Light pseudoscalar masses and decay constants with a mixed action

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Mixed action simulations

Work done with Christopher Aubin and Ruth Van de Water

These simulations use MILC lattices (with 2+1 Asqtad staggered quarks in the sea sector) and domain wall quarks in the valence sector.

Advantages

- A large number of ensembles with different volumes, sea quark masses and lattice spacings exist and are publicly available.
- The existing ensembles have 2+1 flavors of light sea quarks ($m_{strange}/10$ for the lightest quarks)
- The good chiral properties of the valence sector make things much simpler than the staggered case. There are only two additional parameters (over pure domain wall) that appear at one loop in the mixed action ChPT for m_π, f_π, and B_K. They can both be obtained from spectrum calculations.
- NPR can be carried through in the same way as in domain wall.

Mixed action calculations

In 1-loop Mixed Action χPT only two parameters beyond those of domain-wall:

$$m_{dw}^2 = 2\mu_{\rm dw}(m_v + m_{\rm res}) \ , \tag{1}$$

$$m_I^2 = 2\mu_{\rm stag} m_s + a^2\Delta_I \ , \tag{2}$$

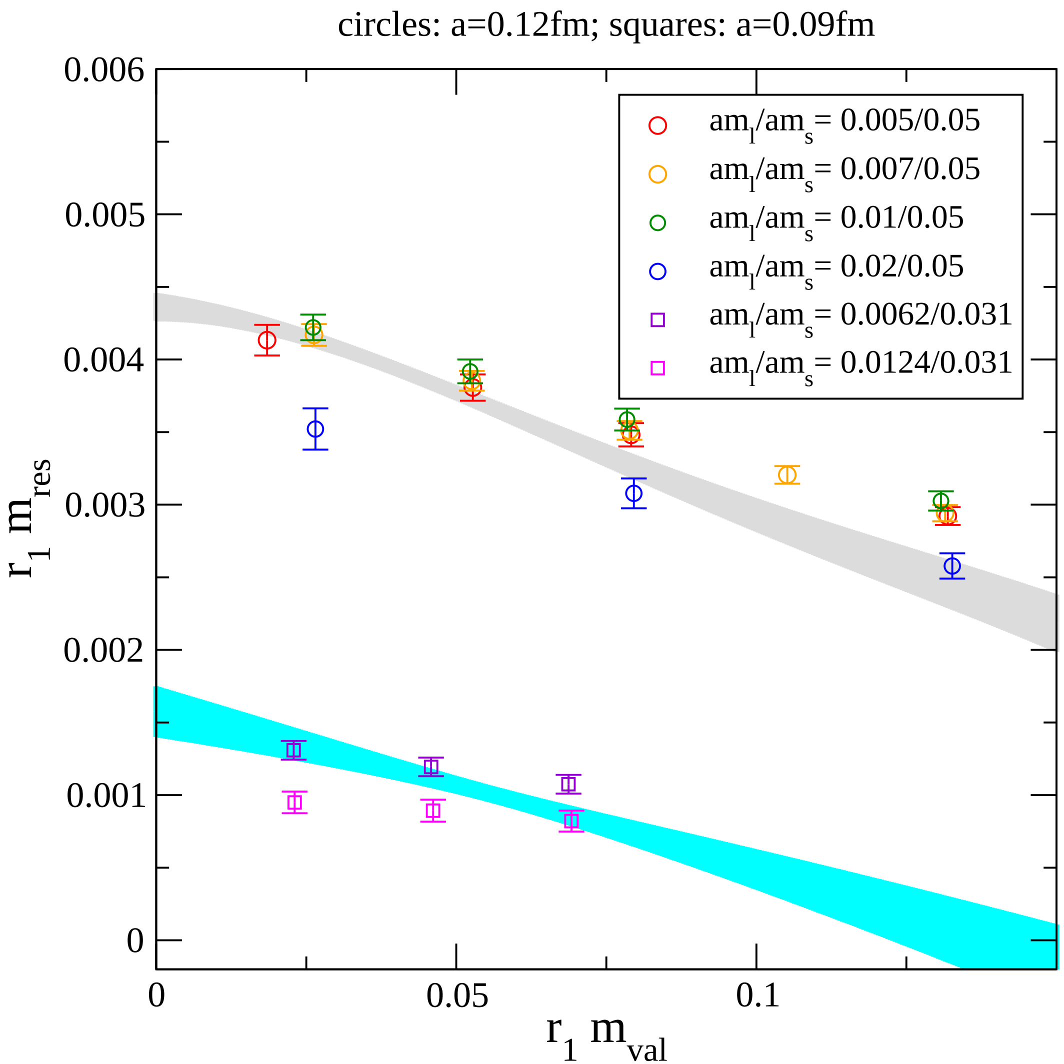
$$m_{mix}^2 = \mu_{\rm dw}(m_v + m_{\rm res}) + \mu_{\rm stag} m_s + a^2\Delta_{\rm mix}, \tag{3}$$

