RHMC simulation of two-dimensional N=(2,2) super Yang-Mills with exact supersymmetry

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Introduction

Simulation of Super Yang-Mills with dynamical fermion

Application is now available

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Rational Hybrid Monte Carlo(RHMC) for two-dim. $N = (2, 2)$ SYM

- model (Sugino model) and simulation detail
- application: observing dynamical SUSY breaking

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- topological twist $\Rightarrow$ scalar Q on a site
- simulation (check of the formulation)

Catterall, Suzuki, Fukaya-Kanamori-Suzuki-Takimi

Sugino model

Sugino, JHEP 01(2004)067

target: 2-dim $\mathcal{N} = (2,2)$ SYM

nilpotent Q (Twisted) SUSY Algebra, continuum

$$Q^2 = \delta_{\phi}^{(\text{gauge})} \quad Q_0^2 = \delta_{\overline{\phi}}^{(\text{gauge})} \quad \{Q, Q_0\} = 2i\partial_0 + 2\delta_{A_0}^{(\text{gauge})}$$

Q-exact Lagrangian (continuum)

$$\mathcal{L} = Q(\dots) = \frac{1}{g^2}\,\mathrm{tr}\Big\{\frac{1}{4}F_{01}^2 + D_\mu\phi D_\mu\overline{\phi} + \frac{1}{4}[\phi, \overline{\phi}]^2 + i\psi_\mu D_\mu \eta + \cdots - \frac{1}{4}\eta[\phi, \eta] + \cdots\Big\}$$

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nilpotent Q Lattice version

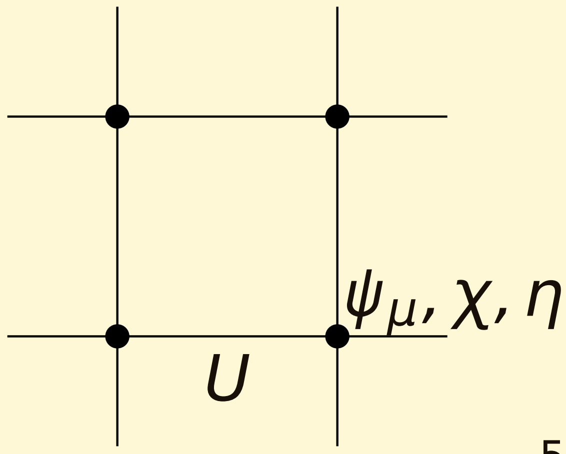
$$Q^2 = \delta_\phi^{(\text{gauge})}$$

Q-exact Lagrangian (lattice)

$$\mathcal{L} = Q(\ldots) = \mathcal{L}[U(x,\mu), \phi(x), \overline{\phi}(x), H(x) \qquad \text{bosons}$$
$$\eta(x), \chi(x), \psi_0(x), \psi_1(x)] \qquad \text{fermions}$$

$$QU(x,\mu) = i\psi_\mu(x)U(x,\mu)$$
$$Q\psi_\mu(x) = i\psi_\mu(x)\psi_\mu(x)$$
$$- i(\phi(x) - U(x,\mu)\phi(x+\hat{\mu})U(x,\mu)^{-1})$$
$$Q\phi = 0$$
$$\vdots$$

fermion: staggered like, taste $\Rightarrow \mathcal{N} = 2$



Lattice Action ($SU(N_C)$)

$$S = Q\frac{1}{a^2g^2}\sum_x \mathrm{tr}\Big[\chi(x)H(x) + \frac{1}{4}\eta(x)[\phi(x),\overline{\phi}(x)] - i\chi(x)\hat{\Phi}(x)$$

$$+\, i\sum_{\mu=0,1}\Big\{\psi_\mu(x)\Big(\overline{\phi}(x) - U(x,\mu)\overline{\phi}(x+a\hat{\mu})U(x,\mu)^{-1}\Big)\Big\}\Big]$$

$$= \frac{1}{a^2g^2}\sum_x \mathrm{tr}\Big[\frac{1}{4}\hat{\Phi}_{\mathrm{TL}}(x)^2 + \ldots \qquad \Big]$$

$$i\hat{\Phi}(x) = \frac{U(x,0,1) - U(x,0,1)^{-1}}{1 - \frac{1}{\epsilon^2}||1 - U(x,0,1)||^2} \sim 2iF_{01}$$

with $||1 - U(x,0,1)|| < \epsilon$

To suppress lattice artifact “vacua”, we need:

$0 < \epsilon < 2\sqrt{2}$ for $N_C = 2, 3, 4$

$0 < \epsilon < 2\sqrt{N_C}\sin(\pi/N_C)$ for $N_C > 5$

Algorithm & Parameters

- RHMC

$$\int Df \exp(-S_{\text{fermion}}) = \text{Pf}(D) = \int DF \exp(-F^\dagger (D^\dagger D)^{-1/4} F)$$

$$= \int DF \exp\left(-F^\dagger \left[a_0 + \sum_{i=1}^{n} \frac{a_i}{D^\dagger D + b_i}\right] F\right)$$

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- lattice size: 3×6 – 30×10

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- anti-periodic/periodic boundary condition in time direction
- Computer: RIKEN Super Combined Cluster(RSCC)

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- The correct target thoery? [in progress, with H. Suzuki]
  supercharges: exact Q + the remainings
  Q_0, Q_1, $\tilde{Q}$: check of the restoration

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  Motivation in physics

  - not broken in tree level $\Rightarrow$ not broken in all orders
  - Witten index: *Not* available in this system

  [Maybe broken? Hori-Tong]

Method for observing ~~SUSY~~

Kanamori-Suzuki-Sugino Phys.Rev.D77 091502(2008)

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Requirement for the lattice model:

- Nilpotent Q
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Advantage

- 1 point function, easy to measure

How to make Hamiltonian?

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$\mathcal{H} = 0$: SUSY $\mathcal{H} > 0$: ~~SUSY~~

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$\Downarrow$

Use the algebra! $\{Q, Q_0\} = 2i\partial_0$

$Q\mathcal{J}_0^{(0)} = 2\mathcal{H}$

Discretized "Noether current" for Q_0:

$$\mathcal{J}_0^{(0)}(x) = \frac{1}{a^4 g^2} \mathrm{tr}\Big\{ \eta(x)[\phi(x), \overline{\phi}(x)]^2 + 2\chi(x)H(x)$$

$$- 2i\psi_0(x)\big(\overline{\phi}(x) - U(x,0)\overline{\phi}(x+a\hat{0})U(x,0)^{-1}\big)$$

$$+ 2i\psi_1(x)\big(\overline{\phi}(x) - U(x,1)\overline{\phi}(x+a\hat{1})U(x,1)^{-1}\big)\Big\}$$

Anti-periodic boundary condition

The conjugate applied field: temperature $Z = \operatorname{tr} e^{-\beta H}$

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- ~~SUSY~~ by the boundary condition(or temperature)
- We need $\beta \to \infty$

We should measure

the ground state energy:

$$\mathcal{E}_0 \equiv \lim_{\beta\to\infty} \langle \mathcal{H} \rangle_{\alpha\mathrm{PBC}} = \lim_{\beta\to\infty} \langle Q \mathcal{J}_0^{(0)}/2 \rangle_{\alpha\mathrm{PBC}} \begin{cases} = 0 & \text{SUSY} \\ > 0 & \cancel{\text{SUSY}} \end{cases}$$

Using

- Nilpotent Q
- Q-exact action (= Q-invariant)

