Search for the Charmoinum Dissociation Temperature with Variational Analysis in Lattice QCD

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Plan of this talk

1. Introduction
2. Our approach
3. Meson correlator & wave function
4. Charmonium or scattering state?
5. Multi-state variational analysis
6. Numerical results
7. Conclusion and future plan

Introduction

- **According to the sequential J/Ψ suppression scenario,**
  - not only the dissociation temperature of J/ψ

    but also the dissociation temperatures of excited charmonia are important.

- **Current lattice studies for the charmonia dissociation temperatures**
  - Most of them investigated charmonia spectral functions with MEM.

    e.g. A. Jackovac et al., Phys. Rev. **D75**, 014506 (2007)

    →S wave states (η_c, J/Ψ) seem to survive up to 1.5 T_C

    but may dissolve at very high temperature.

    There are ambiguities of MEM in terms of default model at high temperature etc., so it is necessary to check the results with other methods.

    →P wave states (χ_c) seem to dissolve just above T_C.

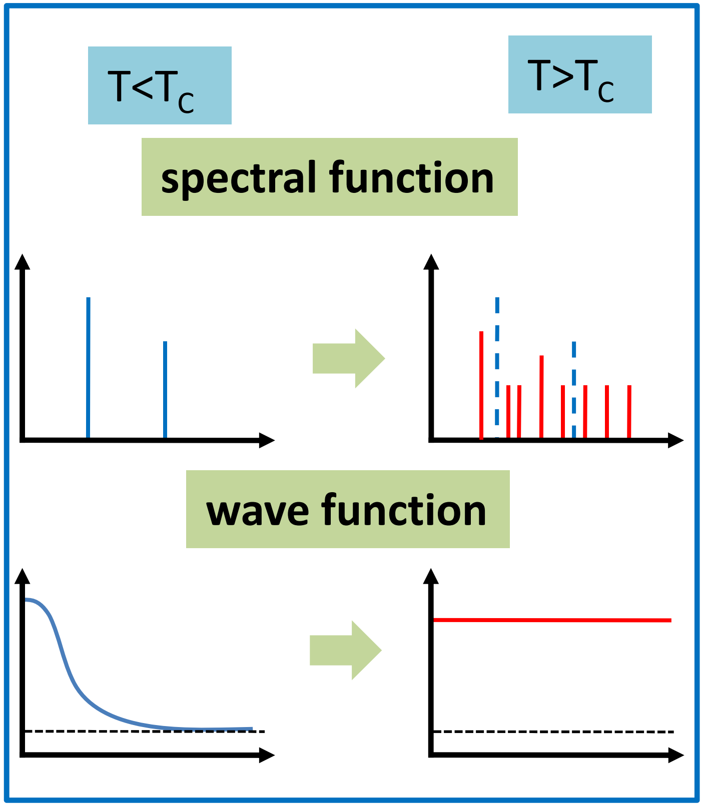
    This is misreading
    T. Umeda, Phys. Rev. **D75**, 094502 (2007)
    because the constant mode effect of meson correlator was NOT taken care.

  - Excited charmonia (e.g. Ψ') have NOT been investigated well yet.

    We should investigate excited charmonia too.

Our approach

- We investigate the chamonia dissociation temperature with following considerations.



- On a finite volume lattice, spectral function consists of discrete spectra only.
- When all charmonia fully dissolved above T_C, we naively expect:
  - **Charmona peaks should vanish and peaks of some scattering states may appears above T_C.**
  - **Wave function for scattering states extend to large distances.**
- To examine the expectations, we
  - **study both effective masses and wave functions**
  - **adopt mult-state variatonal analysis**

    to extract excited states and is well-suited for discrete spectra
  - **subtract the constant mode from the moesn correlators**

We test with quenched approximation

Meson correlator & wave function

- Meson correlator : to extract charmonia mass

$$C(t) \equiv \sum_{\vec{x}} \langle \mathcal{O}_\Gamma(\vec{x},t)\mathcal{O}_\Gamma^\dagger(\vec{0},0)\rangle$$: meson correlator with zero momentum

$$\mathcal{O}_\Gamma(\vec{x},t) \equiv \bar{q}(\vec{x},t)\Gamma q(\vec{x},t)$$: meson operator

$$\Gamma = \begin{cases} \gamma_5, & (\mathrm{Ps}) \\ \gamma_i, & (\mathrm{V}) \\ \mathbf{1}, & (\mathrm{S}) \\ \gamma_5\gamma_i, & (\mathrm{Av}) \end{cases}$$

(Ps)	$J^{PC}=0^{-+}$
(V)	$J^{PC}=1^{--}$
(S)	$J^{PC}=1^{++}$
(Av)	$J^{PC}=0^{++}$

$$C(t) = A_0 \cosh[m_0(t - L_t/2)] + A_1 \cosh[m_1(t - L_t/2)] + \cdots$$

L_t : lattice size of temporal direction

- Wave function

$$BS(\vec{r},t) \equiv \sum_{\vec{x}} \langle \bar{q}(\vec{x},t)\Gamma' q(\vec{x}+\vec{r},t)\mathcal{O}_\Gamma^\dagger(\vec{0},0)\rangle$$: Bethe-Salpeter amplitude

$$= \psi_0(\vec{r}) \cosh[m_0(t - L_t/2)] + \psi_1(\vec{r}) \cosh[m_1(t - L_t/2)] + \cdots$$

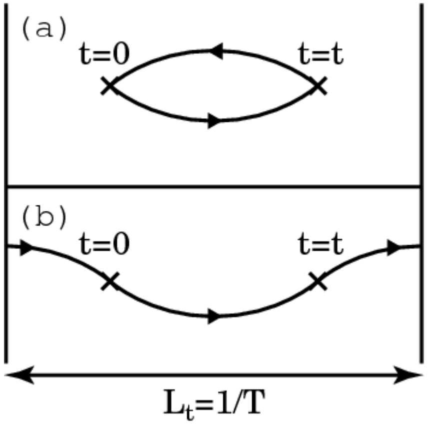
$$\Gamma' = \begin{cases} \gamma_5, & (\mathrm{Ps}) \\ \gamma_i, & (\mathrm{V}) \\ \gamma_i, & (\mathrm{S}) \\ \gamma_i, & (\mathrm{Av}) \end{cases}$$

$$\Gamma = \begin{cases} \gamma_5, & (\mathrm{Ps}) \\ \gamma_i, & (\mathrm{V}) \\ \sum_i \gamma_i \overleftrightarrow{\partial}_i, & (\mathrm{S}) \\ \sum_{i,j} \epsilon_{ijk}\gamma_i \overleftrightarrow{\partial}_j, & (\mathrm{Av}) \end{cases}$$

(Ps)	$J^{PC}=0^{-+}$
(V)	$J^{PC}=1^{--}$
(S)	$J^{PC}=1^{++}$
(Av)	$J^{PC}=0^{++}$

Separation of constant mode

- Constant mode of meson correlator T. Umeda, Phys. Rev. **D75**, 094502 (2007)



Because of temporal BC, there are wraparound contributions (b)

↓

Constant mode

$$C(t) = A_0 + A_1 \cosh[m_1(t - L_t/2)] + A_2 \cosh[m_2(t - L_t/2)] + \cdots$$

  – Because wraparound contributions come from single quark propagations, constant mode effects strongly appear in deconfined phase,

    ← which leads misreading as if the P wave chamonia dissolved just above T_C.

