Lambda-nucleon force from lattice QCD

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for PACS-CS Collaboration

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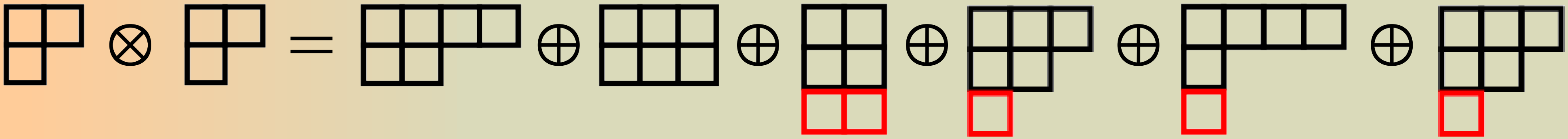

Introduction:

- Study of hyperon-nucleon (*YN*) and hyperon-hyperon (*YY*) interactions is one of the important subjects in the nuclear physics.
  - Structure of the neutron-star core,
    - Hyperon mixing, softning of EOS, inevitable strong repulsive force,
  - H-dibaryon problem,
    - To be, or not to be,
- The project at J-PARC:
  - Explore the multistrange world,
- However, the phenomenological description of *YN* and *YY* interactions has large uncertainties, which is in sharp contrast to the nice description of phenomenological *NN* potential.

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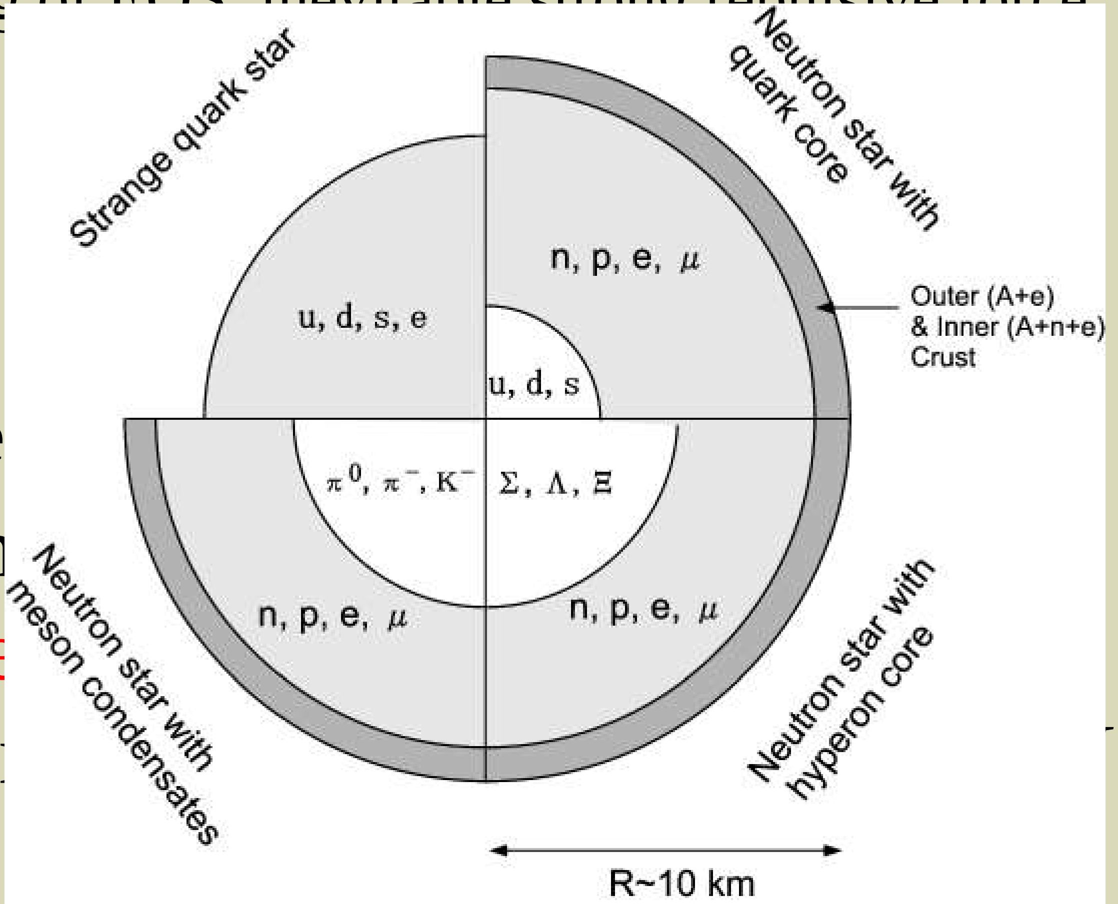

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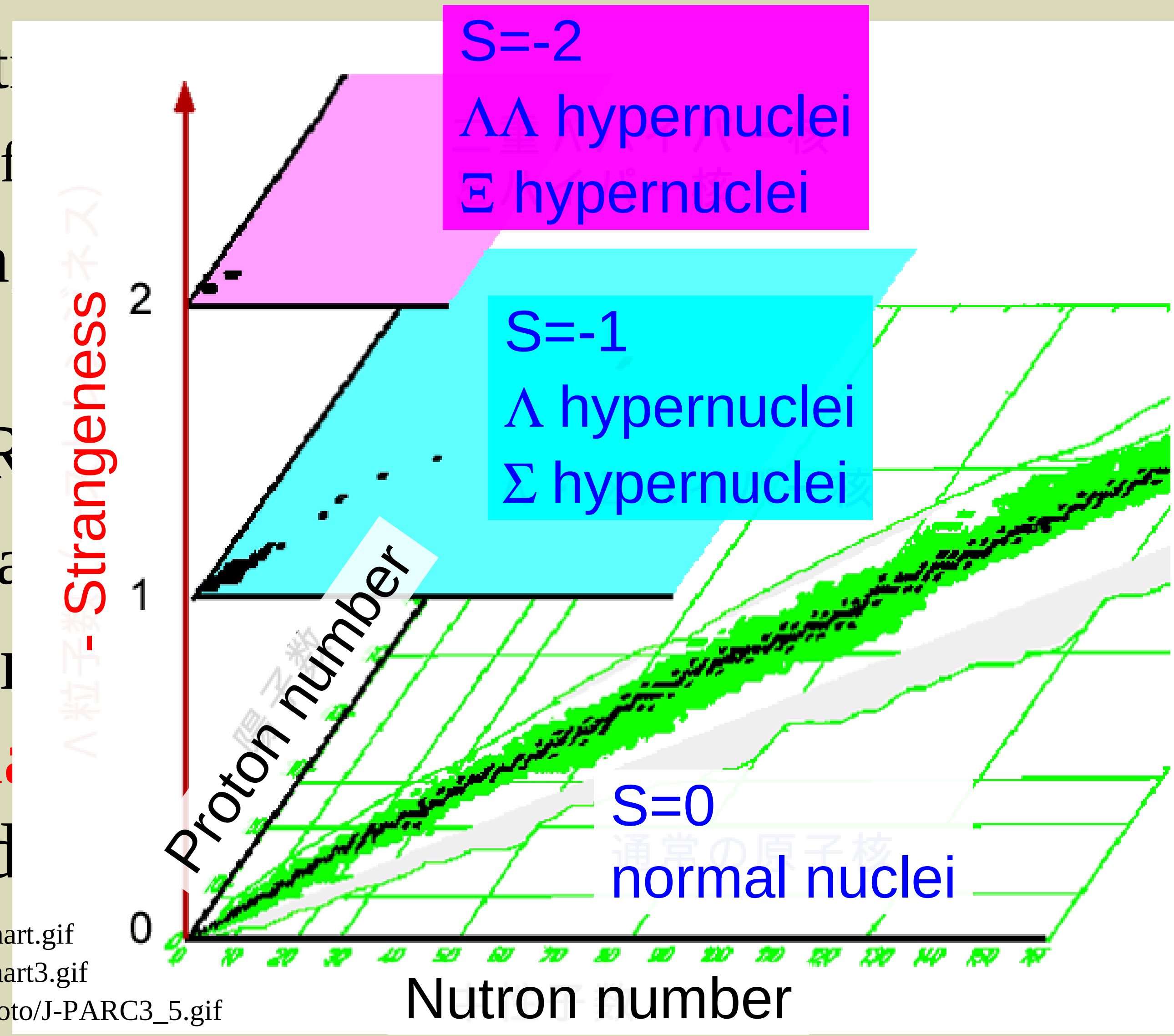
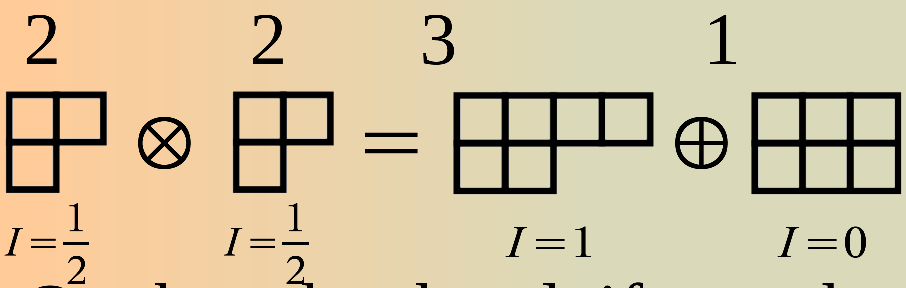


