Heavy baryon mass spectrum from Lattice QCD with 2+1 dynamic sea quark flavors

Heechang Na
with Steven Gottlieb

Indiana University

July-16-2008

Lattice 2008, Williamsburg, VA

Outline

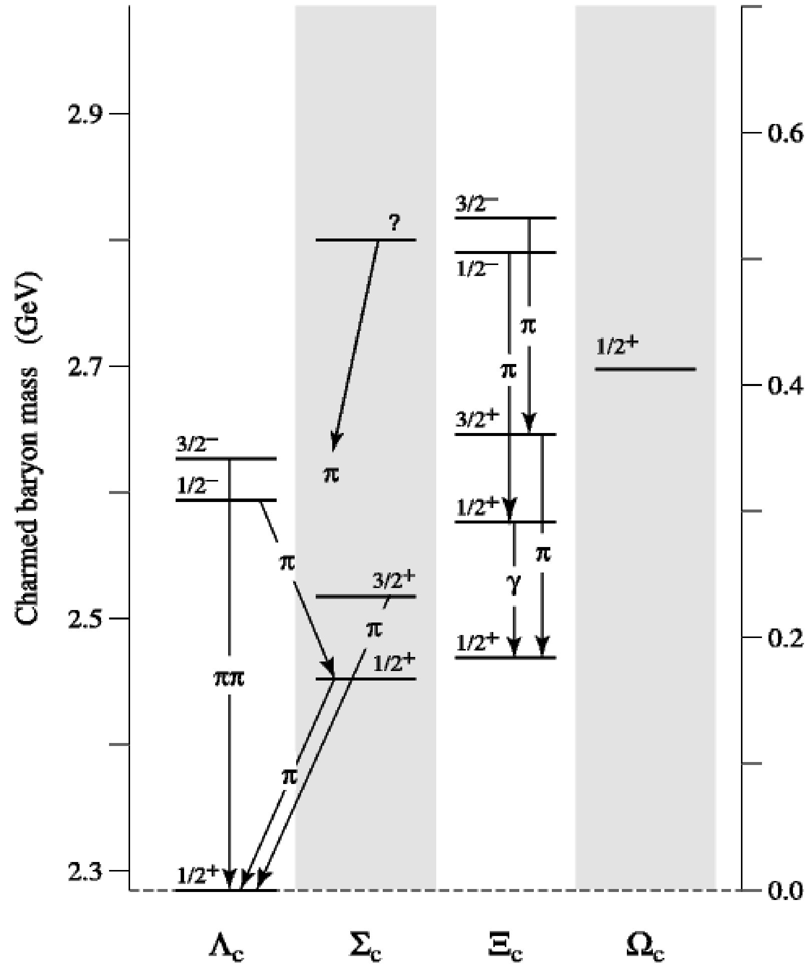
- Introduction
- Formalism
  - Operators
  - Two point function
  - Cancellations of copy indices
- Lattices and propagators
- Data analysis
  - Ratio of propagators
  - Chiral extrapolations
- Results
  - Charmed heavy baryons
  - Bottom heavy baryons
  - Doubly heavy baryons
- Future study

• Heavy baryons in Lattice QCD

- Singly and doubly charmed heavy baryons
- Singly and doubly bottom heavy baryons :

$$\Lambda_H, \Sigma_H, \Sigma^*_H, \Xi_H, \Xi'_H, \Xi^*_H, \Omega_H, \Omega^*_H$$
$$\Xi_{HH}, \Xi^*_{HH}, \Omega_{HH}, \Omega^*_{HH}$$

- Lattice QCD with 2+1 flavors



PDG, J. Phys. G 33, 1 (2006)

- Operators (K.C. Bowler et al., PRD 54, 3619 (1996))

$$O_5 = \varepsilon_{abc}(\psi_1^{aT} C\gamma_5 \psi_2^{b})\Psi_H^{c}, \quad O_\mu = \varepsilon_{abc}(\psi_1^{aT} C\gamma_\mu \psi_2^{b})\Psi_H^{c}$$

	J^p	s^π	Content	Baryon
O_5	1/2+	0^+	**l l h**	Λ_h
			l s h	Ξ_h
O_μ		1^+	**l l h**	Σ_h
			l s h	Ξ'_h
			s s h	Ω_h
			l h h	Ξ_{hh}
			s h h	Ω_{hh}
	3/2+		**l l h**	Σ^*_h
			l s h	Ξ^*_h
			s s h	Ω^*_h
			l h h	Ξ^*_{hh}
			s h h	Ω^*_{hh}

• Formalism: Construction of two point function

- Two point function with a Staggered light quark and a Wilson heavy quark
  - Conversion between a Naive propagator and a Staggered propagator!

$$G_{\Psi}(x;y) = \Omega(x) G_{\Phi} \Omega(y)^{+}$$
$$G_{\Phi} = \hat{I}_4 G_{\chi}(x,y)$$

$$G_{\psi}(x,y) = \Omega(x)\Omega^{+}(y) G_{\chi}(x,y)$$
$$\text{where } \Omega(x) = \prod_{\mu} (\gamma_{\mu})^{x_{\mu}/a}$$

  - Now, we can write the Heavy-Light correlator

$$\sum_x e^{ip\cdot x} < W^{+}_{\Gamma_{sc}}(x) W_{\Gamma_{sk}}(0) > = \sum_x e^{ip\cdot x} Tr\left[\Gamma_{sc} G_{\Psi}(0;x) \Gamma^{+}_{sk} G_H(x;0)\right]$$
$$= \sum_x e^{ip\cdot x} \sum_{c,c'} \left[\mathrm{tr}\{\Gamma_{sc} \Omega^{+}(x) \Gamma^{+}_{sk} G^{c'c}_H(x;0)\} G^{cc'}_{\chi}(0;x)\right]$$
$$\text{where } W_{\Gamma} = \overline{\Psi}_H(x) \Gamma \Psi(x)$$

- (M. Wingate et al. PRD67, 054505 (2003))

- Two point functions for the heavy baryons

$$C_5(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \left\langle O_5(\vec{x},t)\overline{O}_5(\vec{0},0)\right\rangle$$
$$= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \varepsilon_{abc}\varepsilon_{a'b'c'} \mathrm{tr}[G_1^{aa'T}(x,0)C\gamma_5 G_2^{bb'}(x,0)(C\gamma_5)^+]G_H^{cc'}(x,0)$$

$$C_{ij}(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \left\langle O_i(\vec{x},t)\overline{O}_j(\vec{0},0)\right\rangle$$
$$= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \varepsilon_{abc}\varepsilon_{a'b'c'} \mathrm{tr}[\Omega^T(x)C\gamma_i\Omega(x)(C\gamma_j)^+]G_{1\chi}^{aa'}(x,0)G_{2\chi}^{bb'}(x,0)G_H^{cc'}(x,0)$$

$$\Omega(x) = \gamma_0^t \gamma_1^{x_1} \gamma_2^{x_2} \gamma_3^{x_3}$$

Therefore,

$$C_5(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} 4\varepsilon_{abc}\varepsilon_{a'b'c'} G_{1\chi}^{aa'}(x,0)G_{2\chi}^{bb'}(x,0)G_H^{cc'}(x,0)$$

$$C_{ij}(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} 4(-1)^{x_i} \delta_{ij}\varepsilon_{abc}\varepsilon_{a'b'c'} G_{1\chi}^{aa'}(x,0)G_{2\chi}^{bb'}(x,0)G_H^{cc'}(x,0)$$

