Neutral Kaon Mixing beyond the Standard Model with Domain Wall Fermions

Jan Wennekers

for the RBC and UKQCD collaborations

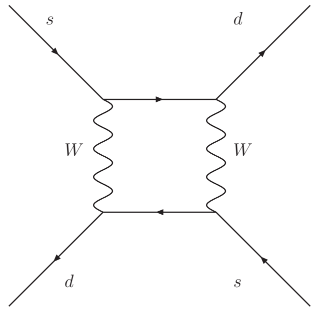
Lattice 2008 Williamsburg

Outline

$K - \bar{K}$ Mixing

- In the Standard Model: effective operator with $(V - A)(V - A)$ structure from integrating out Ws in box diagram
- FCNCs from theories beyond the SM constrained to be small
- model independent studies use the *mass insertion approximation*
- Lattice QCD can provide matrix elements for a full operator basis

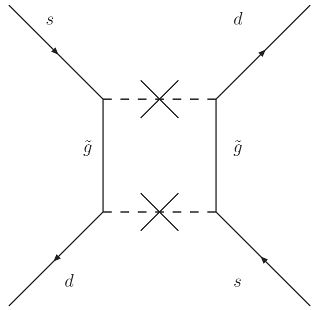
SM: W and t in box diagram



$K - \bar{K}$ Mixing

- In the Standard Model: effective operator with $(V - A)(V - A)$ structure from integrating out Ws in box diagram
- FCNCs from theories beyond the SM constrained to be small
- model independent studies use the *mass insertion approximation*
- Lattice QCD can provide matrix elements for a full operator basis

SUSY: $\tilde{g}$ and $\tilde{q}$ in box diagram



Operators and RG running

- operator basis

$$Q_1 = \bar{s}^a \gamma_\mu P_L d^a \bar{s}^b \gamma^\mu P_L d^b$$
$$Q_2 = \bar{s}^a \gamma_\mu P_L d^a \bar{s}^b \gamma^\mu P_R d^b$$
$$Q_3 = \bar{s}^a P_L d^a \bar{s}^b P_R d^b$$
$$Q_4 = \bar{s}^a P_L d^a \bar{s}^b P_L d^b$$
$$Q_5 = \bar{s}^a \sigma_{\mu\nu} P_L d^a \bar{s}^b \sigma^{\mu\nu} P_L d^b$$

- related to “SUSY” basis by Fierz identities
- continuum QCD: only Wilson coefficients of Q_2, Q_3 and Q_4, Q_5 mix

$$\mu \frac{d}{d\mu} \vec{C}(\mu) = \begin{pmatrix} \gamma_1 & & & & \\ & \gamma_{22} & \gamma_{23} & & \\ & \gamma_{32} & \gamma_{33} & & \\ & & & \gamma_{44} & \gamma_{45} \\ & & & \gamma_{54} & \gamma_{55} \end{pmatrix} \vec{C}(\mu)$$

Operators and RG running

- operator basis

$$
\begin{aligned}
Q_1 &= \bar{s}^a \, \gamma_\mu P_L \, d^a \bar{s}^b \, \gamma^\mu P_L \, d^b \\
Q_2 &= \bar{s}^a \, \gamma_\mu P_L \, d^a \bar{s}^b \, \gamma^\mu P_R \, d^b \\
Q_3 &= \bar{s}^a \, P_L \, d^a \bar{s}^b \, P_R \, d^b \\
Q_4 &= \bar{s}^a \, P_L d^a \bar{s}^b \, P_L \, d^b \\
Q_5 &= \bar{s}^a \, \sigma_{\mu\nu} P_L \, d^a \bar{s}^b \, \sigma^{\mu\nu} P_L \, d^b
\end{aligned}
$$

- related to “SUSY” basis by Fierz identities
- continuum QCD: only Wilson coefficients of Q_2, Q_3 and Q_4, Q_5 mix
- chiral symmetry essential for reduced mixing
- Domain Wall Fermions well suited for this problem

