Challenges in Hadronic Form-Factor Calculations

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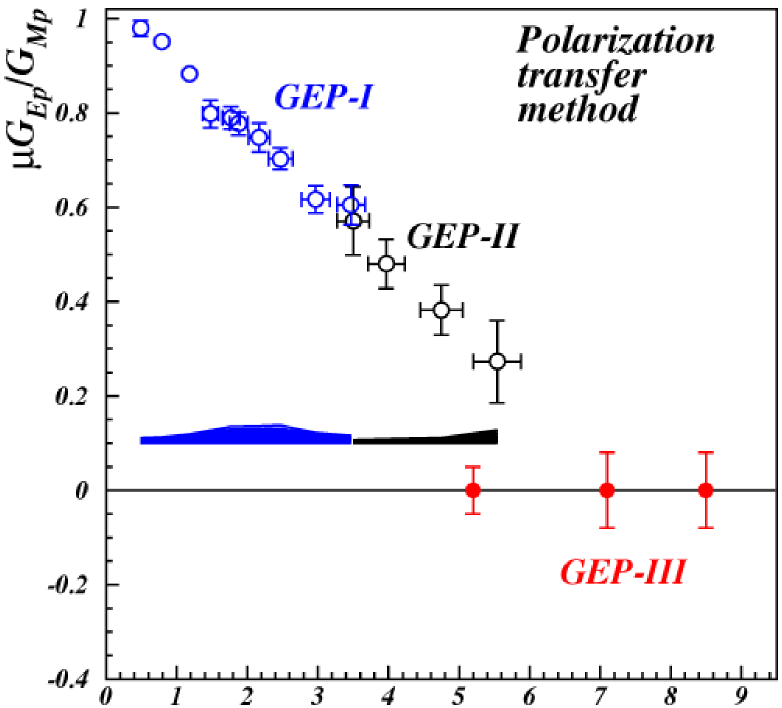
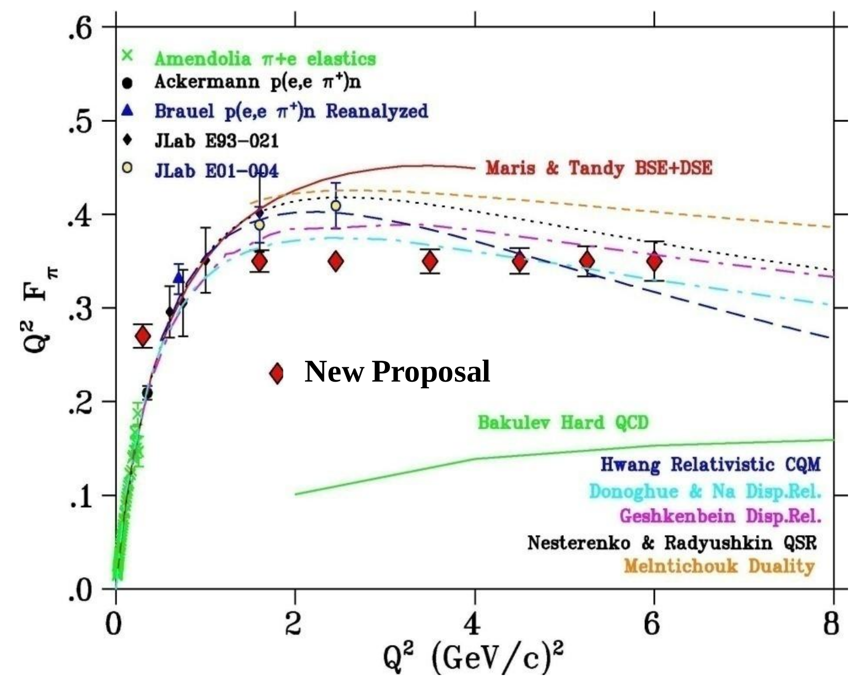
Higher-Q^2 Form Factor

- Higher-Q^2 data will help us to understand hadrons and challenge QCD-based models
- Experiments are looking...
  (JLab 12-GeV upgrade will provide more promising data)

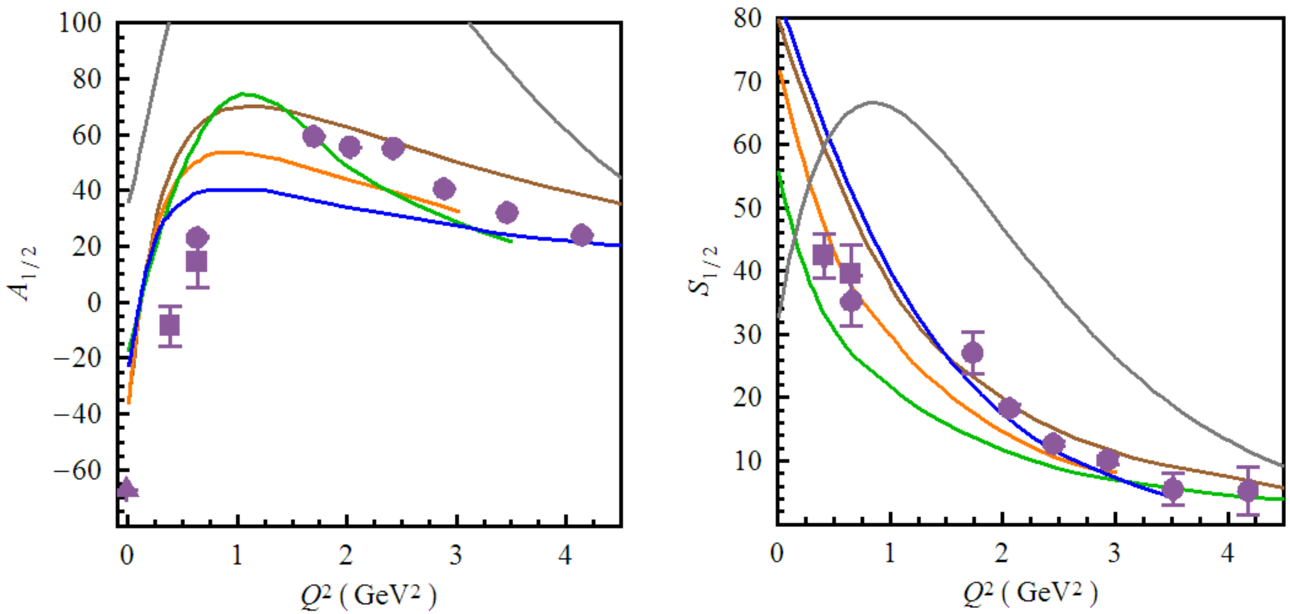




- Want to know: does G_E^p cross zero at large Q^2? If so, where? What's the pion form factor's behavior?

