Dominance of Sign-Coherent Geometry in Topological Vacuum and its Consequences

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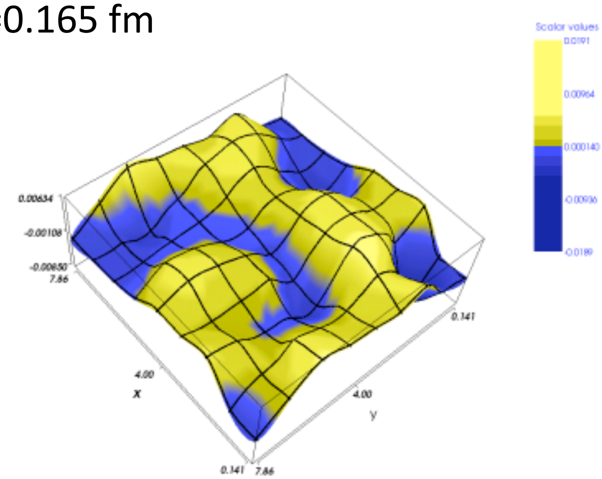
with A. Alexandru and T. Streuer

Fundamental Topological Structure

a=0.165 fm



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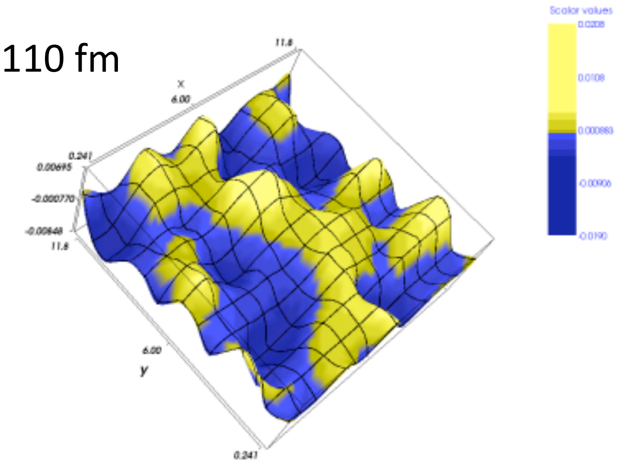

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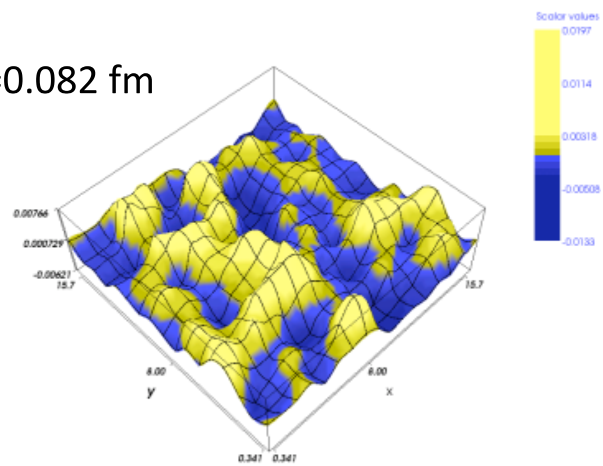

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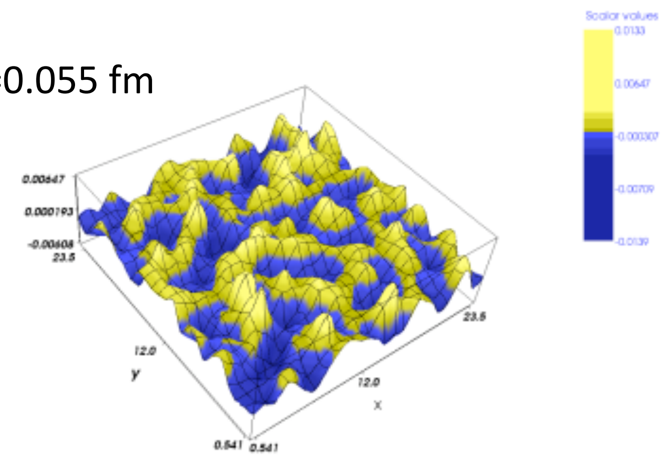


I.H. et al 2002,2003

Fundamental topological Structure II

(i) Low-Dimensional

(ii) Inherently Global

(iii) Space-Filling

(iv) ???????????