- **Midpoint subtracted correlator**

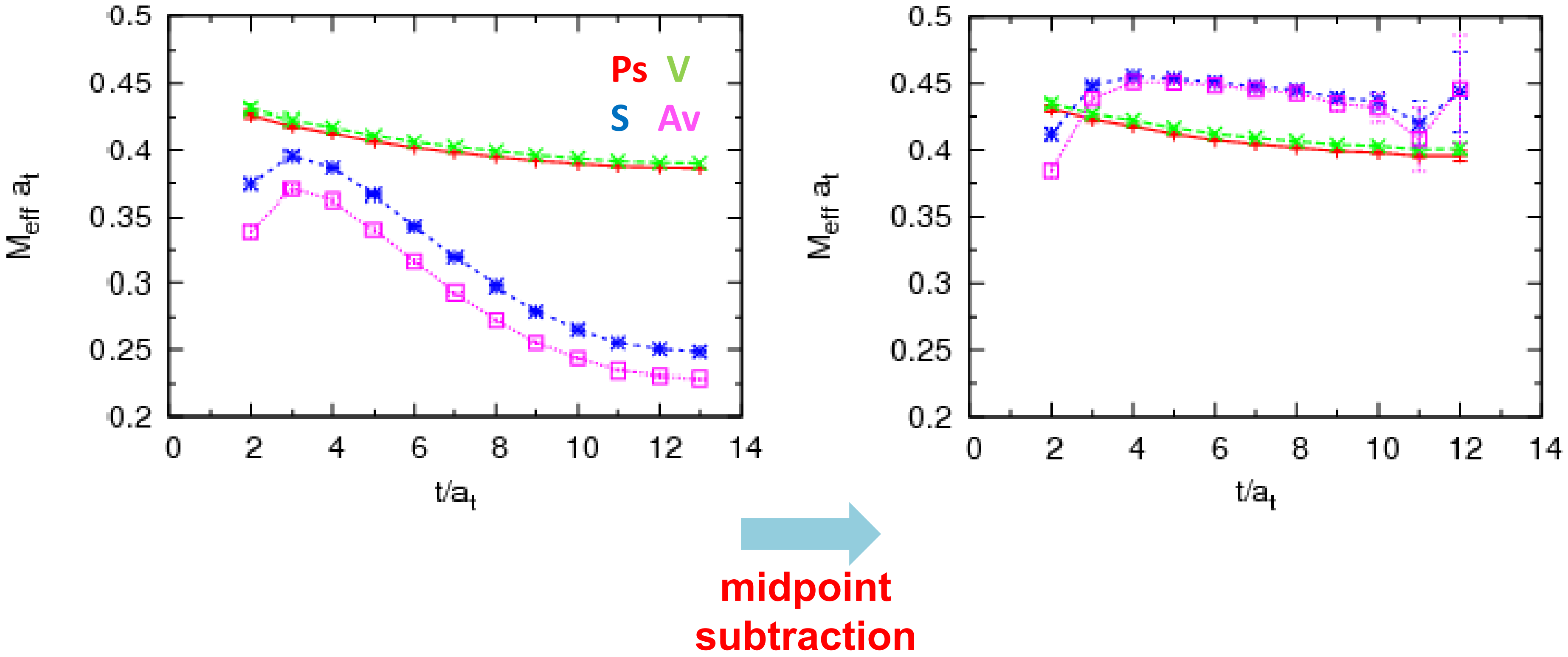
$$\bar{C}(t) \equiv C(t) - C(L_t/2)$$

$$= 2A_1 \sinh^2\left[\frac{m_1}{2}\left(t - \frac{L_t}{2}\right)\right] + 2A_2 \sinh^2\left[\frac{m_2}{2}\left(t - \frac{L_t}{2}\right)\right] + \cdots$$

  – **Constant mode is separated from meson correlator.**

Constant mode effect for effective masses

- Effective masses at $1.1T_C$



There is large constant mode effect for P wave above T_C.

Constant mode should be subtracted from P wave meson correlaors.

Multi-state variational analysis

- Smeared meson operator

$$\mathcal{O}^i_\Gamma \equiv \sum_{\vec{y},\vec{z}} \omega_i(\vec{y})\omega_i(\vec{z})\bar{q}(\vec{x}+\vec{y},t)\Gamma q(\vec{x}+\vec{z},t)$$

$$\omega_i(\vec{x}) \equiv N\ e^{-A_i|\vec{x}|^2}$$

: smearing function
$i=1,2,\ldots,N_{state}$

The parameters A_i

A_1	A_2	A_3	A_4	A_5	A_6
0.02	0.05	0.10	0.15	0.20	0.25

- Effective mass

$$C_{ij}(t) = \sum_{\vec{x}} \langle \mathcal{O}^i_\Gamma(\vec{x},t)\mathcal{O}^{j\dagger}_\Gamma(\vec{0},0)\rangle$$

  - Midpoint subtraction

$$\bar{C}_{ij}(t) = C_{ij}(t) - C_{ij}(L_t/2)$$

  - General eigenvalue equation

$$\bar{C}(t)v_k = \lambda_k(t;t_0)\bar{C}(t_0)v_k$$

$$\lambda_k(t;t_0) = \frac{\sinh^2\left[\frac{M_k^{eff}}{2}\left(t-\frac{L_t}{2}\right)\right]}{\sinh^2\left[\frac{M_k^{eff}}{2}\left(t_0-\frac{L_t}{2}\right)\right]}$$

- Wave function

$$BS_i(\vec{r},t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x},t)\Gamma' q(\vec{x}+\vec{r},t)\mathcal{O}^{i\dagger}_\Gamma(\vec{0},0)\rangle$$

  - Midpoint subtraction

$$\overline{BS}_i(\vec{r},t) = BS(\vec{r},t) - BS(\vec{r},L_t/2)$$

$$\sum_i \overline{BS}_i(\vec{r},t)V_{ik} = \psi_k(\vec{r})\sinh^2\left[\frac{M_k}{2}\left(t-\frac{L_t}{2}\right)\right]$$