- Cancellations of copy indices

$$O_\mu = \varepsilon_{abc}(\psi_1{}^{aT} C\gamma_\mu \psi_2{}^{b})\Psi_H{}^{c} \quad \Rightarrow \quad D_\mu = (\psi_1{}^{T}(x) C\gamma_\mu \psi_2(x))$$

$$\psi^{\alpha'}(x) = \Omega^{\alpha' a}(x)\chi^{a}(x)$$

$$q^{\alpha\, i,a}(y) = \frac{1}{8}\sum_{\xi}\Omega^{\alpha\, i}(\xi)\chi^{a}(y+\xi)$$

$$x = y + \xi$$

$$\chi^{a}(y+\xi) = 2\Omega^{+\, i\alpha}(\xi)q^{\alpha\, i,a}(y)$$

$\psi^{\alpha'}$: Naive quark

χ^{a} : 4 copies of staggered quark

$q^{\alpha\, i,a}$: Staggered quark in taste basis

$\Omega(x) = \gamma_0^{x_0}\gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}$

a : Copy index

α : Staggered spin index

α' : Naive spin index

i : Taste index

$$\psi^{\alpha'}(x) = \Omega^{\alpha' a}(\xi)\chi^{a}(y+\xi) = \Omega^{\alpha' a}(\xi)2\Omega^{+\, i\alpha}(\xi)q^{\alpha\, i,a}(y)$$

- Di-quark operator

$$D_\mu = (\psi_1^T(x) C\gamma_\mu \psi_2(x)) \quad \Rightarrow \quad D_\mu^{conti}(y) = \sum_\xi (\psi_1^T(x) C\gamma_\mu \psi_2(x))$$

$$D_\mu^{conti}(y) = \sum_\xi 2\Omega^{+\,i\alpha}(\xi) q^{\alpha\,i,a}(y) \Omega^{T a\alpha'}(\xi) (C\gamma_\mu)^{\alpha'\beta'} \Omega^{\beta' b}(\xi) 2\Omega^{+\,j\beta}(\xi) q^{\beta\,j,b}(y)$$

$$= \sum_\xi 4\Omega^{+\,i\alpha}(\xi) q^{\alpha\,i,a}(y) (-1)^{\xi_\mu} (C\gamma_\mu)^{ab} \Omega^{+\,j\beta}(\xi) q^{\beta\,j,b}(y)$$

$$\sum_\xi \Omega^{+\,i\alpha}(\xi)(-1)^{\xi_\mu} \Omega^{+\,j\beta}(\xi) = 4(C\gamma_\mu)_{\alpha\beta} \otimes (\gamma_\mu C^{-1})_{ij}$$

$$D_\mu^{conti}(y) = 16 q^{\alpha\,i,a}(y) (C\gamma_\mu)_{\alpha\beta} \otimes (\gamma_\mu C^{-1})_{ij} q^{\beta\,j,b}(y) (C\gamma_\mu)_{ab}$$

$$D_5^{conti}(y) = 16 q^{\alpha\,i,a}(y) (C\gamma_5)_{\alpha\beta} \otimes (\gamma_5 C^{-1})_{ij} q^{\beta\,j,b}(y) (C\gamma_5)_{ab}$$

Overlap with 1^+ and 0^+ spin state with single taste

K. Nagata et al., arXiv:0707.3537

a, b	:	Copy index	i, j	:	Taste index
α, β	:	Staggered spin index	α', β'	:	Naive spin index

- Two-point function of the di-quark operator

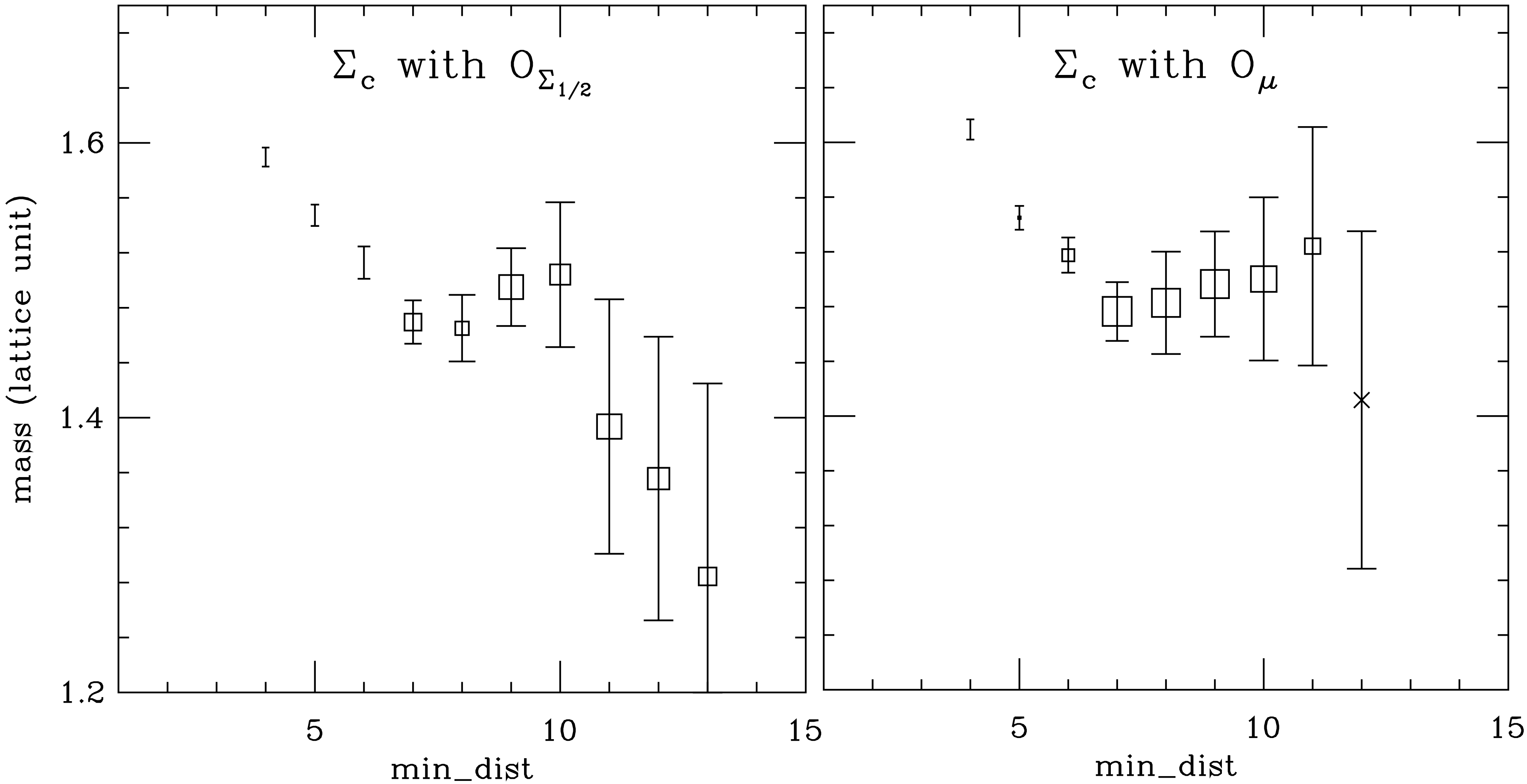
$$
\begin{aligned}
C_{\mu\nu}^{conti}(y;0) &= < D_{\mu}^{conti}(y)\overline{D}_{\nu}^{conti}(0) > \\
&= 16^2 \mathrm{Tr}[G_1(y,0)(C\gamma_\mu)\otimes(C\gamma_\mu)^+ G_2(y,0)(C\gamma_\nu)^+ \otimes (C\gamma_\nu)] \\
&\qquad \times (C\gamma_\mu)_{\mathrm{ab}} \otimes (C\gamma_\mu)^+_{\mathrm{b'a'}} \delta_{bb'}\delta_{aa'} \\
&= 16^2 \mathrm{Tr}[G_1(y,0)(C\gamma_\mu)\otimes(C\gamma_\mu)^+ G_2(y,0)(C\gamma_\nu)^+ \otimes (C\gamma_\nu)] \\
&\qquad \times \mathrm{Tr}[(C\gamma_\mu)(C\gamma_\nu)^+]
\end{aligned}
$$

