Lattice 2008

# Matching the Bare and $\overline{MS}$ Charm Quark Mass using Weak Coupling Simulations

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# Outline

- $m_c$ and HIS-Quarks
- High $\beta$
- Calculating $c_2$
  - ASQTAD
  - HISQ
- Conclusions, $m_c$ and Outlook

# $m_c$

- Quark Masses + $\alpha_s$ + SU(3) = QCD
- Quark masses needed to evaluate some important Matrix Elements
- Confinement complicates extraction must look at hadronic quantities → Lattice
- Extraction for heavy quarks ($m_c$ in particular) has been difficult, we want to address this...

**Highly Improved Staggered Quarks on the Lattice, with Applications to Charm Physics.**

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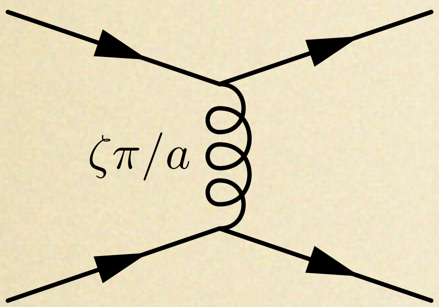
(HPQCD, UKQCD)




$$\mathcal{F}_\mu \equiv \prod_{\rho\neq\mu}\left(1+\frac{a^2\delta_\rho^{(2)}}{4}\right) - \sum_{\rho\neq\mu}\frac{a^2(\delta_\rho)^2}{4} \qquad \mathcal{F}_\mu^{HISQ} = \mathcal{F}_\mu\mathcal{U}\mathcal{F}_\mu$$

- AsqTad: Remove all tree level $\mathcal{O}(a^2)$ errors
  - Remaining $\alpha_S a^2$ errors still too large even for typical $am_c$
- HISQ: Taste changing errors further suppressed by factor of 3 or so + dispersion relation corrected by Naik coeff.