Run parameters

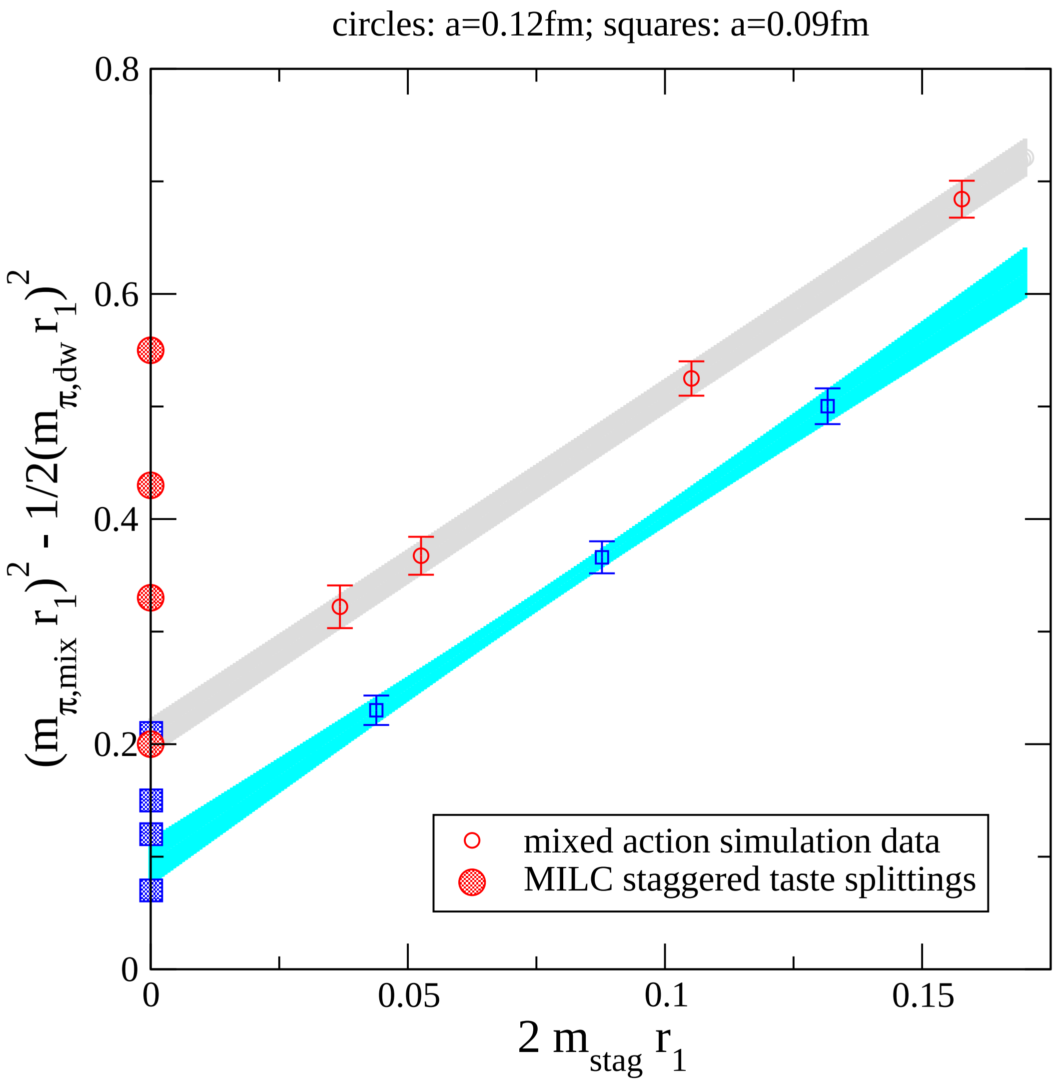
Table 1: Lattice parameters

a(fm)	$a\widehat{m}'/am_s'$	L(fm)	$m_\pi L$	$10/g^2$	Lat Dim	# Confs
≈ 0.12	$0.02/0.05$	2.4	6.2	6.79	$20^3 \times 64$	117
≈ 0.12	$0.01/0.05$	2.4	4.5	6.76	$20^3 \times 64$	220
≈ 0.12	$0.007/0.05$	2.4	3.8	6.76	$20^3 \times 64$	268
≈ 0.12	$0.005/0.05$	2.9	3.8	6.76	$24^3 \times 64$	216
≈ 0.12	$0.01/0.03$	2.4	4.5	6.76	$20^3 \times 64$	160
≈ 0.09	$0.0124/0.031$	2.4	5.8	7.11	$28^3 \times 96$	198
≈ 0.09	$0.0062/0.031$	2.4	4.1	7.09	$28^3 \times 96$	210
≈ 0.09	$0.0031/0.031$	3.4	4.2	7.08	$40^3 \times 96$	38