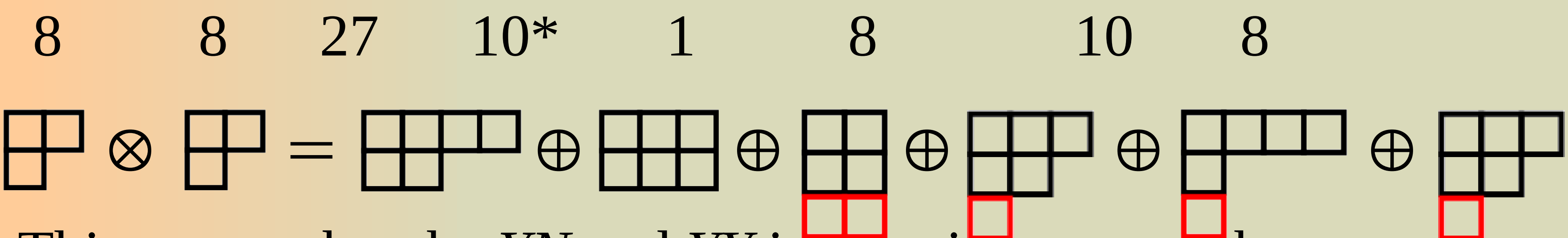
http://www.rarf.riken.go.jp/RIBF/nuclear_chart.gif
http://wwwa1.kph.uni-mainz.de/Hyp2006/chart3.gif
http://www.kek.jp/newskek/2005/marapr/photo/J-PARC3_5.gif

Extension from NN to YN and YY:

- If we take only non-strange sector,
  there are only 2 representations for isospin space.



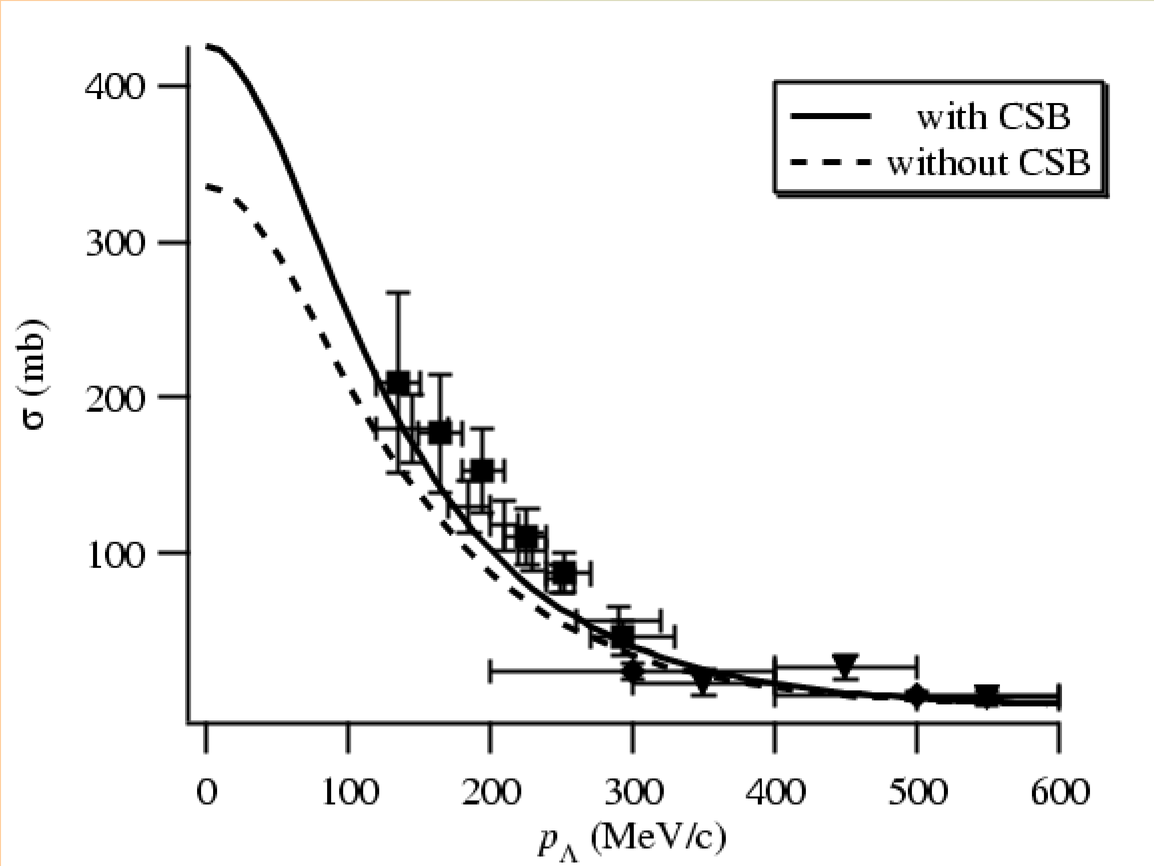
- On the other hand, if we take account of strange degree of freedom, other representations should be included.

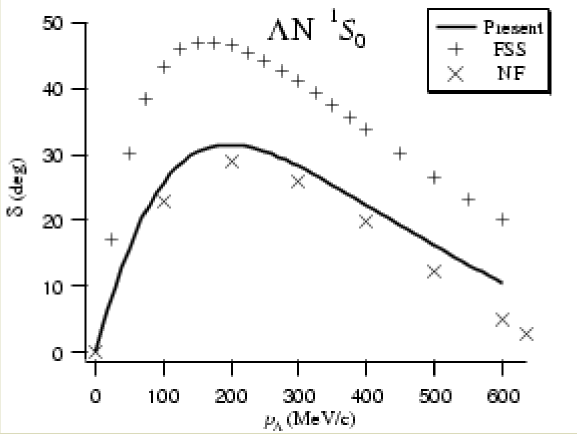


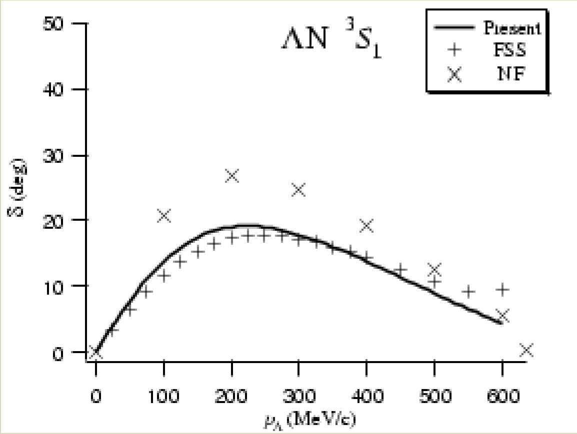
- This means that the *YN* and *YY* interactions cannot be determined from the precise *NN* experimental data even if we assume the flavor SU(3) symmetry.
- Lattice QCD is desirable for the study of the *YN* and *YY* interaction, because this is *ab initio* numerical simulation.

Experimental data for ΛN interaction:

- Only total corss section.
- No phase shift analysis is avairable.
- Spin-dependence is unclear







Recent impressive works of lattice QCD:

- S. Aoki, *et al.*, PRD**71**, 094504 (2005);
  π-π scattering length from the wave function.
- N. Ishii, *et al.*, PRL**99**, 022001 (2007); nucl-th/0611096;
  NN potential from the wave function.