$$\psi - \eta_c = 111(5)\,\text{MeV} \qquad m_c(m_c) = 1.269(9)\,\text{GeV}$$

$$f_D = 207(4)\,\text{MeV} \qquad f_{D_s} = 241(3)\,\text{MeV}$$

# Extracting a physical $m_c$

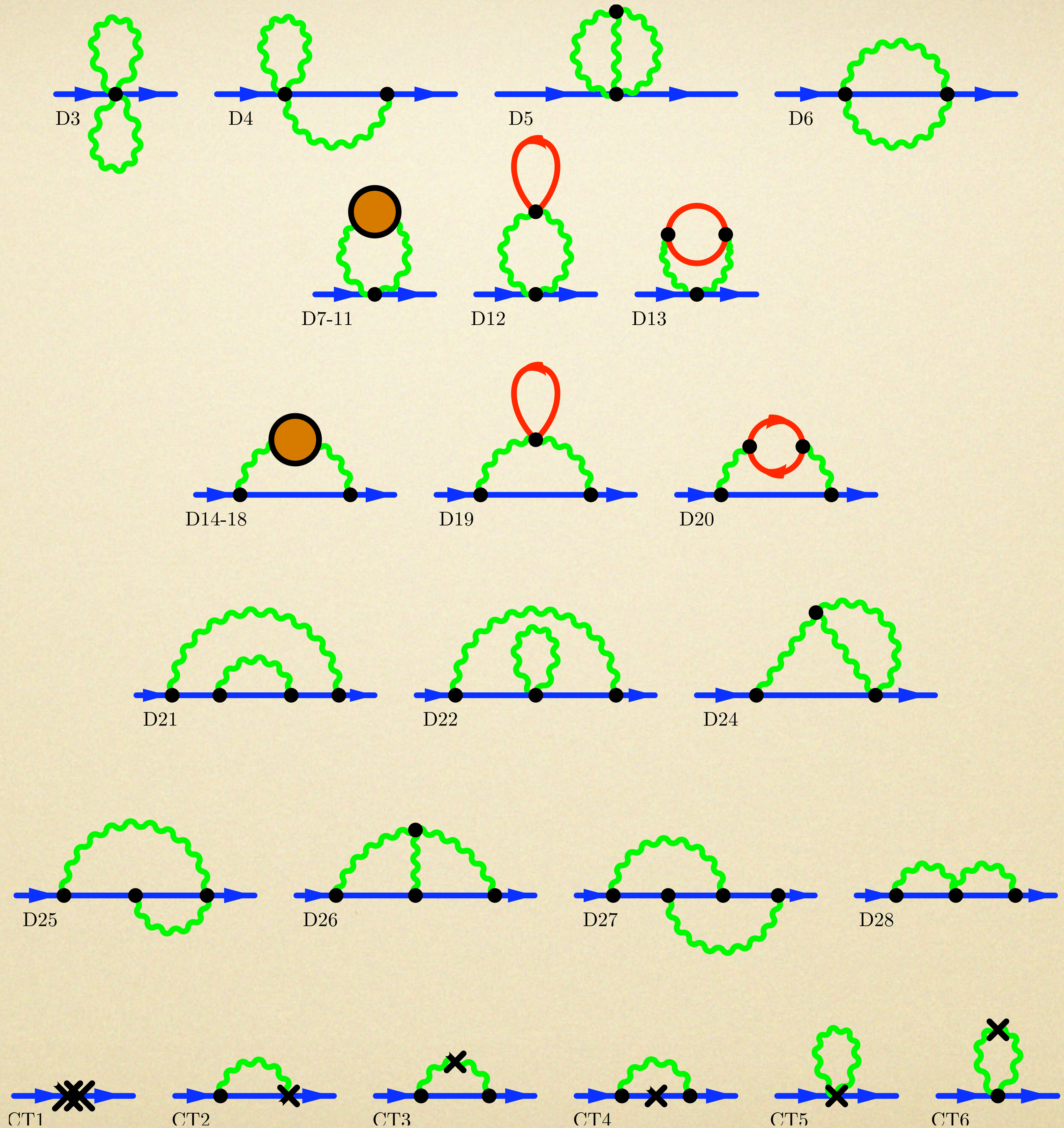
First calculate a lattice $m_c$

$$m_c = \frac{m_c a}{m_{\eta_c} a} \times m_{\eta_c}^{expt}$$ - 4 MILC ensembles

Then match to the $\overline{MS}$ scheme using Pert. Th.

- We want **two loop improvement** for the matching, this is a lot of work, complicated discretisations → complicated P.T.
- We break the problem into gluonic and fermionic pieces, doing $c_{2,q}$ "by hand"

$$M_{pole} = m_q \left(1 + c_1(m_q a)\alpha_V + (c_{2,g}(m_q a) + c_{2,q}(m_q a))\alpha_V^2\right)$$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CT5
CT6

# High $\beta$

- We control the strength of the coupling in simulation
- High Beta = small coupling ➡ large $\pi/a$ so we can **probe the perturbative regime in simulation!**
- Successfully applied extracting $\alpha_s$ from Wilson loops, e.g. the 1x2 wilson loop has expansion

$$-\frac{1}{6}\log W_{1\times 2} = \sum_{n}^{N} c_n \alpha_V^n(q^*_{1\times 2})$$

| | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| Diag. | $1.2039(0)$ | $-1.437(1)$ | $-0.11(9)$ |
| MC | $1.2039(4)$ | $-1.480(28)$ | $-0.28(45)$ |
| MC+ | $--$ | $1.480(10)$ | $-0.28(25)$ |
| MC++ | $--$ | $--$ | $-0.17(10)$ |

- Works excellently as a complement to diagrammatic P.T. ➡ constrain as much as possible diagrammatically then use high beta for the rest.

