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Precision Scale Determination using the Upsilon Spectrum

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The LQCD Scale

- The lattice scale is implicitly set by choice of parameters
  - but **not explicitly known**
- Precise knowledge required to make useful, physical predictions
  - uncertainty in r_1 physical value dominates errors in e.g.
    - f_K, f_π & f_K / f_π [1]
    - m_c [2]
- Use the Upsilon because spectrum is known to be insensitive to heavy quark mass

[1] E. Follana *et al*, HEP-LAT 0706.1726; [2] I. Allison *et al*, HEP-LAT 0805.2999

The NRQCD Action

- due to high b mass, the quark velocities in the Upsilon meson $v^2 << c^2$
  - NRQCD exploits this property

$$G(\mathbf{x}, t+a) = \left(1 - \frac{aH_0}{2n}\right)^n \left(1 - \frac{a\delta H}{2}\right) U_\mu^\dagger(\mathbf{x}, t) \left(1 - \frac{a\delta H}{2}\right) \left(1 - \frac{aH_0}{2n}\right)^n G(\mathbf{x}, t)$$

$$\begin{aligned}
H_0 &= \frac{-\Delta^{(2)}}{2M} \\
\delta H &= -\frac{(\Delta^{(2)})^2}{8M^3} + \frac{ig}{8M^2}(\mathbf{\Delta}^{(\pm)} \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \mathbf{\Delta}^{(\pm)}) \\
&- \frac{g}{8M^2}\sigma \cdot (\mathbf{\Delta}^{(\pm)} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \mathbf{\Delta}^{(\pm)}) \\
&- \frac{g}{2M}\sigma \cdot \tilde{\mathbf{B}} + \frac{a^2\Delta^{(4)}}{24M} - \frac{a(\Delta^{(2)})^2}{16nM^2}
\end{aligned}$$

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Using the Upsilon Spectrum

- Scale can be set by defining
  $$(2S\text{-}1S)_{latt} = a(2S\text{-}1S)_{expr}$$
- Desire ~1% precision in $(2S\text{-}1S)_{latt}$
  - cf *A.Gray et al*, errors ~2%
- We use Random-Wall (or "Stochastic") sources and source smearing
  - computationally cheap way of increasing lattice usage
  - smearing used to improve signal for 2S state (unchanged from *A. Gray et al*):
    $$(2r_0 - r)e^{(-r/2r_0)}$$

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Random Wall

- Assign every spatial point at start time a complex phase
- Smearing function applied repeatedly: centred on every starting spatial point, and multiplied by corresponding phase
- This pairs the quark centred on a point with the anti-quark on the same point
  - contributions from other combinations cancel on average
- Random wall gives an average propagator over all spatial origins for a given temporal origin

