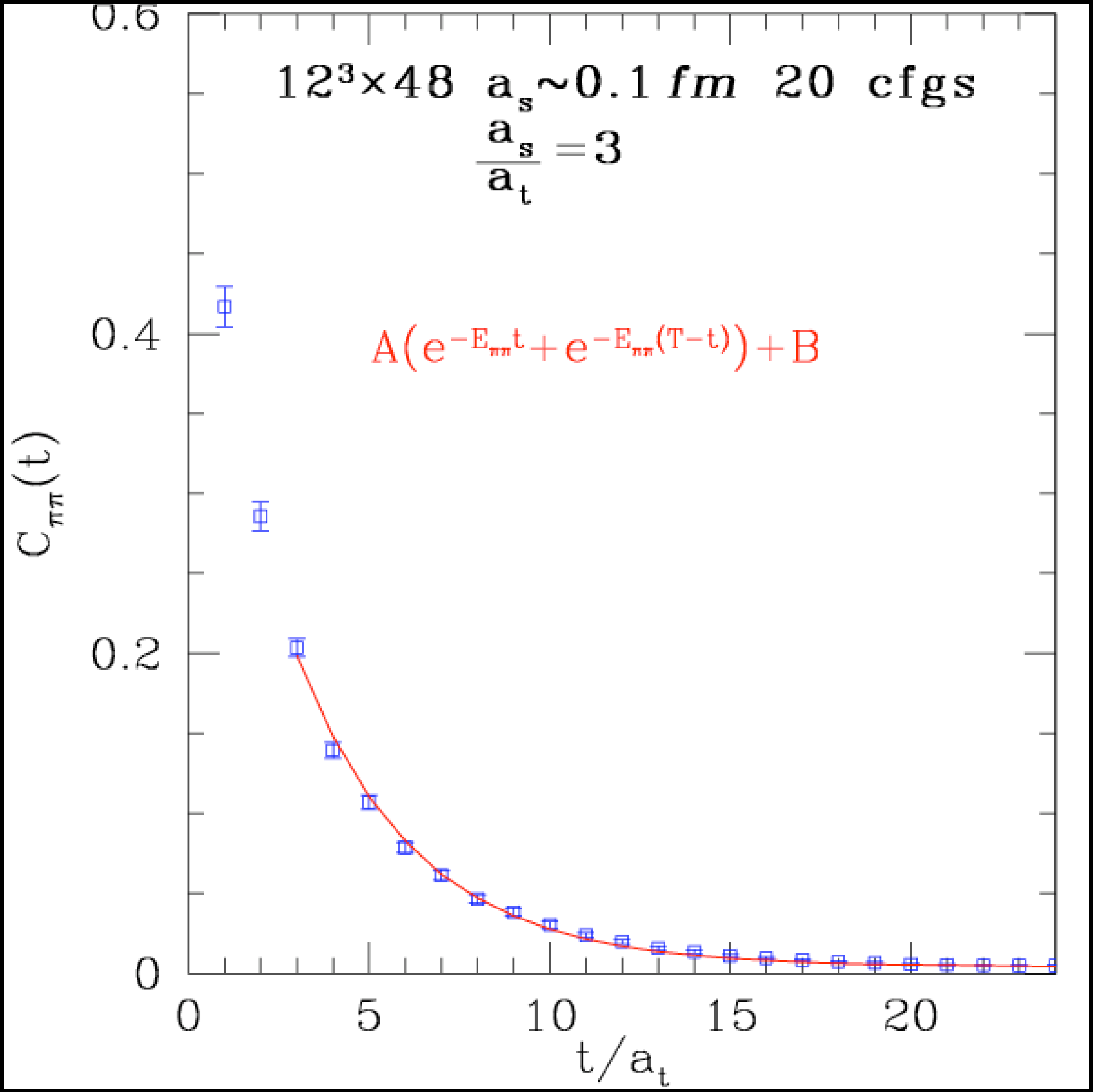
20 configs

I=2 channel

Time+Spin-dilution

looks worse than it really is ...