# The Caveats...

- Not quite as simple as just running simulation code with beta at large values
- Have to worry about the infra-red:
  - Zero Modes
  - Vacuum tunneling, need the correct vacuum
- **Twisted boundary conditions** allow us to avoid zero mode and prevent tunneling between the various $Z_3$ vacuua

# Matching $m_c$ at high-$\beta$

- Evaluate HISQ quark propagators in Coulomb + Axial Gauge at different beta

- Compute an expansion in the strong coupling and fit the coefficients, $c_1$, $c_2$ $\alpha_V(q^*)$

  - Constrained Curve Fitting is *crucial*, we use the free field result a constraint

- We can calculate $q^*$ for each input mass but we need $\alpha_V(q^*)$ ...

Use the three loop expansion of $\log(W_{11})$

$$-\log W_{11} = 3.068393\alpha_V(1 - 0.775\alpha_V - 0.768\alpha_V^2)$$

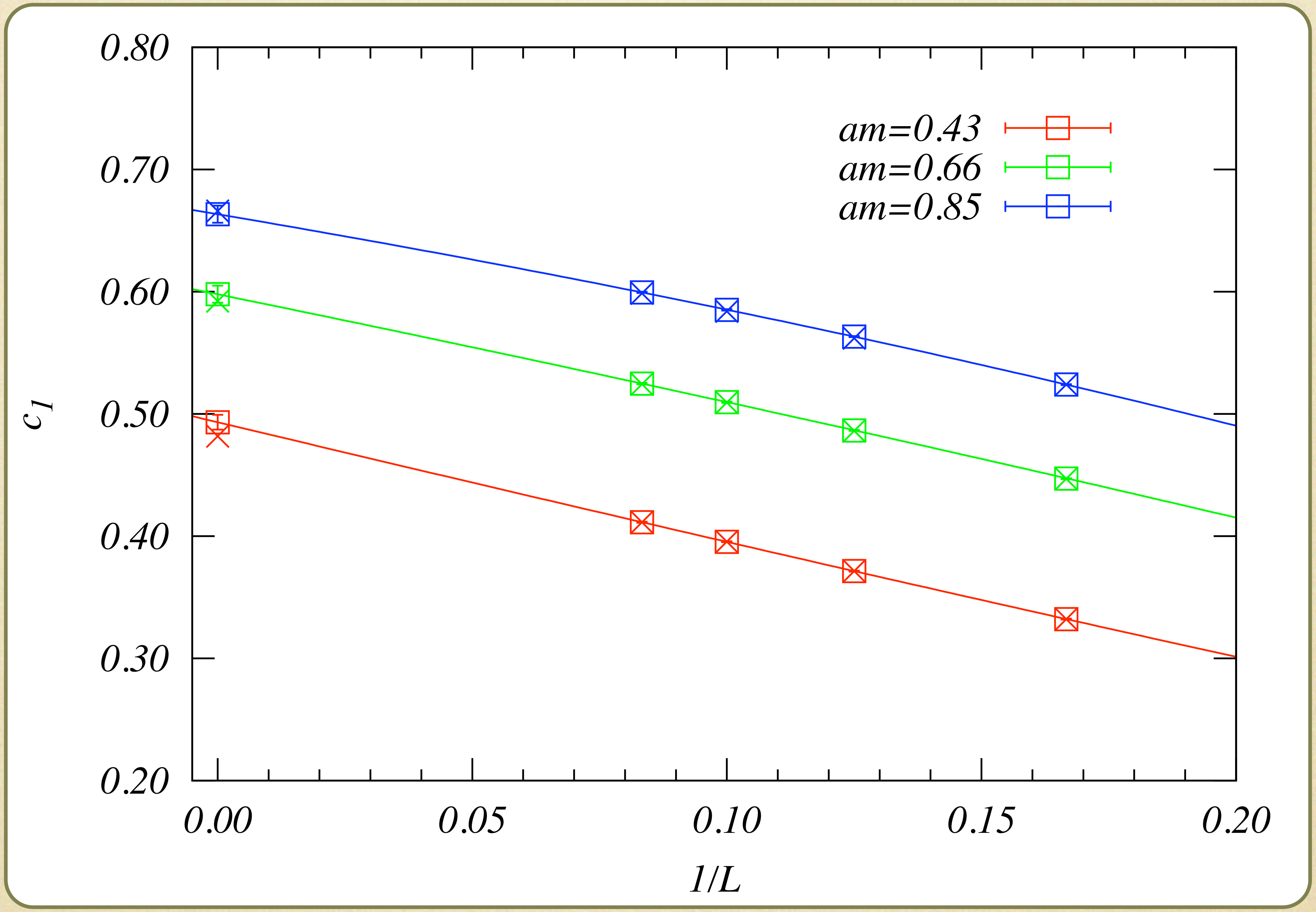
This gives us $\alpha_V(q^*_{1\times 1})$ which we use to solve