Lattice Ensembles

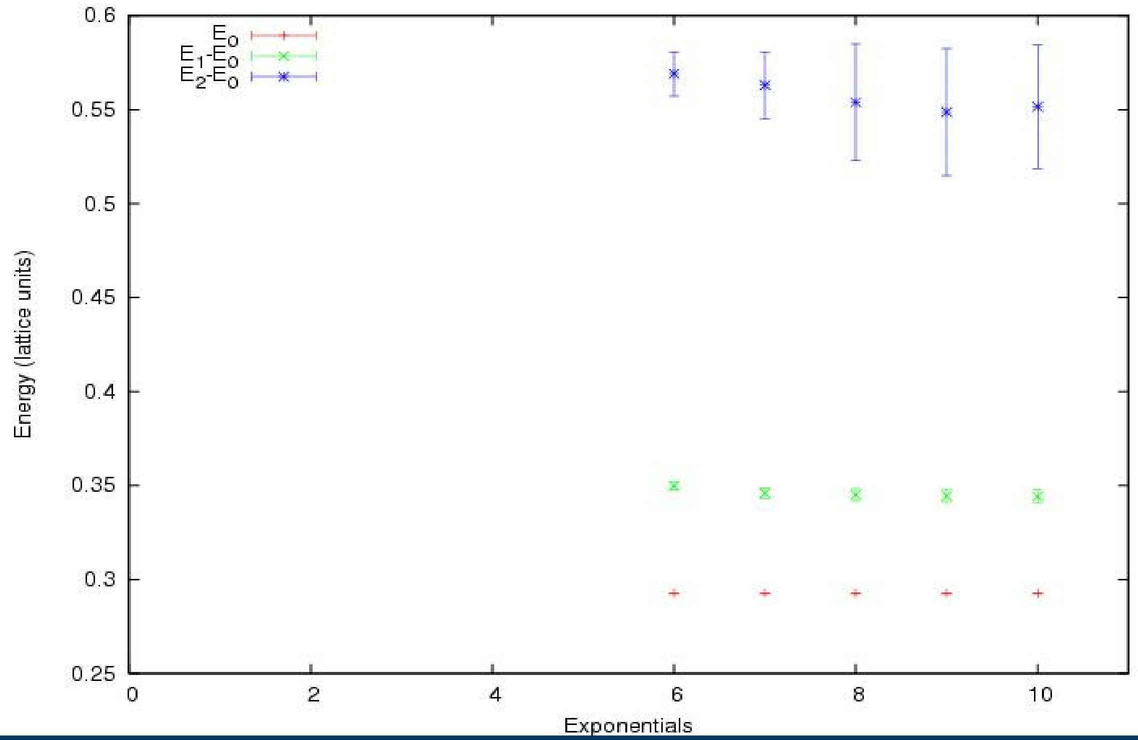
- Ensembles generated by MILC
- 5 used so far
  - plan to use “fine” ensemble between coarse & super-fine

Lattice	Size	n_f	β	am_l, am_s	u_{0L}	aM^0_b	n	Configs	Origins
Very Coarse	16^3 x 48	2+1	6.572	0.0097, 0.0484	0.8218	3.40	4	631	24
	16^3 x 48	2+1	6.586	0.0194, 0.0484	0.8225	3.40	4	631	24
Coarse	20^3 x 64	2+1	6.760	0.010, 0.050	0.8359	2.80	4	595	32
	24^3 x 64	2+1	6.760	0.005, 0.050	0.8362	2.80	4	202	32
Superfine	48^3 x 144	2+1	7.470	0.0036, 0.018	0.8695822	1.34	4	132	8

- Bin over starting temporal origins
  - superfine also binned on pairs of adjacent configurations

Fitting

- Fit using Bayesian method
- Fits should be stable once sufficient exponentials are included in the fit



Results

- PDG: Υ(2S-1S) = 0.56296(40) GeV

Ensemble		E_0	E_1-E_0	a^{-1}
V.Coarse	0097/0484	0.28781(8)	0.4261(40)	1.321(12)
	0194/0484	0.28812(8)	0.4240(42)	1.328(13)
Coarse	010/050	0.292574(59)	0.3443(33)	1.635(16)
	005/050	0.293252(66)	0.3453(36)	1.630(17)
Superfine	0036/018	0.248477(25)	0.1734(34)	3.247(64)

- Random wall gave very accurate ground states
- Excited states do not seem improved by wall, only by our improved statistics