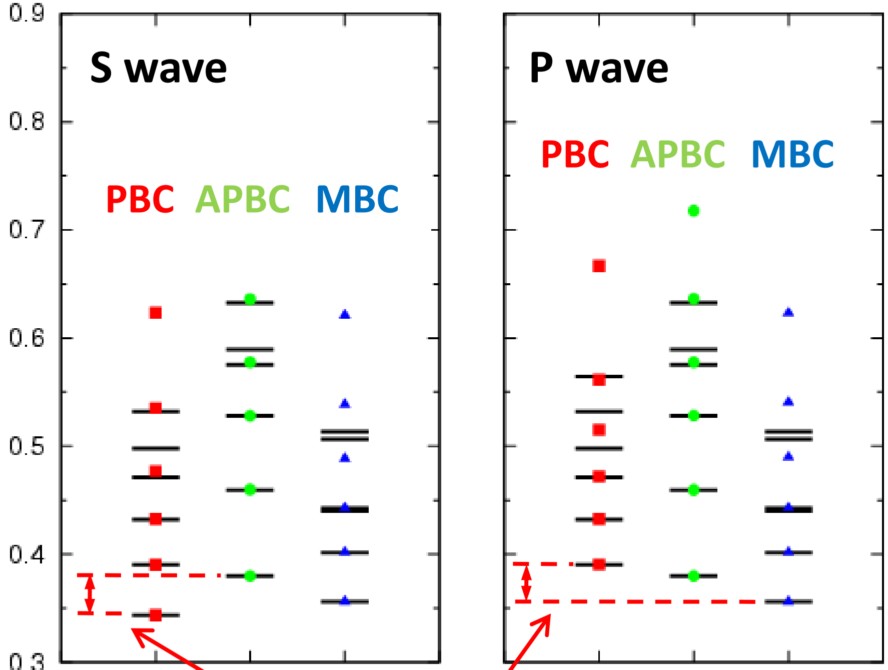
$$V_{ij} = (v_i)_j$$

$$\Psi(\vec{r};\vec{r}_0) \equiv \frac{\sum_i \overline{BS}_i(\vec{r},t)V_{ik}}{\sum_i \overline{BS}_i(\vec{r}_0,t)V_{ik}} = \frac{\psi_k(\vec{r})}{\psi_k(\vec{r}_0)}$$

Variational analysis in free case

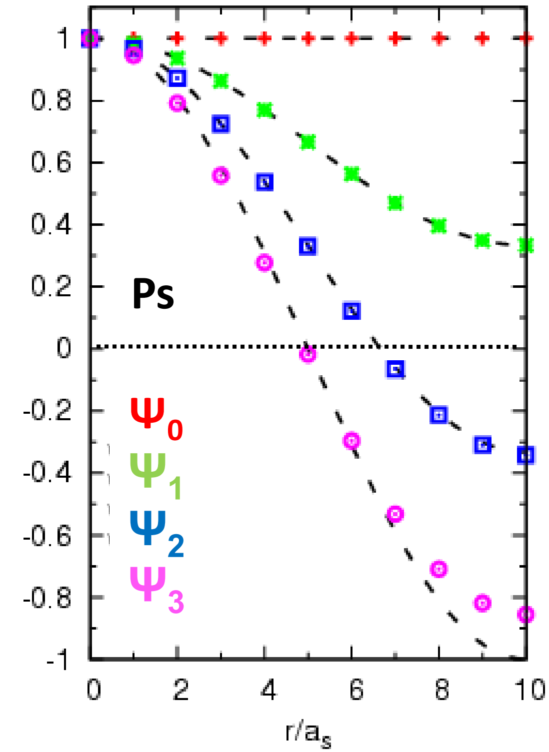
N_{state}=6　$20^3 \times 128$ anisotropic lattice

- Effective mass



Mass shift due to BC

- Wave function



Wave functions extend to large distances.

MBC : Mixed B.C.
(x, y, z)=(AP, P, P)

solid line
: analytical solution

dashed line
: analytical solution

Charmonium or scattering state?

- In finite volume space, scattering states have typical spatial BC dependences.

H. Iida et al.,Phys. Rev. **D74**, 074502 (2006)

Bound states (charmonia)

- Wave function
  - **localized shape**
  - **insensitive to BC**

- Effective mass

$$\text{PBC}\ :\ M_{\text{eff}}^{\text{PBC}} \simeq \text{bound state mass}$$
$$\text{APBC}:\ M_{\text{eff}}^{\text{APBC}} \simeq \text{bound state mass}$$
$$M_{\text{eff}}^{\text{APBC}} - M_{\text{eff}}^{\text{PBC}} \simeq 0$$

Scattering states

- Wave function
  - **extends to large distance**
  - **sensitive to BC**

- Effective mass

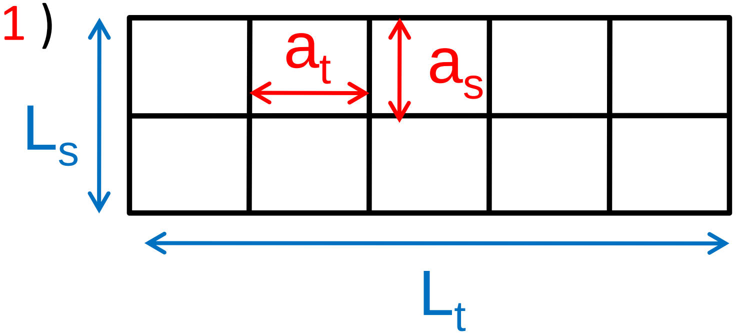
$$\text{PBC}\ :\ M_{\text{eff}}^{\text{PBC}} \simeq 2m_c$$
$$\text{APBC}:\ M_{\text{eff}}^{\text{APBC}} \simeq 2\sqrt{m_c^2 + 3\pi^2/L_s^2}$$
$$M_{\text{eff}}^{\text{APBC}} - M_{\text{eff}}^{\text{PBC}} : \text{finite}$$

m_c : charm quark mass

$$p_{\text{PBC}} = \frac{2n\pi}{L_s}, \quad p_{\text{APBC}} = \frac{(2n+1)\pi}{L_s} \quad n = 0, 1, 2, \ldots$$

Lattice setup

- Action
  - O(a) improved Wilson fermion action (r_s=1)
  - Standard plaquette gauge action
  - Quenched approximation



- Lattice
  - anisotropic lattice → anisotropy $\xi=a_s/a_t=4$
  - L_s=16, 20(, 32)
  - L_t=(8 (3.2T_C),) 12 (2.3T_C), 16 (1.8T_C), 20 (1.4T_C), 26 (1.1T_C), 32 (0.88T_C)
  - a_s=0.0970(5) fm (2.030(13) GeV)

Temperatures are changed in terms of changing temporal lattice size.