Two-Pion Correlation Function



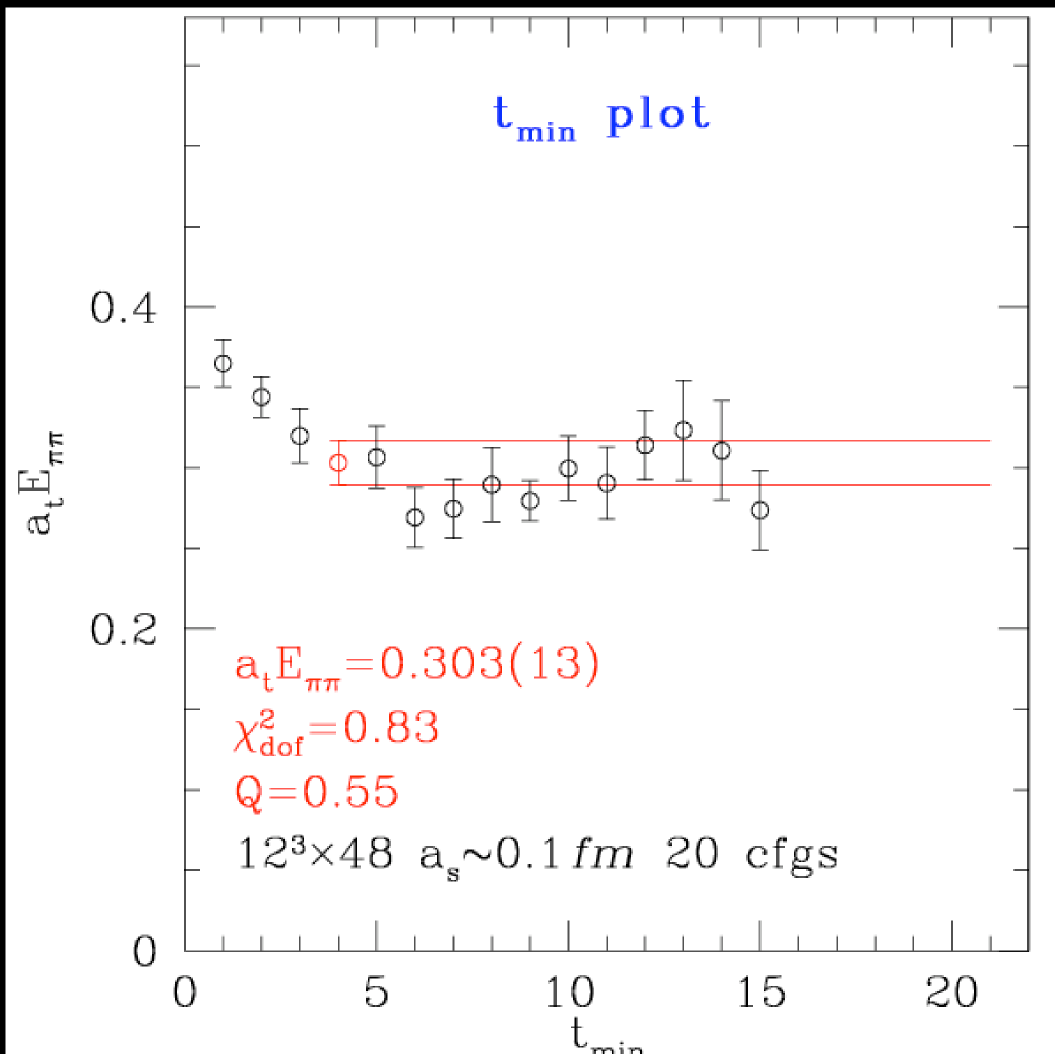
20 configs

I=2 channel

Time+Spin-dilution

- the correlation function is indeed of the expected form with a small positive energy shift

Two-Pion Correlation Function



20 configs

I=2 channel

Time+Spin-dilution

Stable fit out to t-min of 15

~4% errors for the two-particle state

"small" errors like the single pion

a quick summary ...