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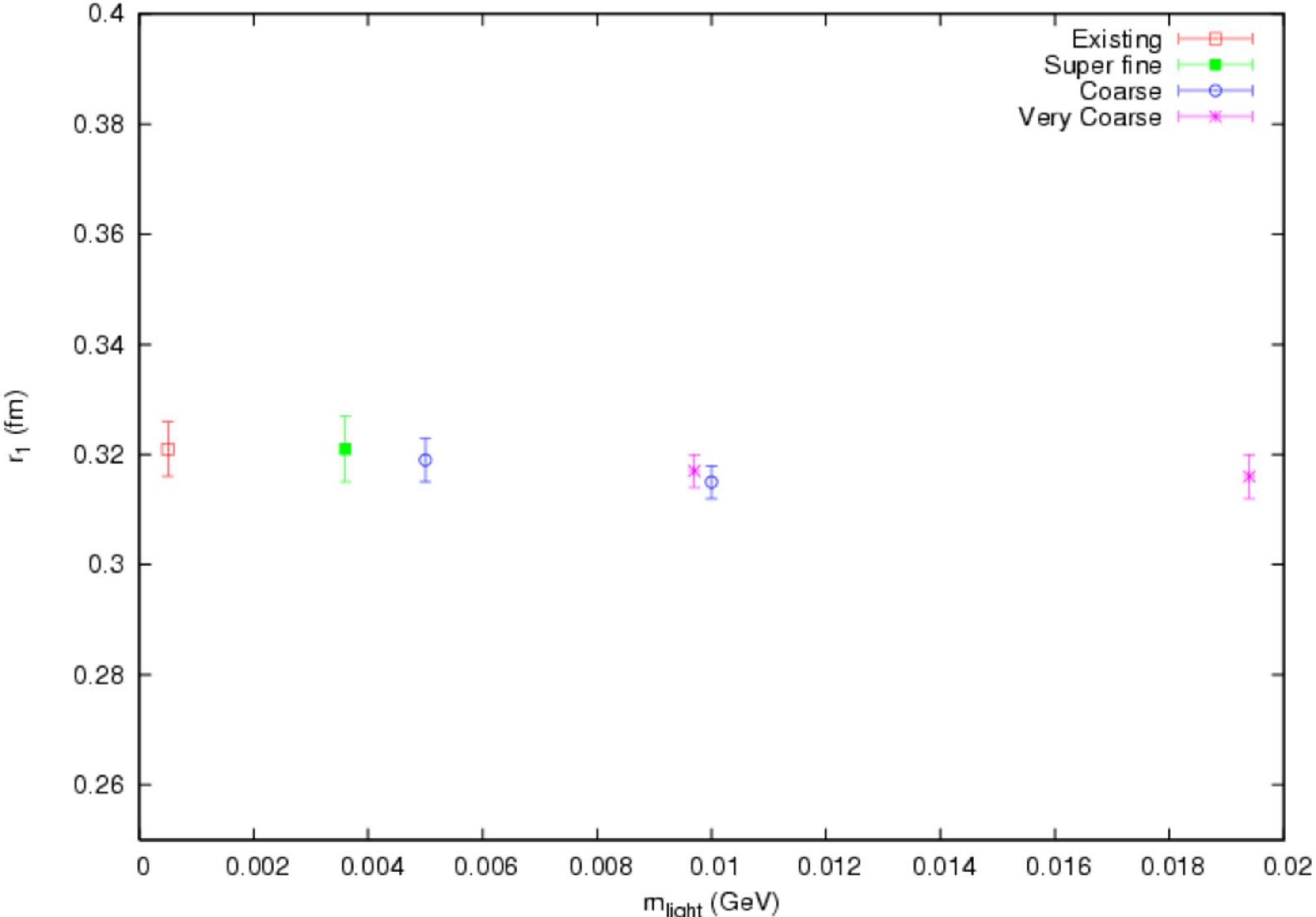
Determination of r_1

- r_1 is defined as the value of *r* at which

$$r^2 F(r) = 1.00,$$

  where *F(r)* is the heavy quark potential gradient
- value of r_1 in lattice units calculated by MILC and converted to physical units via our determination of the Upsilon 2S-1S splitting