$$
\mathrm{Tr}[(C\gamma_\mu)(C\gamma_\nu)^+] = \begin{cases} 0 & \mu \neq \nu \\ 4 & \mu = \nu \end{cases}
$$

- **Consequences of cancellations of the copy indices**
  1. **no taste mixing**
  2. **Impossible to separate 1/2 and 3/2 spin states using the spin projection operators. (the delta function)**

-- The other way to get 1/2 state

$$O_{\Sigma_{1/2}} = \varepsilon_{abc}(\psi_1^{aT} C\gamma_5 \Psi_H^{\ b})\psi_2^{\ c}$$ for $\Sigma_{1/2}$



• Data sets

- MILC coarse lattices
  - $20^3 \times 64$, $a \approx 0.12$ fm
  - 3 ensembles with four different time sources
    - $m_l = 0.007$ $m_s = 0.05$
    - $m_l = 0.01$ $m_s = 0.05$
    - $m_l = 0.02$ $m_s = 0.05$
- Propagators
  - 9 different staggered light valence quarks
    - 0.005 ~ 0.02
  - 3 different staggered strange valence quarks
    - 0.024, 0.03, 0.0415
  - One valence clover heavy quark
    - k = 0.122 (Tuned for charm quark)
      - 007 : 545 confs 010 : 591 confs 020 : 459 confs
    - k = 0.086 (for bottom)
      - 007 : 554 confs 010 : 590 confs 020 : 452 confs

- MILC Fine lattices
  - $28^3 \times 96$, a ≈ 0.09 fm
  - 2 ensembles
    - $m_l = 0.2\ m_s$ with four time sources
      - 557 confs
    - $m_l = 0.4\ m_s$ with four time sources
      - 534 confs
- Propagators
  - 9 different staggered light valence quarks for $0.2\ m_s$
  - 7 different staggered light valence quarks for $0.4\ m_s$
  - One staggered strange valence quark mass
  - One valence clover heavy quark
    - Charm quark: k = 0.127
    - Bottom quark: k = 0.0923

- MILC Medium-coarse lattices
  - $16^3 \times 48$, a ≈ 0.15 fm
  - 3 ensembles
    - $m_l = 0.2\ m_s$ with two time sources
      - 631 confs
    - $m_l = 0.4\ m_s$ with two time sources
      - 631 confs
    - $m_l = 0.6\ m_s$ with two time sources
      - 440 confs
- Propagators
  - 8 different staggered light valence quarks
  - One staggered strange valence quark mass
  - One valence clover heavy quark
    - Charm quark: k = 0.122
    - Bottom quark: k = 0.076