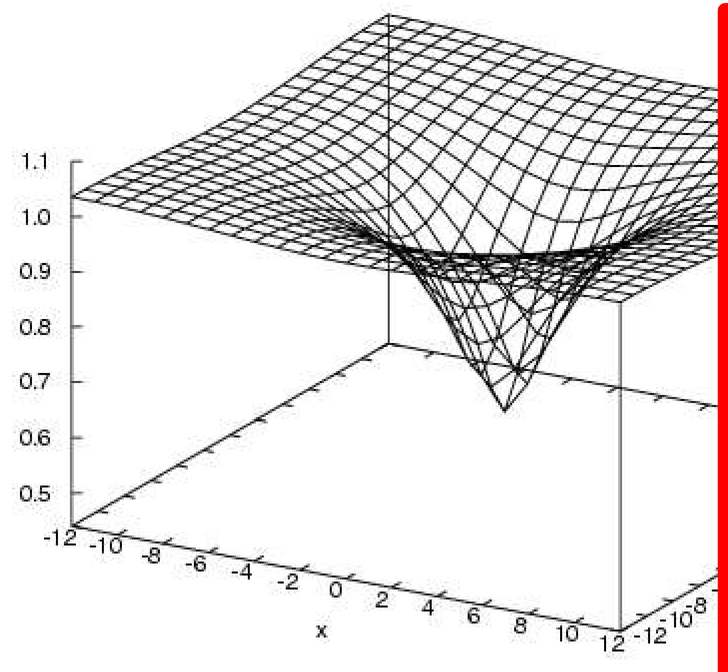
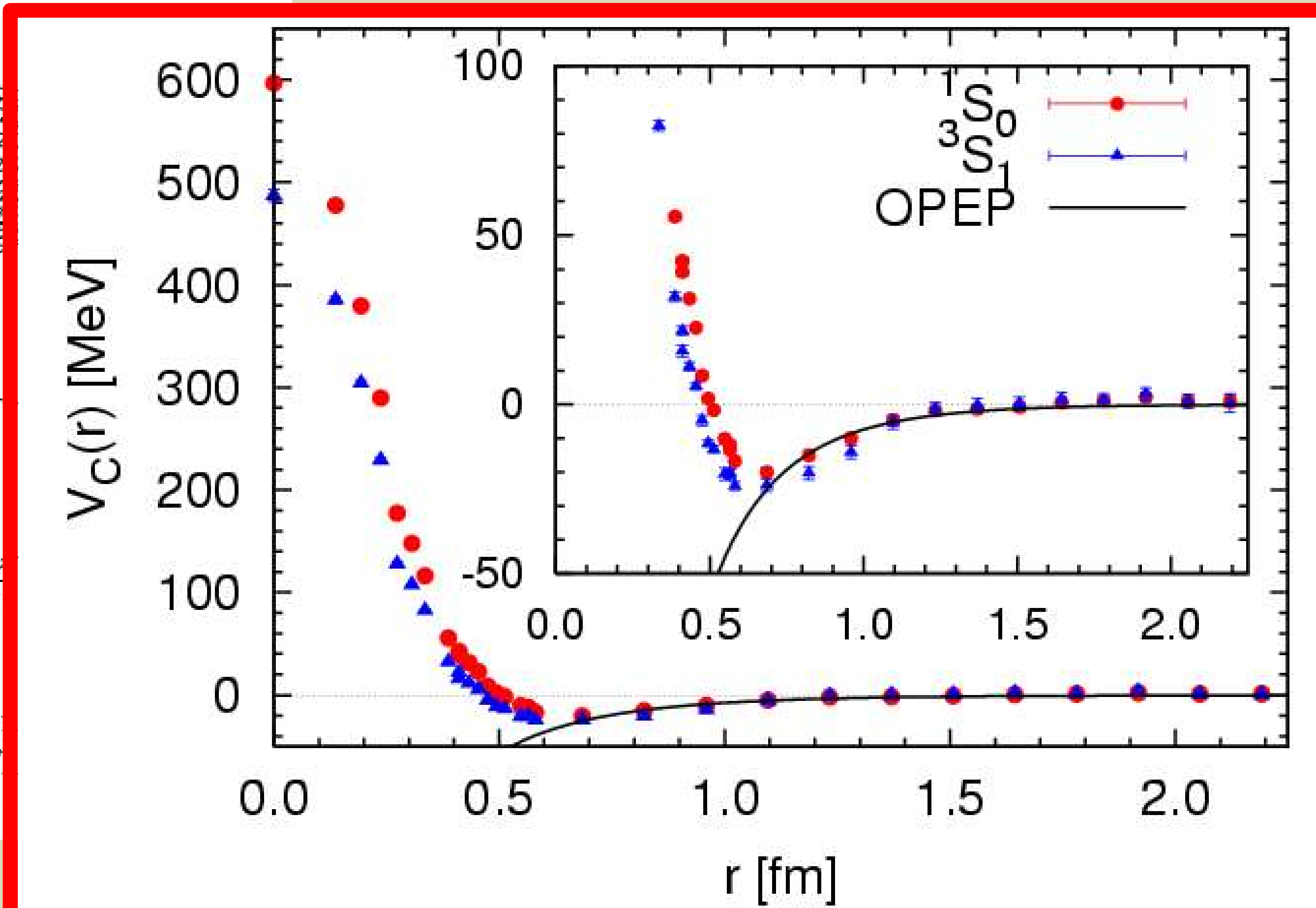


FIG. 1. Two-pion wave function $\phi(\vec{x};k)$ o… $(t,z) = (52,0)$ plane for $m_\pi^2 = 0.273$ GeV^2. T… tor is set at $\vec{x}_0 = (7,5,2)$ ($x_0 = |\vec{x}_0| = 8.832$).

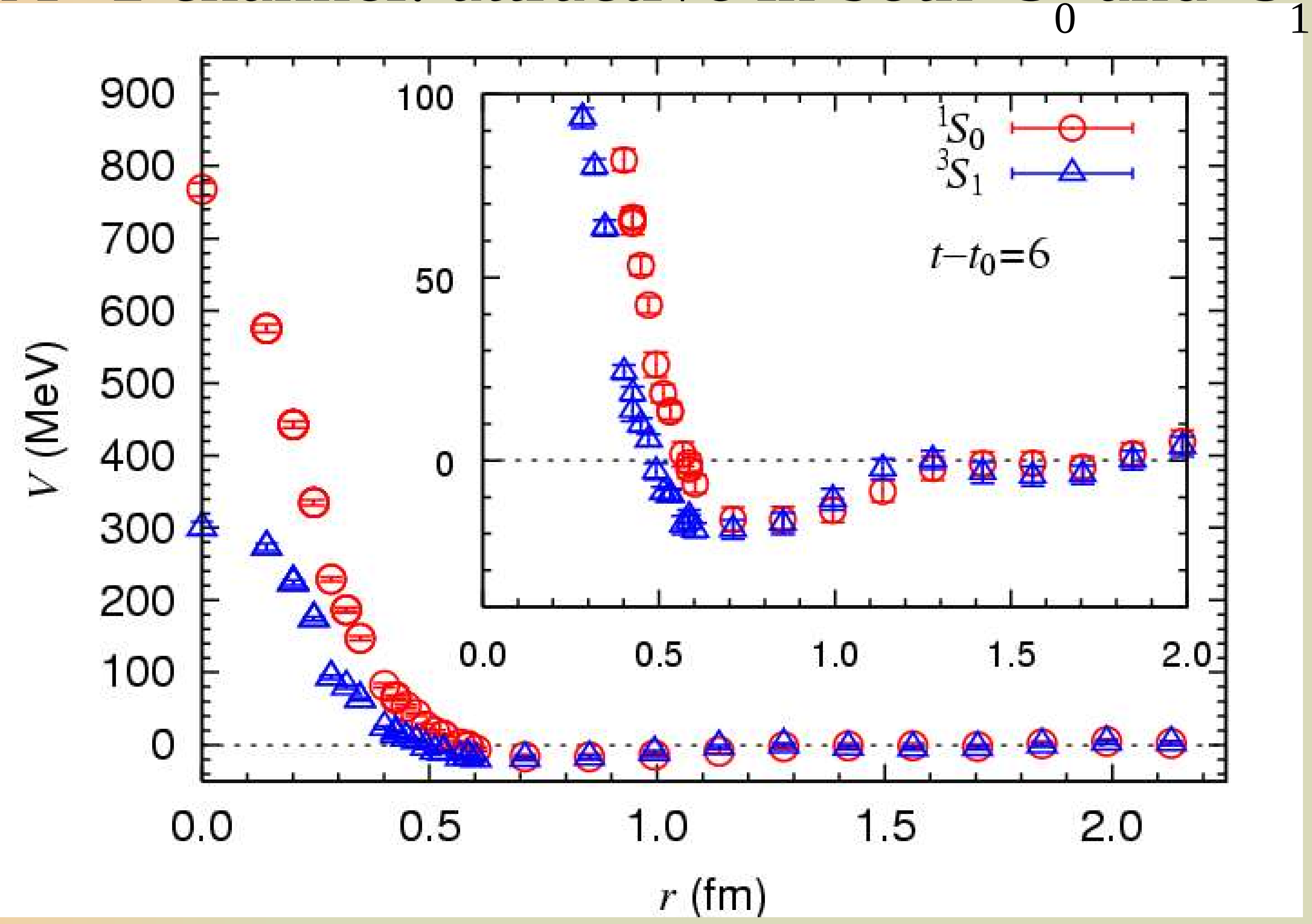


- This work;
  YN and *YY* potentials by applying these techniques.

The first NΞ calculation from lattice QCD

arXiv:0806.1094 [nucl-th]

- Quenched QCD calculation
- Volume: $32^3 \times 32$
- Lattice scale: a=0.14fm (L ~ 4.5 fm)
- Isospin I=1 channel: attractive in both $^1S_0$ and $^3S_1$.