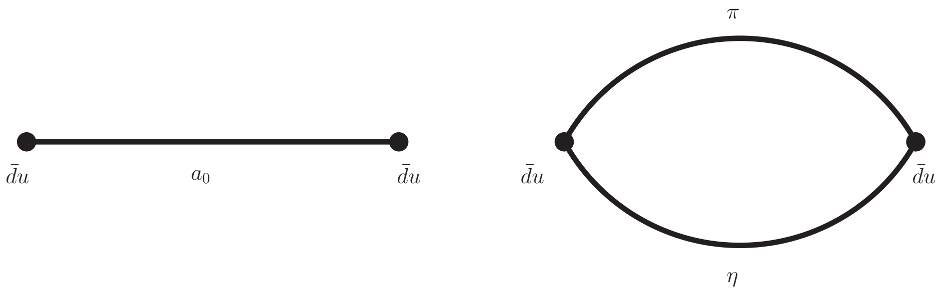
The residual mass



Determining the splitting Δ_{mix}



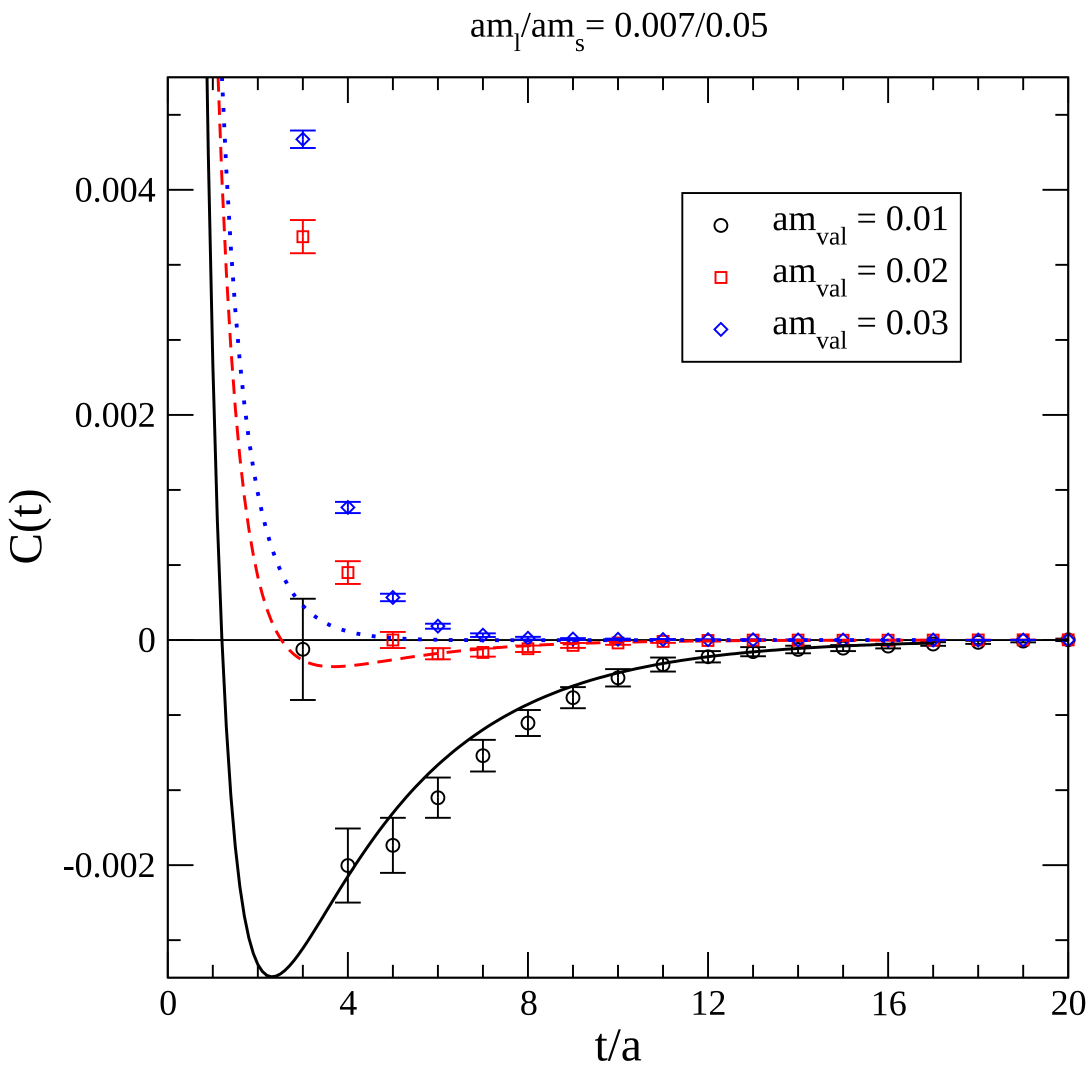
Scalar bubble prediction



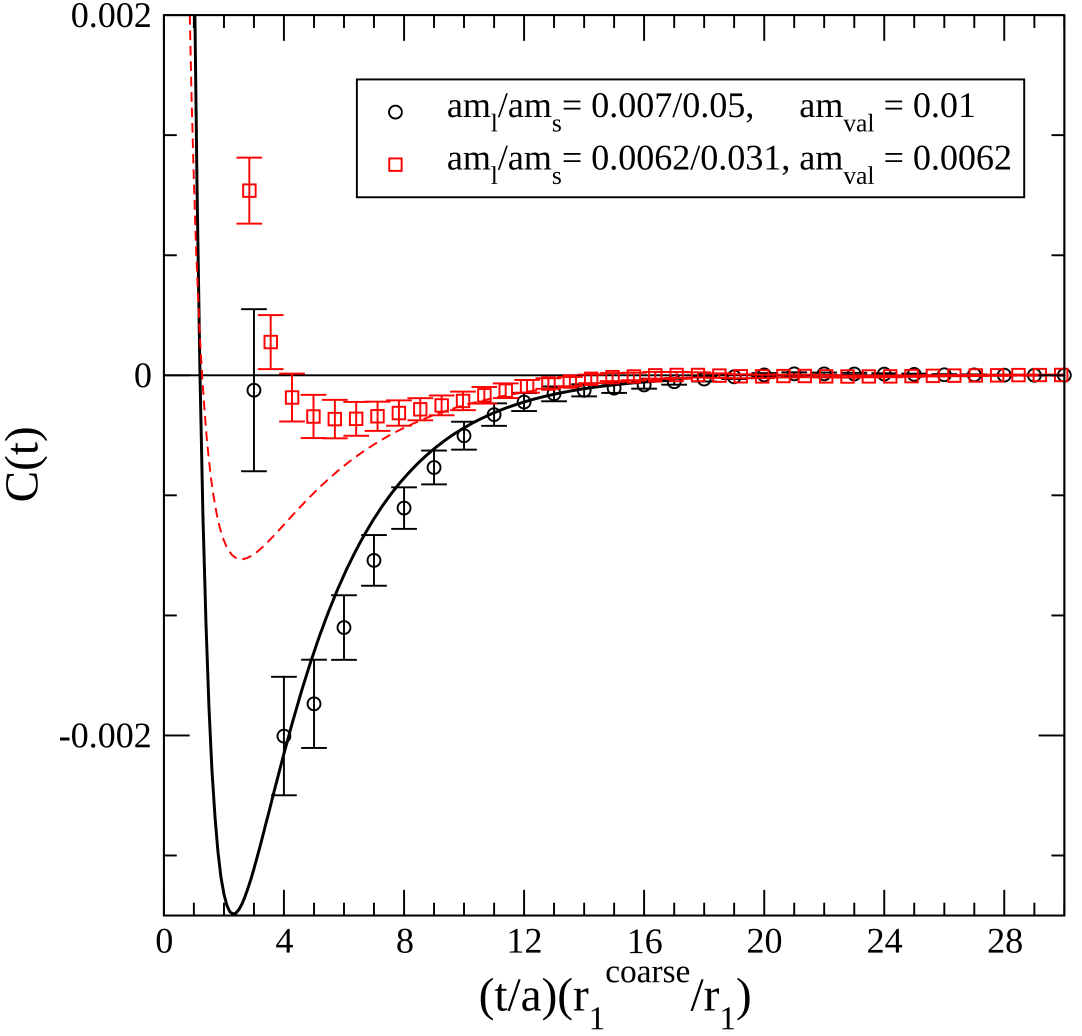
$$B(t) = \frac{\mu^2}{3L^3}\sum_{\mathbf{k}}\Bigg[\frac{2}{9}\frac{e^{-(\omega_{vv}+\omega_{\eta_I})t}}{\omega_{vv}\omega_{\eta_I}}\frac{(m_{S_I}^2-m_{U_I}^2)^2}{(m_{vv}^2-m_{\eta_I}^2)^2}$$
$$-\frac{e^{-2\omega_{vv}t}}{\omega_{vv}^2}\left[\frac{3m_{vv}^2(m_{vv}^2-2m_{\eta_I}^2)+2m_{S_I}^4+m_{U_I}^4}{3(m_{\eta_I}^2-m_{vv}^2)^2}\right]$$