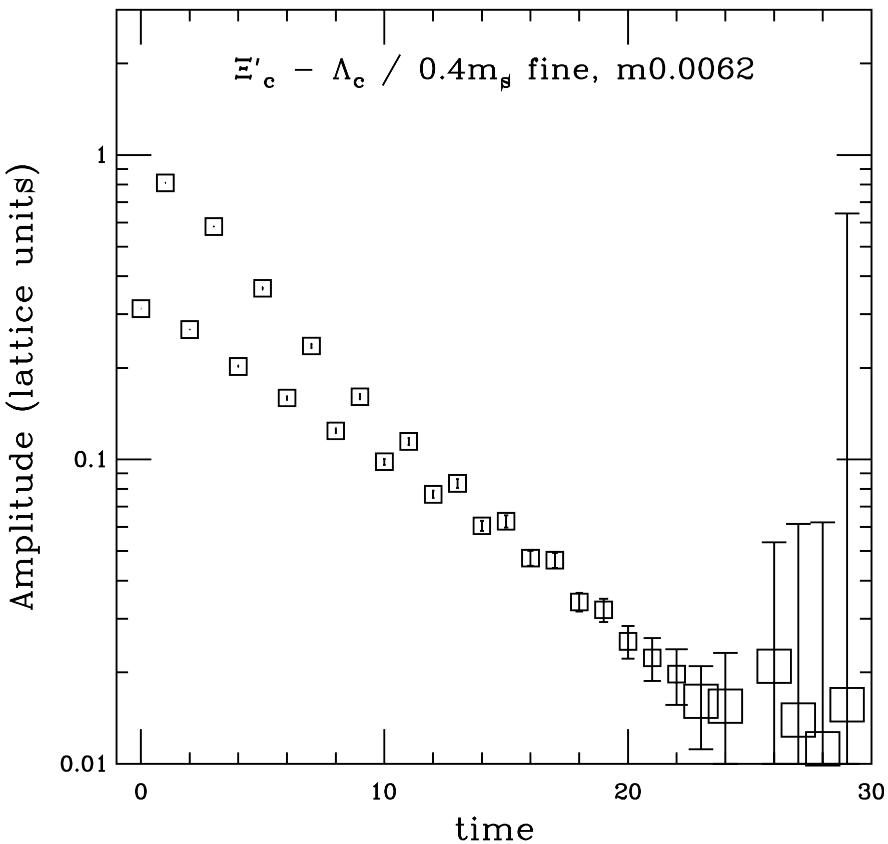
• Data analysis

- Ratio of propagators for mass differences

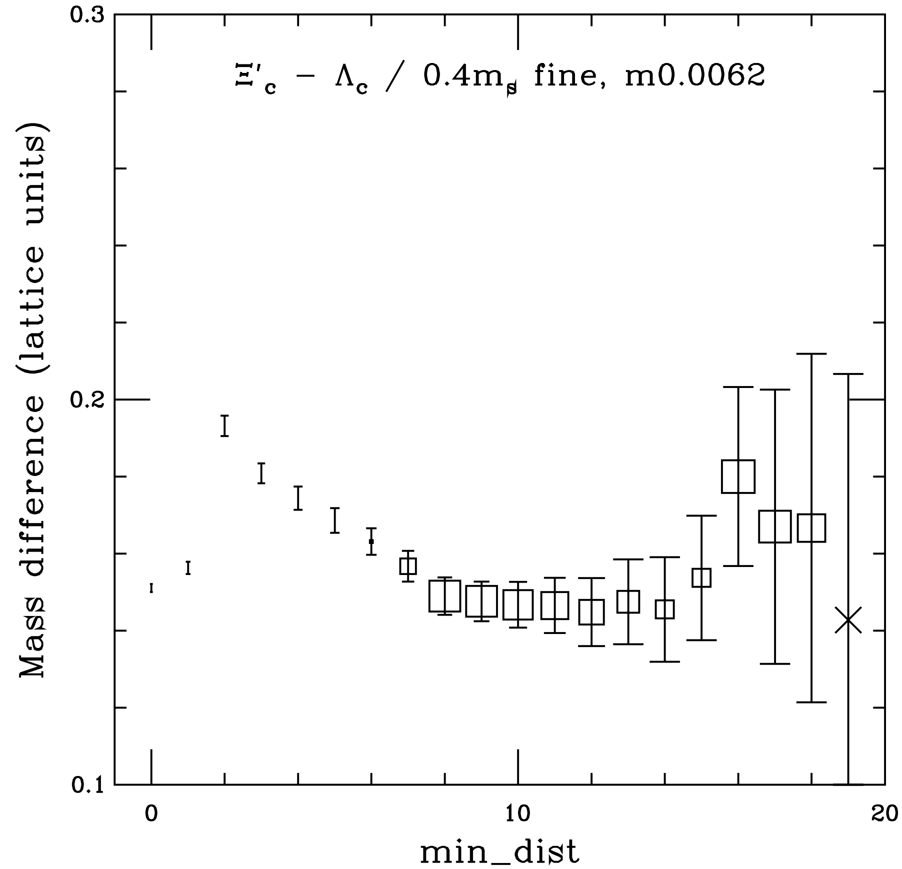
$$P(t) = \frac{A_2 e^{-m_2 t} + \cdots}{A_1 e^{-m_1 t} + \cdots} = A e^{-(m_2 - m_1)t} + \cdots$$

- Correlated least squares fit
- Error estimation
  - 1000 / 500 bootstrap samples
- Simultaneous quadratic chiral extrapolation