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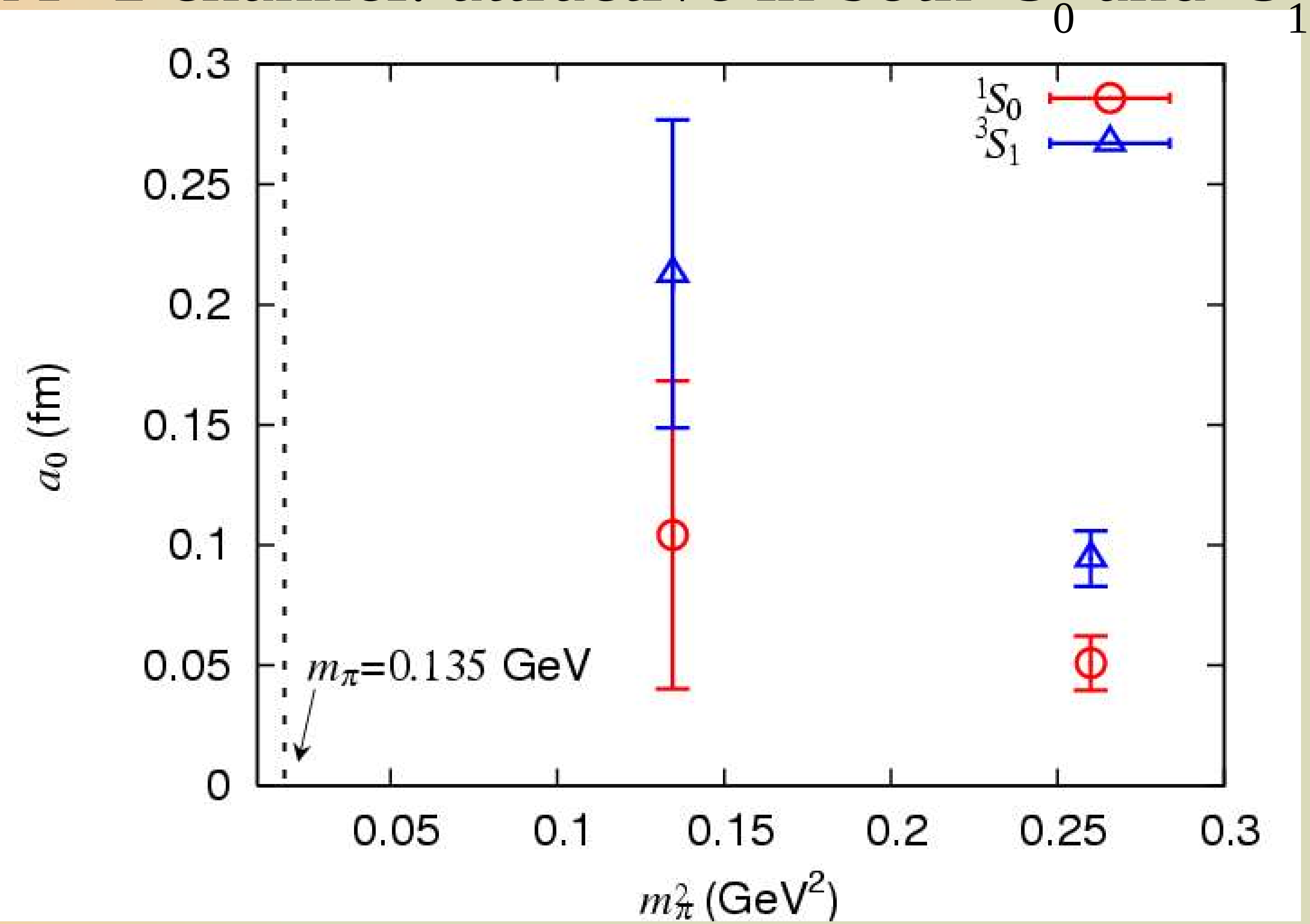

The purpose of this work

- $N\Lambda$ force from lattice QCD

- Spin dependence (Scattering lengths)
- Potential (explore the flavor dependence of baryon potentials)

- Numerical calculation is twofold:
  - Full lattice QCD by using N_F=2+1 PACS-CS full QCD gauge configurations with the spatial lattice volume (2.86 fm)^3
  - Quenched lattice QCD with larger spatial lattice volume (4.5 fm)^3

A recipe for NΛ potential:

- More accurate explanation, see, e.g., arXiv:0805.2462[hep-ph].

- Start from an effective Schroedinger eq for the equal-time Bethe-Salpeter wave funciton:

$$-\frac{1}{2\mu}\nabla^2\phi(\vec{r})+\int d^3r' U(\vec{r},\vec{r}')=E\phi(\vec{r})$$

$$U(\vec{r},\vec{r}')=V_{N\Lambda}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

- A general expression of the potential:

$$V_{N\Lambda}=V_0(r)+V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_\Lambda)$$
$$+V_T(r)S_{12}+V_{LS}(r)(\vec{L}\cdot\vec{S}_+)$$
$$+V_{ALS}(r)(\vec{L}\cdot\vec{S}_-)+O(\nabla^2)$$

A recipe for NΛ potential:

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- The equal time BS wave function in the *S*-wave on the lattice,

$$\phi(\vec{r})=\frac{1}{24}\sum_{R\in O}\frac{1}{L^3}\sum_{\vec{x}}P^{\sigma}_{\alpha\beta}\left\langle 0\left|p_\alpha(R[\vec{r}]+\vec{x})\Lambda_\beta(\vec{x})\right|p\Lambda;k\right\rangle$$

$$p_\alpha(x)=\varepsilon_{abc}(u_a(x)C\gamma_5 d_b(x))u_{c\alpha}(x),$$

$$\Lambda_\alpha(x)=\varepsilon_{abc}\left\{(d_a C\gamma_5 s_b)u_{c\alpha}+(s_a C\gamma_5 u_b)d_{c\alpha}-2(u_a C\gamma_5 d_b)s_{c\alpha}\right\}$$

- The 4-point *N*Λ correlator on the lattice,

$$F_{p\Lambda}(\vec{x},\vec{y},t;t_0)=\left\langle 0\left|p_\alpha(\vec{x},t)\Lambda_\beta(\vec{y},t)\overline{J_{p\Lambda}}(t_0)\right|0\right\rangle$$
$$=\sum_n A_n\left\langle 0\left|p_\alpha(\vec{x})\Lambda_\beta(\vec{y})\right|n\right\rangle e^{-E_n(t-t_0)}$$

$\overline{J_{p\Lambda}}(t_0)$

wall source at $t=t_0$

A recipe for NΛ potential:

- More accurate explanation, see, e.g., arXiv:0805.2462[hep-ph].

- Calculate the 4-point $N\Lambda$ correlator on the lattice,

$$\phi_{N\Lambda}(x-y)e^{-E(t-t_0)} \propto \langle p_\alpha(x,t)\Lambda_\beta(y,t)\overline{\Lambda_{\beta'}}(0,t_0)\overline{p_{\alpha'}}(0,t_0)\rangle$$

  - Which has the physical meanings of,
    - Create a $N\Lambda$ state and making imaginary time evolution, in order to have the lowest state of the $N\Lambda$ system.
    - Take the amplitude $\phi(\boldsymbol{x}-\boldsymbol{y})$, which can be understood as a wave function of the non-relativistic quantum mechanics.

- Obtain the effective central potential from the effective Schroedinger equation.

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2+V(r)\right)\phi(r)=E\phi(r)$$

$$\rightarrow \quad V(r)=E+\frac{\hbar^2}{2\mu}\frac{\nabla^2\phi(r)}{\phi(r)}$$

Full QCD calculations by using N_F=2+1 PACS-CS gauge configurations:

- S. Aoki, et al., (PACS-CS Collaboration), arXiv:0807.1661 [hep-lat].
- Iwasaki gauge action at β=1.90 on $32^3 \times 64$ lattice
- O(a) improved Wilson quark action
- $1/a$ = 2.17 GeV (a = 0.0907 fm)
- $\kappa_{ud} = 0.13770$, $\kappa_s = 0.13640$
- $m_\pi = 0.30$GeV, $m_K = 0.59$GeV,
  $m_\rho = 0.84$GeV, $m_{K*} = 0.97$GeV
  $m_p = 1.1$GeV, $m_\Lambda = 1.2$GeV, $m_\Sigma = 1.3$GeV

Quenched calculation with larger spatial volume:

- Plaquette gauge action and Wilson fermion action
- Gauge coupling β=5.7
- Volume: $32^3 \times 48$ ($L \sim 4.5$ fm).
- Lattice spacing: $a \sim 0.14$ fm. ($1/a \sim 1.4$ GeV.)
- The lattice calculations were performed by using KEK Blue Gene/L supercomputer.
- The main results are obtained with
  - $\kappa_{ud} = 0.1665$ for the u and d quarks, and
  - $\kappa_s = 0.1643$ for s quark.

☞ Meson masses:

$m_\pi \sim 0.511.2(6)$ GeV

$m_\rho \sim 0.861(2)$ GeV

$m_K \sim 0.605.3(5)$ GeV

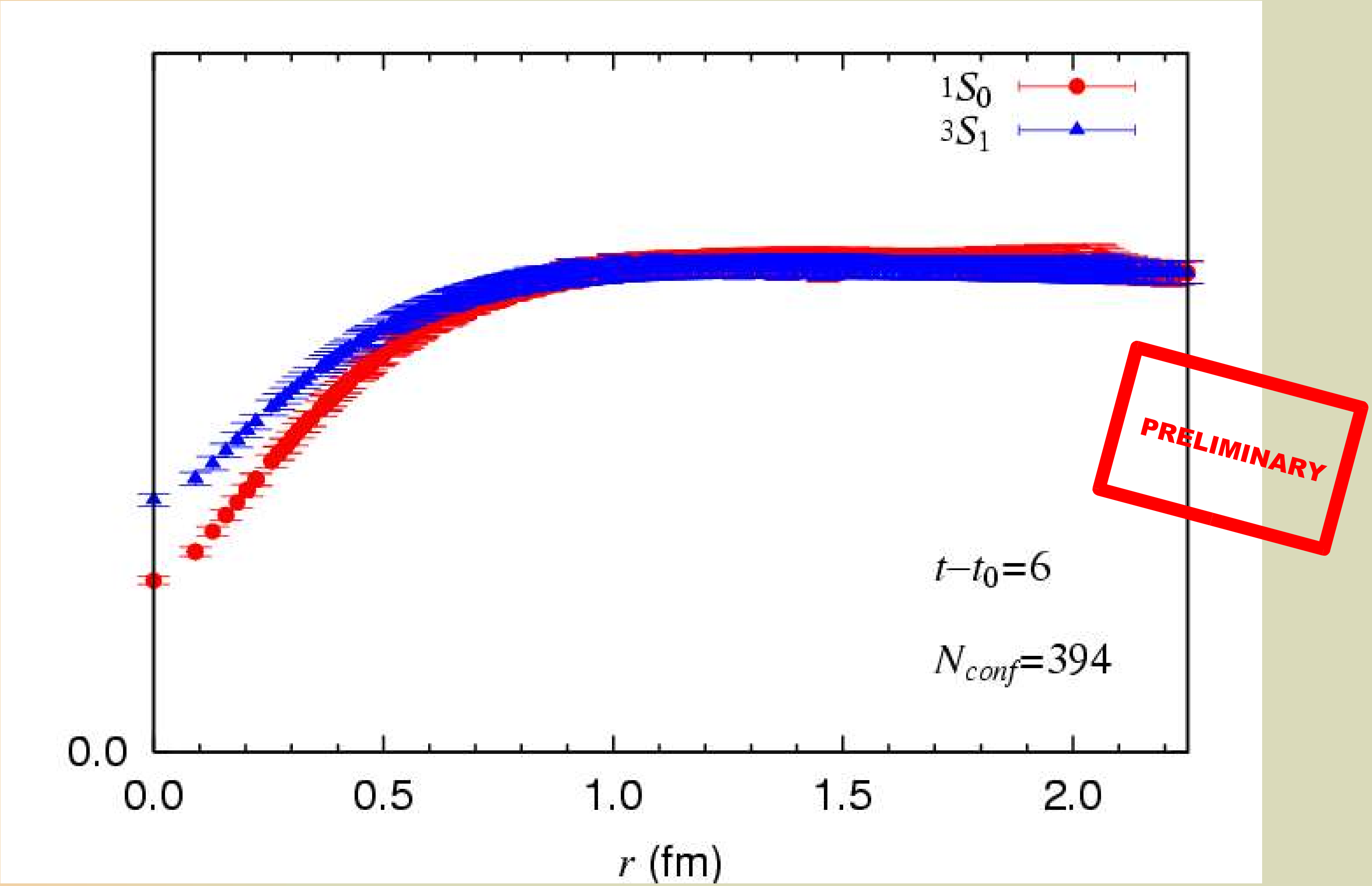
$m_{K^*} \sim 0.904(2)$ GeV

Results

Nf=2+1 Full QCD by using PACS-CS

Results —— wave function

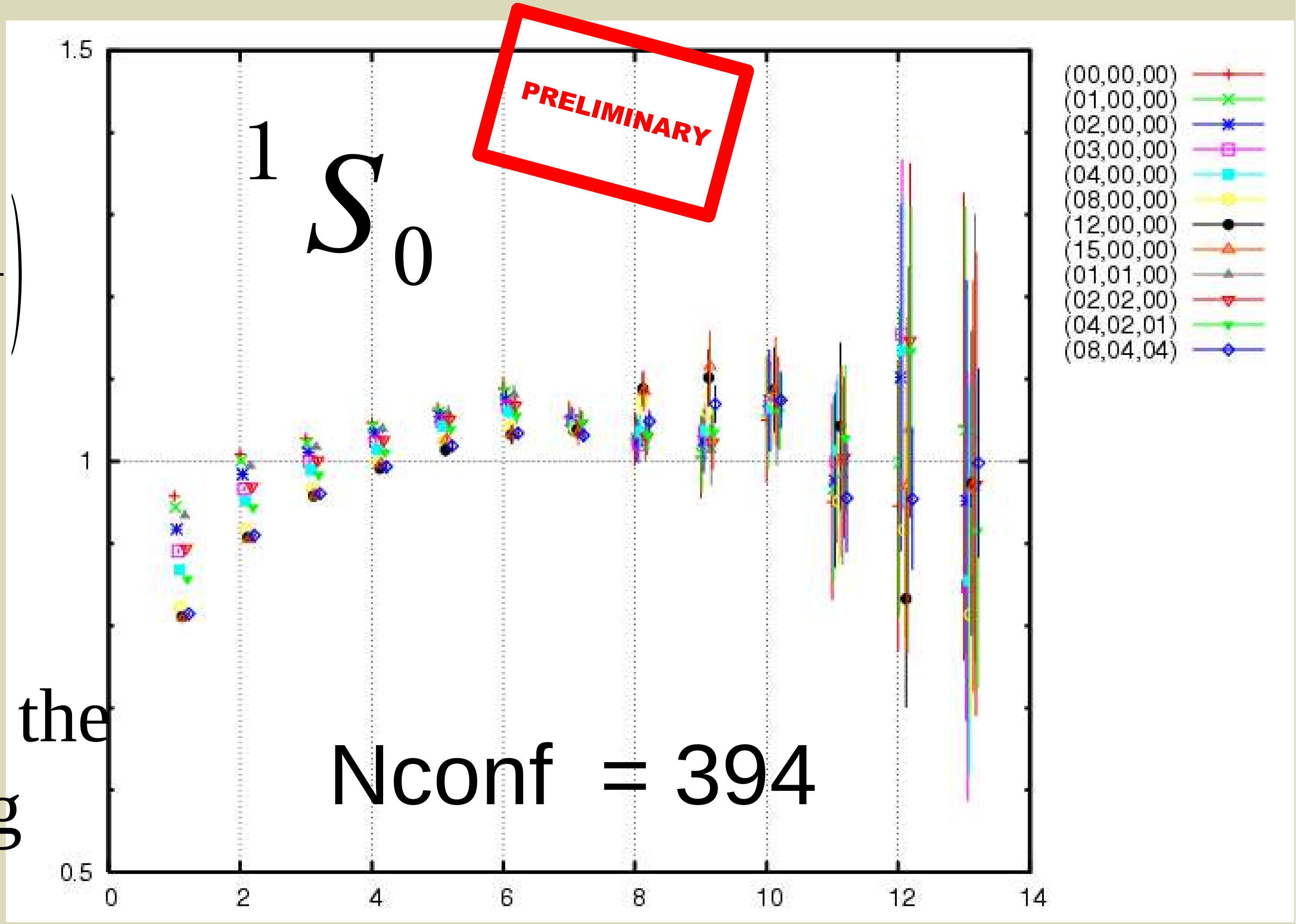
- Suggests the repulive core in short range for both spin S=0 and 1.



Results —— “effective mass”

- Time dependence of 4-point correlator to find the ground state (plateaux in the effective mass)

$$m_{eff}(t-t_{0,}\vec{r}=\vec{x}-\vec{y}) \equiv \log\left(\frac{F_{p\Lambda}(\vec{x},\vec{y},t;t_0)}{F_{p\Lambda}(\vec{x},\vec{y},t+1;t_0)}\right)$$



$t-t_0$

- Still hardly find the plateaux starting in the $^1S_0$ channel.