N-P_{11} Form Factor

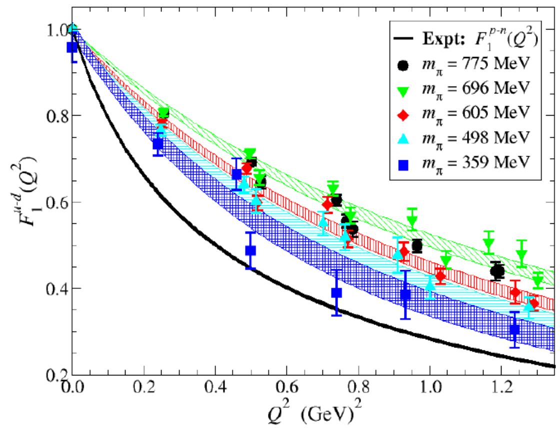
- Experiments at Jefferson Laboratory (**CLAS**), MIT-Bates, LEGS, Mainz, Bonn, GRAAL, and Spring-8
- *Helicity amplitudes* are measured (in 10^{-3} GeV$^{-1/2}$ units)
- Many models disagree (a selection are shown below)

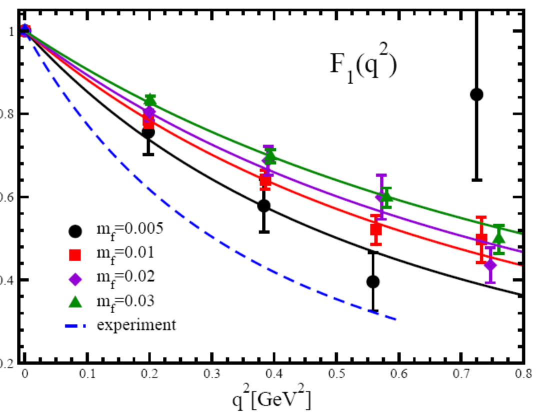


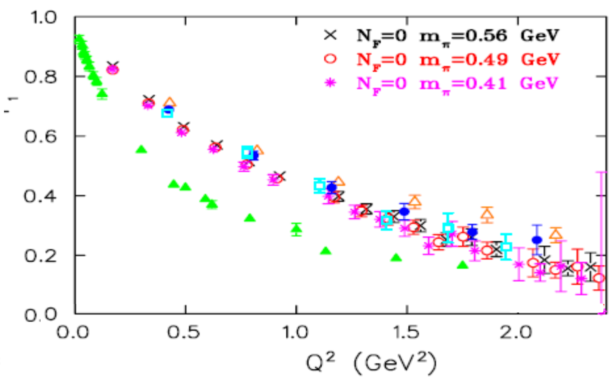
- If the Roper is the first radially excited state of nucleon, this is the data to compare with

Focusing on Two Challenges

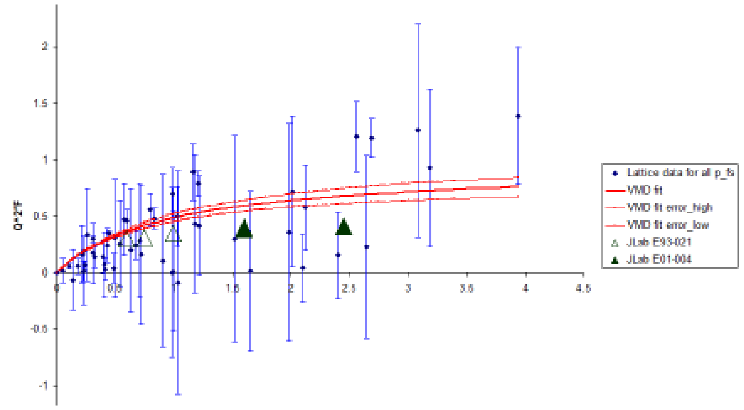
- Large-Q^2 calculations
  - Typical Q^2 range for nucleon form factors is < 3.0 GeV^2







  - Higher-Q^2 calculations suffer from poor noise-to-signal ratios

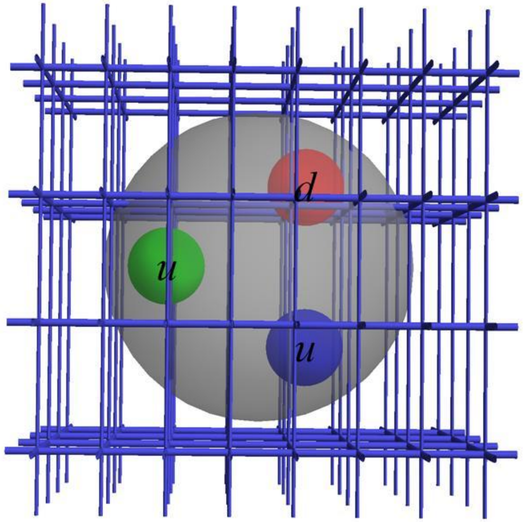


Focusing on Two Challenges

- Large-Q^2 calculations
  - Typical Q^2 range for nucleon form factors is < 3.0 GeV^2
  - Higher-Q^2 calculations suffer from poor noise-to-signal ratios
- Radially excited transition form factor calculations
  - A different way to understand the properties of excited hadrons
  - Euclidean space: signal falls exponentially with time
  - Help from anisotropic lattices