$$-\frac{e^{-2\omega_{vv}t}}{2\omega_{vv}^4}(\omega_{vv}t+1)\frac{(m_{U_I}^2-m_{vv}^2)(m_{S_I}^2-m_{vv}^2)}{m_{\eta_I}^2-m_{vv}^2}+\frac{3}{2}\frac{e^{-2\omega_{vu}t}}{\omega_{vu}^2}+\frac{3}{4}\frac{e^{-2\omega_{vs}t}}{\omega_{vs}^2}\Bigg]$$

Prelovsek, PR D73, 014506 (2006), hep-lat/0510080

Scalar bubble prediction vs. data



Scalar bubble prediction vs. data



Approach to chiral fits

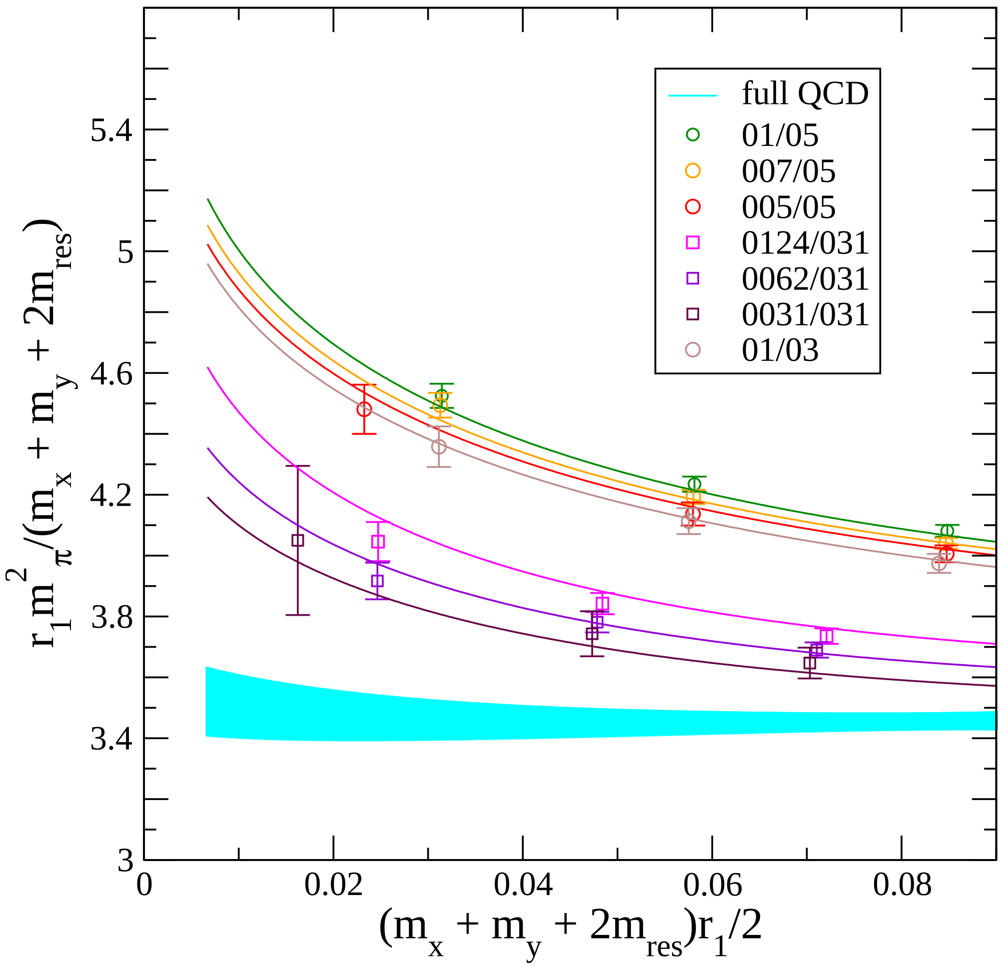
We have generated data with relatively high statistics so that we can resolve a correlation matrix and obtain reliable confidence levels in fits.