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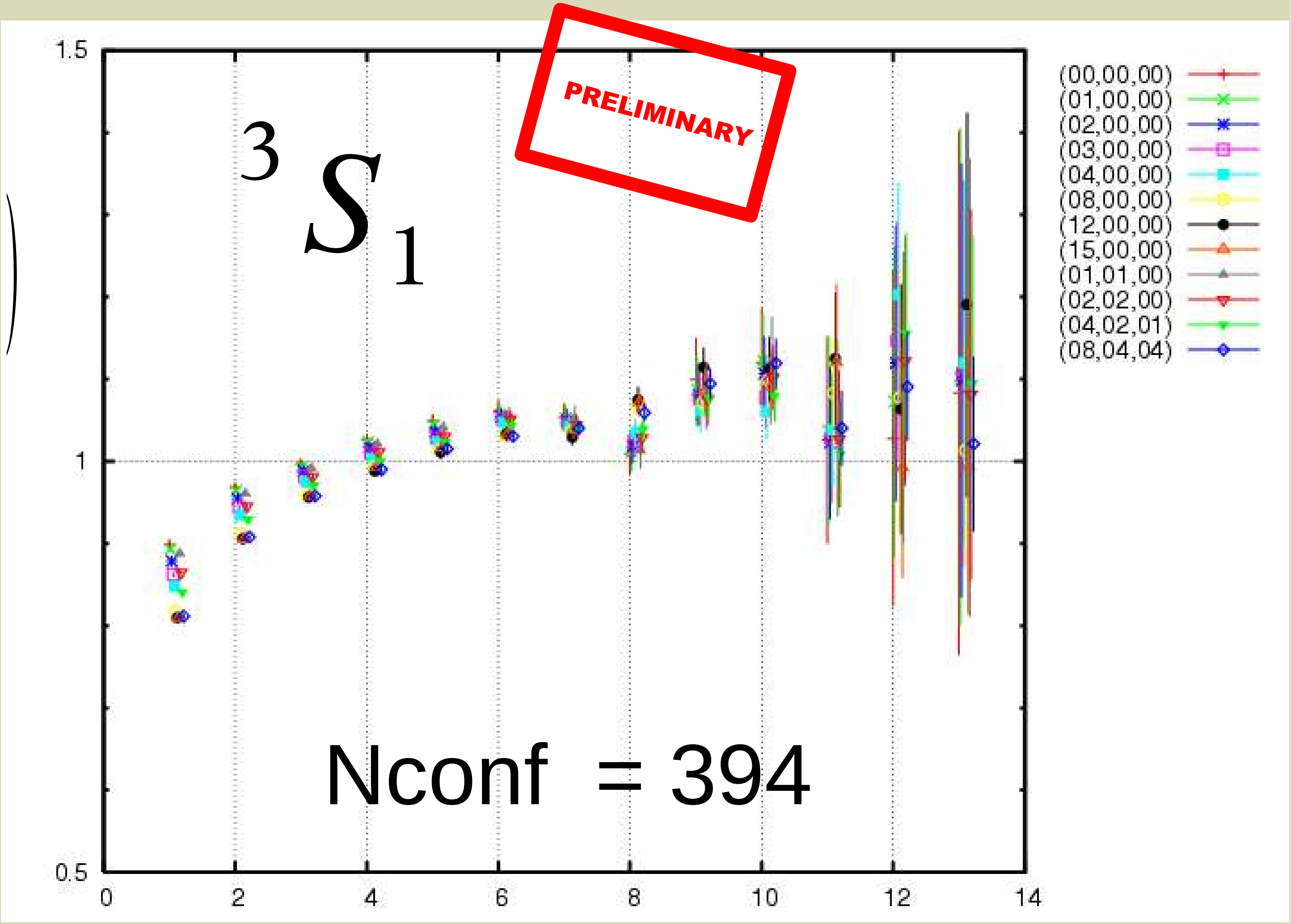
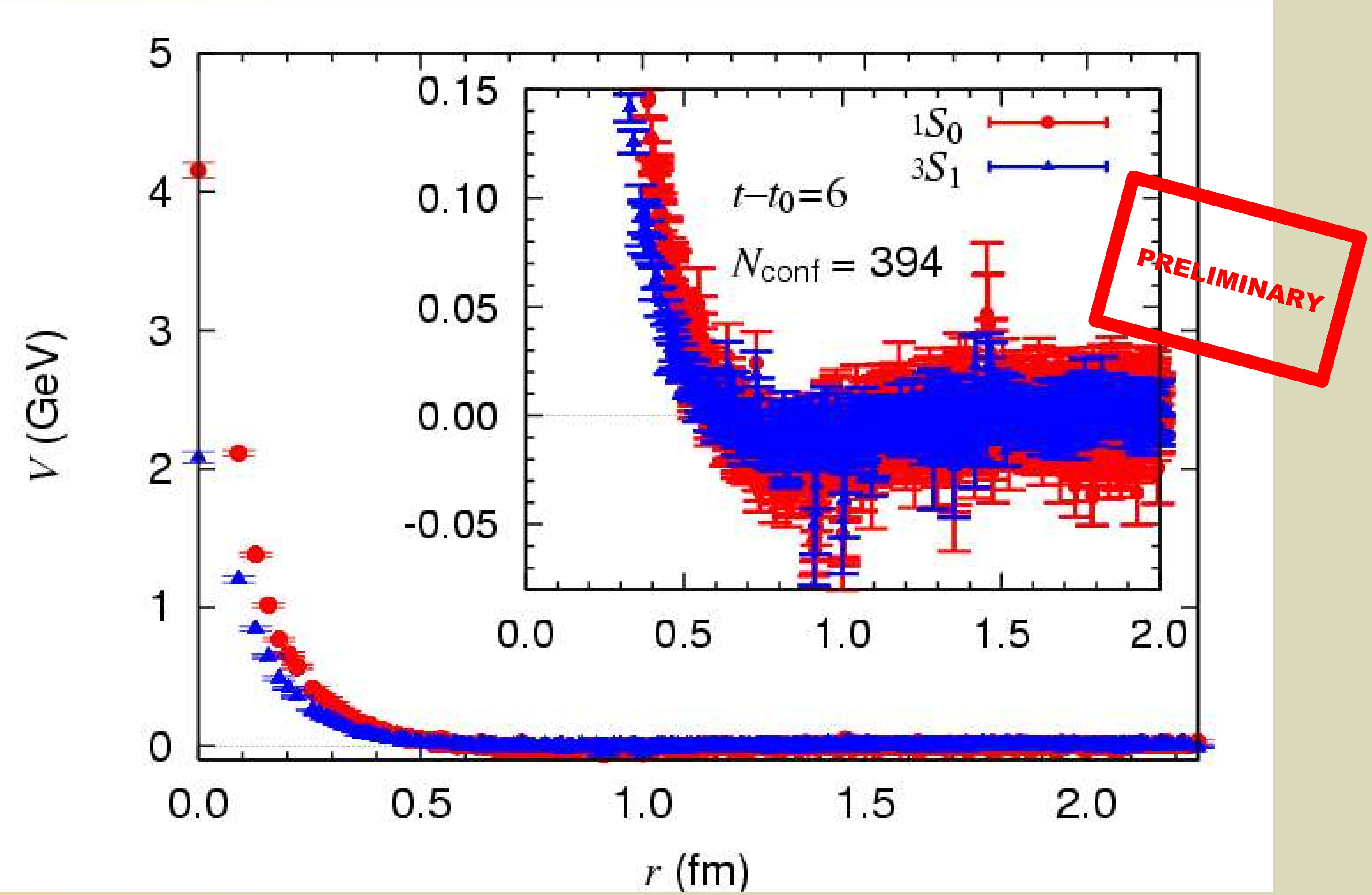


$t-t_0$

- The plateaux starting would appear at $t-t_0 = 6$.

Results —— potential

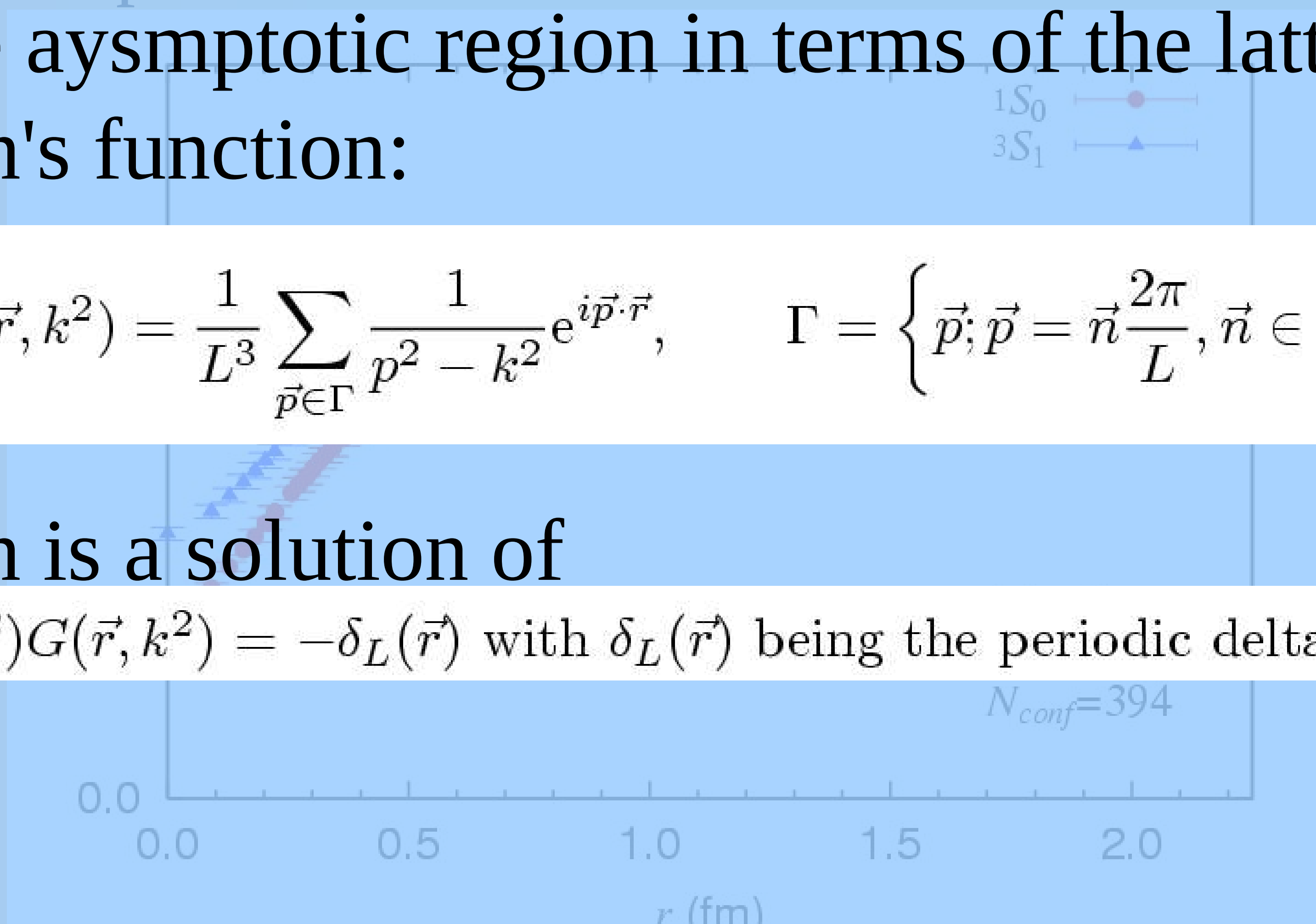
- NΛ potential, from lattice QCD for the first time.



- Strong repulsive core in spin S=0 channel.
- Spin dependence.

Results —— wave function

- Suggests the repulsive core in short range and attractive force in medium range (0.5fm < r < 1fm) for both spin S=0 and 1.



The non-relativisitc energy $E=k^2/(2\mu)$ can be accurately determined by fitting the wave funciton in the aysmptotic region in terms of the lattice Green's function:

$$G(\vec{r},k^2) = \frac{1}{L^3}\sum_{\vec{p}\in\Gamma}\frac{1}{p^2-k^2}e^{i\vec{p}\cdot\vec{r}}, \qquad \Gamma = \left\{\vec{p};\vec{p}=\vec{n}\frac{2\pi}{L},\vec{n}\in \boldsymbol{Z}^3\right\},$$

which is a solution of

$(\triangle + k^2)G(\vec{r},k^2) = -\delta_L(\vec{r})$ with $\delta_L(\vec{r})$ being the periodic delta function.