Lattice Setup

- "Quenched" for exploratory study
  - $16^3 \times 64$ **an**isotropic lattice, $\xi = 3$
  - Better resolution for excited states
  - Wilson gauge action + clover fermion action
  - $a_t^{-1} \approx 6$ GeV and $a_s \approx 0.125$ fm ($L < 2$ fm)
  - $m_\pi \approx 720$ (480 and 1100) MeV
  - 200 configurations



Anisotropic lattice

- Baryon interpolating field

$$J_\alpha\left(\vec{p}, t\right) = \sum_{\vec{x},a,b,c} e^{i\vec{p}\cdot\vec{x}} \epsilon^{abc} \left[u_a^T(y_1,t) C\gamma_5 d_b(y_2,t)\right] u_{c,\alpha}(y_3,t)\phi(y_1 - x)\phi(y_2 - x)\phi(y_3 - x)$$

- No disconnected contributions
- Multiple Gaussian smearings

Form Factors

- The form factors are buried in the amplitudes

$$\Gamma^{(3),T}_{\mu,AB}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = a^3 \sum_n \sum_{n'} \frac{1}{Z_j} \frac{Z_{n',B}(p_f) Z_{n,A}(p_i)}{4E'_n(\vec{p}_f) E_n(\vec{p}_i)} e^{-(t_f - t)E'_n(\vec{p}_f)} e^{-(t - t_i)E_n(\vec{p}_i)}$$
$$\times \sum_{s,s'} T_{\alpha\beta} u_{n'}(\vec{p}_f, s')_\beta \langle N_{n'}(\vec{p}_f, s') \left| j_\mu(0) \right| N_n(\vec{p}_i, s) \rangle \overline{u}_n(\vec{p}_i, s)_\alpha$$

- Nucleon form factor ($n = n' = 0$)

$$\langle N \left| V_\mu \right| N \rangle(q) = \overline{u}_N(p') \left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2m} \right] u_N(p) e^{-iq\cdot x}$$

- Nucleon-Roper form factor ($n = 0$, $n' = 1$ or $n = 1$, $n' = 0$)

$$\langle N_2 \left| V_\mu \right| N_1 \rangle_\mu(q) = \overline{u}_{N_2}(p') \left[F_1(q^2) \left(\gamma_\mu - \frac{q_\mu}{q^2} \not{q} \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) e^{-iq\cdot x}$$

- Need best possible input from two-point correlators

Variational Method

- Generalized eigenvalue problem:

[C. Michael, Nucl. Phys. B 259, 58 (1985)]
[M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)]

  - Construct the matrix

$$C_{i\ j}(t) = \langle 0 \mid \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) \mid 0 \rangle$$

  - Solve for the generalized eigensystem of

$$C(t_0)^{-1/2} C(t) C(t_0)^{-1/2} v = \lambda(t, t_0) v$$

with eigenvalues

$$\lambda_n(t, t_0) = e^{-(t-t_0)E_n}\left(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)})\right)$$

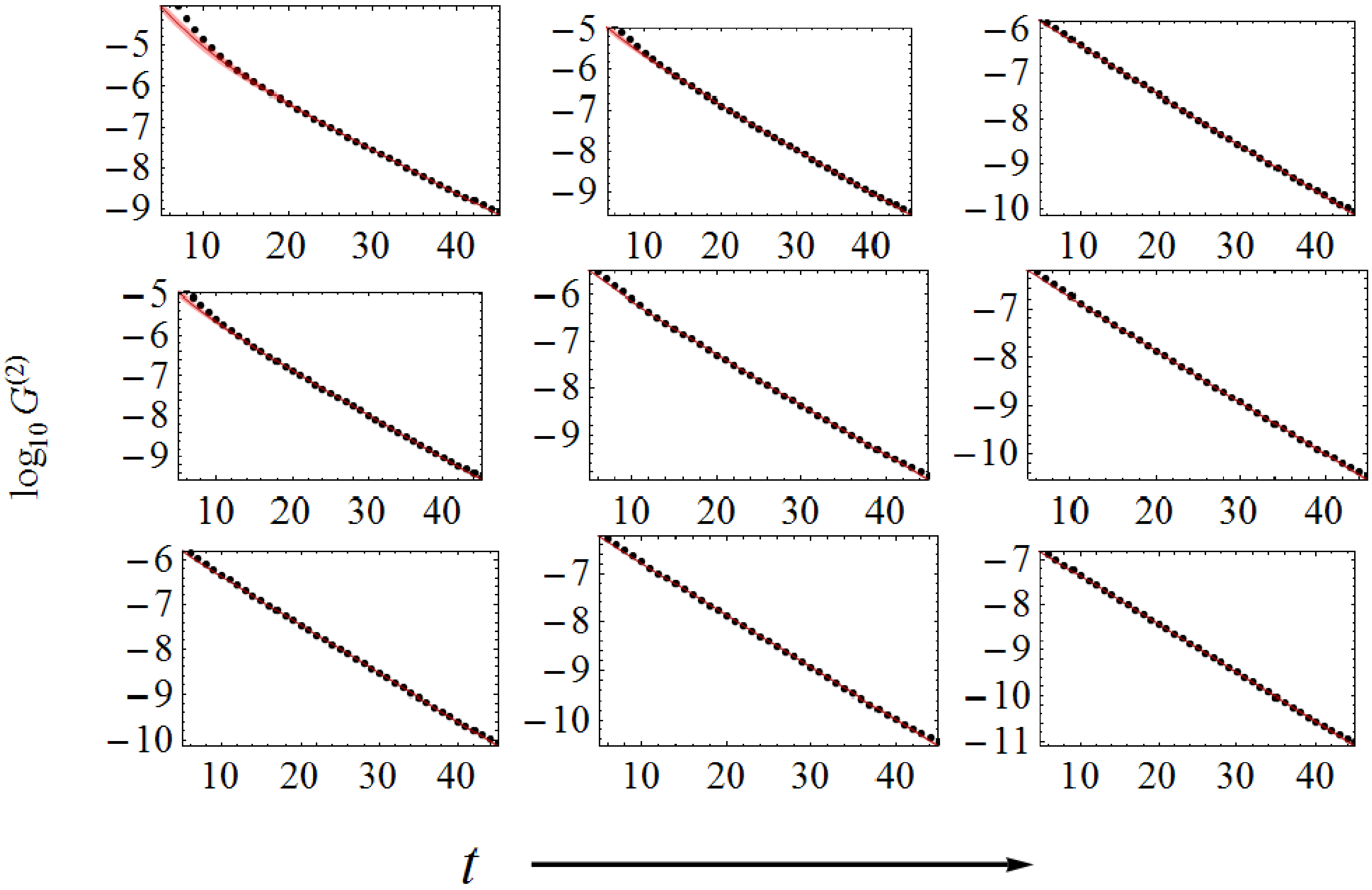
Now the original correlator matrix can be approximated by