Example: Supersymmetric Quantum Mechanics

(known): form of the potential $\Rightarrow$ broken or not

lattice SQM: a model by Catterall

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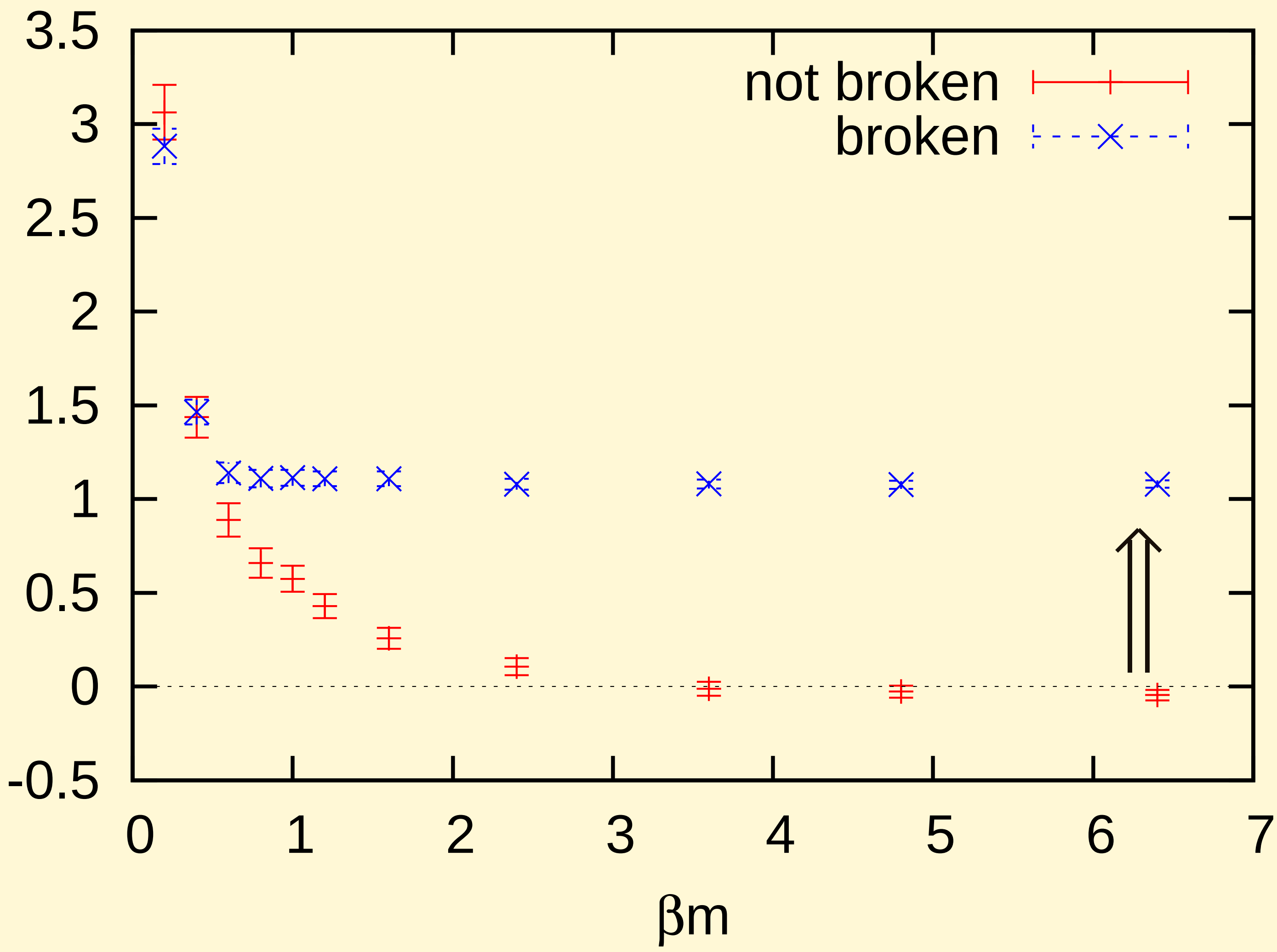
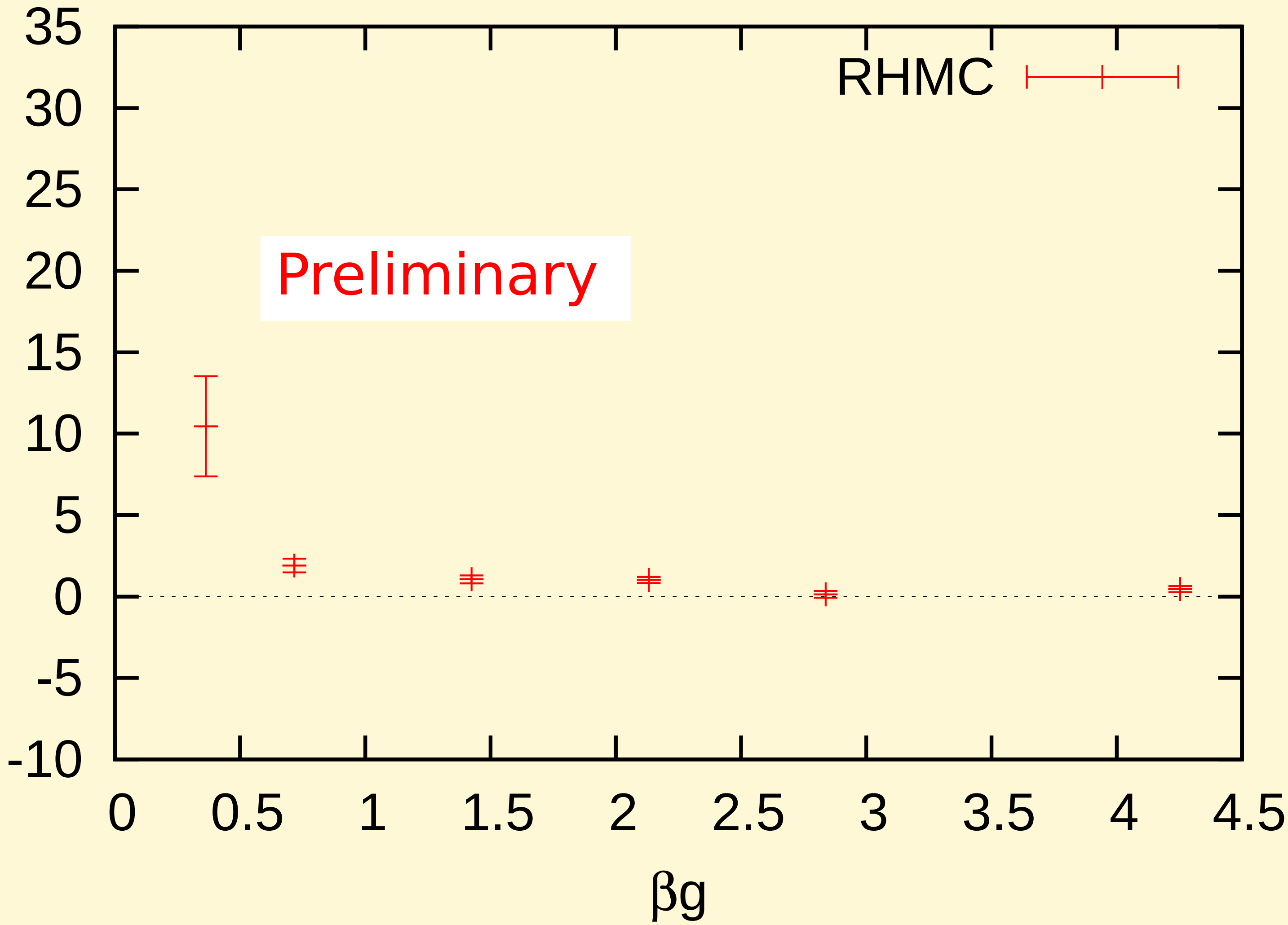


