Clover improvement for stout-smeared 2+1 flavour SLiNC fermions: perturbative results

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QCDSF collaboration

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Outline

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Total action

Current simulations with $2+1$ flavours require a careful choice of lattice representations of fermions and gluons

QCDSF collaboration:

$$S^{\rm total}(U, \mathbf{U}, \psi; c_{SW}, \kappa, c_i) = S_{\rm SLiNC} + S_G(U; c_i)$$

SLiNC action=Stout Link Non-perturbative Clover

$$S_{\rm SLiNC} = S_F(U, \mathbf{U}, \psi; c_{SW}, \kappa)$$

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Fermionic part

Clover action with stout smeared links **U** in the hopping term

$$
\begin{aligned}
S_F(U,\mathbf{U},\psi;c_{SW},\kappa) \;=\; \sum_x \Big\{ & \overline{\psi}(x)\psi(x) \\
& -\kappa\overline{\psi}(x)\mathbf{U}_\mu^\dagger(x-\hat{\mu})[1+\gamma_\mu]\psi(x-\hat{\mu}) \\
& -\kappa\overline{\psi}(x)\mathbf{U}_\mu(x+\hat{\mu})[1-\gamma_\mu]\psi(x+\hat{\mu}) \\
& +\frac{i}{2}\,\kappa\, c_{SW}\overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(U,x)\psi(x)\Big\}
\end{aligned}
$$

Fermionic part

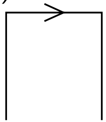
where *[Morningstar/Peardon]*

$$\mathbf{U}_\mu(x) = \exp\{iQ_\mu(x)\}\, U_\mu(x)$$

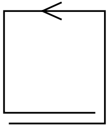
$$Q_\mu(x) = \frac{\omega}{2i}[VU^\dagger - UV^\dagger - \frac{1}{N_c}\mathrm{Tr}(VU^\dagger - UV^\dagger)]_\mu$$

V_μ is the sum of all staples around U_μ.

Terms at $O(\omega)$:



$VU^\dagger U$



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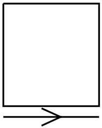

$\mathrm{Tr}()U$

Fermionic part

Benefits:
- UV-filtering $\rightarrow$ improving chiral behavior of clover fermions
- UV-filtering $\rightarrow$ suppressing unwanted tadpole contributions
- Stout smearing $\rightarrow$ fat link remains automatically in the gauge group

Choices:
- $\omega \approx 0.1$ $\rightarrow$ mild smearing
- $F_{clover}(U)$ unsmeared $\rightarrow$ fermionic matrix remains not too extended

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Gauge part

Symanzik improved gauge action:

$$S_G(U;c_i) = \frac{6}{g^2}\left[c_0 \sum_{\text{plaquette}} \frac{1}{3}\,\mathrm{Re\,Tr}\,(1 - U_{\text{plaquette}}) + c_1 \sum_{\text{rectangle}} \frac{1}{3}\,\mathrm{Re\,Tr}\,(1 - U_{\text{rectangle}})\right]$$

with $c_1 = -1/12\,,\ c_0 + 8c_1 = 1\,,\ \beta = \dfrac{6}{g^2}\,c_0$

Gauge part

Benefits:

- Six-link gauge actions $\rightarrow \mathcal{O}(a^2)$ improvement
- Six-link gauge actions $\rightarrow$ better phase behavior for 2+1 *JLQCD*
- Tree-level Symanzik $\rightarrow \Lambda^{\overline{MS}}/\Lambda^{latt} \approx \mathcal{O}(1)$
- One-loop corrections $\Delta c_i^{(1)}$ to the $c_i \rightarrow \Delta c_1^{(1)} \approx -0.01$, $\Delta c_2^{(1)} \approx -0.00006$ *Zhao et al. [2007]*

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Parameters of SLiNC

Summary of parameters:
c_i, ω, c_{SW}, number of smearing steps (n_{smear})

- c_i, ω, n_{smear}: certain freedom

but:
c_{SW} has to be tuned to cancel $\mathcal{O}(a)$ scaling violation
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Perturbative O(a) improvement

First determinations of c_{SW} in one-loop have been published by:

Wohlert[1987] (twisted antiperiodic b.c., plaquette action)
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This talk:
off-shell Green function from SLiNC action

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qqg-Vertex

Looking for quantity → one-loop information for c_{sw}

Quark-quark-gluon-vertex (V_μ): it contains to lowest order the improvement parameter c_{sw} → one-loop calculation sufficient

$$\begin{aligned} V_\mu(p_1, p_2, c_{SW}) &= -i\, g\, \gamma_\mu - g\, \tfrac{1}{2}\, a\, \mathbf{1}(p_1 + p_2)_\mu \\ &\quad + c_{SW}\, i\, g\, \tfrac{1}{2}\, a\, \sigma_{\mu\alpha}(p_1 - p_2)_\alpha + \mathcal{O}(a^2)\,. \end{aligned}$$

with

$$c_{SW} = 1 + g^2\, c_{SW}^{(1)}$$

Strategy: Calculate the full three-point function V^μ_{qqg} to one-loop and demand that all $\mathcal{O}(a)$ terms cancel → $c_{SW}^{(1)}$

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Off-shell "benefit"

Calculating the qqg-vertex off-shell → additional improvement of the quark field is necessary:

$$\psi_\star(x) = \left(1 + a\, c_D\, \overrightarrow{\not{D}} + a\, i\, g\, c_{NGI} \not{A}(x)\right)\, \psi(x)$$

$c_D = -\frac{1}{4}\left(1 + g^2\, c_D^{(1)}\right) + \mathcal{O}(g^4)$
QCDSF [2001]

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Feynman diagrams

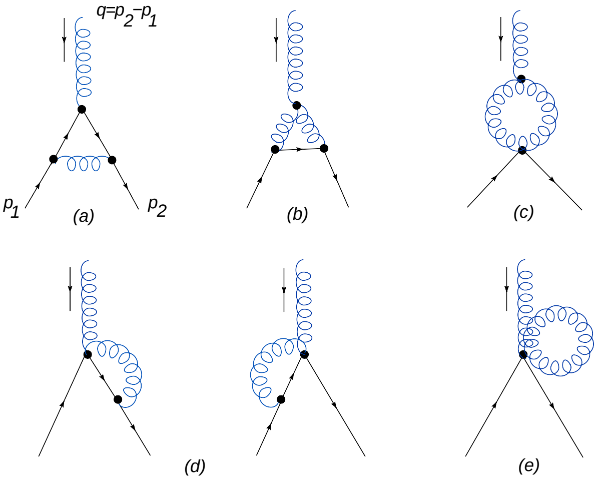

Figure: One-loop diagrams contributing to the amputated quark-quark-gluon vertex

Example: qqggg-Vertex and stout smearing

$$V^{abc}_{\alpha\beta\gamma}(p_2,p_1,k_1,k_2,k_3,\omega) = \frac{1}{6}\,a^2 g^3 \sum_\mu \Big\{ W_{1\mu}(p_2,p_1)\left[F^{abc}_{\alpha\beta\gamma\mu}(k_1,k_2,k_3) + \text{cyclic perm.}\right] - 6\,\omega\, W_{2\mu}(p_2,p_1)\left[T^{abc}_{sa}\, V_{\alpha\mu}(k_1)\, g_{\beta\gamma\mu}(k_2,k_3) + \text{cyclic perm.}\right]\Big\}.$$

$$F^{abc}_{\alpha\beta\gamma\mu}(k_1,k_2,k_3) = T^{abc}_{ss} f^{(1)}_{\alpha\beta\gamma\mu}(k_1,k_2,k_3) + T^{abc}_{aa}\left(f^{(2)}_{\alpha\beta\gamma\mu}(k_1,k_2,k_3) - f^{(2)}_{\alpha\gamma\beta\mu}(k_1,k_3,k_2)\right) + \left(T^{abc}_{ss} - \frac{1}{N_c} d^{abc}\right) f^{(3)}_{\alpha\beta\gamma\mu}(k_1,k_2,k_3)\,,$$

$$f^{(1)}_{\alpha\beta\gamma\mu}(k_1,k_2,k_3) = \frac{1}{2}\, V_{\alpha\mu}(k_1,\omega)\, V_{\beta\mu}(k_2,\omega)\, V_{\gamma\mu}(k_3,\omega)\,,$$