Using SU(3) chiral perturbation theory in order to interpolate about the strange quark mass and extrapolate in the light quark mass. We are using one-loop SU(3) mixed action χPT and higher order analytic terms.

Plans to investigate SU(2) χPT and 2-loop corrections.

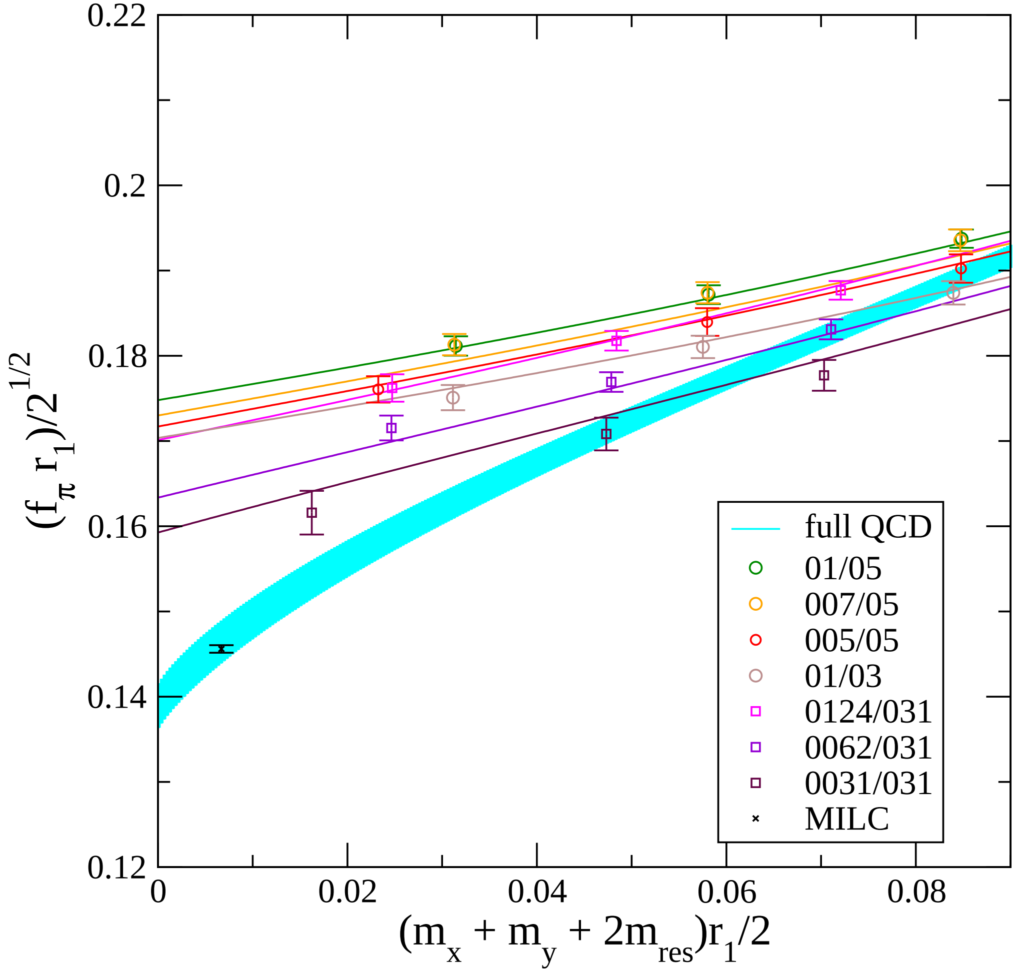
Separate fits to m_π^2/m_q and f_π, where leading order μ is taken from linear fits to m_π^2 data, evaluated in region of data, rather than chiral limit. f_π evaluated at physical pion point.