• Ratio of propagators

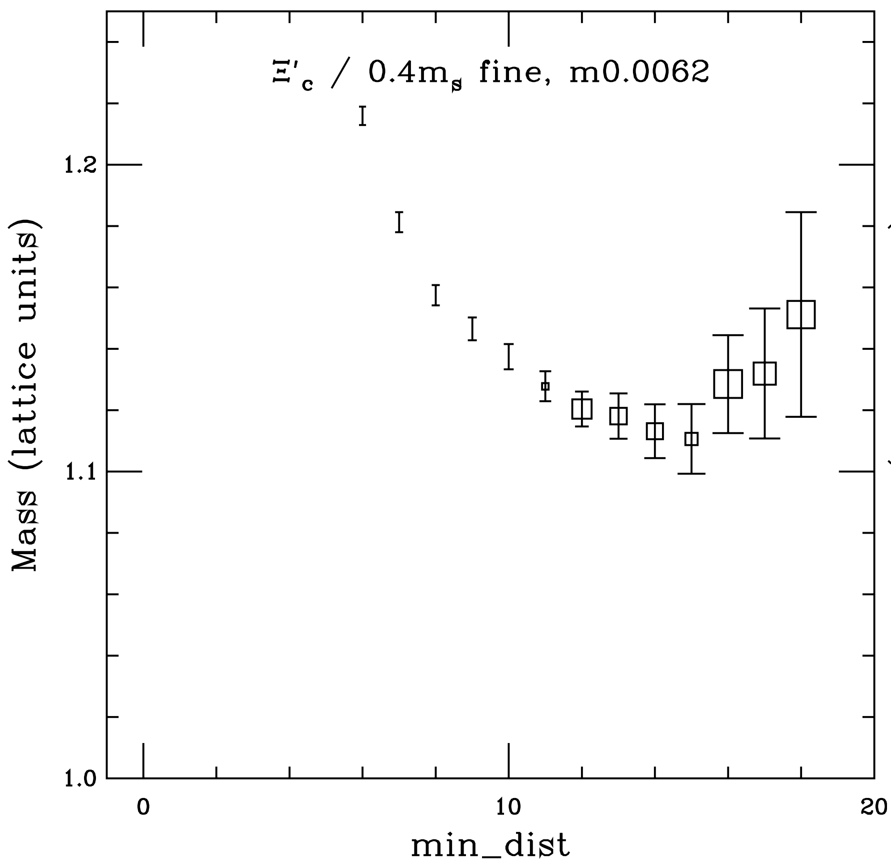


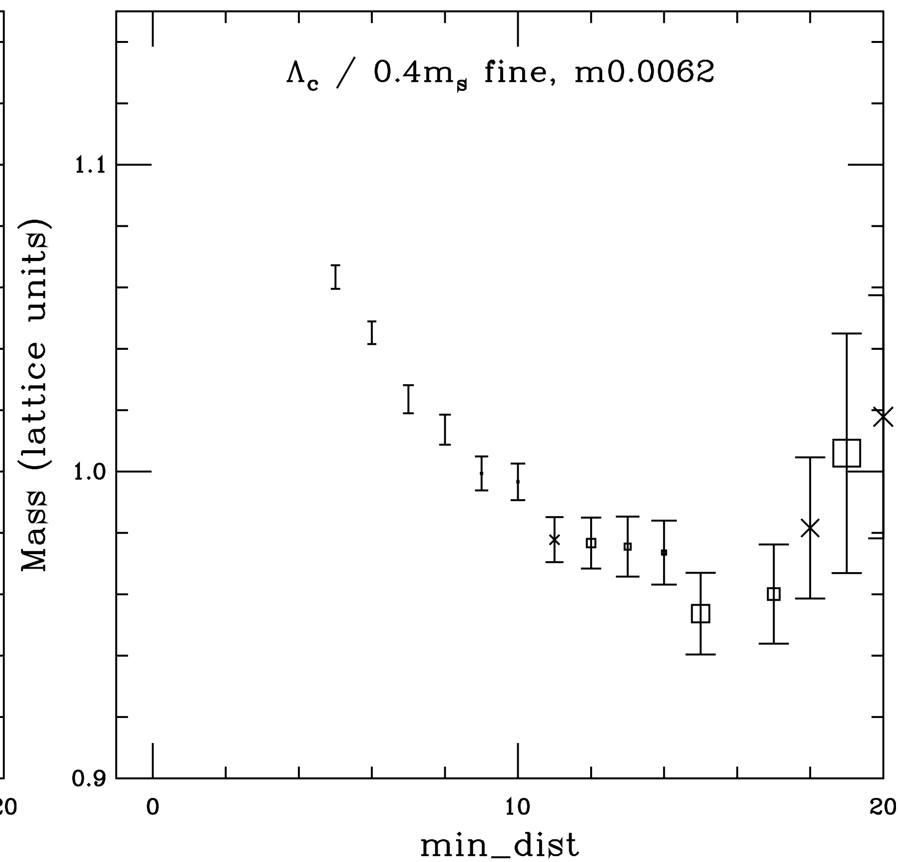
Ratio



Fitting result of the ratio

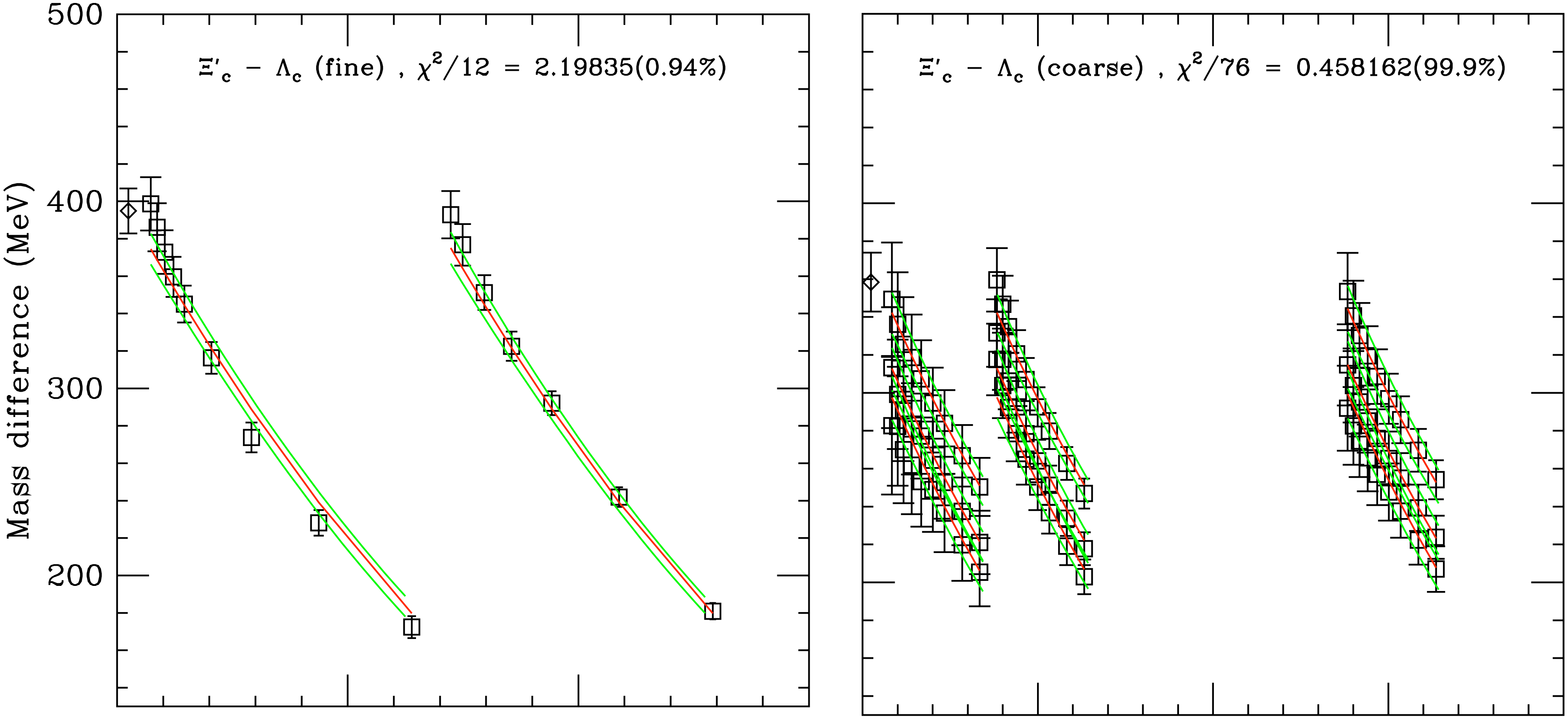
• Ratio of propagators (continue..)