Determination of r_1

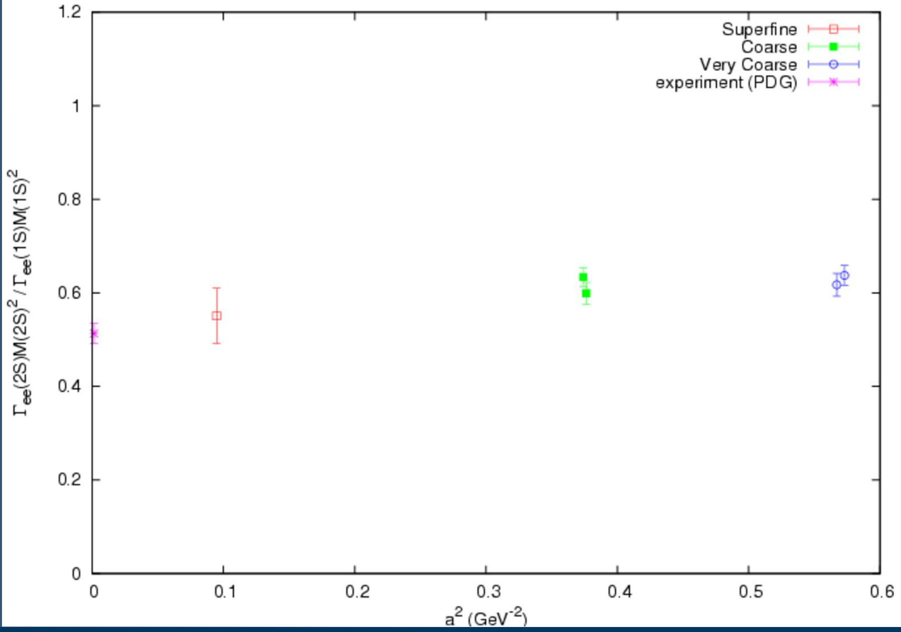


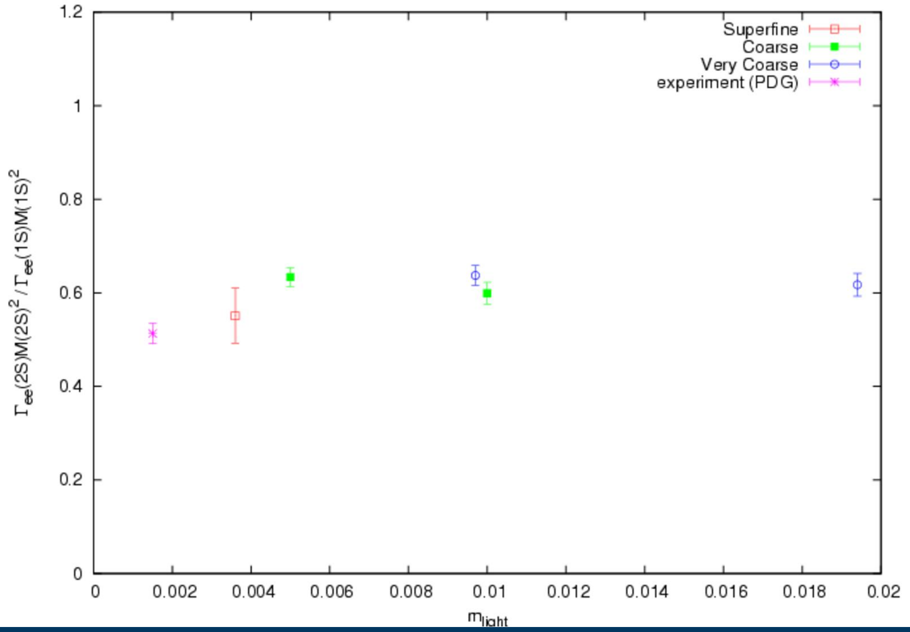
Υ Leptonic Width

- Calculate ratios of leptonic widths to leading order using amplitudes

$$\frac{\Gamma_{ee}(2S)\,M^2_{\Upsilon(2S)}}{\Gamma_{ee}(1S)\,M^2_{\Upsilon(1S)}} = \frac{|\Psi_2(0)|^2}{|\Psi_1(0)|^2}$$