- *elevates sign-coherence to dynamical role*
- *elevates geometry to dominating dynamical role*
- *gives fundamental structure an analytic aspect*
- *connects order (geometry) to predictions*

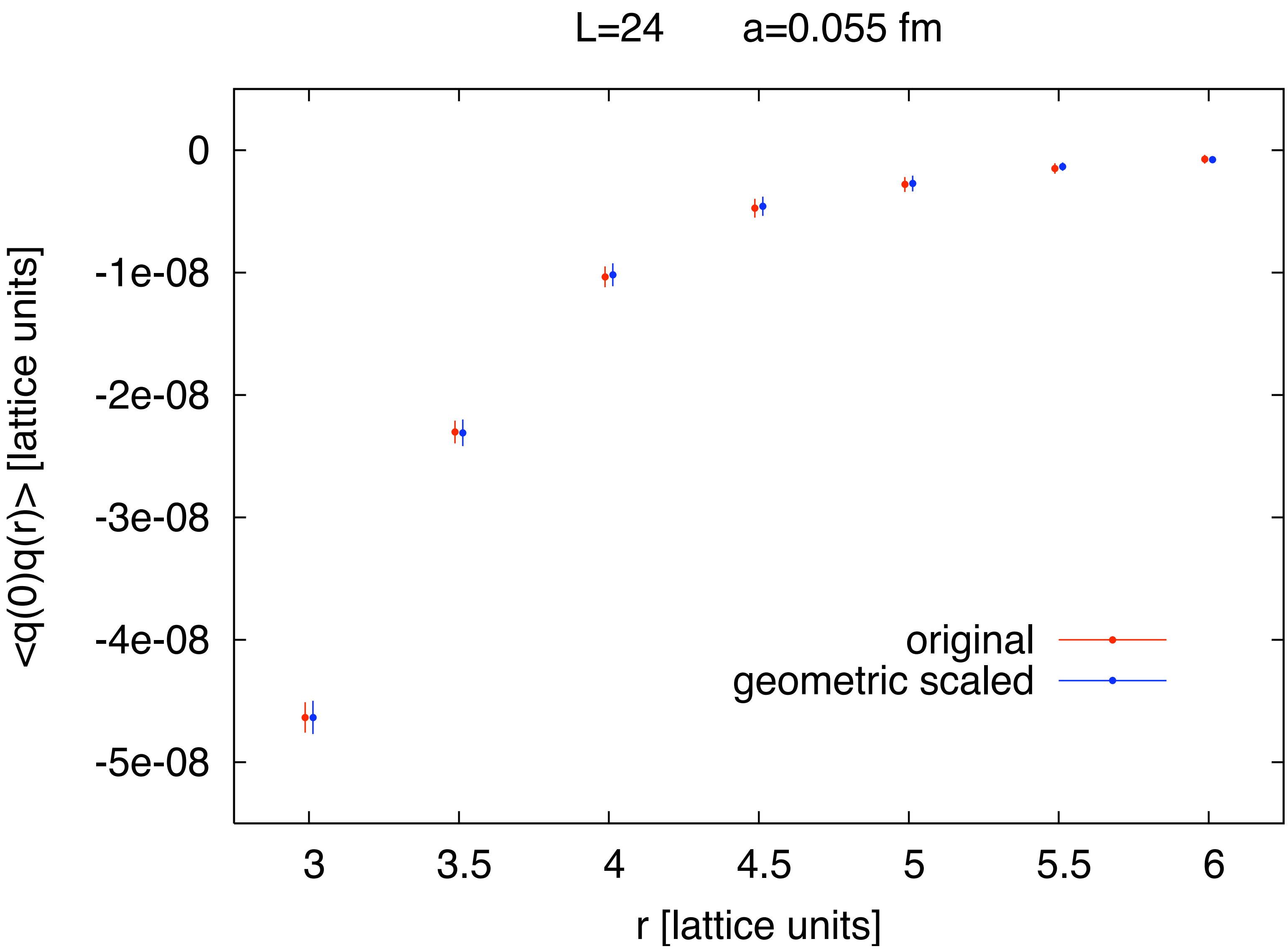
Basic Observation

$$\mathcal{C} \;\longrightarrow\; \bar{\mathcal{C}}$$

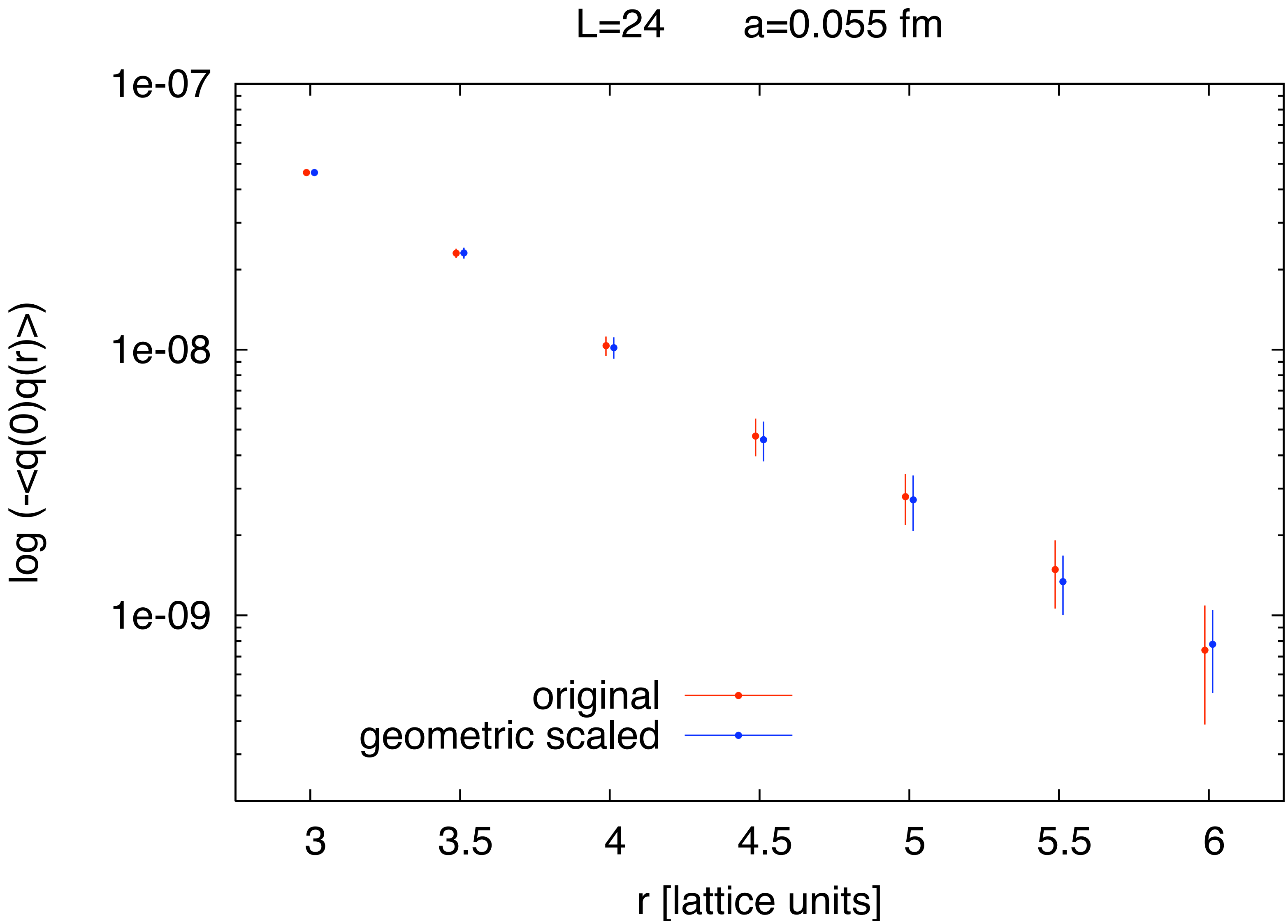