Result for 2-dim $\mathcal{N} = (2, 2)$ SYM

$SU(2)$, ground state energy seems small



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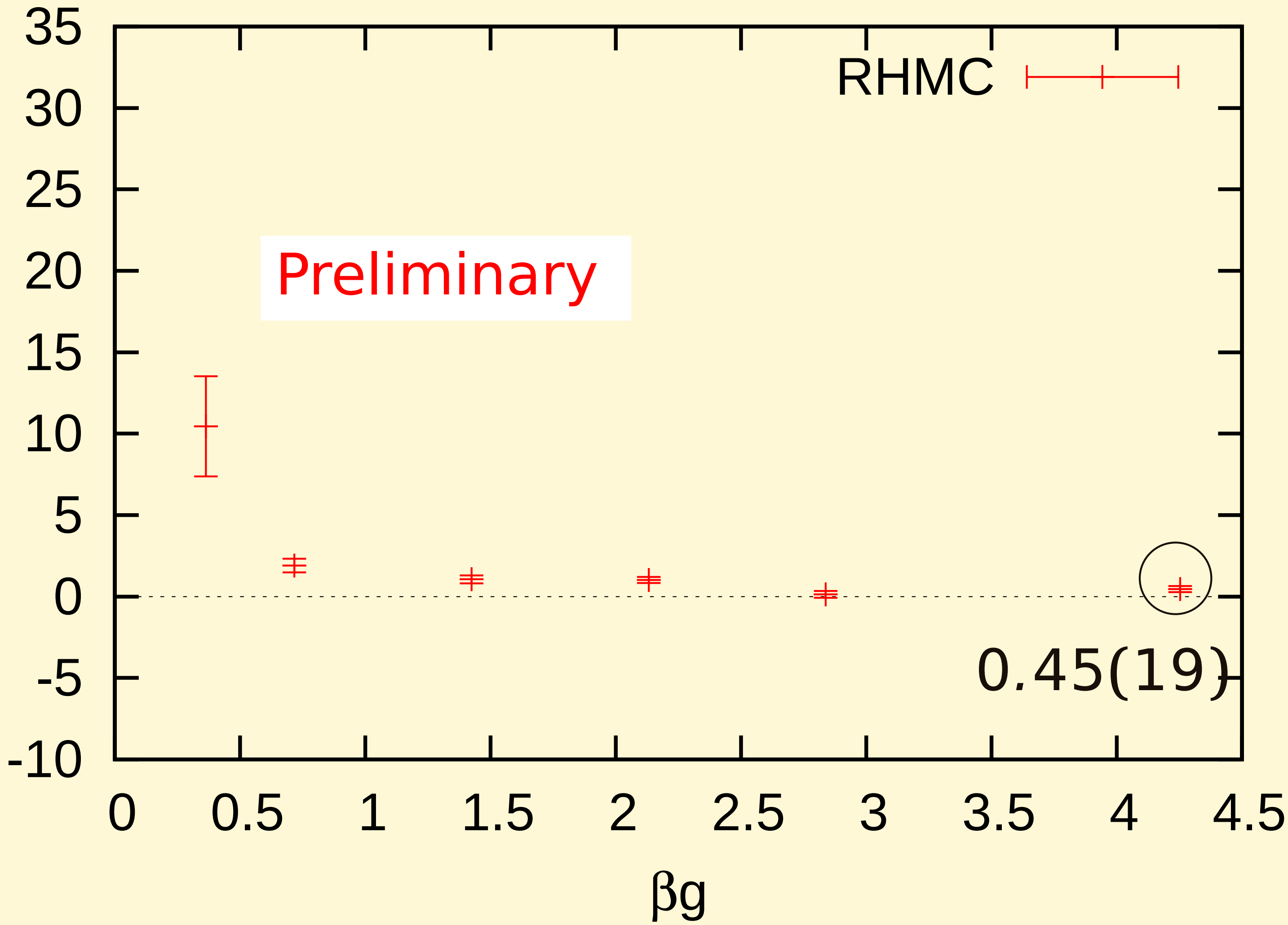
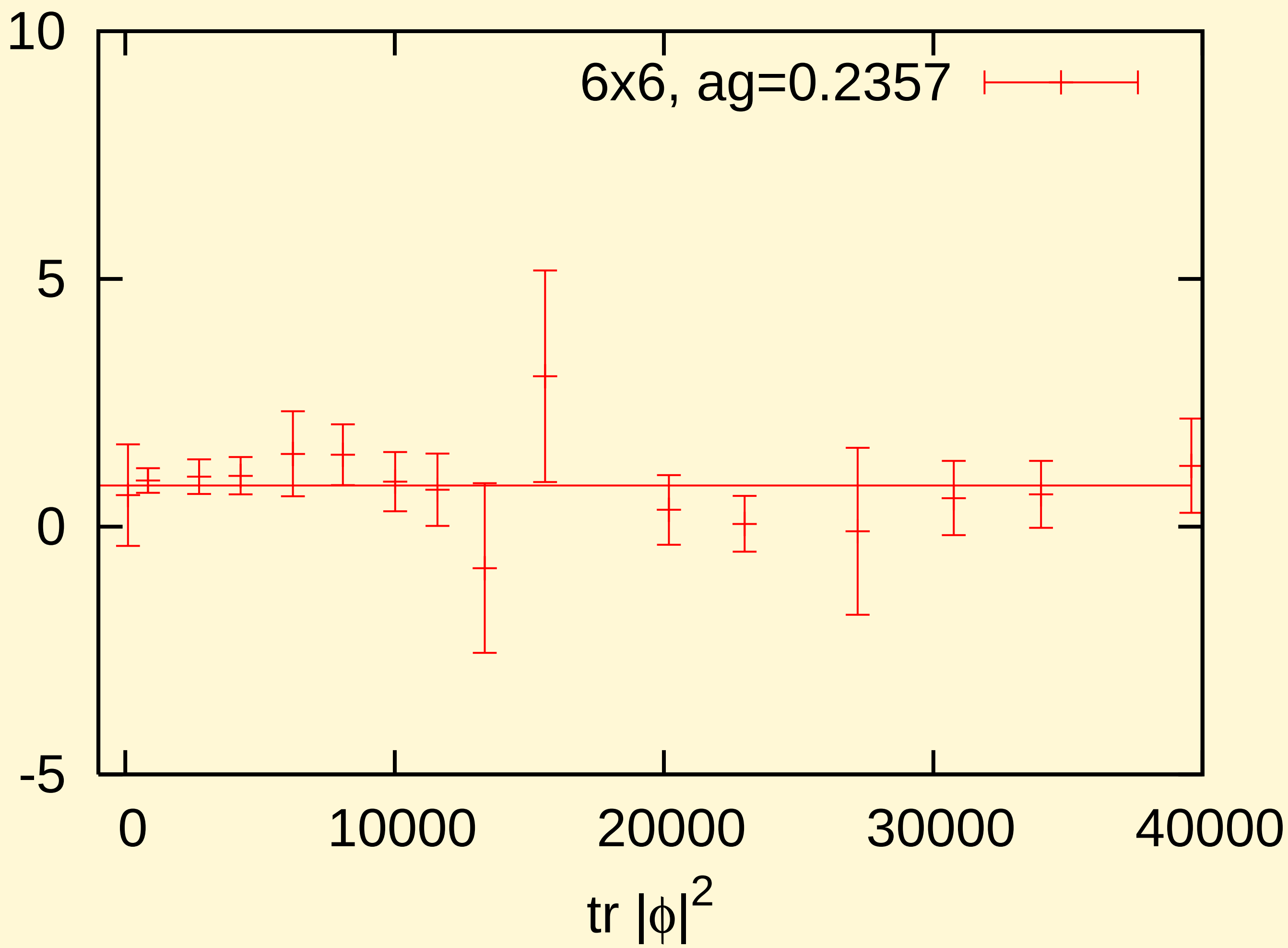