$$f^{(2)}_{\alpha\beta\gamma\mu}(k_1,k_2,k_3) = \frac{1}{2}\, V_{\alpha\mu}(k_1,\omega)\, V_{\beta\mu}(k_2,\omega)\,\delta_{\gamma\mu} - \frac{1}{2}\,\delta_{\alpha\mu}\delta_{\beta\mu}\, V_{\gamma\mu}(k_3,\omega) + 6\,\omega\,\delta_{\alpha\beta}\left[\mathrm{c}_\mu(k_1-k_2)\,\mathrm{c}_\beta(2k_3+k_1+k_2)\,\delta_{\gamma\mu} + \mathrm{s}_\mu(k_3)\,\mathrm{s}_\gamma(k_3+2k_1)\,\delta_{\beta\mu}\right]$$

$$f^{(3)}_{\alpha\beta\gamma\mu}(k_1,k_2,k_3) = 2\,\omega\,\delta_{\beta\gamma}\Big[\left(3\,w_{\alpha\mu}(k_1,k_2+k_3) + v_{\alpha\mu}(k_1+k_2+k_3)\right)\delta_{\alpha\beta} + 12\,\mathrm{s}_\beta(k_1)\,\mathrm{s}_\alpha(k_2)\,\mathrm{s}_\alpha(k_3)\left(\mathrm{s}_\beta(k_1+k_2+k_3)\,\delta_{\alpha\mu} - \mathrm{s}_\alpha(k_1+k_2+k_3)\,\delta_{\beta\mu}\right)\Big]$$

Example: qqggg-Vertex and stout smearing

Notation:

$$T^{abc}_{ss} = \{T^a, \{T^b, T^c\}\}\,, \quad T^{abc}_{aa} = [T^a, [T^b, T^c]]\,, \quad T^{abc}_{sa} = \{T^a, [T^b, T^c]\}$$

$$\mathrm{s}_\mu(k) = \sin\left(\frac{a}{2}k_\mu\right), \quad \mathrm{c}_\mu(k) = \cos\left(\frac{a}{2}k_\mu\right), \quad s^2(k) = \sum_\mu \mathrm{s}^2_\mu(k)\,,$$

$$s^2(k_1, k_2) = \sum_\mu \mathrm{s}_\mu(k_1 + k_2)\,\mathrm{s}_\mu(k_1 - k_2) \equiv s^2(k_1) - s^2(k_2)$$

$$\begin{aligned} W_{1\mu}(p_2, p_1) &= \mathrm{i}\,\mathrm{c}_\mu(p_2 + p_1)\,\gamma_\mu + r\,\mathrm{s}_\mu(p_2 + p_1) \\ W_{2\mu}(p_2, p_1) &= \mathrm{i}\,\mathrm{s}_\mu(p_2 + p_1)\,\gamma_\mu - r\,\mathrm{c}_\mu(p_2 + p_1) \end{aligned}$$

$$\begin{aligned} V_{\alpha\mu}(k, \omega) &= \delta_{\alpha\mu} + 4\,\omega\,v_{\alpha\mu}(k) \\ v_{\alpha\mu}(k) &= \mathrm{s}_\alpha(k)\,\mathrm{s}_\mu(k) - \delta_{\alpha\mu}\,s^2(k) \\ g_{\alpha\beta\mu}(k_1, k_2) &= \delta_{\alpha\beta}\,\mathrm{c}_\alpha(k_1 + k_2)\,\mathrm{s}_\mu(k_1 - k_2) - \\ &\quad \delta_{\alpha\mu}\,\mathrm{c}_\alpha(k_2)\,\mathrm{s}_\beta(2k_1 + k_2) + \delta_{\beta\mu}\,\mathrm{c}_\beta(k_1)\,\mathrm{s}_\alpha(2k_2 + k_1) \\ w_{\alpha\mu}(k_1, k_2) &= \mathrm{s}_\alpha(k_1 + k_2)\,\mathrm{s}_\mu(k_1 - k_2) - \delta_{\alpha\mu}\,s^2(k_1, k_2)\,, \quad w_{\alpha\mu}(k, 0) = v_{\alpha\mu}(k) \end{aligned}$$