$$\bar{\mathcal{C}} \;\equiv\; \{\; \bar{q}(x) \equiv \mathrm{sgn}\big(q(x)\big),\; q(x) \in \mathcal{C} \;\}$$

Replaces values of topological density with their signs!

Basic Observation II



Basic Observation III



Quantitative Measure (overlap)

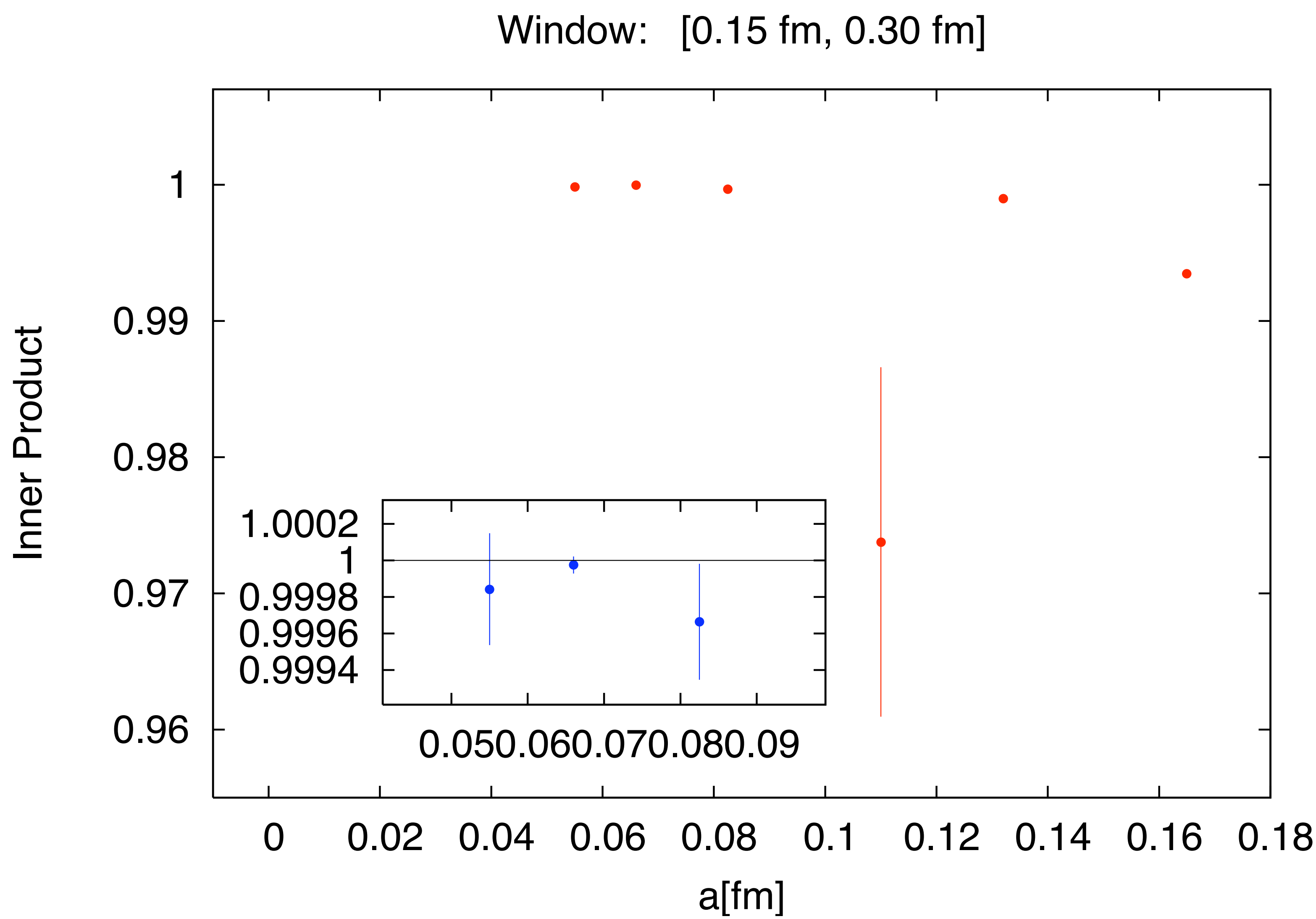
$$f_1 \cdot f_2 \equiv \sum_x f_1(x) f_2(x)$$