Outline

RI-MOM Scheme for Four-Fermion Operators

- 5×5 matrix needed to renormalise operator basis

$$\Lambda_{ij} \equiv (\Gamma_i)^{ABCD}_{\alpha\beta\gamma\delta} (P_j)^{BADC}_{\beta\alpha\delta\gamma}$$

- Γ amputated four-point vertex function, P projector
- renormalisation condition

$$\frac{1}{Z_q^2} Z(\mu) = \Lambda_{\text{tree}} \cdot \Lambda^{-1}(p^2 = \mu^2)$$

- trade Z_q for Z_A:

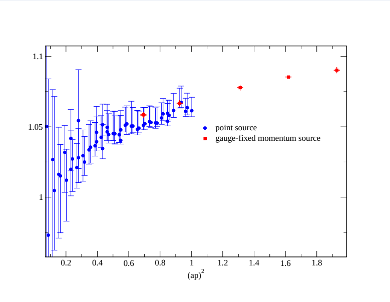
$$\frac{1}{Z_A^2} Z = \Lambda_{\text{tree}} \cdot \Lambda^{-1} / \Lambda_A$$

Gauge-fixed Momentum Sources

- Landau gauge fixed configurations
- source: 4-d volume source with phase factor $e^{2\pi i p \cdot x}$
- vertices calculated at sink position

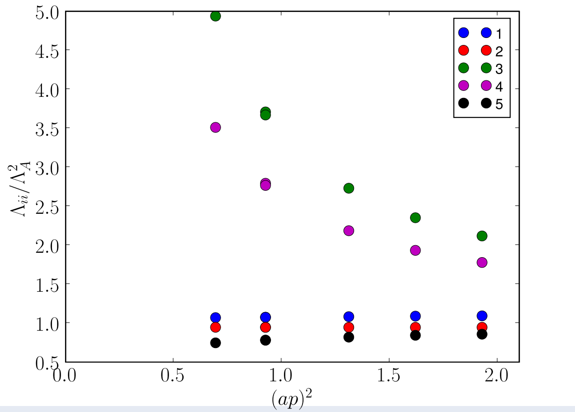
- much smaller statistical errors than point sources
- small number of momenta
- different $O((ap)^4)$ errors for momenta with same p^2 $\Rightarrow$ Dirk Brömmel's talk on Friday afternoon