$$C_{ij} = \sum_{n=1}^{r} (C(t_0)^{1/2} v_n^*)_i (v_n C(t_0)^{1/2})_j \lambda_n(t, t_0) = \sum_n \frac{E_n + m}{2E_n} Z_{i,n} Z_{j,n} e^{-E_n t}$$

  - Three smearings (*i*,*j*) are chosen for this work
    - 2nd excited state is contaminated by remaining states

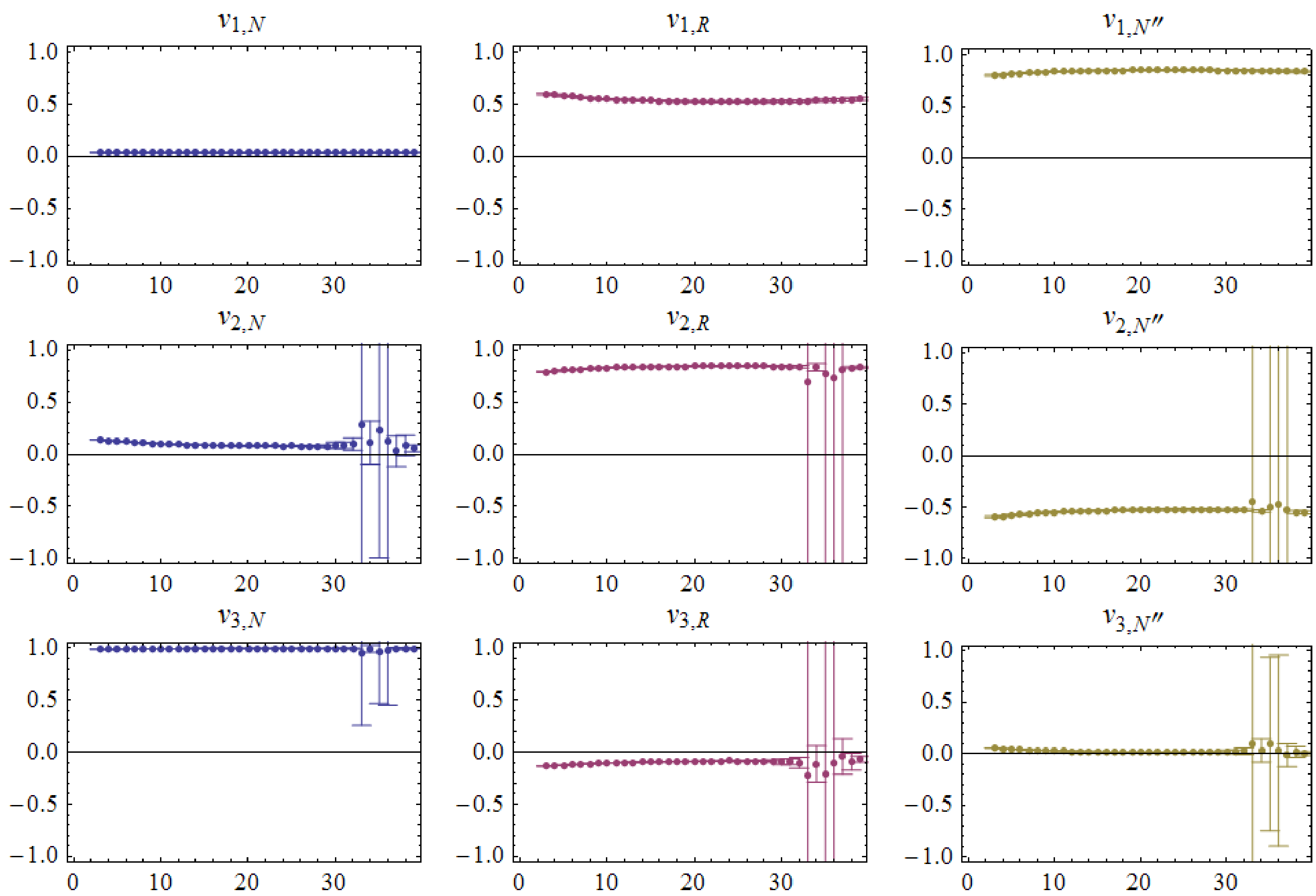
Variational Method

- Reconstruct two-point correlators from Z and λ



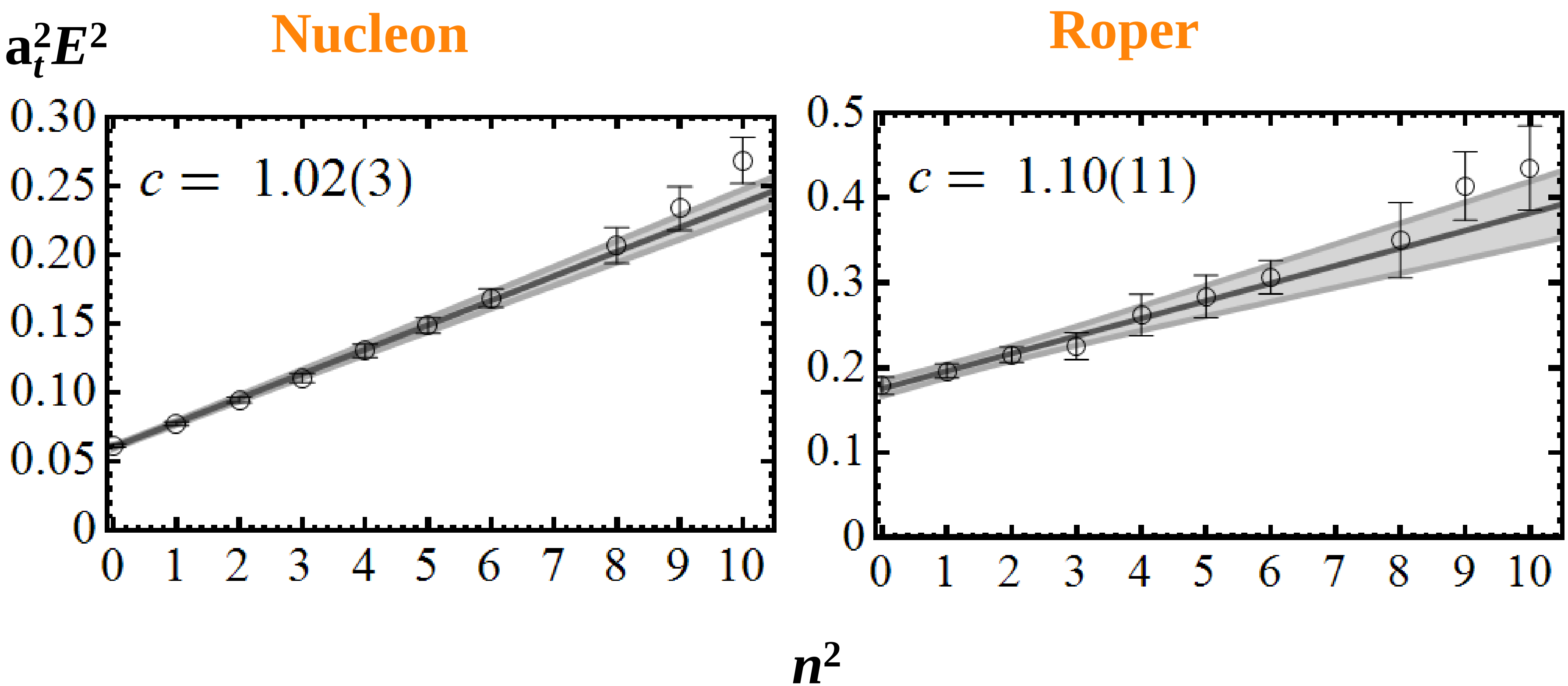
Variational Method