$$\mathcal{O}[f_1, f_2] \equiv \frac{f_1 \cdot f_2}{|f_1|\,|f_2|}$$

$$\bar{\mathcal{O}} \equiv \mathcal{O}\left[\,1, \frac{f_2}{f_1}\,\right]$$

"relative overlap"

Quantitative Measure II



Conjecture

There exist computable configuration–based reductions $\mathcal{C} \longrightarrow \bar{\mathcal{C}}$ *with* $\bar{\mathcal{C}}$ *containing homogeneous sign–coherent regions such that*

$$\lim_{a \to 0} \bar{\mathcal{O}}\big[G(r,a), \bar{G}(r,a), \mathcal{I}(r_1^p, r_2^p, a)\big] \;=\; 1$$

for arbitrary $\mathcal{I}(r_1^p, r_2^p, a)$. *Here* G *and* $\bar{G}$ *are lattice two–point functions of the original and the reduced ensemble respectively,* r_1^p, r_2^p *are physical distances and*

$$\mathcal{I}(r_1^p, r_2^p, a) \;\equiv\; \Big[\, \frac{r_1^p}{a}, \, \frac{r_2^p}{a} \,\Big]$$

Fundamental topological Structure II

(i) Low-Dimensional

(ii) Inherently Global

(iii) Space-Filling

(iv) Homogeneous

- *elevates sign-coherence to dynamical role*
- *elevates geometry to dominating dynamical role*
- *gives fundamental structure an analytic aspect*
- *connects geometry (order) to predictions*