point/gf mom source comparison



Mixing Matrix

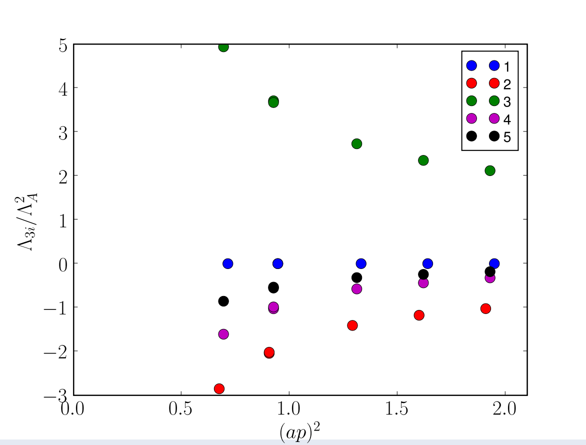
diagonal elements



- larger p^2 dependence for some operators

Mixing Matrix

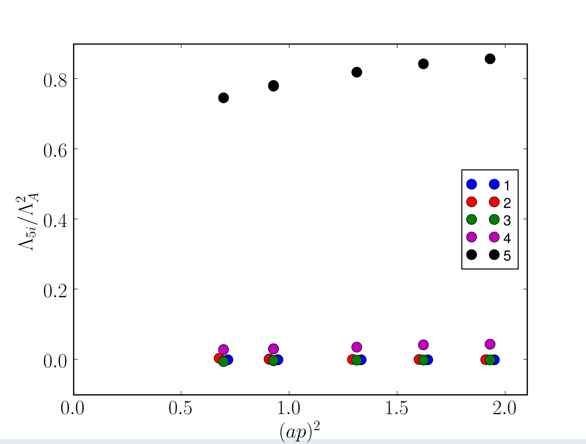
off-diagonal elements, Q_3



- block diagonal structure like in the continuum

Mixing Matrix

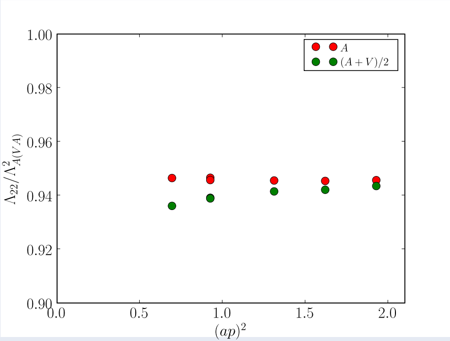
off-diagonal elements, Q_5



- block diagonal structure like in the continuum

Mixing Matrix

$\Lambda_A - \Lambda_V$ splitting



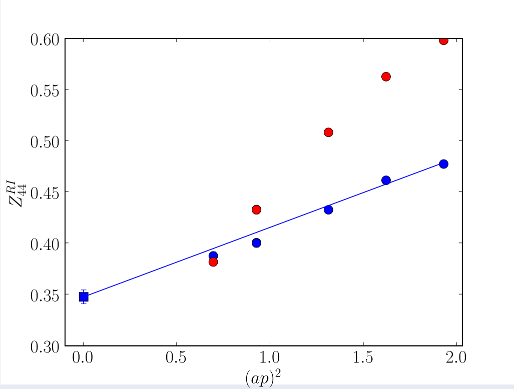
- difference between $\Lambda_A(p^2)$ and $\Lambda_V(p^2)$ leads to systematic error
- way out: RI-MOM with non-exceptional momentum configurations
  $\Rightarrow$ Chris Kelly's talk in a few minutes
  $\Rightarrow$ Yasumich Aoki's talk on Wednesday afternoon

Removing the Perturbative Running

- RG evolution of the Wilson coefficients, $\vec{C}(\mu_2) = U(\mu_2, \mu_1)\vec{C}(\mu_1)$
- U depends on anomalous dimension (matrix) γ, known at NLO

[Ciuchini et al 1998, Buras et al 2001]

diagonal element



Outline

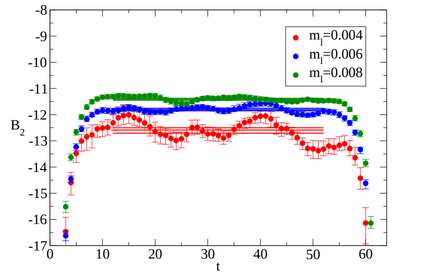
DWF Lattices

- Iwasaki gauge action with $\beta = 2.25$, Domain Wall fermion action
- $32^3 \times 64 \times 16$ lattices (new!)
- lattice spacing: $a^{-1} = 2.42(4)\,\text{GeV} \times \frac{0.47\,\text{fm}}{r_0}$ (statistical error only)
- $m_{\text{res}} = 0.00066(2)$ (preliminary)

m_l	m_s	m_π	Renorm.	Matrix Elements
0.004	0.03	$\sim 300\,\text{MeV}$	0.004	0.002, 0.004, 0.006, 0.008 0.025, 0.03
0.006	0.03	$\sim 365\,\text{MeV}$	0.006	”
0.008	0.03	$\sim 420\,\text{MeV}$	(0.008)	”

Raw Data

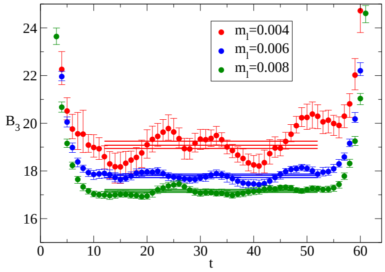
B_2, unitary masses



- gauge-fixed wall sources
- use of fermionic boundary conditions: $\mathrm{p}+\mathrm{a}$ at $t=0$, $\mathrm{p}-\mathrm{a}$ at $t=$ ”64”
- currently $O(100)$ measurements for each mass, will go up to 200
- statistical errors $O(1\%)$ at lowest kaon mass