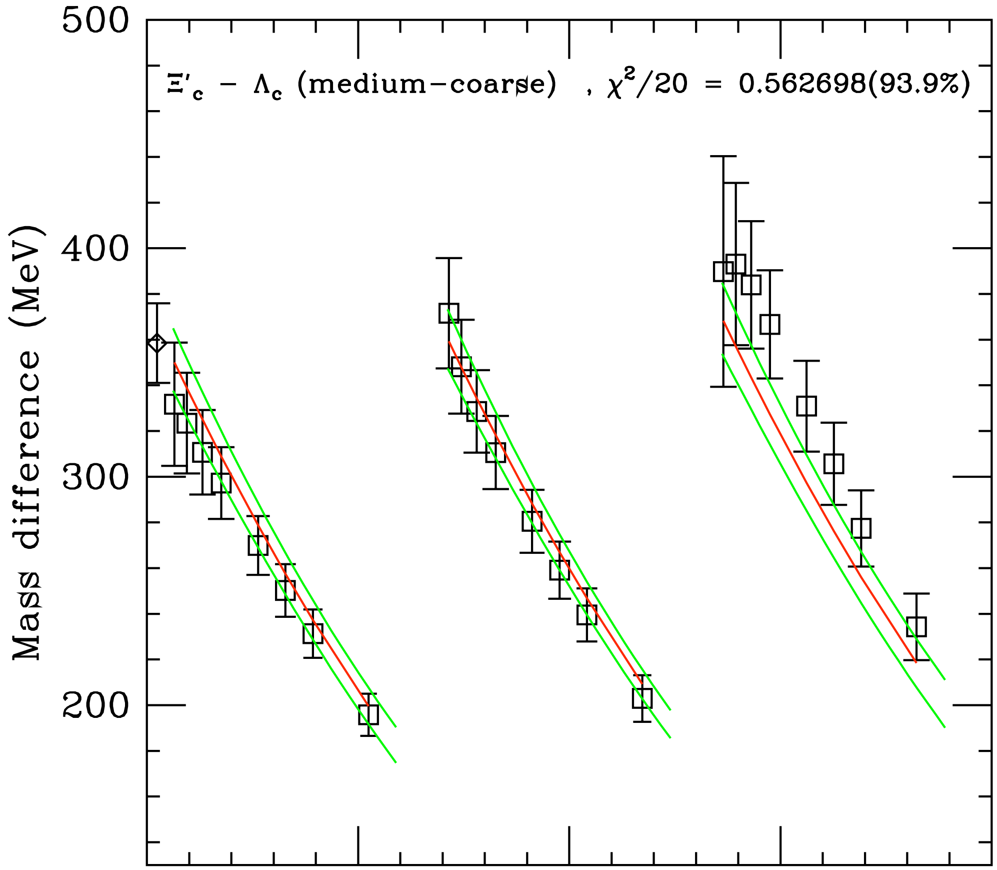
Fitting results of each particle

• Simultaneous quadratic chiral extrapolation

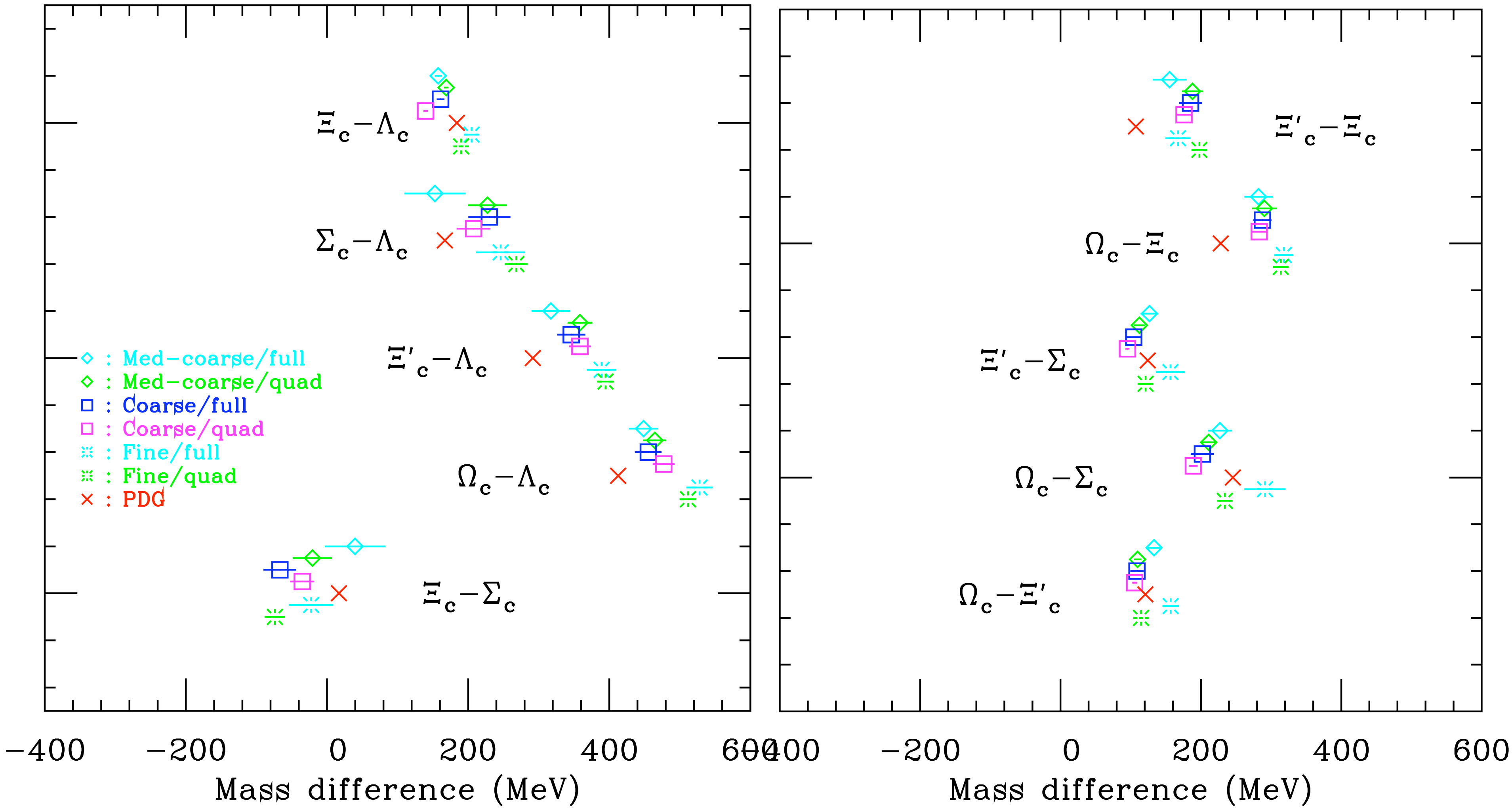


Fit model function: $c_0 + c_1 m_v + c_2 m_v^2 + c_3 m_s + c_4 m_{sea}$

• Simultaneous quadratic chiral extrapolation (continue..)