More on Homogeneity

$$q(x) = \bar{q}\, s(x) + \eta(x) \qquad s \in \{-1, 0, 1\}$$

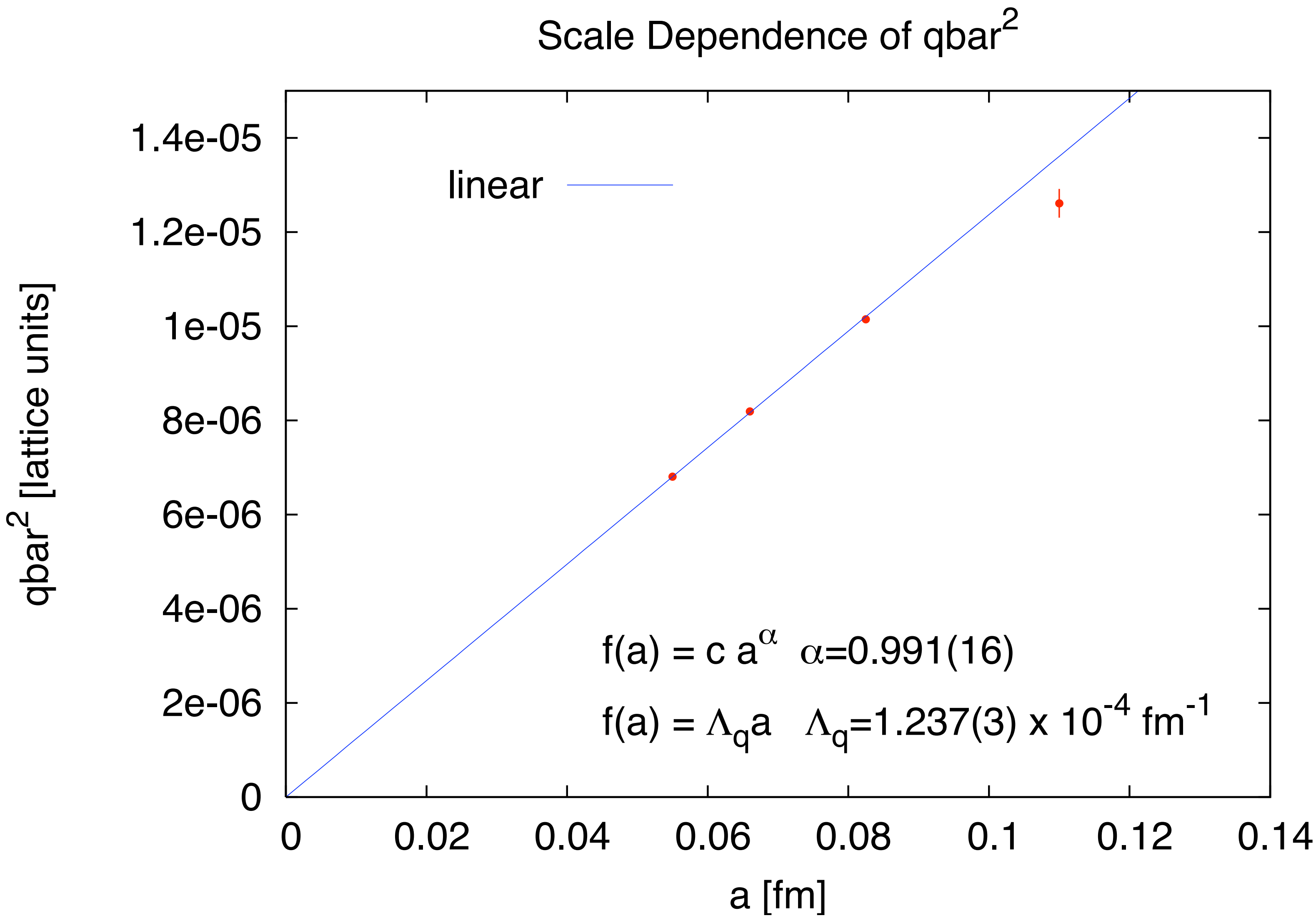
$$\lim_{a \to 0} \langle\, \eta(0)\, \eta(r_p/a)\, \rangle \;=\; 0$$

"unphysical noise"

$$G(r,a) \quad \Leftrightarrow \quad \bar{q}^2(a)\, \bar{G}(r,a)$$

Proportionality constant measures the density of "real stuff" !

Measuring the "Real Stuff"



Scaling of the “Real Stuff”

$$\bar{q}(a) \;\propto\; a^{\frac{1}{2}}$$