Results: c_{SW}

$$c_{SW} = 1 + g^2\, c_{SW}^{(1)}$$

$$\begin{aligned} c_{SW}^{(1)} &= C_F\left(0.116185 + 0.828129\,\omega - 2.455080\,\omega^2\right) \\ &\quad + N_c\left(0.013777 + 0.015905\,\omega - 0.321899\,\omega^2\right) \end{aligned}$$

coincides for $\omega = 0$ with *Aoki,Kuramashi [2003]*

Results: κ_c

Additive mass renormalization

$$am_0 = \frac{1}{2\kappa_c} - 4 = \frac{g^2 C_F}{16\pi^2} \frac{\Sigma_0}{4} \quad \rightarrow \quad \kappa_c = \frac{1}{8}\left(1 - \frac{g^2 C_F}{16\pi^2} \frac{\Sigma_0}{4}\right)$$

SLiNC action + quark self energy ($\Sigma(p=0)$) $\rightarrow \kappa_c$:

$$\kappa_c = \frac{1}{8}\Big[1 + g^2 C_F \left(0.037730 - 0.662090\,\omega + 2.668543\,\omega^2\right)\Big] .$$

$\omega = 0.088689 \rightarrow \kappa_c = \frac{1}{8}$

Quark field improvement results

$$\psi_\star(x) = \left(1 + a\, c_D\, \overrightarrow{\not{D}} + a\, i\, g\, c_{NGI}\, \not{A}(x)\right)\, \psi(x)$$

$$c_D = -\frac{1}{4}\left(1 + g^2\, c_D^{(1)}\right) + \mathcal{O}(g^4)$$

$$c_D^{(1)} = C_F\, (0.037614 + 0.011755\, \xi - 0.835571\, \omega + 3.418757\, \omega^2)$$

(ξ - covariant gauge parameter)

$$c_{NGI} = g^2\, c_{NGI}^{(1)} + \mathcal{O}(g^4)$$

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Mean field improvement

Bare coupling constant g^2 leads to a poor approximation:
- g^2 is large in most quantities
- perturbative series converges poorly

Two ideas combined

(1) Calculate each quantity in a simple mean field approximation

$\rightarrow$ Re-express the perturbative result as the mean field result multiplied by a perturbative correction factor

$\rightarrow$ One-loop correction term should be small

(2) Bare coupling $g^2 \rightarrow$ "boosted" coupling constant g^2_{MF}

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Mean field improvement and stout smearing

Express with two mean fields:
u_0 - a mean value for the unsmeared link
u_S - a mean value for smeared links

$$\kappa_c(g^2) \to \kappa_c^{MF}({g_{MF}}^2, u_S) = \frac{{u_S}^{pert}({g_{MF}}^2)}{u_S}\,\kappa_c({g_{MF}}^2)$$

$$c_{SW}(g^2) \to c_{SW}^{MF}({g_{MF}}^2, u_S, u_0) = \frac{u_S}{{u_0}^4}\,\frac{{u_0}^{pert,4}({g_{MF}}^2)}{{u_S}^{pert}({g_{MF}}^2)}\,c_{SW}({g_{MF}}^2)$$

with

$$ {u_S}^{pert} = 1 - \frac{g^2\,C_F}{16\pi^2}\,k_S(\omega)\,, \qquad {u_0}^{pert} = 1 - \frac{g^2\,C_F}{16\pi^2}\,k_S(\omega = 0)$$

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$$\Downarrow$$

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c_{SW}^{\mathrm{Sym},MF} &= \frac{u_S}{u_0^4} \Big\{ 1 + g_{MF}^2 \times \\
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$\beta = 6.0,\ u_S = 0.9497,\ u_0 = 0.8644:$

$c_{SW}^{\mathrm{Sym}} = 1.4484 \rightarrow c_{SW}^{\mathrm{Sym},MF} = 1.8678 \leftrightarrow c_{SW}^{\mathrm{Sym},NP} = 2.137$

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$$\Downarrow$$

$$\kappa_c^{\mathrm{Sym},MF} = \frac{1}{8u_S}\Big[1 + g_{MF}^2\, C_F \times \left(-0.008053 + 0.0500781\,\omega - 0.471784\,\omega^2\right)\Big]$$