2-quark operators (mesons)

- noisy effective masses
  (cured by using a modified definition)
- small fit errors (3% with 20 configs)

3-quark operators (baryons)

- effective mass errors under control
- fit errors are roughly the same
  as the effective mass errors
  ("proper fit" to be done in the future)

4-quark operators (pi-pi)

- very much like the 2-quark operator
  operator case (4% errors with 20 configs)

Future/currently running work

Time-Spin-Colour dilution

... errors are reduced by roughly 20%

(need higher statistics to confirm)

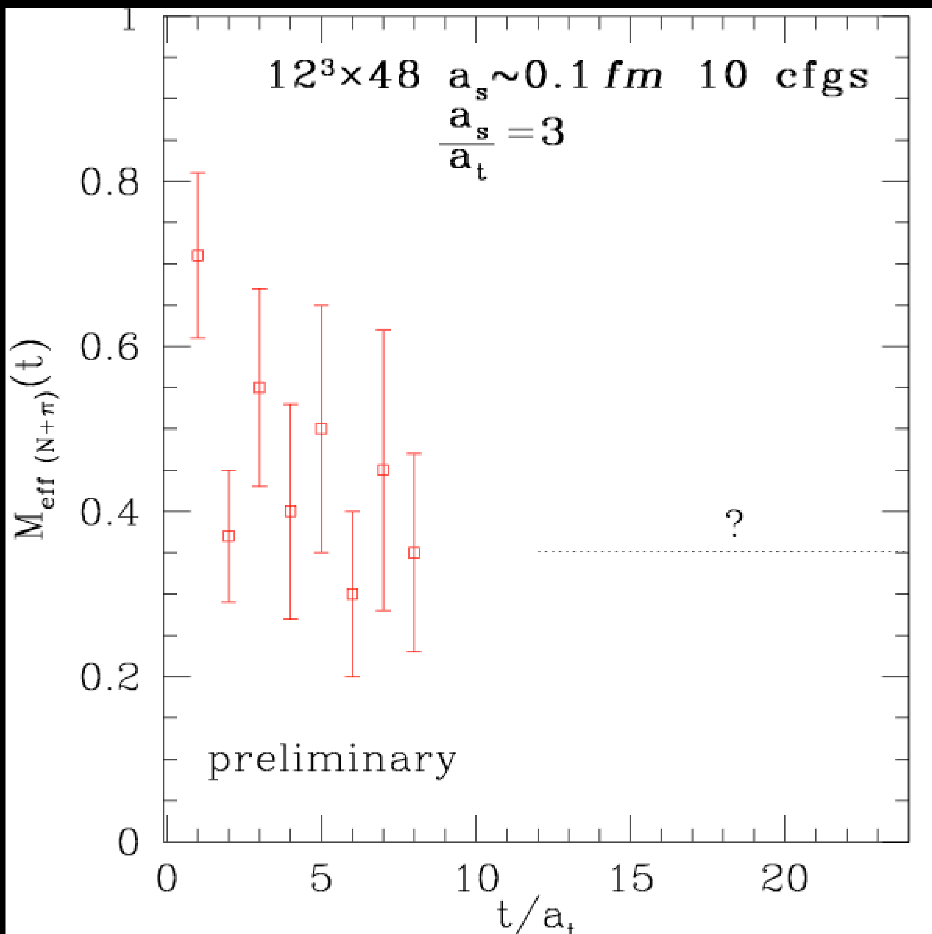
Nucleon-Pion (5-quark) operator

...first indications are that the fluctuations are comparable to that of pi-pi

A new approach to calculating all-to-all quark propagators are being investigated as well

very preliminary (test stage)

proton+pion state



but errors are probably not realistic ...

Conclusions

- The noise dilution method of estimating all-to-all propagators works well for multi-hadron operators
  - pion-pion energy can be extracted quite easily ...
- Multi-particle operators are constructed from the 3-quark/2-quark building blocks
  - easier to build multi-particle correlators
  - can build a basis of finite momentum operators
- These operators may be essential for spectroscopy on light/large Nf=2 (2+1, 3) configurations
- Simulation of proton-pion state underway ...
- also pursuing new way of computing all-to-all propagators to further reduce the errors