“Half—linear” (fractal?) density.

The fabric of topological structure consists of half-linear filaments (skeleton)!

The Relevant Consequence

$$G_p(x_p) = \lim_{a\to 0} \frac{G(x_p/a, a)}{a^8} = \lim_{a\to 0} \frac{\bar{q}^2(a)\bar{G}(x_p/a, a)}{a^8}$$

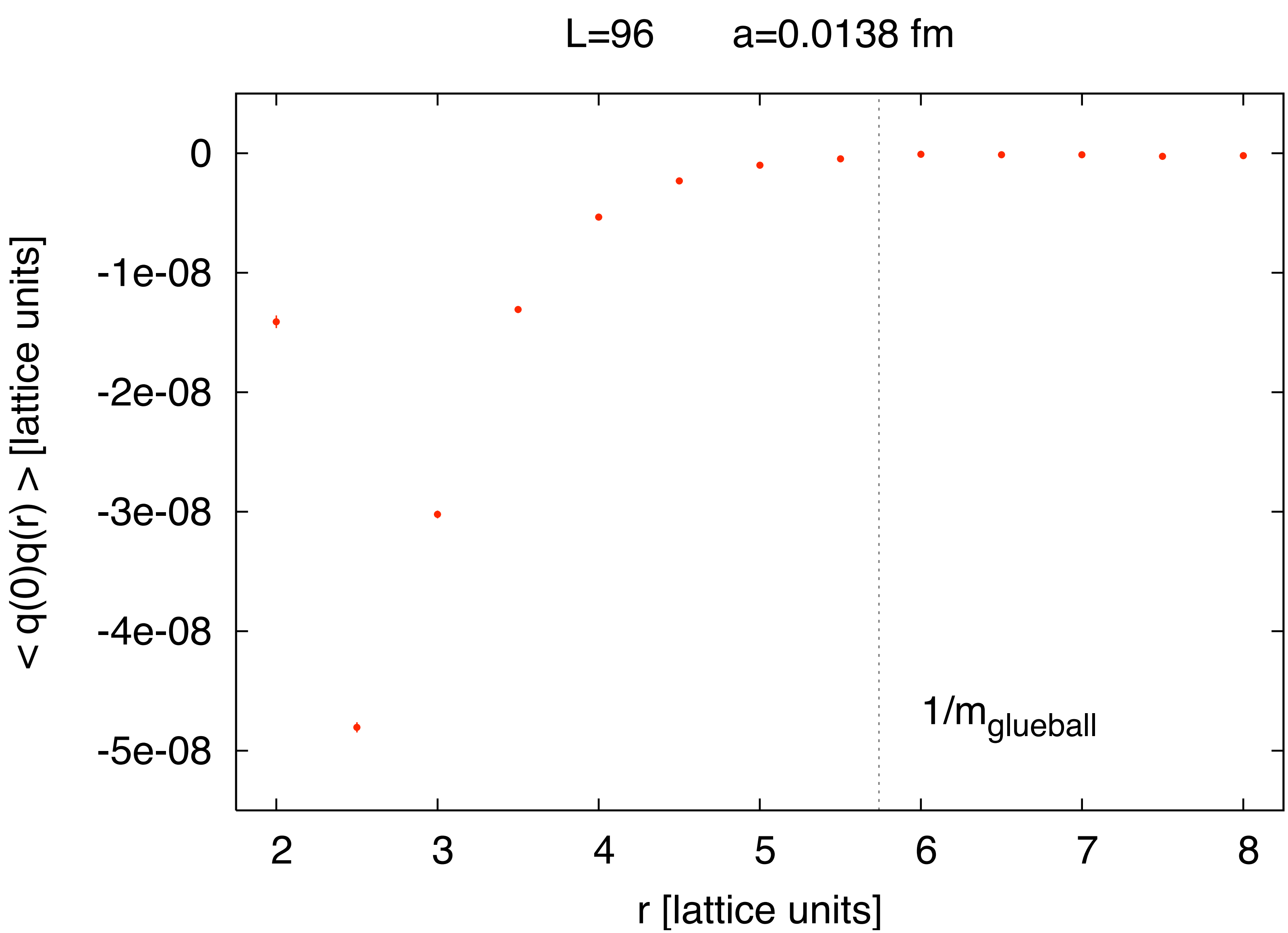
$$= \lim_{a\to 0} \frac{\bar{G}(x_p/a, a)}{a^7} \underset{x_p\to 0}{\longrightarrow} \bar{F}(x_p/a, a)\, \frac{a^{2d}}{{x_p}^{2d}}\, \frac{1}{a^7}$$