$$\alpha(q) = \frac{4\pi}{\beta_0 \tilde{q}}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\log\tilde{q}}{\tilde{q}} + \frac{\beta_1^2}{\beta_0^4\tilde{q}^2}\left(\left(\tilde{q} - \frac{1}{2}\right)^2 + \frac{\beta_2^V\beta_0}{\beta_1^2} - \frac{5}{4}\right)\right]$$

This gives the scale $\Lambda$ from $\tilde{q} = \log(q^2/\Lambda^2)$
and back substitution then gives us $\alpha_V(q^*_m)$

→ Start extracting $c_1$ and $c_2$, $c_1$ first...

- We can check the validity of high beta by comparing to diagrammatic P.T. for $c_1$




$$c_1(L) = c_1(L_\infty) + \frac{X_{c_1,1}}{L} + \frac{X_{c_1,2}}{L^2} + \cdots$$

- To get $c_2$, constrain $c_1$, then take $c_2$ infinite volume

$$c_1(L) = c_1 - X_1 \frac{1}{L} + \cdots$$

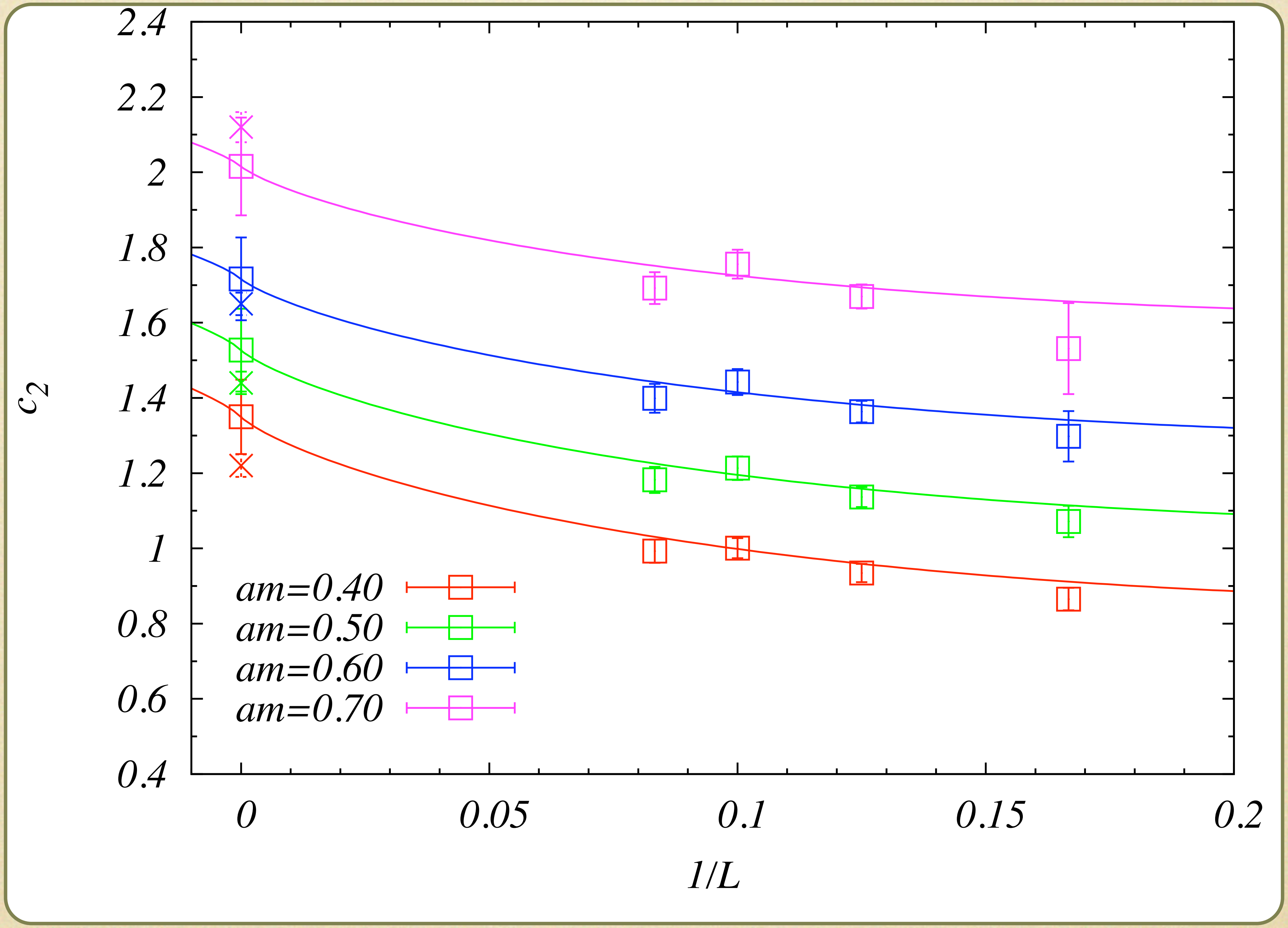
$$am(L) = am - X_1 \frac{\alpha_V(q^*)}{L} + \cdots$$