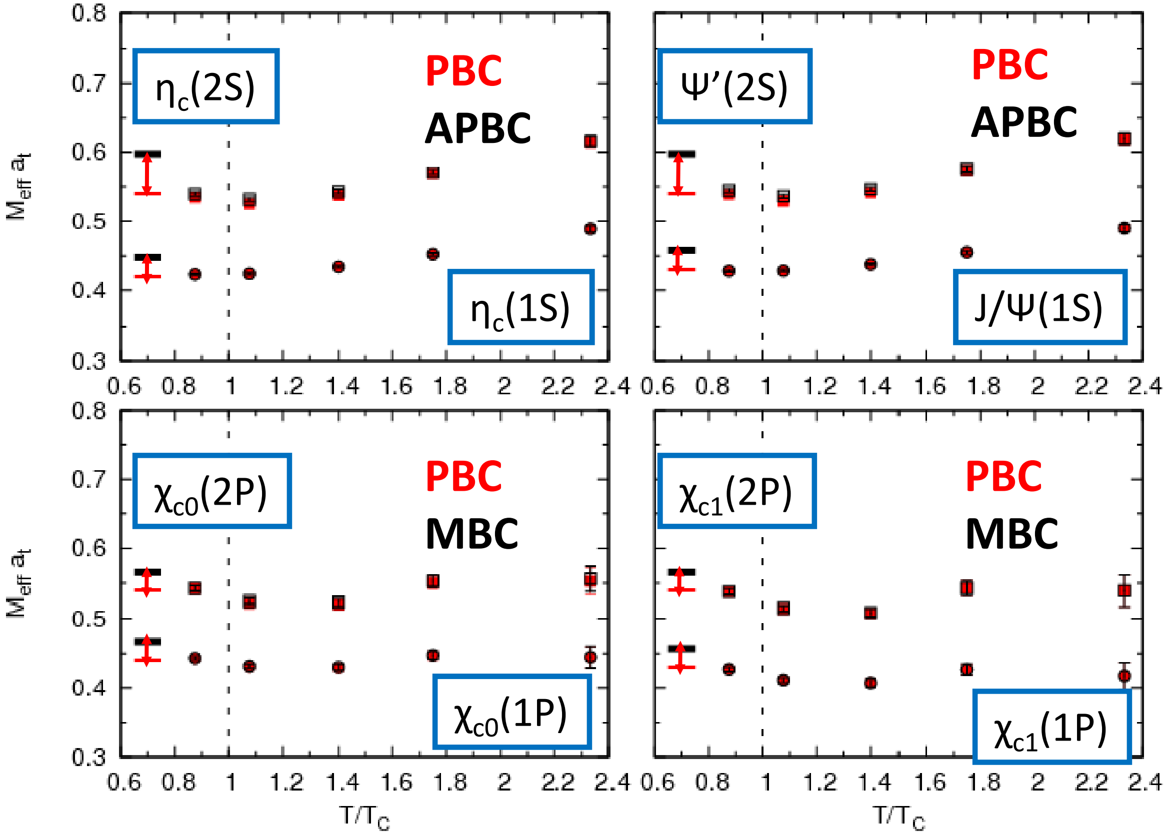
- Gauge configuration

	L_s=16	L_s=20	L_s=32
Local op.	800	800	200
Derivative op.	300	300	200

- Gauge fixing : Coulomb gauge

Numerical results : effective mass

- Temperature and spatial BC dependence



N_{state}=4
(Results with N_{state}=6 consistent)
$20^3 \times L_t$ lattice

○ : the ground state
□ : the first excited state

MBC : Mixed B.C.
(x, y, z)=(AP, P, P)

: mass shift for the free case

There is no clear BC dependence for all charmonium states up to 2.3 T_C.

There seems to be no scattering state contribution up to 2.3T_C.

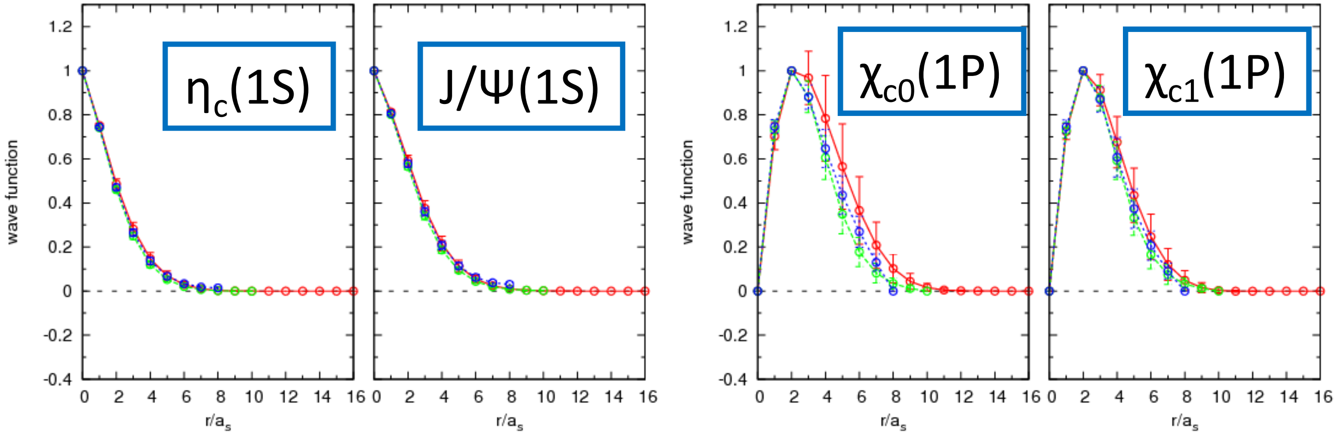
Numerical results : wave function (2.3T_C)