- Eigenvectors (at $p = 0$) show overlap of smearings with states



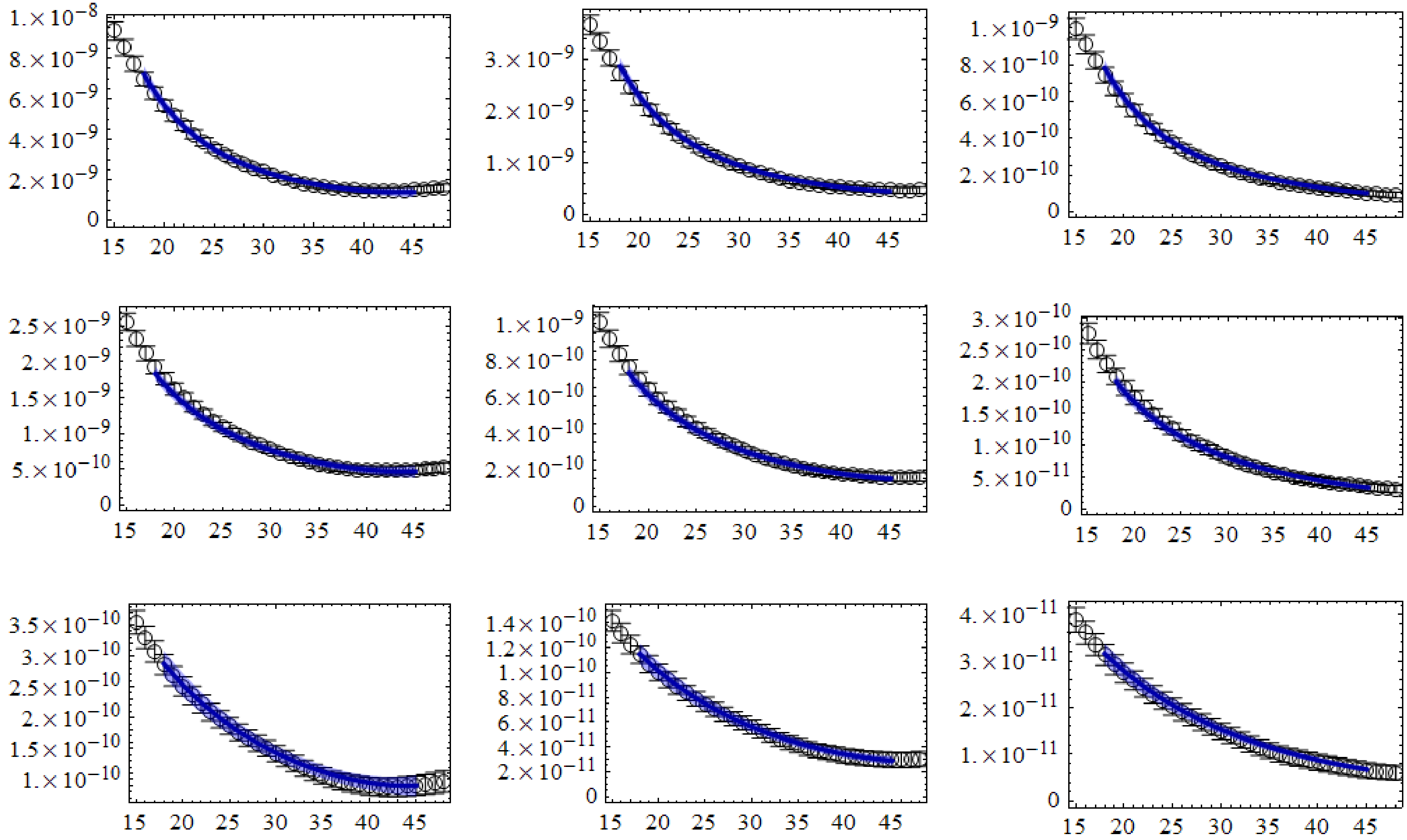
Dispersion Relation

- Example: m_π = 720 MeV



Three-Point Fitting

- Example: $P_f = \{0,0,0\}$, $P_i = \{0,1,1\}$, V_4



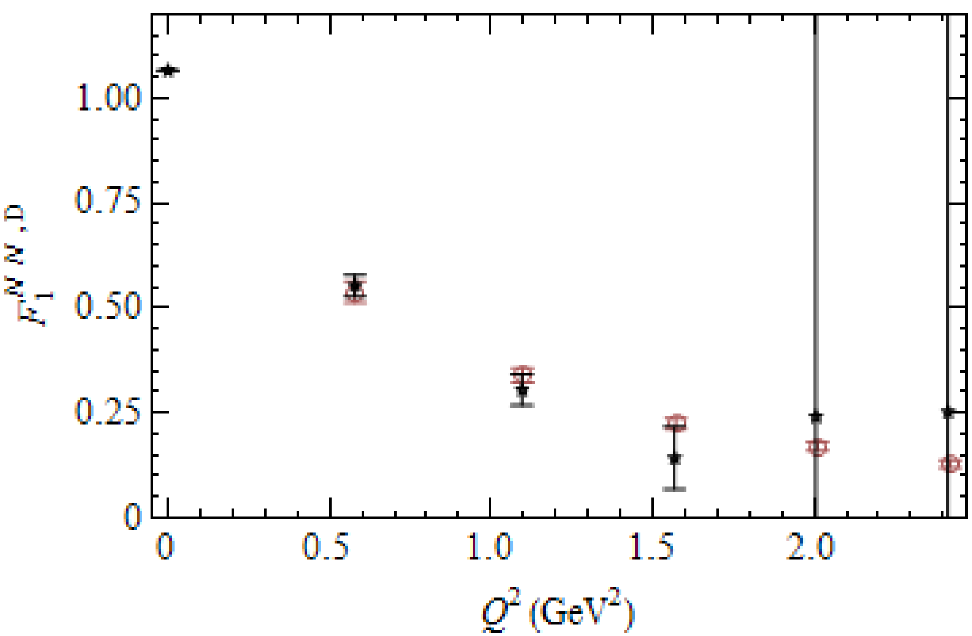
Comparison with Ratio Method

- Compare calculation with "ratio" approach:

$$R_\mu = \frac{\Gamma^{(3),\mathcal{P}}_{\mu,GG}(t_i,t,t_f,\vec{p}_i,\vec{p}_f)}{\Gamma^{(2)}_{GG}(t_i,t_f,\vec{p}_f)} \times \left(\frac{\Gamma^{(2)}_{LG}(t,t_f,\vec{p}_i)\Gamma^{(2)}_{GG}(t_i,t,\vec{p}_f)\Gamma^{(2)}_{LG}(t_i,t_f,\vec{p}_f)}{\Gamma^{(2)}_{LG}(t,t_f,\vec{p}_f)\Gamma^{(2)}_{GG}(t_i,t,\vec{p}_i)\Gamma^{(2)}_{LG}(t_i,t_f,\vec{p}_i)}\right)^{\frac{1}{2}}$$

  and SVD solutions

- Results

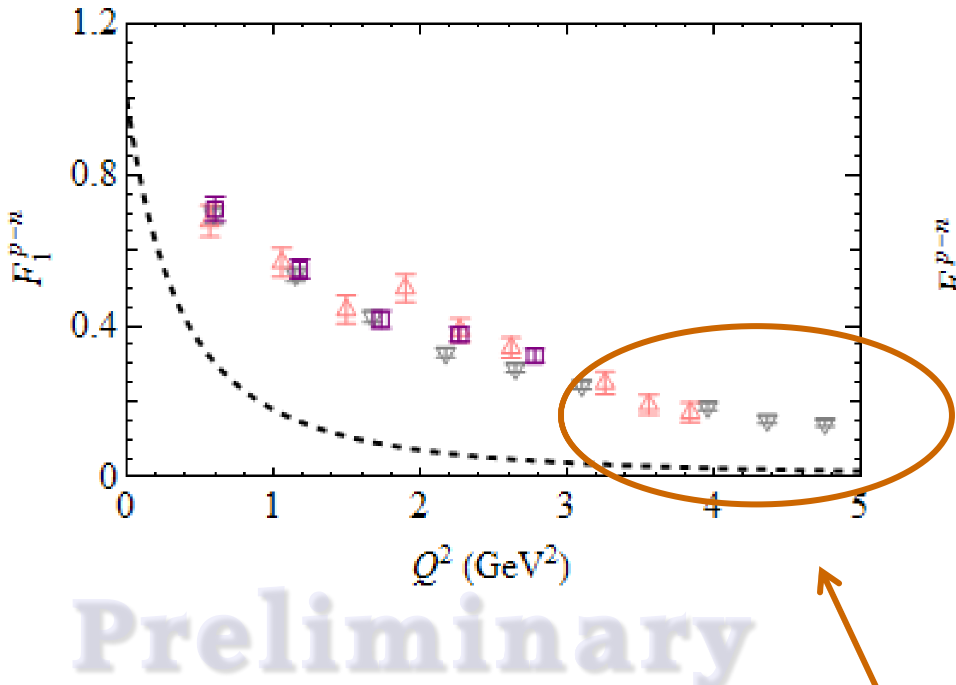


- Consistent with conventional ratio approach
- Smaller errorbar at larger Q^2
- Normally smearings are tuned to eliminate higher-energy contribution