Results —— wave function

- Suggests the repulsive core in short range and

The energies are almost zero. (although slightly negative (attractive)).

Interaction in the ${}^1S_0$ seems to be more attractive than that in ${}^3S_1$.

A very preliminary results for fitting the k^2 (in lattice unit)

${}^1S_0$ channel:

	$t-t_0=5$	$t-t_0=6$	$t-t_0=7$
r=13		-2.2e-4(1.8e-4)	-4.2e-4(2.6e-4)
r=14		-2.9e-4(2.1e-4)	-4.9e-4(3.1e-4)
r=15	-3.0e-4(1.7e-4)	-3.9e-4(2.4e-4)	-4.7e-4(3.7e-4)
r=16	-3.6e-4(1.9e-4)	-5.0e-4(2.7e-4)	-5.2e-4(3.8e-4)

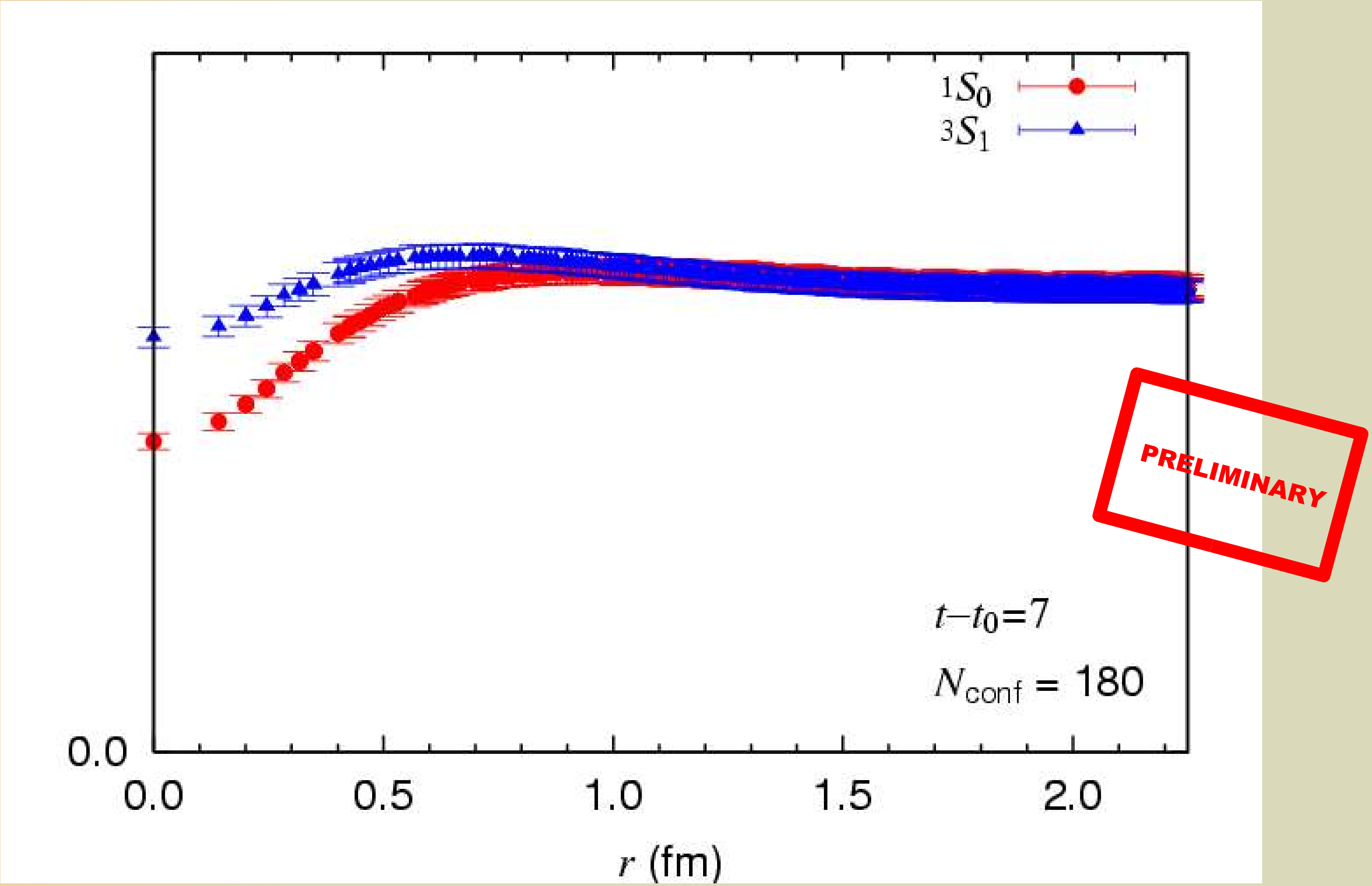
${}^3S_1$ channel:

	$t-t_0=5$	$t-t_0=6$	$t-t_0=7$
r=13		-1.5e-4(1.6e-4)	-2.9e-4(2.2e-4)
r=14		-2.0e-4(1.9e-4)	-3.1e-4(2.6e-4)
r=15	-2.0e-4(1.5e-4)	-2.6e-4(2.1e-4)	-3.1e-4(2.9e-4)
r=16	-2.5e-4(1.7e-4)	-2.9e-4(2.4e-4)	-3.2e-4(3.2e-4)

Quenched QCD with larger spatial volume

Results —— wave function

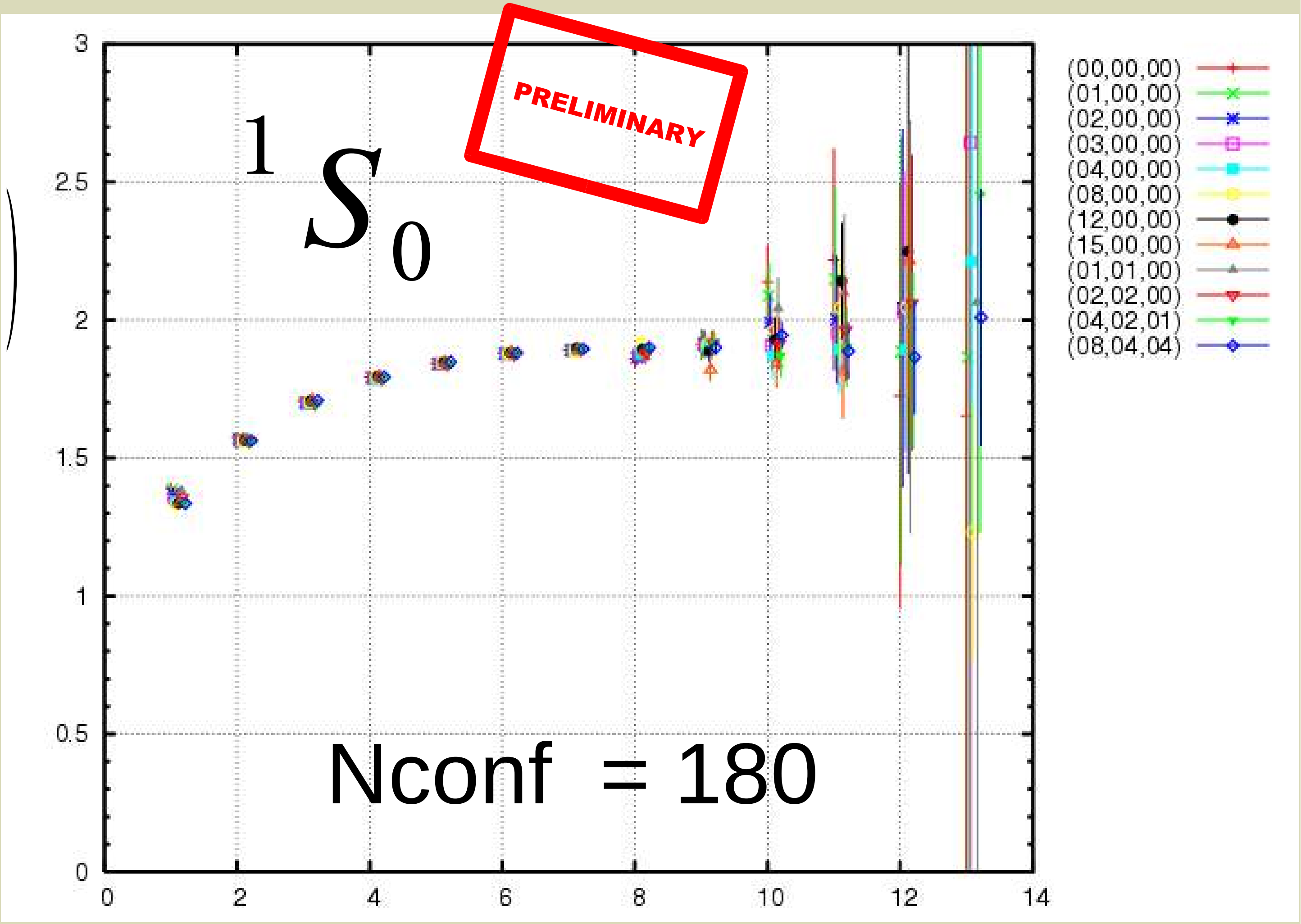
- Suggests the repulive core in short range for both spin S=0 and 1.