- Volume dependence

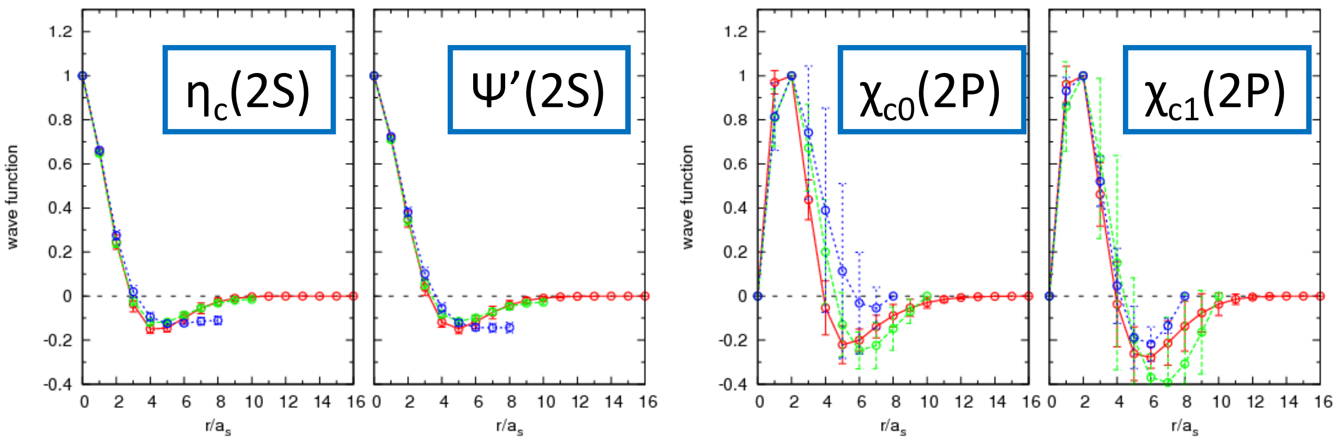
L_s=32
L_s=20
L_s=16

N_{state}=4

The ground state



The first excited state



- No sensible volume dependences
- Spatially localized even at T=2.3T_C for both ground state and 1st excited state
- Same for larger N_{state}

Charmonia still survive at 2.3T_C.

Conclusion & future plan

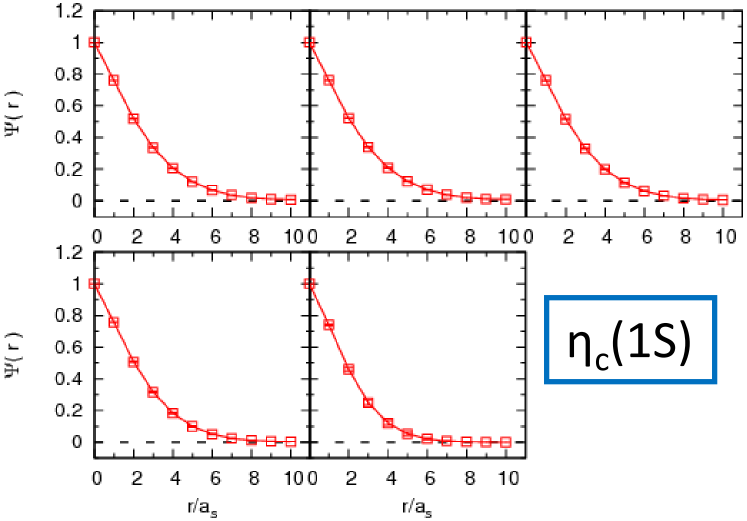
- Charmonia dissociation temperatures are studied with variational analysis in quenched anisotropic lattice QCD.
  - **There is large constant mode effect for P wave above T_C.**
    P wave charmonia still survive up to $2.3T_C$ when constant mode effect is considered.
  - **No scattering state appears up to $2.3T_C$**
  - **We find no clear evidences of dissociation for the all charmonium states (η_c, J/ψ, χ_{c0}, χ_{c1} and their first excited states) up to $2.3T_C$ so far.**
  - **Sequential charmonum dissociation scenario based on the naïve dissociation picture may be more complicated.**

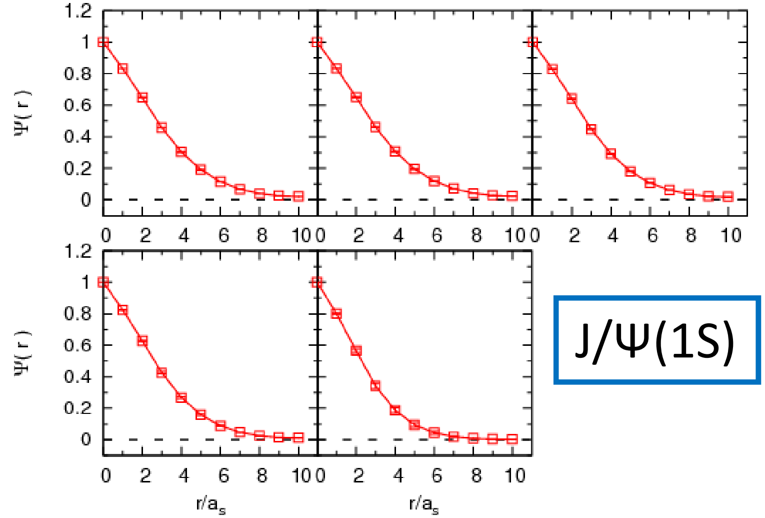
- Future plan
  - **Study to investigate how charmonia really dissolve**
  - **Simulation at higher temperatures**
  - **Full QCD simulation**

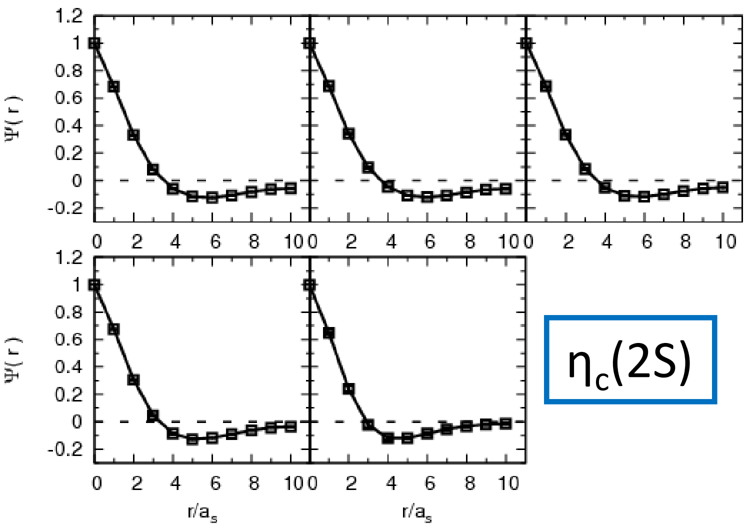
END

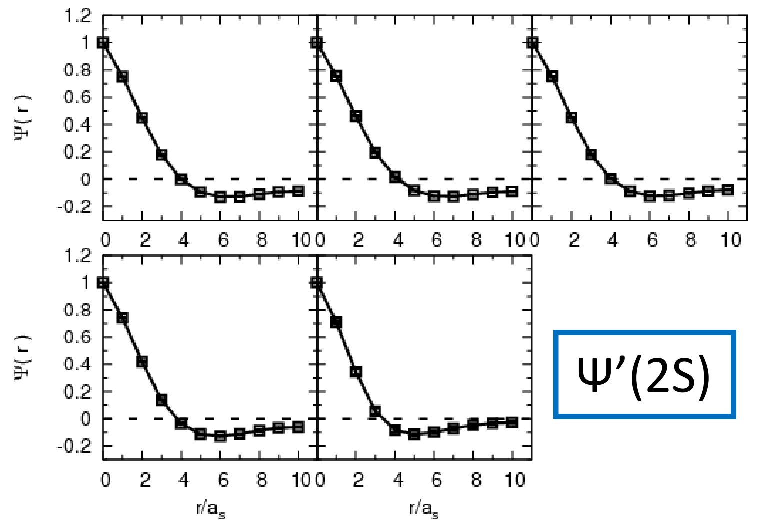
Numerical results : wave function (2)