Raw Data

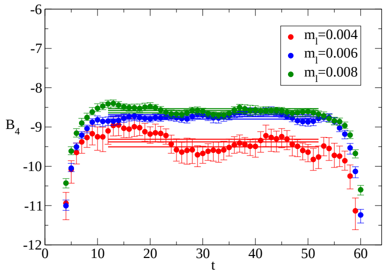
B_3, unitary masses



- gauge-fixed wall sources
- use of fermionic boundary conditions: $\mathrm{p} + \mathrm{a}$ at $t = 0$, $\mathrm{p} - \mathrm{a}$ at $t =$ "64"
- currently $O(100)$ measurements for each mass, will go up to 200
- statistical errors $O(1\%)$ at lowest kaon mass

Raw Data

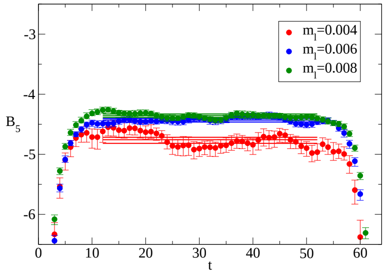
B_4, unitary masses



- gauge-fixed wall sources
- use of fermionic boundary conditions: p $+$ a at $t = 0$, p $-$ a at $t =$ ”64”
- currently $O(100)$ measurements for each mass, will go up to 200
- statistical errors $O(1\%)$ at lowest kaon mass

Raw Data

B_5, unitary masses



- gauge-fixed wall sources
- use of fermionic boundary conditions: p + a at $t = 0$, p − a at $t =$ ”64”
- currently $O(100)$ measurements for each mass, will go up to 200
- statistical errors $O(1\%)$ at lowest kaon mass

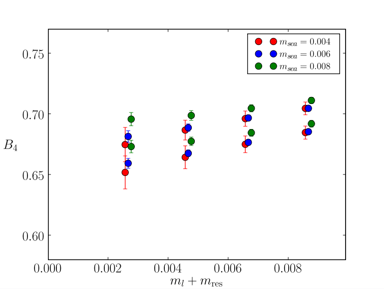
Mass Dependence

- B_K style normalisation quark mass dependent for B_2 - B_5

$$\langle \bar{K}|Q_i|K\rangle = N_i m_K^2 F_K^2 B_i, \quad N_i = \frac{8}{3}, -\frac{4}{3}R, 2R, \frac{5}{3}R, -4R$$

$$R = \left(\frac{m_K}{m_s^r + m_d^r}\right)^2$$

B4



Mass Dependence

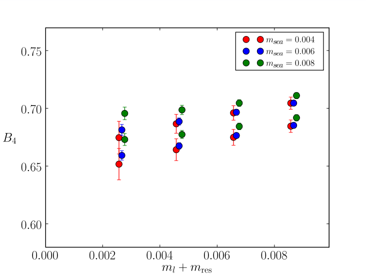
- B_K style normalisation quark mass dependent for B_2 - B_5

$$\langle \bar{K} | Q_i | K \rangle = N_i m_K^2 F_K^2 B_i, \quad N_i = \frac{8}{3}, -\frac{4}{3}R, 2R, \frac{5}{3}R, -4R$$

$$R = \left(\frac{m_K}{m_s^r + m_d^r} \right)^2$$

- plan: use partially quenched Heavy Meson ChPT (SU(2))
- first fix LO LECs from fits in the pion sector
- treat the kaon as heavy relative to the pion

B4



Outline

- status report on project about neutral kaon mixing beyond the SM
- 2+1 flavour DWF with small quark masses and very small chiral symmetry breaking
- renormalisation of the chosen operator basis in the RI-MOM scheme
- reduced operator mixing due to (lattice) chiral symmetry
- preliminary results on the matrix elements from the new 32^3 ensembles
- analysis in framework of partially quenched ChPT is on the way
- scaling study with existing coarser DWF lattices planned