m_π^2/m_q chiral fit



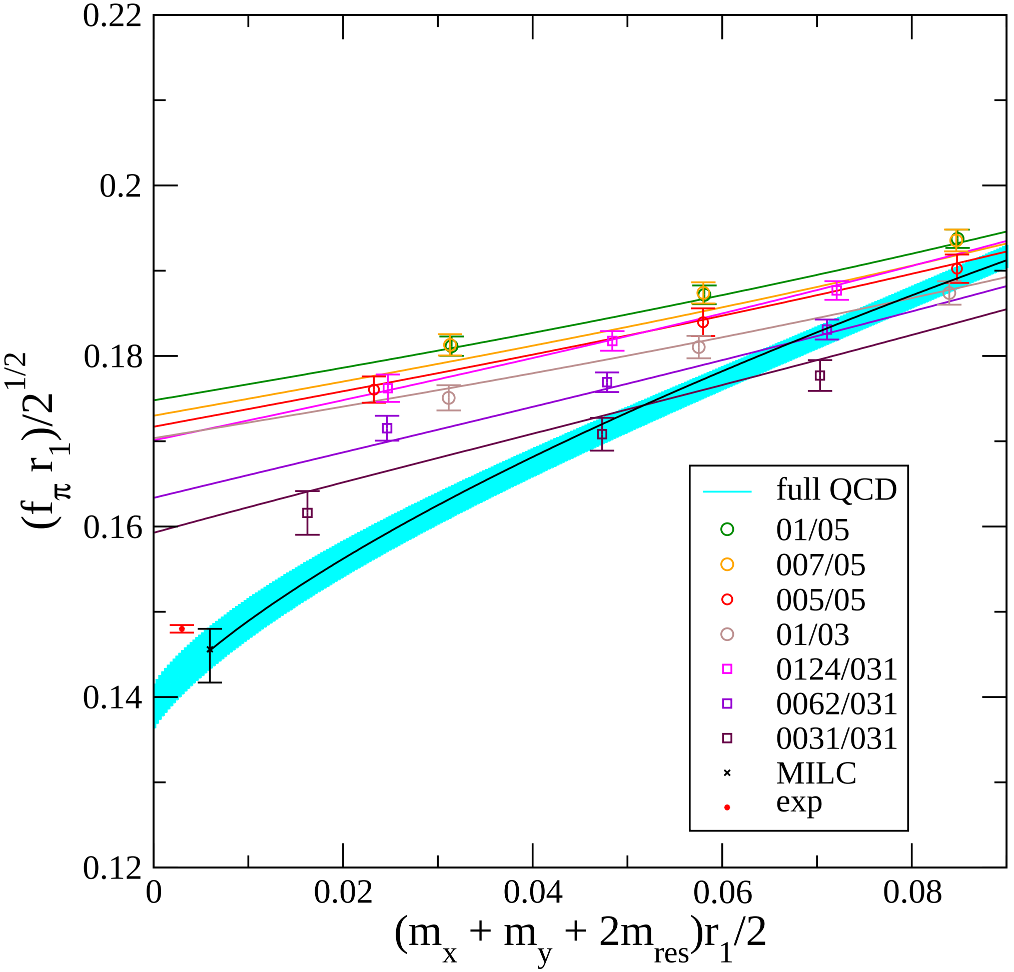
χ^2/dof=0.58, C.L.=0.97

f_π chiral fit



$\chi^2/$dof=1.09, C.L.=0.33

f_π chiral fit (compared w/ MILC)