- Temperature dependence (S wave)





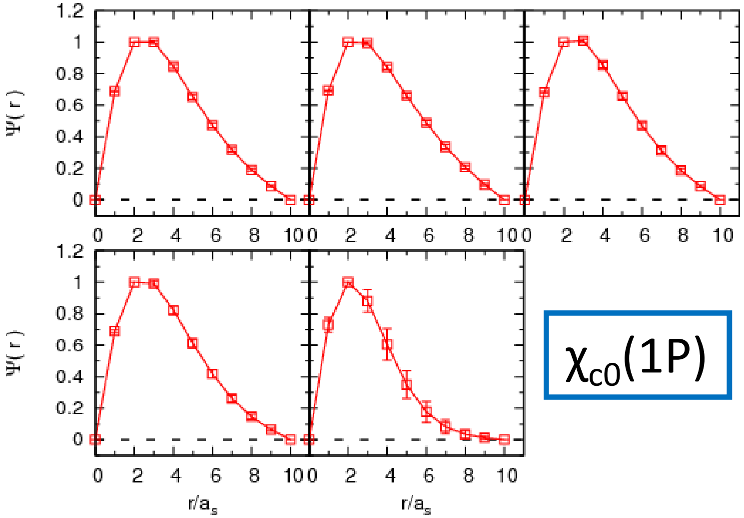




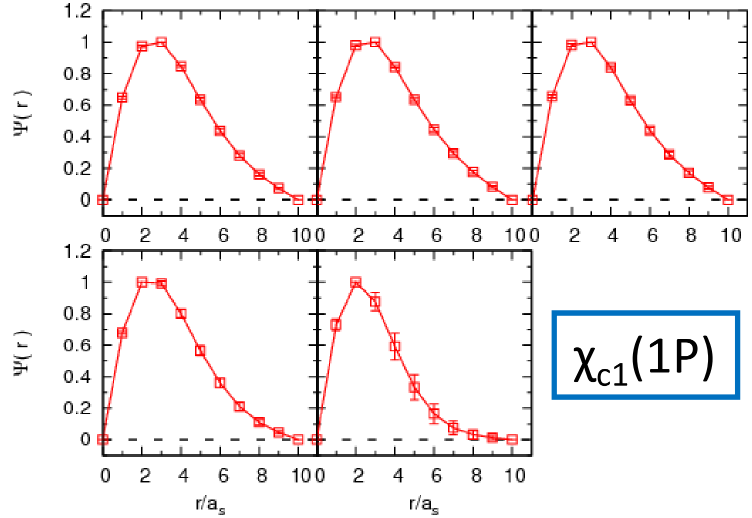
- **Spatially localized up to 2.3 $T_{C.}$**

Numerical results : wave function (3)

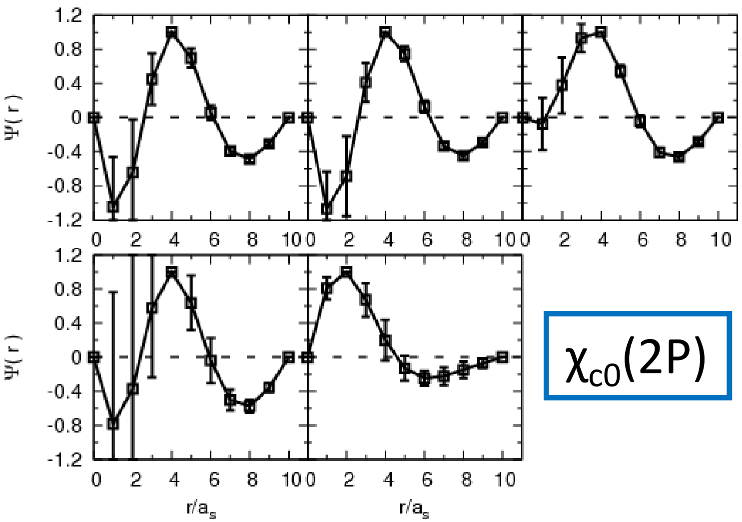
- Temperature dependence (P wave)