$$where: \Psi_n(0) = a(loc, n)$$





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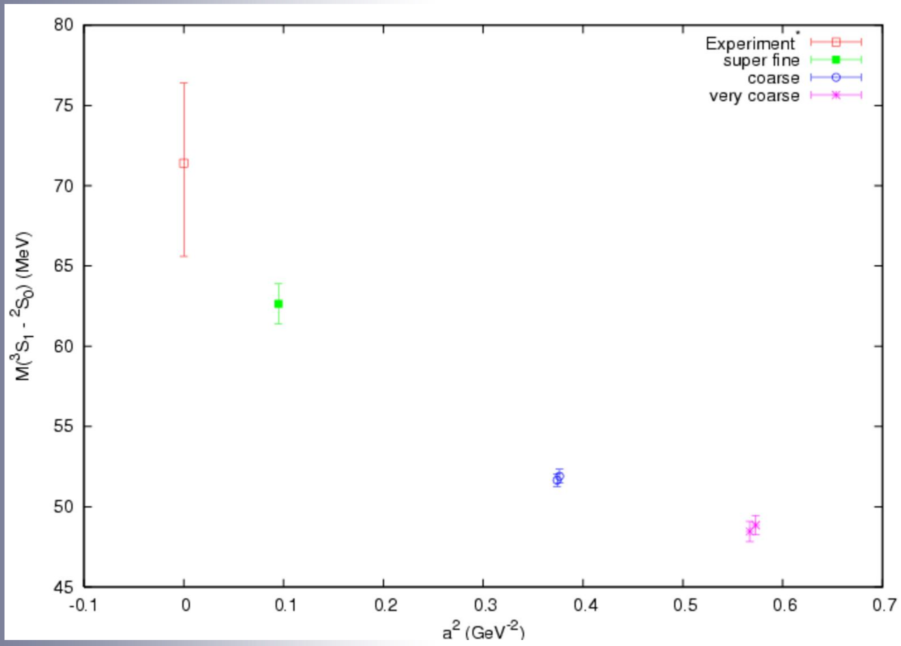
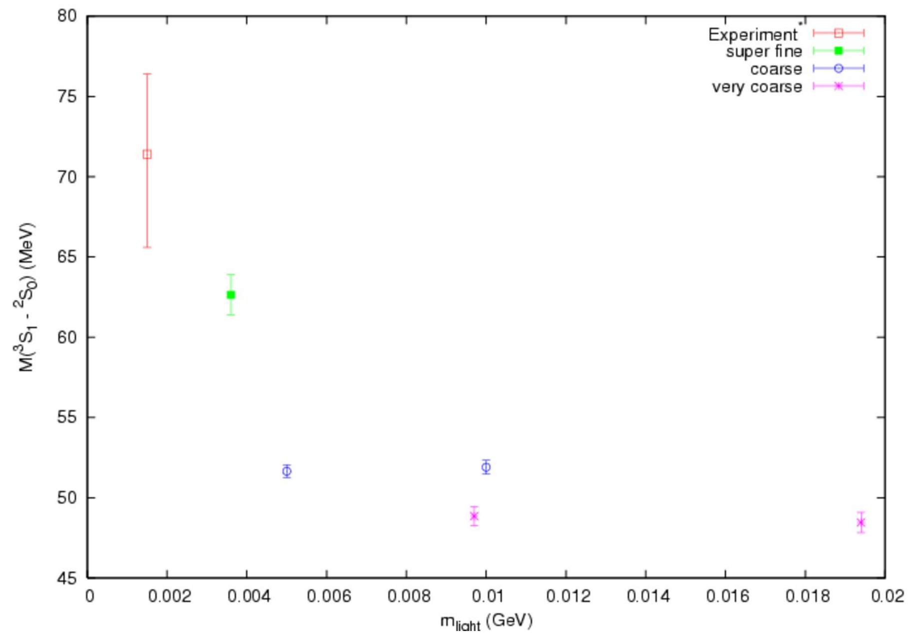
Hyperfine Splitting

- Splitting between the 1^3S_1 and 1^1S_0 energy levels





Errors on theoretical points are only statistical/fitting
Radiative and discretisation errors are expected to be larger

*BaBar - arXiv:0807.1086 [hep-ex]

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The 'Foo' Particle: Checking Systematics

- Imaginary particle contrived to allow us to investigate systematics
- Spin structure removed, quark mass decreased
- Comparison between different lattice spacings will highlight discretisation errors as other more complicated effects are suppressed
- We are also able to use perturbation theory to tune coefficients in the action to improve the test
- Work in progress

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Conclusion

- Υ useful for making precise determinations of certain lattice quantities
- Of particular importance is the lattice scale
  - calculated to the ~1% level
- Random wall sources extremely useful for E_0, but less so for higher states
  - relied on higher statistics instead
- Also able to extract other interesting quantities from our data