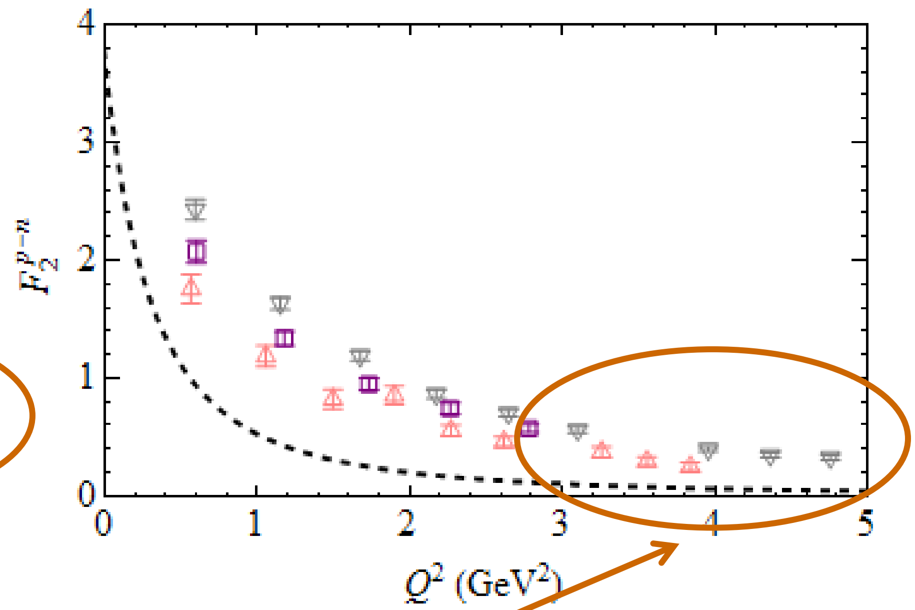
Nucleon Form Factors

- Pion masses around **480**, **720** and **1100** MeV

Isovector F_1



Isovector F_2



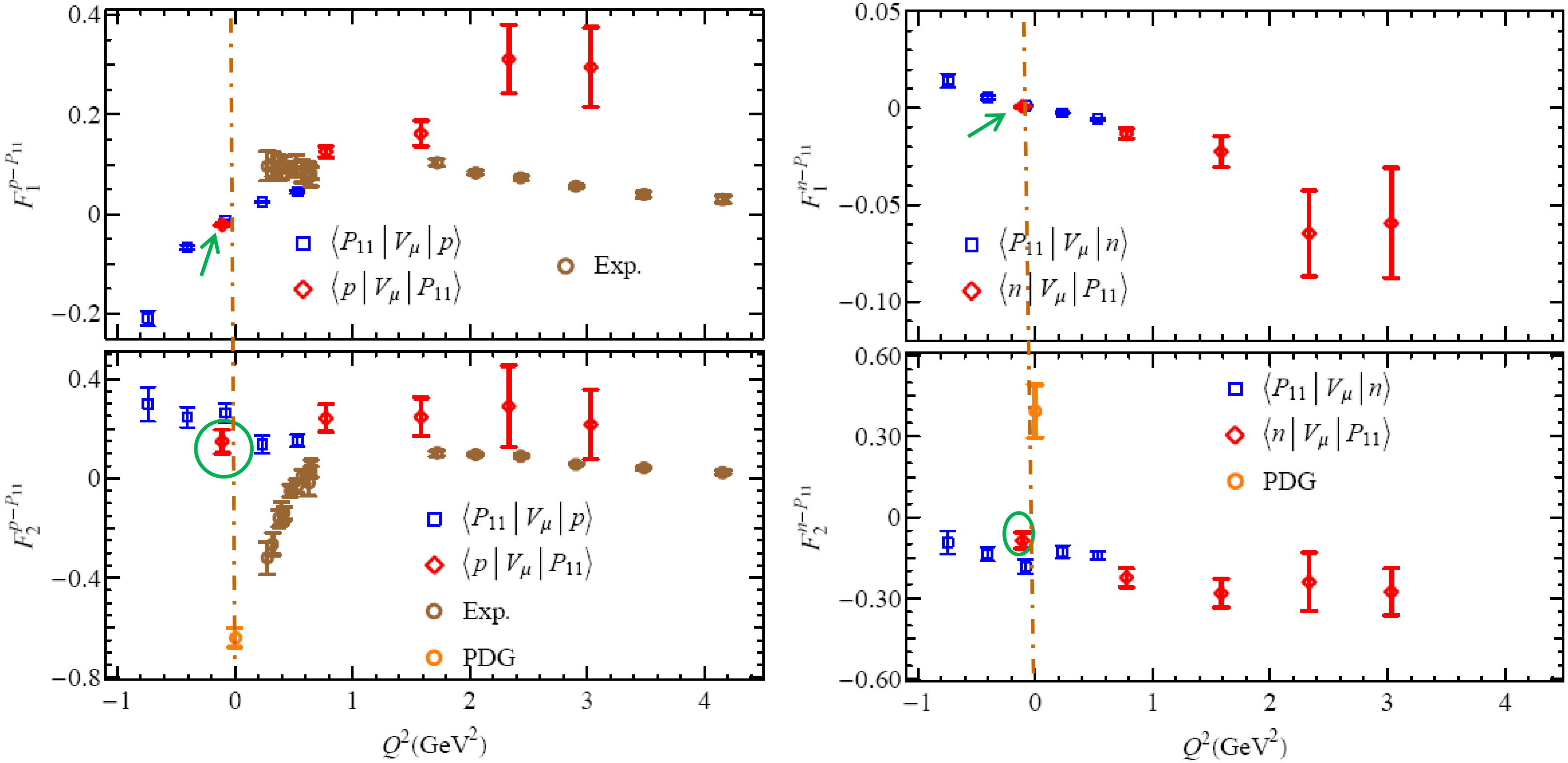
Clean signal for momentum region of $3 < Q^2 < 5$ GeV^2