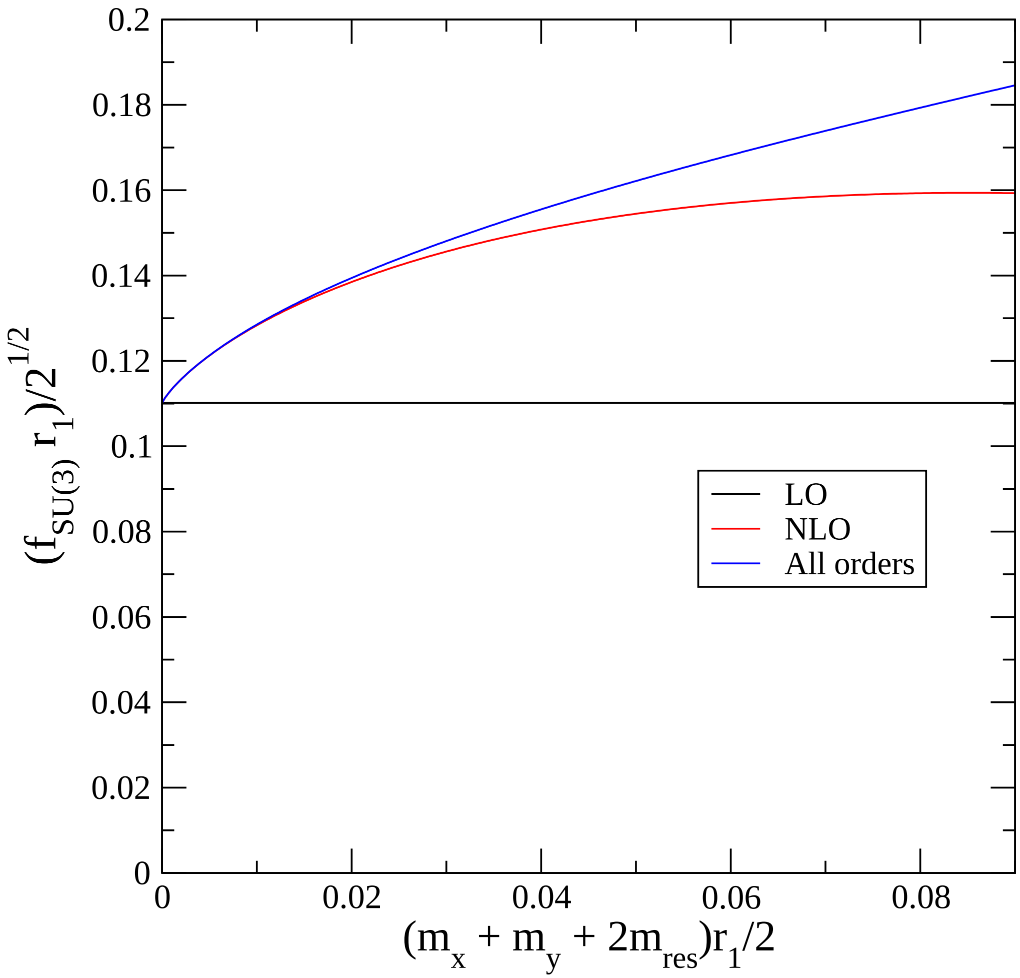


Fit results

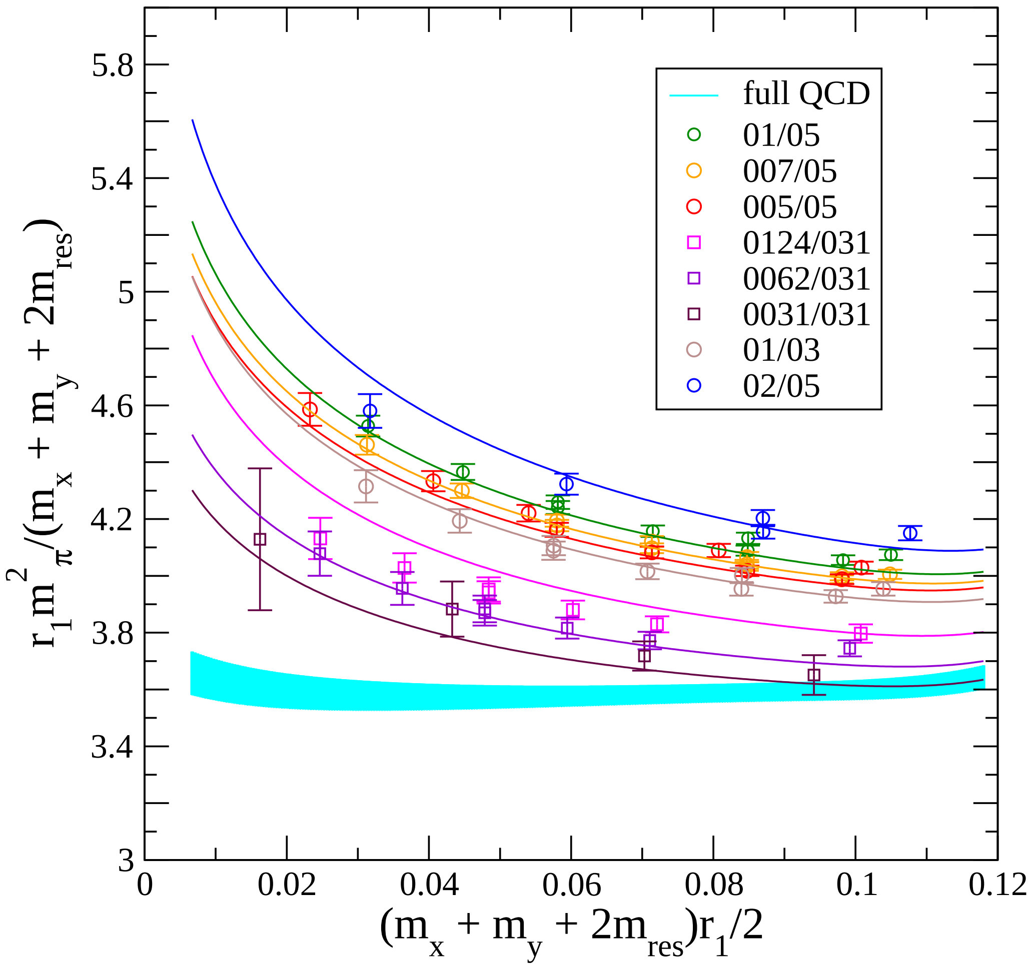
type of f_π fit	$\chi^2/$d.o.f.	C.L.
NNLO analytic	1.09	33%
No NNLO	3.78	3×10^{-14}
No NLO logs	2.43	8×10^{-6}
No FV	1.66	1%
No splittings	1.33	10%

type of m_π^2/m_q fit	$\chi^2/$d.o.f.	C.L.
NNLO analytic	0.58	97%
No NNLO	4.51	7×10^{-19}
No NLO logs	2.06	4×10^{-4}
No FV	0.91	61%
No splittings	0.82	75%