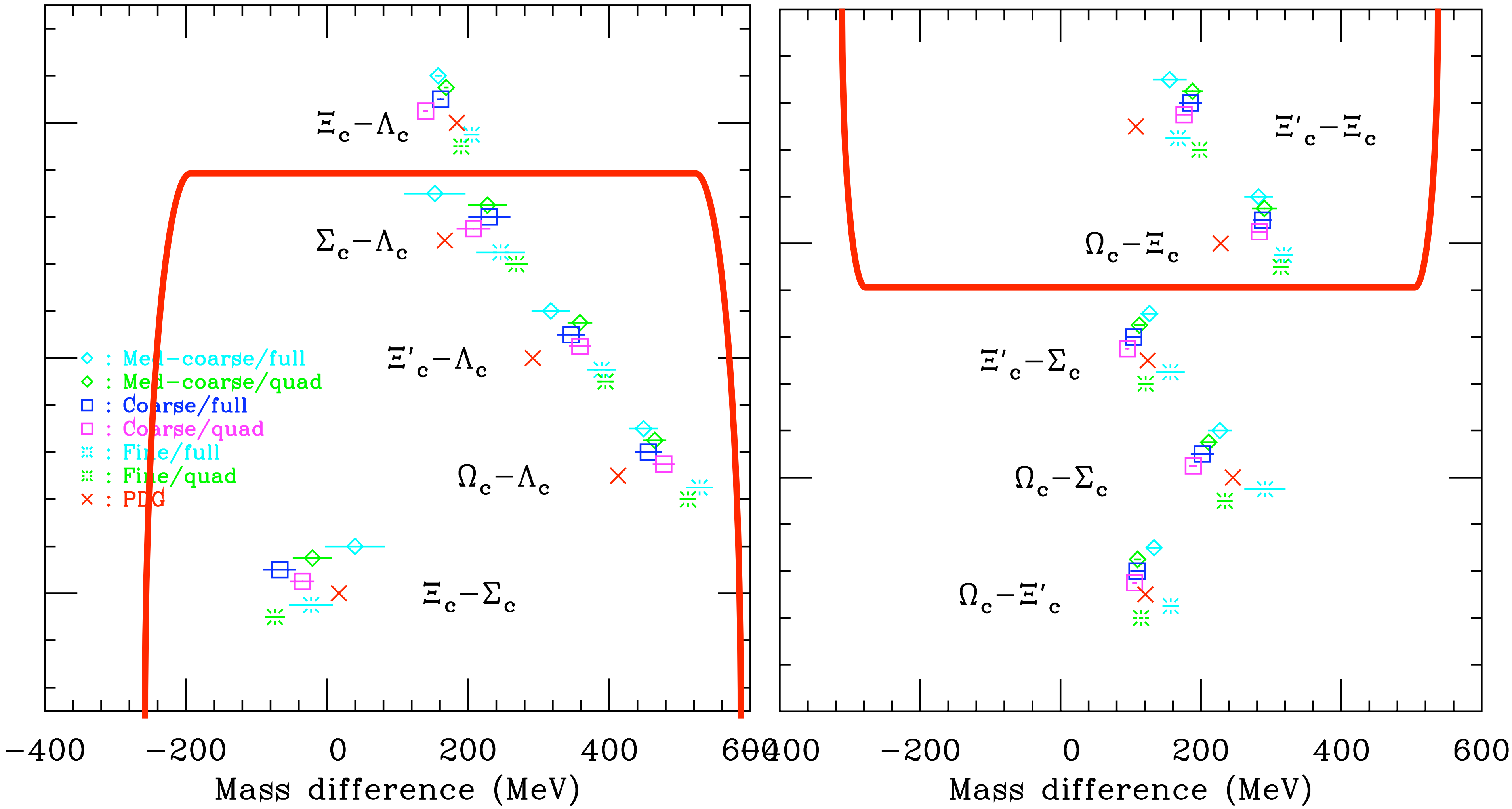


- **Results** (Preliminary)
    - $1/2^+$ singly charmed heavy baryons

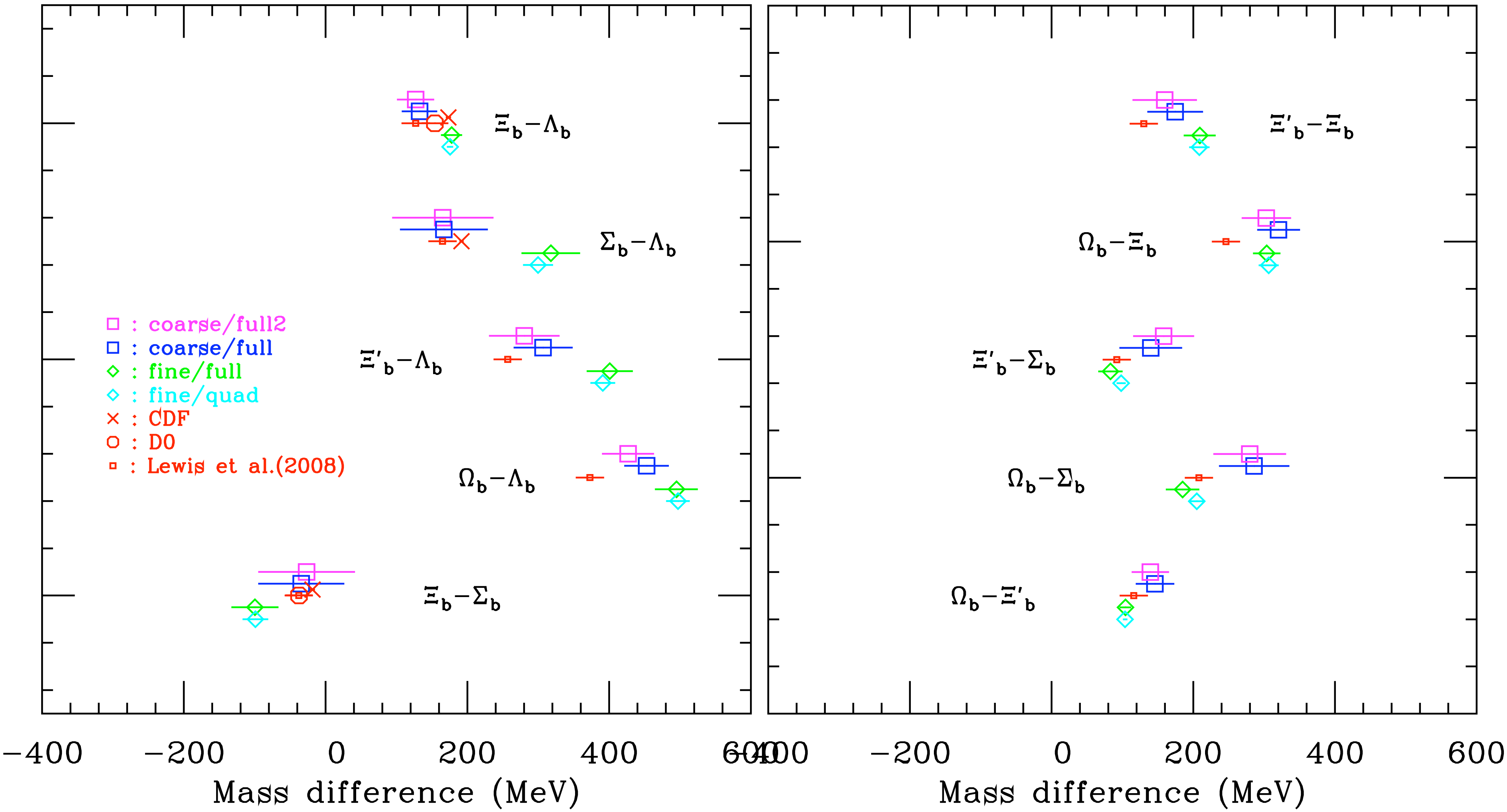


• Results (Preliminary)