- Working on larger-Q^2 regions with non-zero p_f
- N_f = 3 and 2+1 data are on the way

Nucleon-Roper Form Factors

- Completed exploratory study on quenched lattices arXiv:0803.3020

Proton-P_{11} **720 MeV Pion** **Neutron-P_{11}**

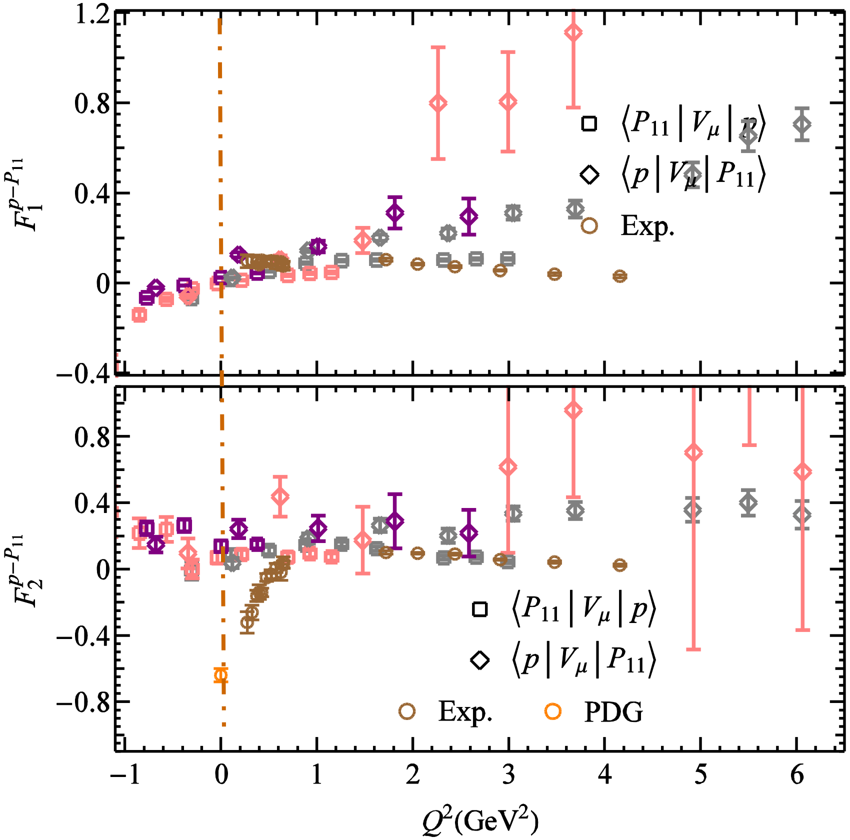


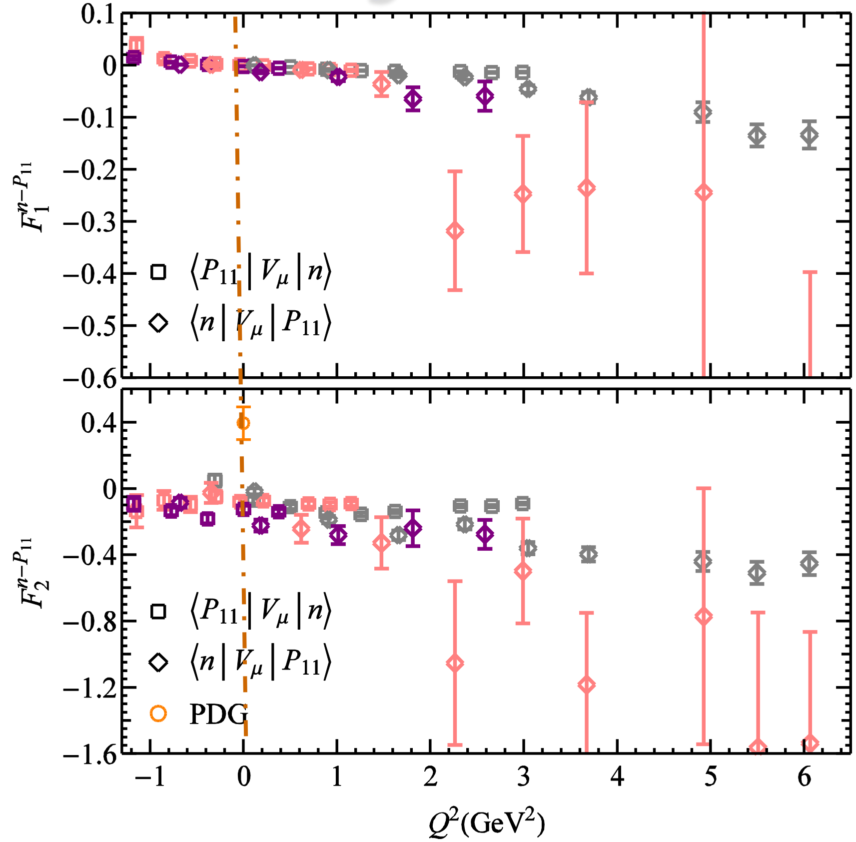
- Possible decaying state (circled above)
- 200 configurations give us reasonable signal
- Lower pion mass will shift the time-like region to space-like region

Nucleon-Roper Form Factors

- Add two more mass points at $m_\pi \sim$ **480** and **1100** MeV

Proton-P_{11} Preliminary **Neutron-P_{11}**





- Need to remove the points with potential decay kinematics
- May want to calculate on a second volume

Summary and Outlook

Solving the form-factor challenges in Lattice QCD calculation…

- Large-Q^2 momentum N-N form factors; no sign of negative G_E for 5.5 GeV^2
- We demonstrate a method to determine N-N^* form factors
- Test case is in a small "quenched" box with large pion mass

Further along our roadmap…

- Starting full-QCD anisotropic lattice calculations this summer
- Search over low and larger-Q^2 regions
- Implement group theory operators for baryons
- Other N-N^* form factors. The methodology developed can be applied to many other excited-nucleon form factors.