Convergence of SU(3) χPT

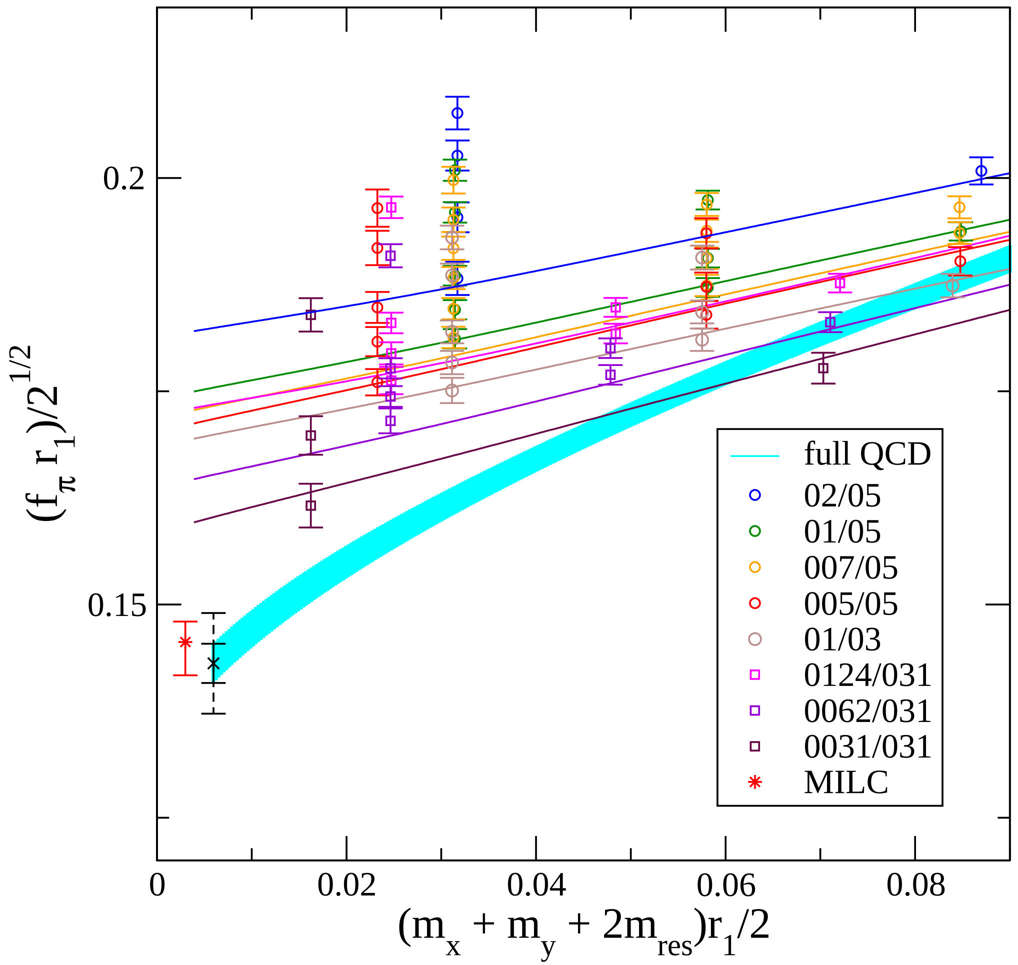


m_π^2/m_q with a higher cut on masses



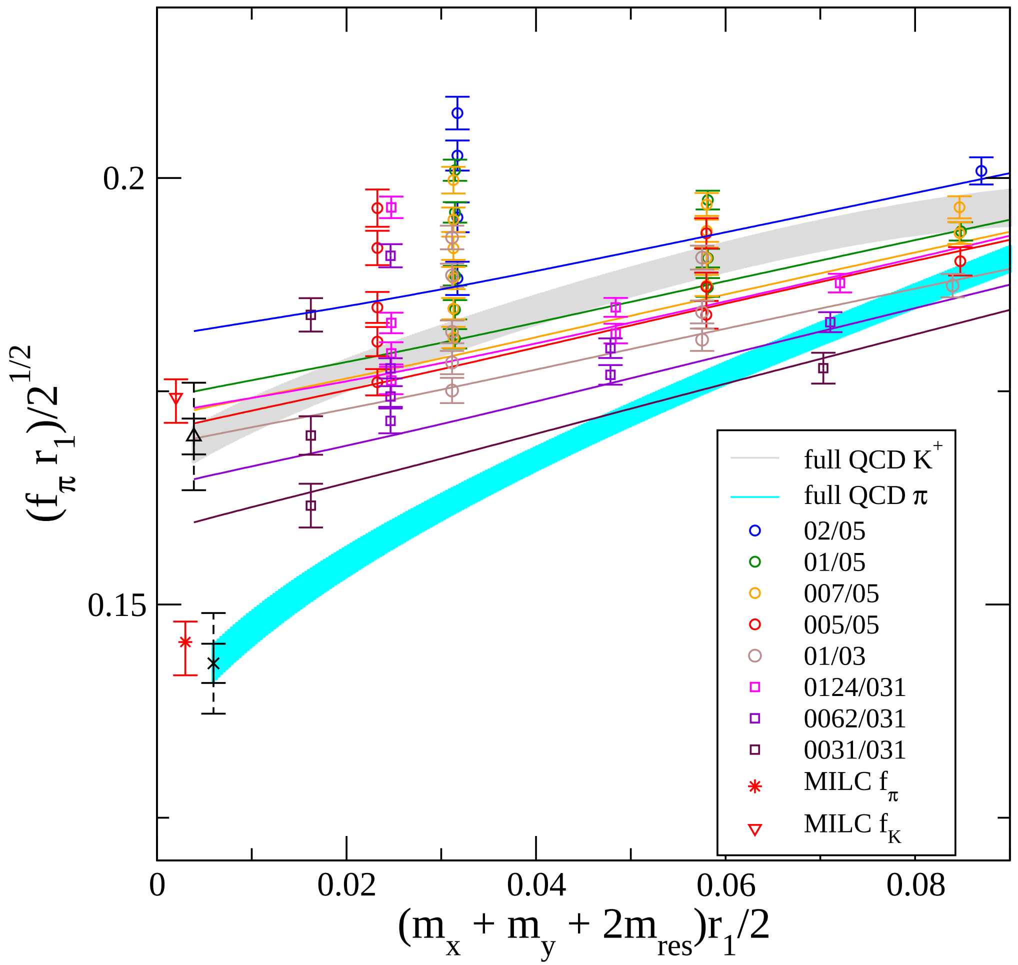
χ^2/dof=1.14, C.L.=0.24

f_π with a higher cut on masses



χ^2/dof=1.25, C.L.=0.12

f_K determination



Preliminary decay constant error budget

source	f_K	f_π	f_K/f_π
statistics	1.2	1.6	1.5
input r_1	1.6	2.0	0.3
chiral/continuum extrap	3.1	3.1	1.4
finite volume	0.3	0.9	0.9
total error	3.7	4.1	2.3

Preliminary error budget for the decay constants and their ratio. Uncertainties are quoted as a percentage. The total combines systematic errors with statistical errors in quadrature.

Preliminary result for $|V_{us}|$

$f_K/f_\pi = 1.185(18)(20)$ where the first error is statistical and the second is systematic. The MILC value is $f_K/f_\pi = 1.197(3)(^{+6}_{-13})$.

Following Marciano [PRL 93 (2004) 231803] and MILC [PR D71 (2005) 034504, arXiv:0710.1118] we can obtain $|V_{us}|$ from the kaon leptonic branching fraction [KLOE Collaboration, Phys. Lett. B 632, 76 (2006)],

$|V_{us}| = 0.2269(51)$

This is consistent with other recent determinations of $|V_{us}|$ and with unitarity constraints, given the latest $|V_{ud}|$.

For the future

Investigate alternative fits: SU(2) χPT, SU(3) 2-loop logarithms in order to study convergence of the chiral expansion, and potentially decrease systematic errors.

Move on to B_K. We have B_K data on the same lattices, and the analysis is in progress.