$\beta = 6.0,\ u_S = 0.9497,\ u_0 = 0.8644$:

$\kappa_c^{\mathrm{Sym}} = 0.1245 \rightarrow \kappa_c^{\mathrm{Sym},MF} = 0.1276 \leftrightarrow \kappa_c^{\mathrm{Sym},NP} = 0.124356$

$$\kappa_c^{\rm Sym} = \frac{1}{8}\Big[1 + g^2\, C_F \times \left(0.037730 - 0.662090\,\omega + 2.668543\,\omega^2\right)\Big]$$

$$\Downarrow$$

$$\kappa_c^{{\rm Sym},MF} = \frac{1}{8u_S}\Big[1 + g_{MF}^2\, C_F \times \left(-0.008053 + 0.0500781\,\omega - 0.471784\,\omega^2\right)\Big]$$

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Choice of g^2_{MF} or $\frac{\Lambda^{MF}_{\rm lat}}{\Lambda_{\overline{MS}}}$

The natural choice

$$g^2_{MF} = \frac{g^2}{u_0^4} \, .$$

We have the relation (e.g. *Kawai, Seo [1981]*)

$$\begin{aligned} \frac{1}{g^2_{\overline{MS}}(\mu)} - \frac{1}{g^2_{MF}(a)} &= 2b_0 \left(\log \frac{\mu}{\Lambda_{\overline{MS}}} - \log \frac{1}{a\Lambda^{MF}_{\rm lat}} \right) \\ &= 2b_0 \log(a\mu) + d_g + N_f \, d_f + \frac{k_u}{3\pi^2} \end{aligned}$$

giving

$$\frac{\Lambda^{MF}_{\rm lat}}{\Lambda_{\overline{MS}}} = \exp\left(\frac{d_g + N_f \, d_f + k_u/3\pi^2}{2b_0} \right) \, .$$

$(k_u = k_S(\omega = 0))$

Choice of g^2_{MF} or $\frac{\Lambda^{MF}_{\mathrm{lat}}}{\Lambda_{\overline{MS}}}$

We have

$d_g = -0.2361$ (*Hasenfratz et al.[1980]*)

$d_f = 0.0314917$ (*Booth et al.[2001]*), independent of ω

$k_u = 0.732525\,\pi^2$

$\longrightarrow \quad \frac{\Lambda^{MF}_{\mathrm{lat}}}{\Lambda_{\overline{MS}}} = 2.459$

SLiNC and point operators

Expected that renormalization (Z-) factors closer to unity

Z-factors for $\mathcal{O} = \bar{\psi}\, 1\, \psi,\ \bar{\psi}\, \gamma_5\, \psi,\ \bar{\psi}\, \gamma_\mu\, \psi,\ \bar{\psi}\, \gamma_5 \gamma_\mu\, \psi$

General one-loop form

$$Z_{\mathcal{O}} = 1 - \frac{g^2\, C_F}{16\pi^2} \left(\gamma_{\mathcal{O}} \log(a^2\mu^2) + \mathcal{B}_{\mathcal{O}} \right)$$

Mean field improving program:

$$Z_{\mathcal{O}}^{MF} = u_S(1 - \frac{g_{MF}^2\, c_F}{16\pi^2} \left(\gamma_{\mathcal{O}} \log(a^2\mu^2) + \mathcal{B}_{\mathcal{O}} - k_S(\omega) \right)$$

$\mathcal{O} = \bar{\psi}\, 1\, \psi = S$

$$\mathcal{B}_S = 15.0747 - 168.341\omega + 242.254\omega^2$$

$$\stackrel{\omega=0}{=} 15.0747 \quad \text{unsmeared}$$

$$\stackrel{\omega=0.1}{=} 0.663069 \quad \text{smeared}$$

Mean field improvement:

$\beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1$:

$Z_S = 0.9907 \to Z_S^{MF} = 0.9564$

$\mathcal{O} = \bar{\psi}\, 1\, \psi = S$

$$\mathcal{B}_S = 15.0747 - 168.341\omega + 242.254\omega^2$$
$$\stackrel{\omega=0}{=} 15.0747 \quad \text{unsmeared}$$
$$\stackrel{\omega=0.1}{=} 0.663069 \quad \text{smeared}$$