Results —— "effective mass"

- Time dependence of 4-point correlator to find the ground state (plateaux in the effective mass)

$$m_{eff}(t-t_{0},\vec{r}=\vec{x}-\vec{y}) \equiv \log\left(\frac{F_{p\Lambda}(\vec{x},\vec{y},t;t_0)}{F_{p\Lambda}(\vec{x},\vec{y},t+1;t_0)}\right)$$



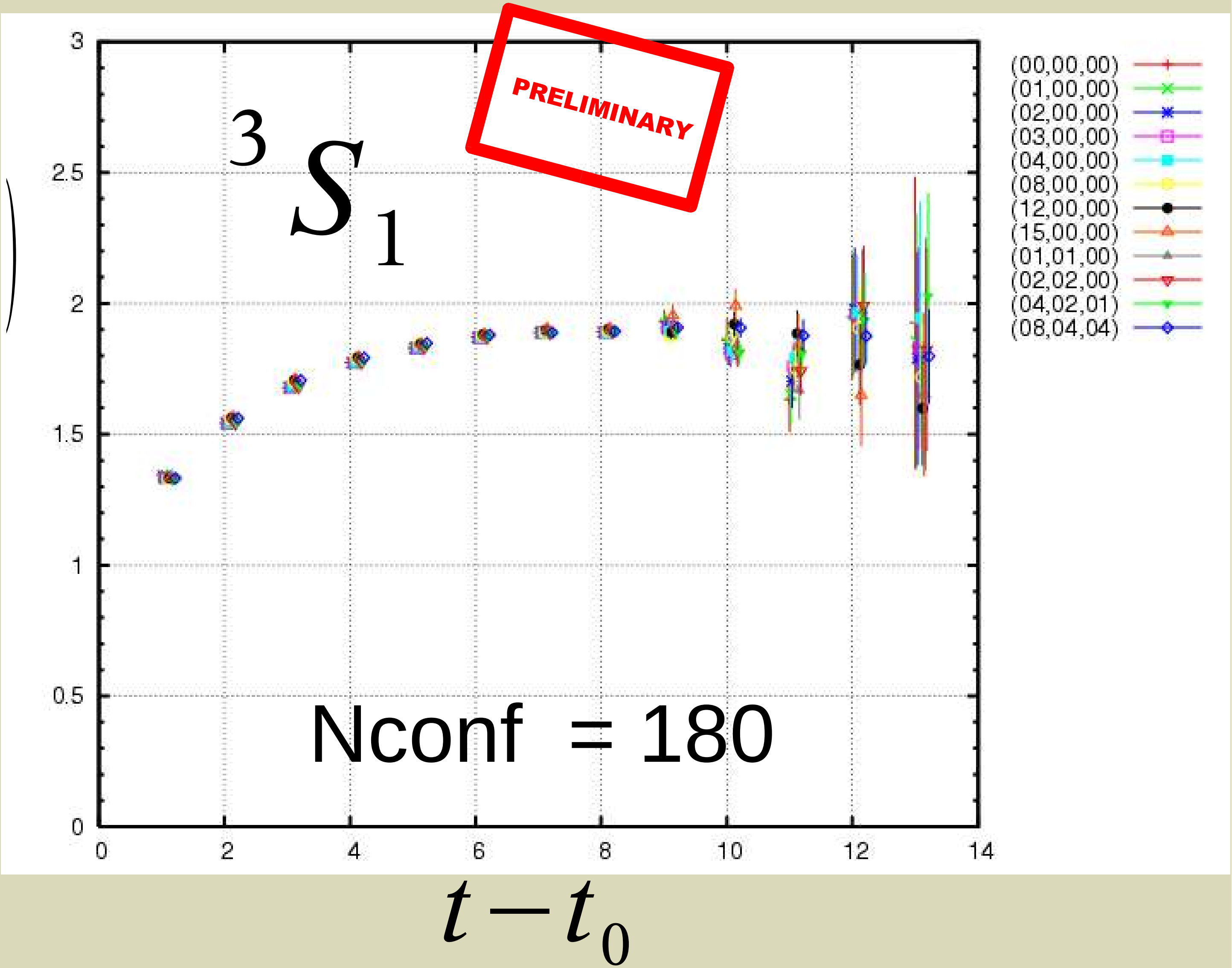
$t-t_0$

- The plateaux starting appear at $t-t_0=7$.

Results —— "effective mass"

- Time dependence of 4-point correlator to find the ground state (plateaux in the effective mass)

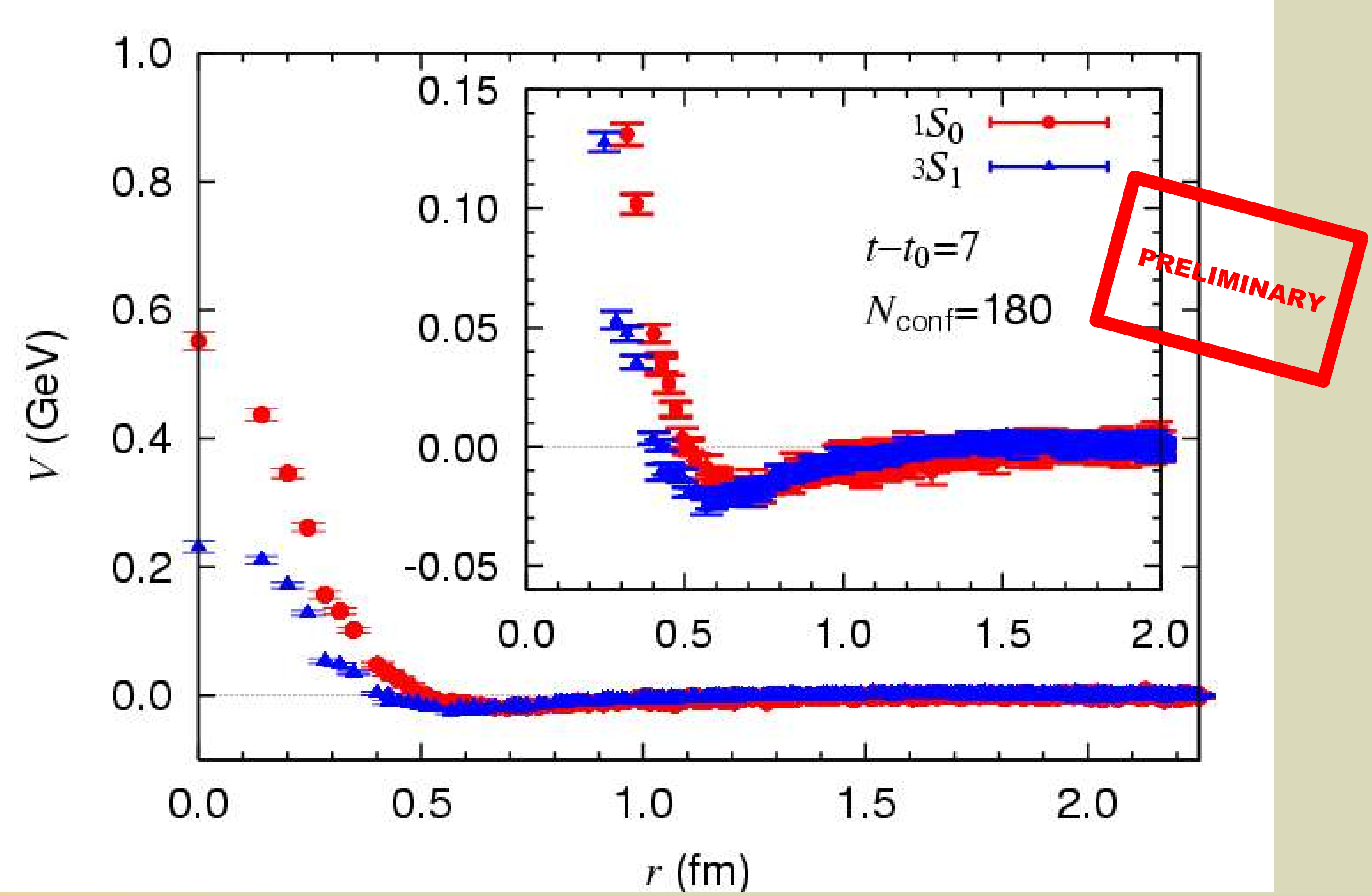
$$m_{eff}(t-t_{0,}\vec{r}=\vec{x}-\vec{y}) \equiv \log\left(\frac{F_{p\Lambda}(\vec{x},\vec{y},t;t_0)}{F_{p\Lambda}(\vec{x},\vec{y},t+1;t_0)}\right)$$



- The plateaux starting would appear at $t-t_0 = 7$.

Results —— potential

- NΛ potential, from lattice QCD for the first time.



- Strong repulsive core in spin S=0 channel.
  (but relatively weaker than that from the full QCD)
- Spin dependence.

Summary:

- Study the $N\Lambda$ force by using lattice QCD.
- N_f=2+1 Full QCD with PACS-CS:
  - Strong repulsive core in $^1S_0$.
  - Spin dependence in short distance region.
  - Scattering lengths will be small and attractive (~ 0.1fm),
  - We need to more statistics and to check volume dependence to see the spin-dependence.
- Quenched QCD with larger spatial volume:
  - Results are qualitatively similar to those from full QCD.
- We will study further with:
  - Energy dependence (coupled channel with ΣN)
  - Tensor force
  - Spin-orbit (LS) frce, antisymmetric LS force.