$$c_2(L) = c_2(L_\infty) + \frac{1}{L}(X_{c_2,1} + Y_{c_2,1} \log(L^2)) + \frac{1}{L^2}(X_{c_2,2} + Y_{c_2,2} \log(L^2)) + \cdots$$

$$Y_{c_2,1} = \frac{11}{4\pi} X_{c_1,1}$$

- $X_{c_1,1}$ comes from the $c_1$ infinite volume fits and is vital in constraining the fit form

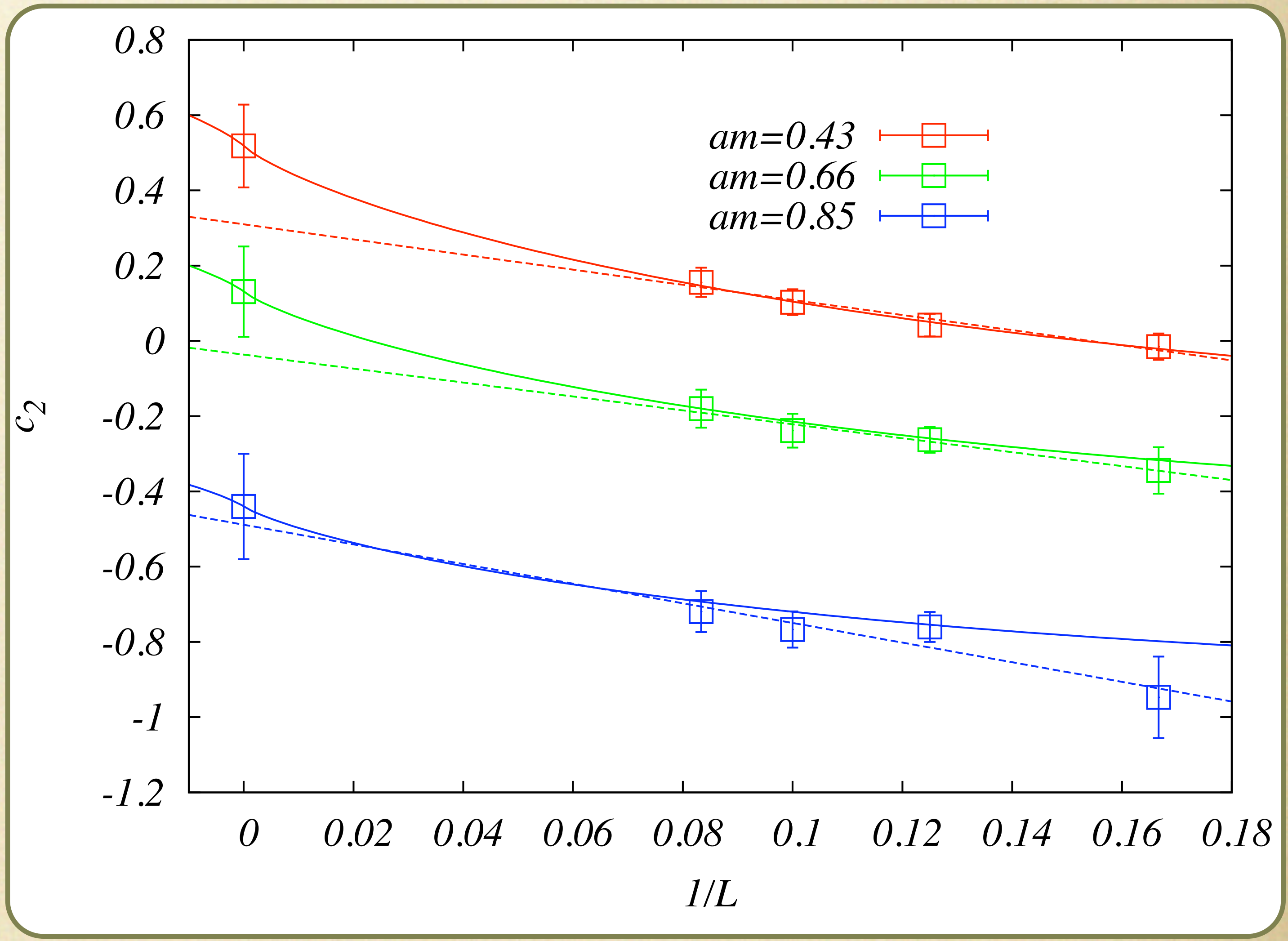
- For AsqTad, gluonic $c_2$ is already know




- **Agrees with diagrammatic P.T.** again, but the logs are needed in the fits

# $c_2$ Infinite Volume - HISQ

- Simple linear extrapolation would give a different answer
- Same analysis for AsqTad showed how important this is




Our High-β result for $c_2$ ...

| mass | 0.30 | 0.43 | 0.50 | 0.66 | 0.85 |
|---|---|---|---|---|---|
| $c_2(L=\infty)$ | 0.29(11) | 0.52(11) | 1.21(11) | 0.13(12) | $-0.44(14)$ |