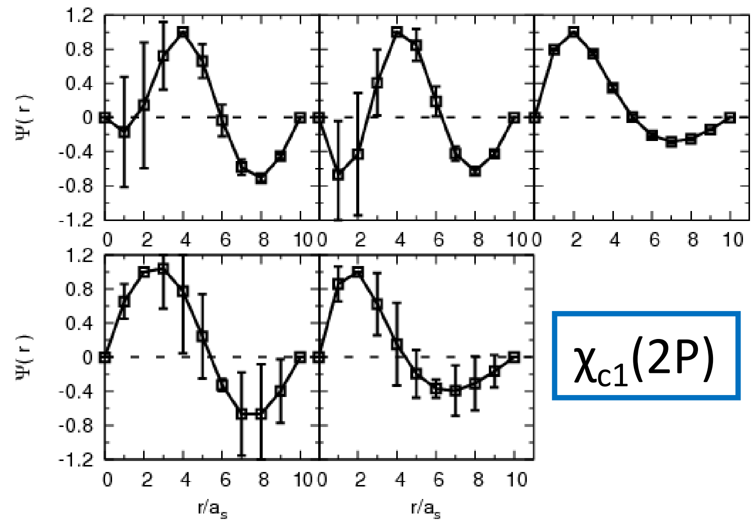
$\chi_{c0}(1P)$



$\chi_{c1}(1P)$



$\chi_{c0}(2P)$

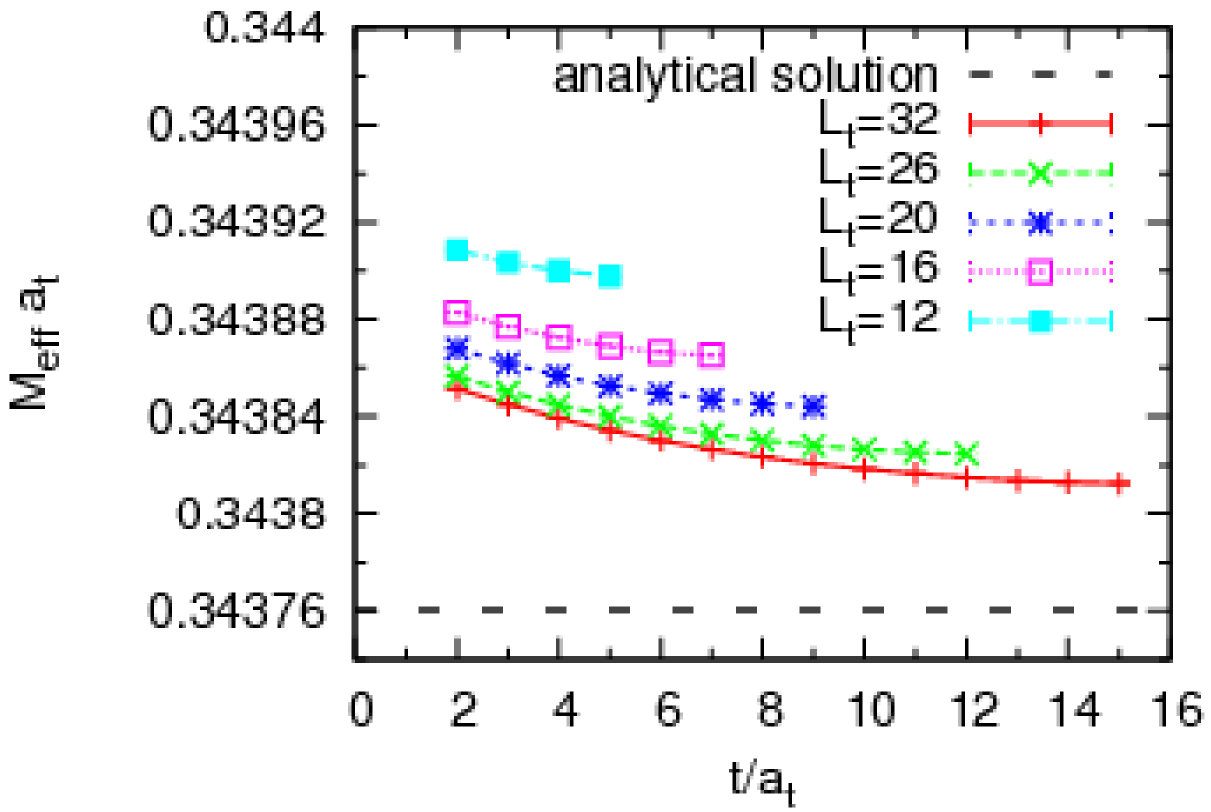


$\chi_{c1}(2P)$

- **Spatially localized up to 2.3 $T_{C.}$**

Systematic error of varational anaylsis

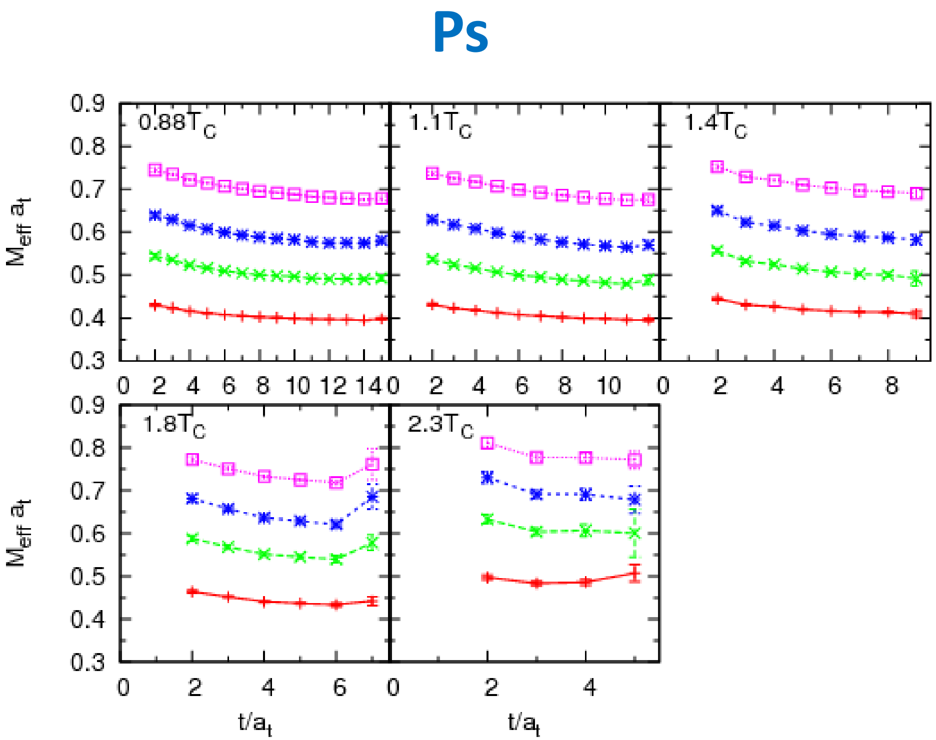
- L_t dependence of effective mass in free case

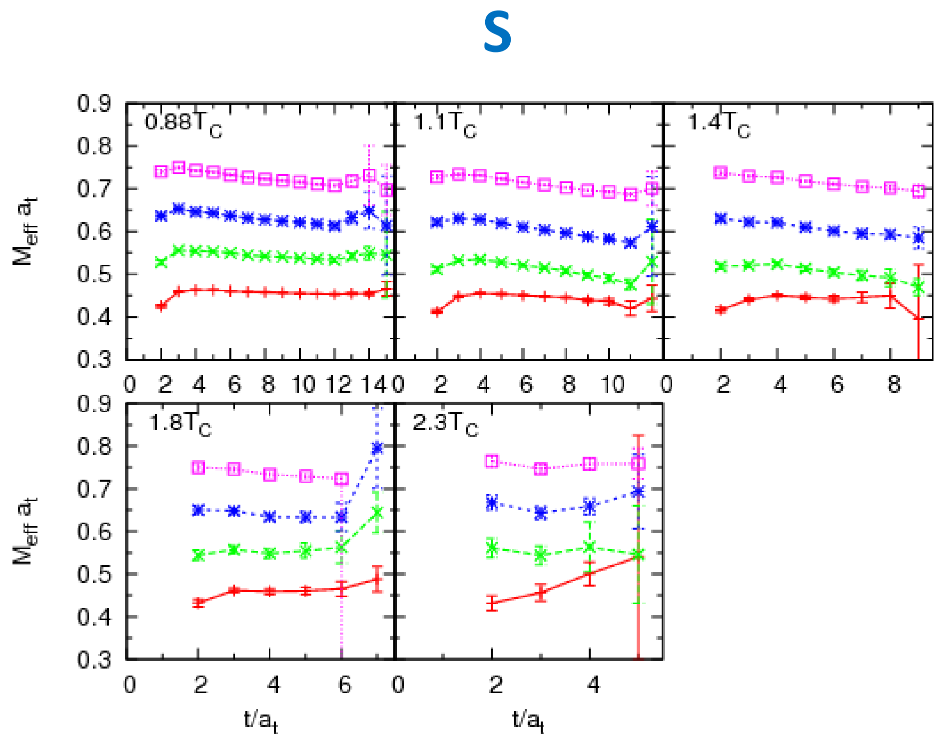


Effective mass shifts up side as L_t becomes smaller.