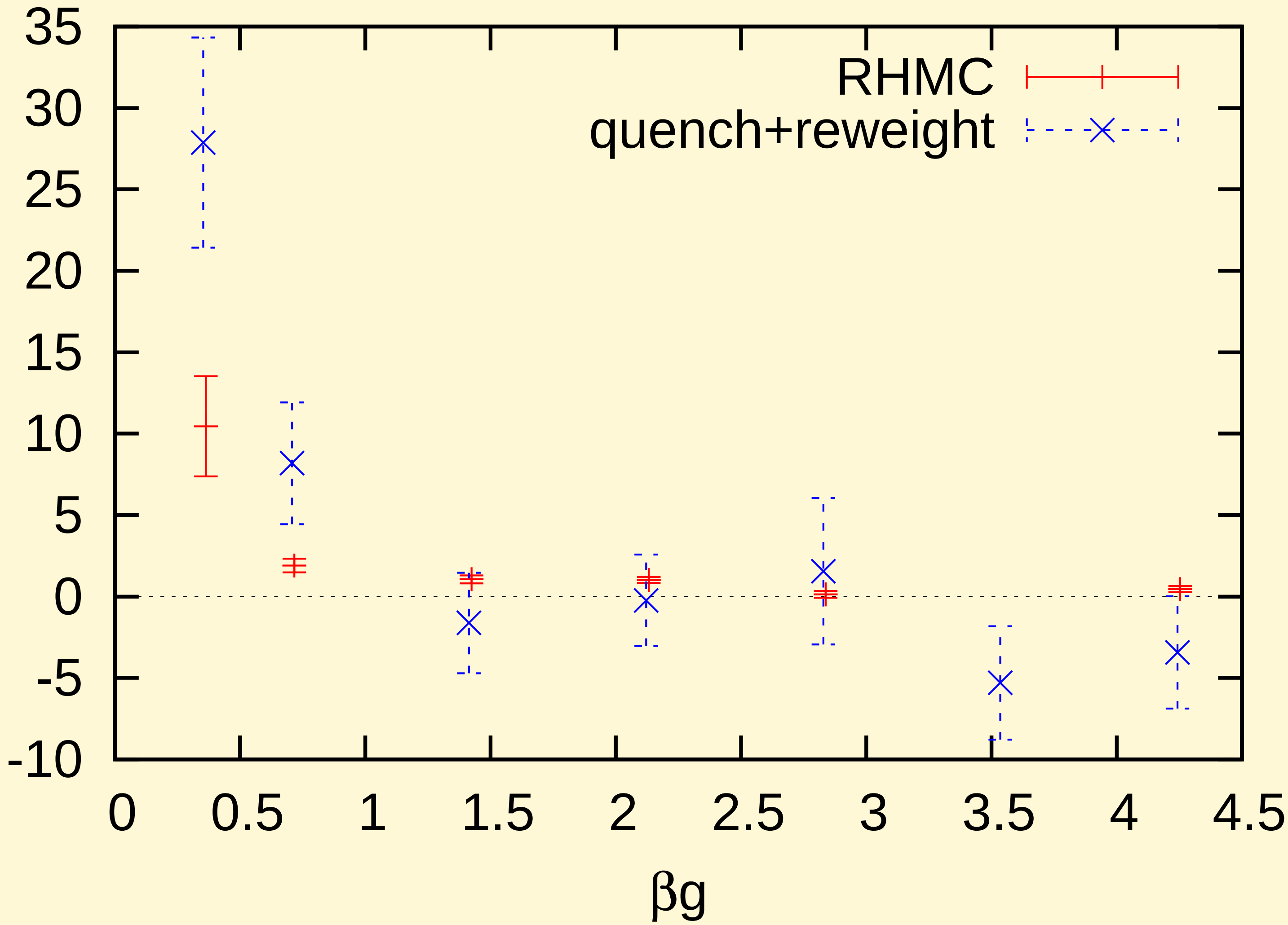
No dependence on the flat direction

- flat direction: $[\bar{\phi}, \phi] = 0$
- Hamiltonian does not depend on $\mathrm{tr}\,|\phi|^2$



Result for 2-dim $\mathcal{N} = (2,2)$ SYM

$SU(2)$, compare with quench + reweighting



Conclusion

Simulation of Super Yang-Mills with dynamical fermion

Application is now available

RHMC for two-dim. $N = (2,2)$ SYM (Sugino Model)

- model and simulation detail
- application:
  - observing dynamical SUSY breaking
    measurement of the ground state energy
  - [in progress]
    check of the correct target theory in the continuum,
    we can check it

Thank you.

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Under the periodic condition: simulation does NOT work

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$$(\text{Witten index}) = Z_{\text{PBC}} = (\operatorname{tr} e^{-\beta H})_{\text{PBC}}$$

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	SUSY	~~SUSY~~
correct $\langle H \rangle_{\text{PBC}}$	$= 0$	> 0
simulation of "$\langle H \rangle_{\text{PBC}}$"	$= 0$	$= 0$

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Our hamiltonian: $\langle Q\text{-exact} \rangle_{\text{PBC}} = 0 \propto -\frac{\partial}{\partial\beta}(\text{Witten index})$