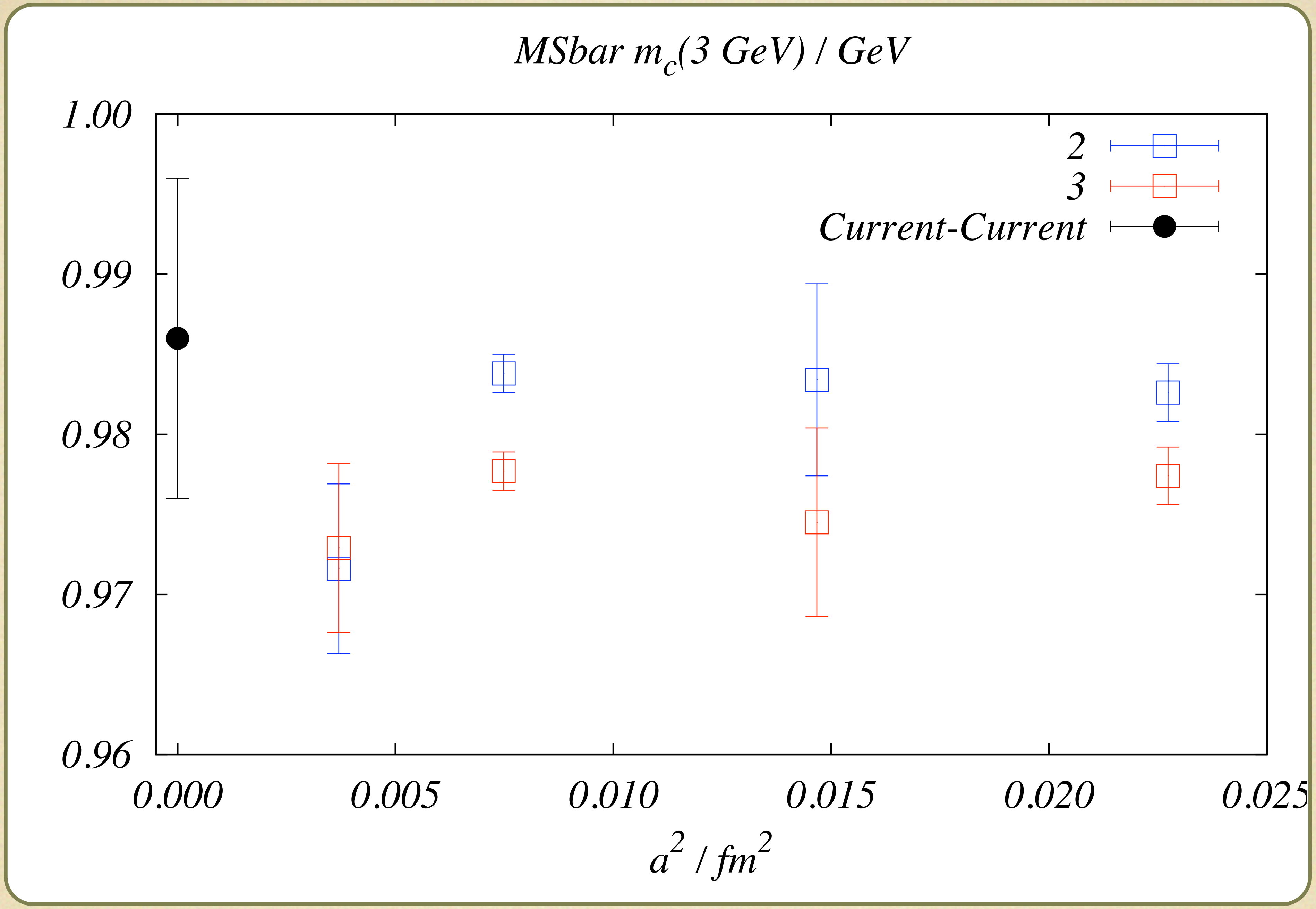
# $m_c$

- We match $m_{latt}$ and $m_{\overline{MS}}$ converting $\alpha_{latt}$ and $\alpha_{\overline{MS}}$ to $\alpha_V$

$$m_{\overline{MS}}(\mu) = Z_m m_{latt} = (1 + Z_1 \alpha_V(aq^*) + Z_2 \alpha_V^2(aq^*) + Z_3 \alpha_V^3(aq^*) + \cdots) m_{latt}$$

$$Z_2 = Z_{22}\bar{l}^2 + Z_{21}\bar{l} + Z_{20} \qquad : \qquad \bar{l} = \log(a\mu)$$

- $c_2$ feeds into the the coefficients $Z_{xx}$ as $A_{X0}(am)$
- Match to $\overline{MS}$ at $\mu = 3$ GeV, which requires calculating a $q^*$ for $\alpha_V$




$$m_c^{\overline{MS}}(\mu = 3\,\mathrm{GeV}) = 0.9830(64)(49)(255)\,\mathrm{GeV}$$

Errors: (fitting)(scale setting)(perturbative matching)

# Conclusions

- High beta works. Especially in combination with traditional P.T.

- We have a second consistent $m_{c}$....

$$\text{This Work:} \quad m^{\overline{MS}}(3\,\text{GeV}) = 0.983(25)\,\text{GeV}$$

$$\text{Current-Current:} \quad m^{\overline{MS}}(3\,\text{GeV}) = 0.988(10)\,\text{GeV}$$

- The same formulation for s and c make $m_c/m_s$ an obvious target for investigation next