$$\Longrightarrow \quad d = \frac{7}{2}$$

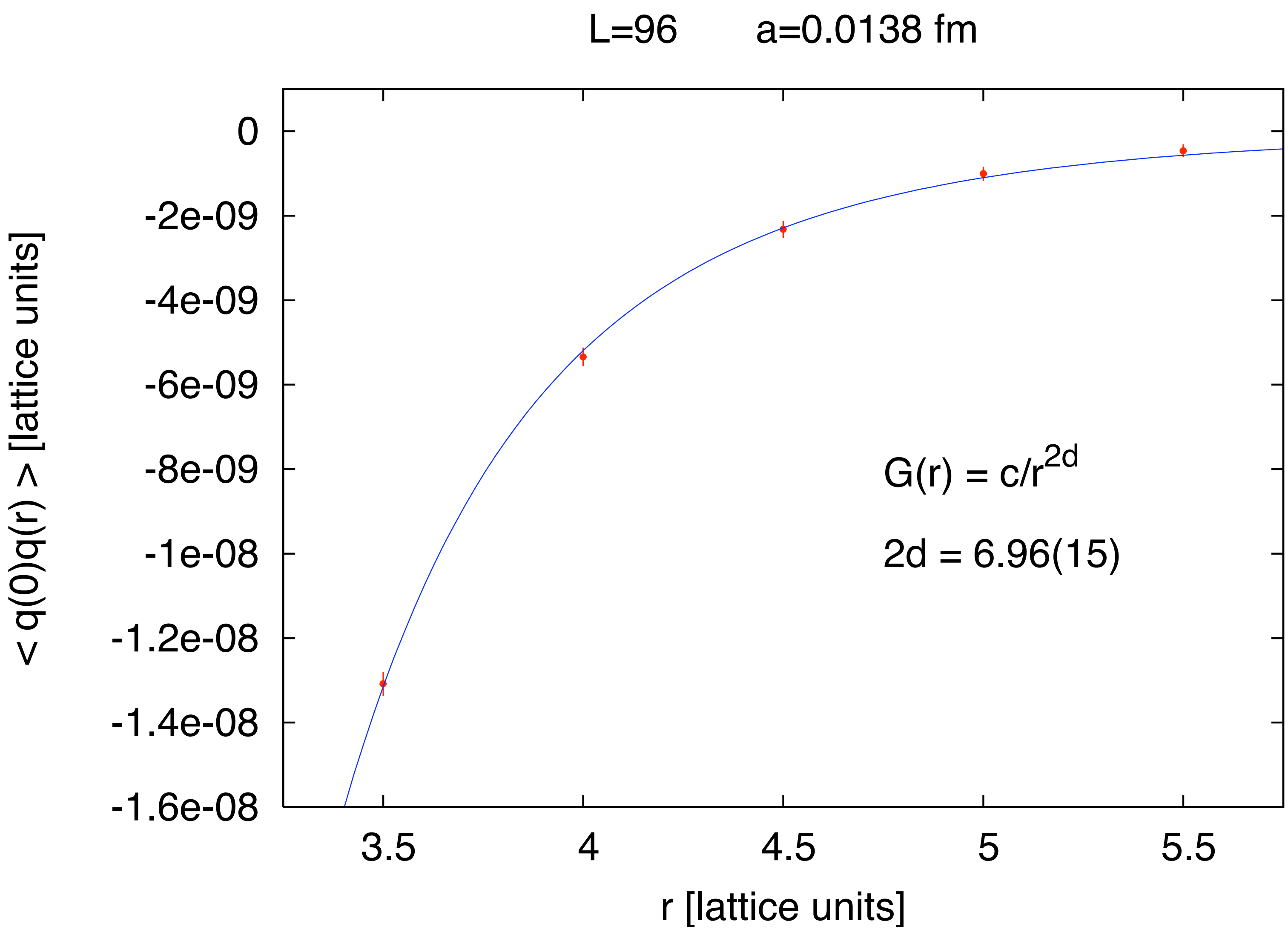
Relevant Consequence II

Does topological density have anomalous dimension?

L=96 Test


L=96   a=0.0138 fm
0
-1e-08
-2e-08
-3e-08
-4e-08
-5e-08
< q(0)q(r) > [lattice units]
2
3
4
5
6
7
8
r [lattice units]
1/m_glueball

L=96 Test II


L=96 a=0.0138 fm
0
-2e-09
-4e-09
-6e-09
-8e-09
-1e-08
-1.2e-08
-1.4e-08
-1.6e-08
< q(0)q(r) > [lattice units]
3.5
4
4.5
5
5.5
r [lattice units]
G(r) = c/r2d
2d = 6.96(15)

Relevant Consequence III

We should be open to the possibility that anomalous dimension for q(x) is there for real!!!

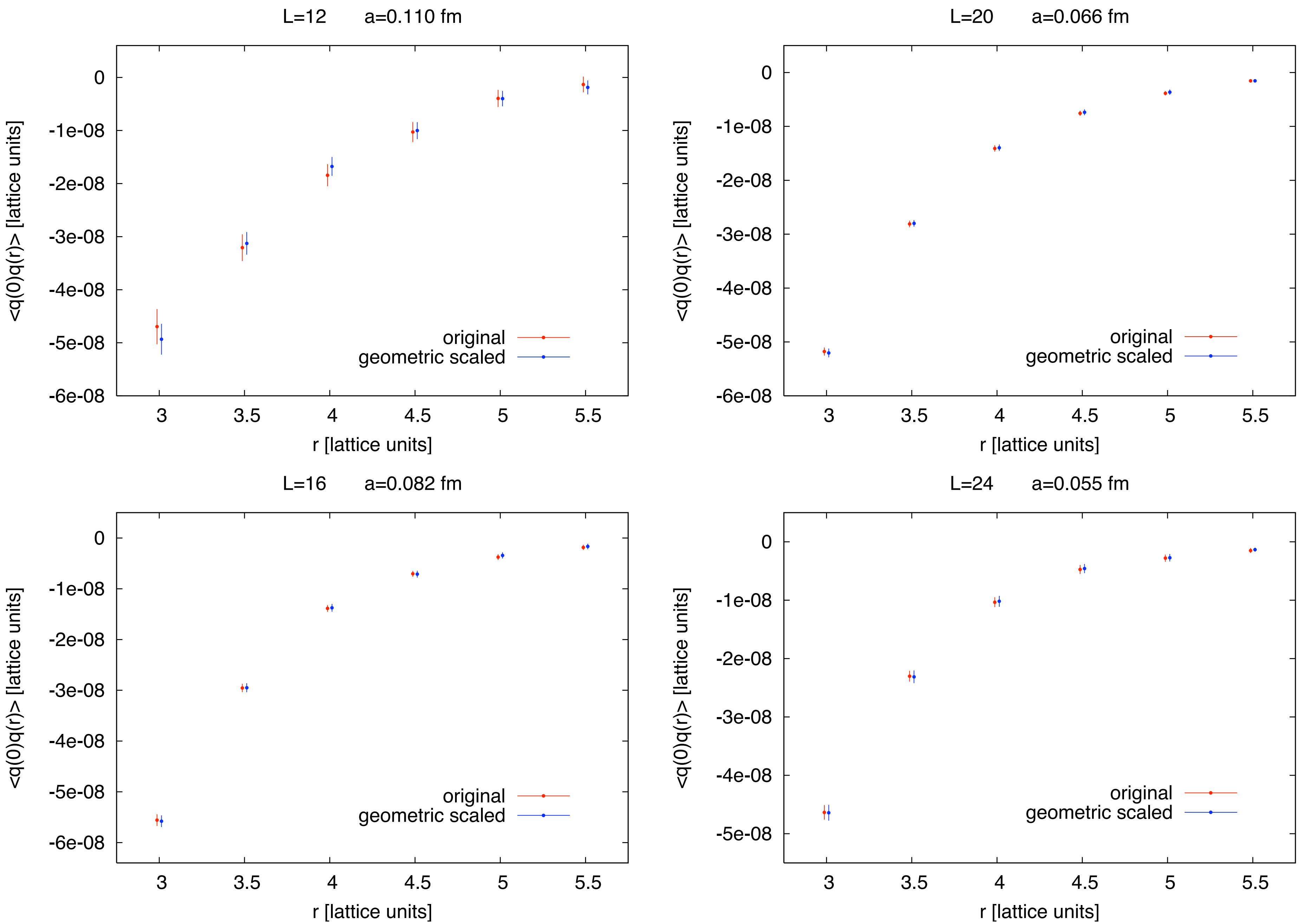
- questions about exact nature of asymptotic freedom