- 1/2$^+$ singly charmed heavy baryons

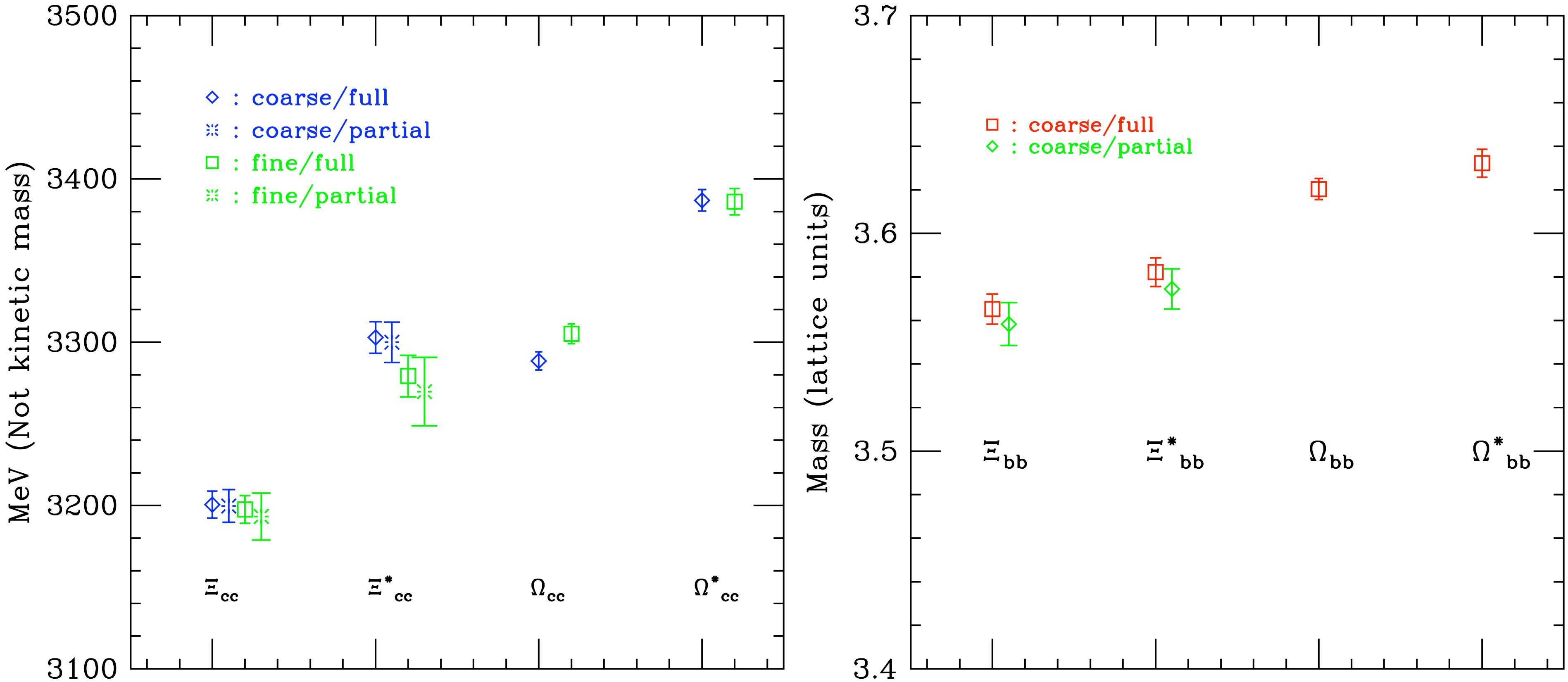


- $1/2^+$ singly bottom heavy baryons with the coarse lattice



Recent measurements from CDF and D0, and dynamic calculations by Lewis and Woloshyn.

- Doubly heavy baryons



Future study

- Extrapolations with ChiPT
- Increase statistics
- Super-fine lattice (a~0.06fm)
- 3/2 and 1/2 spin states
  - Bayesian fit
  - Variational method
- Constant mass shift
- More about error analysis
- Discretization errors

Backup slides

-- Surprisingly, C_{ij} is a diagonal matrix for i and j indices

$$C_{ij}(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \varepsilon_{abc}\varepsilon_{a'b'c'} \mathrm{tr}[\Omega^T(x)C\gamma_i\Omega(x)(C\gamma_j)^+] \times G_{1\chi}^{aa'}(x,0)G_{2\chi}^{bb'}(x,0)G_H^{cc'}(x,0)$$

$$\Omega(x) = \gamma_0^t\gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}$$

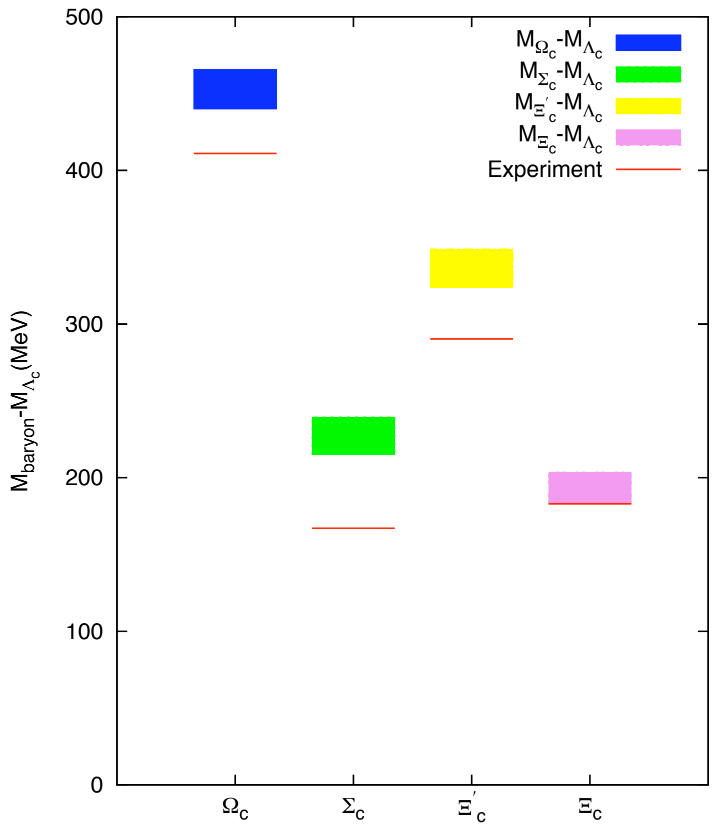
$$\mathrm{tr}[\Omega^T(x)C\gamma_i\Omega(x)(C\gamma_j)^+] = 4(-1)^{x_i}\delta_{\mathrm{ij}}$$

$$C_{ij}(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} 4(-1)^{x_i}\delta_{ij}\varepsilon_{abc}\varepsilon_{a'b'c'}G_{1\chi}^{aa'}(x,0)G_{2\chi}^{bb'}(x,0)G_H^{cc'}(x,0)$$

$$C_{ij}(t) = P_{ij}^{3/2}C_{3/2}(t) + P_{ij}^{1/2}C_{1/2}(t)$$

$$= (\delta_{ij} - \frac{1}{3}\gamma_i\gamma_j)C_{3/2}(t) + \frac{1}{3}\gamma_i\gamma_jC_{1/2}(t)$$

$$C_{3/2}(t) \propto e^{-m_{3/2}t}, \quad C_{1/2}(t) \propto e^{-m_{1/2}t}$$



Poster presented by Liuming Liu in Lattice 2008