Mean field improvement:

$$\beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1 :$$

$$Z_S = 0.9907 \rightarrow Z_S^{MF} = 0.9564$$

$\mathcal{O} = \bar{\psi}\,\gamma_5\,\psi = P$

$$\mathcal{B}_P = 19.1500 - 267.462\omega + 1065.55\omega^2$$
$$\stackrel{\omega=0}{=} 19.1500 \quad \text{unsmeared}$$
$$\stackrel{\omega=0.1}{=} 3.0593 \quad \text{smeared}$$

Mean field improvement:

$\beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1$:

$Z_P = 0.9569 \to Z_P^{MF} = 0.8990$

$\mathcal{O} = \bar{\psi}\,\gamma_5\,\psi = P$

$$\mathcal{B}_P = 19.1500 - 267.462\omega + 1065.55\omega^2$$

$$\stackrel{\omega=0}{=} 19.1500 \quad \text{unsmeared}$$

$$\stackrel{\omega=0.1}{=} 3.0593 \quad \text{smeared}$$

Mean field improvement:

$\beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1$:

$Z_P = 0.9569 \rightarrow Z_P^{MF} = 0.8990$

$$\mathcal{O} = \bar{\psi}\,\gamma_\mu\,\psi = V$$

$$\begin{aligned} \mathcal{B}_V &= 11.9106 - 170.763\omega + 754.029\omega^2 \\ &\overset{\omega=0}{=} 11.9106 \quad \text{unsmeared} \\ &\overset{\omega=0.1}{=} 2.37464 \quad \text{smeared} \end{aligned}$$

Mean field improvement:

$\beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1$:

$Z_V = 0.9666 \to Z_V^{MF} = 0.9154 \leftrightarrow Z_V^{NP} = 0.889$

$\mathcal{O} = \bar{\psi}\,\gamma_\mu\,\psi = V$

$$\mathcal{B}_V = 11.9106 - 170.763\omega + 754.029\omega^2$$
$$\stackrel{\omega=0}{=} 11.9106 \quad \text{unsmeared}$$
$$\stackrel{\omega=0.1}{=} 2.37464 \quad \text{smeared}$$

Mean field improvement:

$$\beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1 :$$

$$Z_V = 0.9666 \to Z_V^{MF} = 0.9154 \leftrightarrow Z_V^{NP} = 0.889$$

$$\mathcal{O} = \bar{\psi}\,\gamma_5\gamma_\mu\,\psi = A$$

$$\begin{aligned} \mathcal{B}_A &= 10.7165 - 127.200\omega + 342.380\omega^2 \\ &\overset{\omega=0}{=} 10.7165 \quad \text{unsmeared} \\ &\overset{\omega=0.1}{=} 1.42034 \quad \text{smeared} \end{aligned}$$

Mean field improvement:

$\beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1$:

$Z_A = 0.9800 \rightarrow Z_A^{MF} = 0.9383$

$\mathcal{O} = \bar{\psi}\gamma_5\gamma_\mu\psi = A$

$$
\begin{aligned}
\mathcal{B}_A &= 10.7165 - 127.200\omega + 342.380\omega^2 \\
&\overset{\omega=0}{=} 10.7165 \quad \text{unsmeared} \\
&\overset{\omega=0.1}{=} 1.42034 \quad \text{smeared}
\end{aligned}
$$

Mean field improvement:

$\beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1$:

$Z_A = 0.9800 \to Z_A^{MF} = 0.9383$

Summary

- We have introduced the SLiNC action as a base for future 2+1 simulations
- Using standard perturbation theory we have calculated one-loop non-amputated Green's function related to the qqg-vertex with SLiNC fermions
- The result is used to determine the improvement coefficient c_{SW} including stout smearing
- We determined the quark field improvement coefficients c_D and c_{NGI}
- Using SLiNC and quark self energy we determined κ_c also
- On-shell we have reproduced earlier results for non-smeared links
- Mean field improvement for smeared links has been discussed
- With SLiNC fermions we calculated the one-loop corrections to point operators

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