- Θ is a dimensionfull parameter

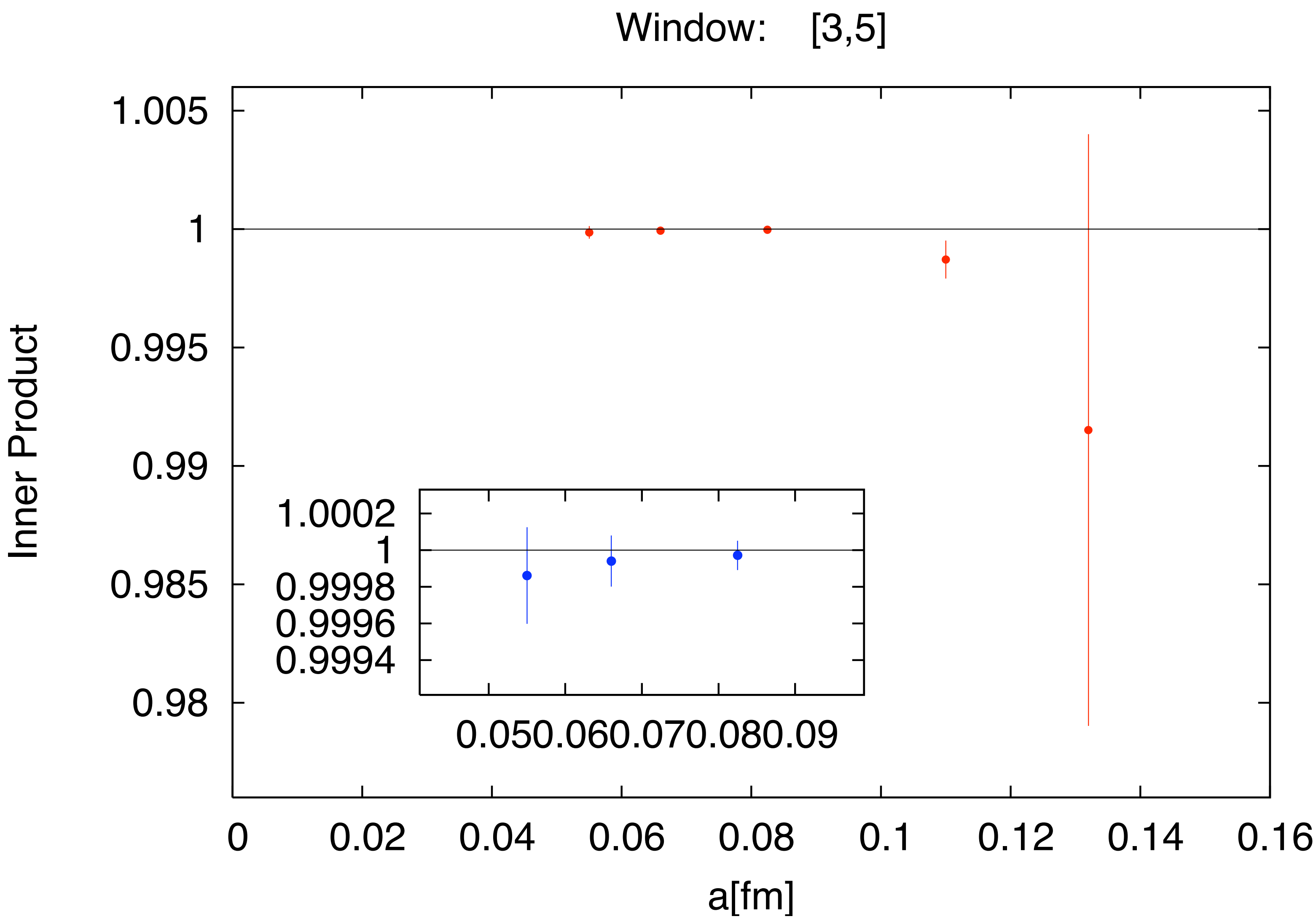
- strong CP problem is not really a problem, etc

Lesson

This is an example of how thinking in terms of fundamental structure can be useful!!!


L=12 a=0.110 fm
L=20 a=0.066 fm
L=16 a=0.082 fm
L=24 a=0.055 fm
<q(0)q(r)> [lattice units]
r [lattice units]
original
geometric scaled
0
-1e-08
-2e-08
-3e-08
-4e-08
-5e-08
-6e-08
3
3.5
4
4.5
5
5.5

Quantitative Measure III



Quantitative Measure IV