Stability of effective masses

- t dependence of effective mass

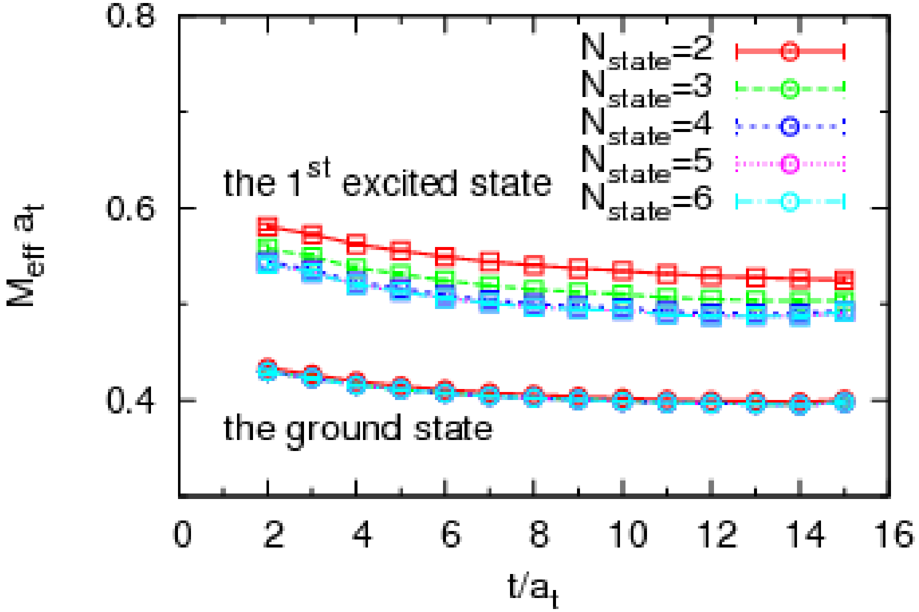




Effective masses are stable up to 2.3 T_C.

N_{state} dependence : effective mass

Ps



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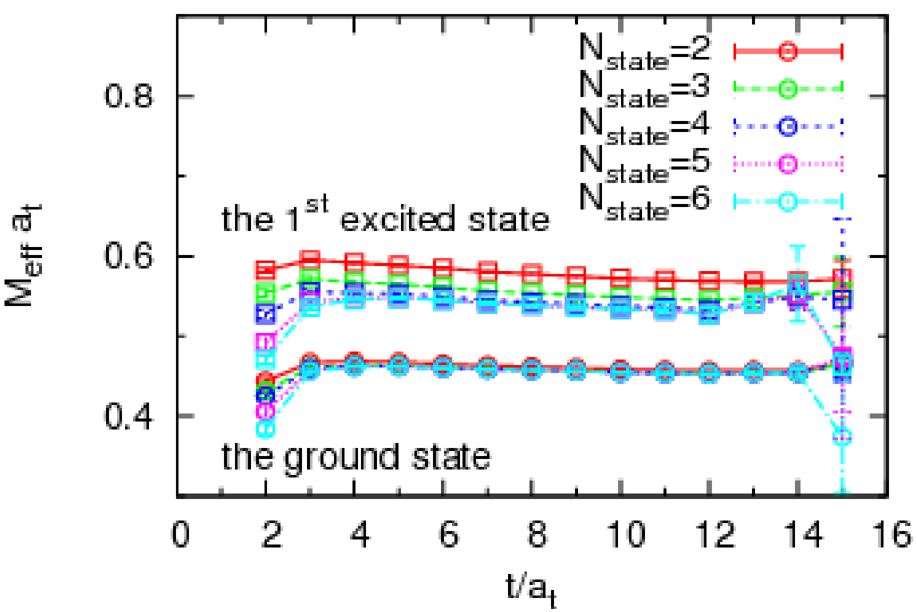
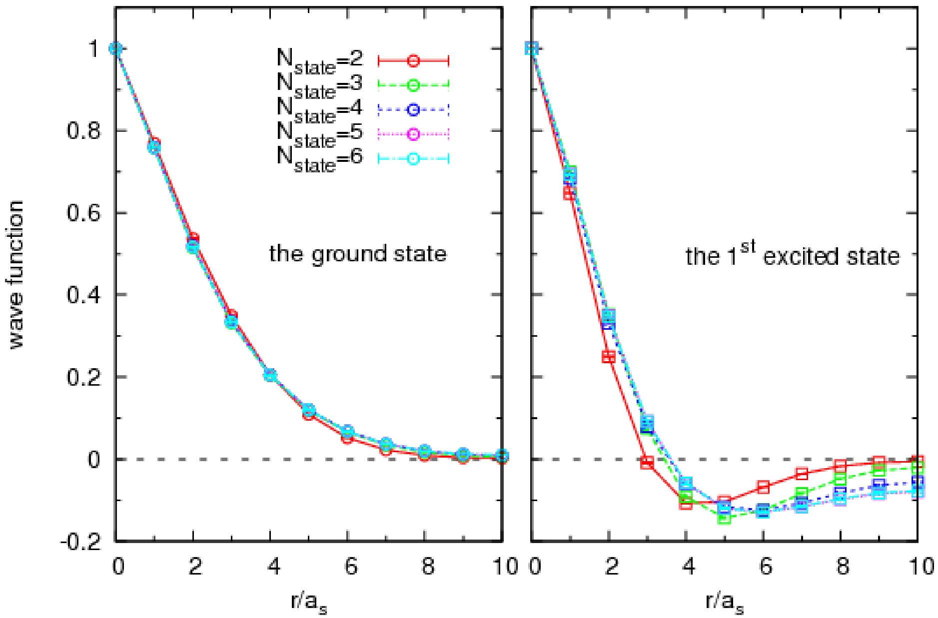


N_{state} dependence : wave function

Ps



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