Hunting for the strangeness content of the nucleon

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- Introduction
- How to calculate Δq
- Disconnected contributions: stochastic methods
- Results
- Outlook

Work in progress, results preliminary.

Contribution of the quark spin to the spin of the nucleon.

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

Along with the quark angular momentum, L_q and gluon total angular momentum ΔG:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G$$

Extraction of $\Delta\Sigma$ from experiment:

- measure the spin structure function $g_1(x, Q^2)$ for neutron, proton.
- extract $\Delta q(x, Q^2)$ from g_1's and integrate over x:

$$\Delta q(Q^2) = \int_0^1 dx\, \Delta q(x, Q^2).$$

- Assumptions required to extrapolate data to $x < 10^{-3}$.

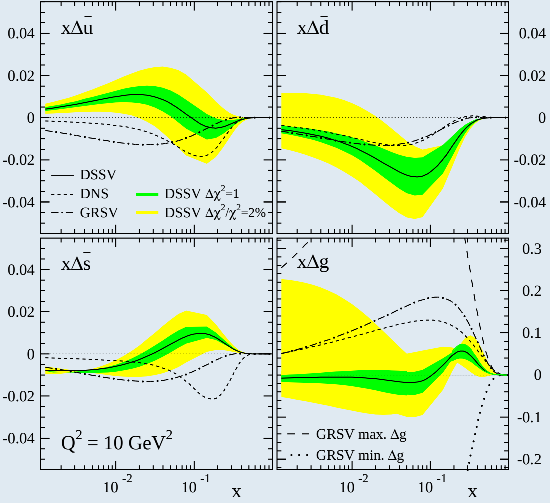
Hermes 2006 (COMPASS very similar): (at $Q^2 = 5\,\mathrm{GeV}^2$)

$$\begin{aligned}\Delta u &= 0.842(4)(8)(9)\\ \Delta d &= -0.427(4)(8)(9)\\ \Delta s &= -0.085(13)(8)(9)\end{aligned}$$

Errors are (theoretical)(experimental)(evolution).
However,

- data only for $x > 0.004$,
- restrict integration to range where $\exists$ data $\Rightarrow \Delta s \approx 0$.

Global fit to experimental data

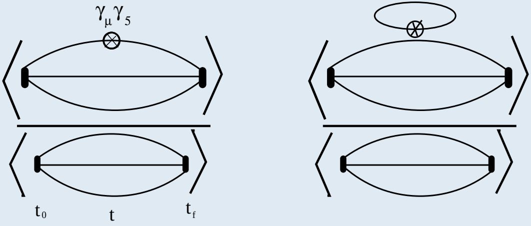


D de Florian, R Sassot, M Stratmann, W Vogelsang 2008

Leading twist:

$$\langle N, p, s | \bar{q}\gamma_\mu\gamma_5 q | N, p, s\rangle = 2M_N s_\mu \frac{\Delta q}{2}$$

On the lattice extracted using



(a) connected (b) disconnected

For Δs $\exists$ only the disconnected contribution.

Extract matrix element from ratio (at zero momentum)

$$R^{con}(t,t_f) = \frac{\langle \Gamma^{\alpha\beta}_{\rm pol} C^{\beta\alpha}_{3pt}(t_0,t,t_f)\rangle}{\langle \Gamma^{\alpha\beta}_{\rm unpol} C^{\beta\alpha}_{2pt}(t_0,t_f)\rangle}$$

$$R^{dis}(t,t_f) = -\frac{\langle \Gamma^{\alpha\beta}_{\rm pol} C^{\beta\alpha}_{2pt}(t_0,t_f)\sum_{\mathbf{x}} \mathrm{Tr}(\gamma_j\gamma_5 M^{-1}(\mathbf{x},t;\mathbf{x},t))\rangle}{\langle \Gamma^{\alpha\beta}_{\rm unpol} C^{\beta\alpha}_{2pt}(t_0,t_f)\rangle}$$

- polarized in the j-direction.
- $\Gamma_{\rm pol} = i\gamma_j\gamma_5(1+\gamma_4)/2$, $\Gamma_{\rm unpol} = (1+\gamma_4)/2$.
- We smear C_{3pt} and C_{2pt} at source and sink.
- (Not yet) renormalised.

For $t_f \gg t \gg t_0$:

$$R^{con}(t,t_f) + R^{dis}(t,t_f) \to 2\frac{\langle N,\vec{0},s|(\bar{q}\gamma_j\gamma_5 q)^{latt}|N,\vec{0},s\rangle}{2M_N} = \Delta q^{latt}$$

The disconnected contribution requires all-to-all propagators.
$\Rightarrow$ stochastic methods:

Generate a set of random noise vectors $|\eta_\ell\rangle$, $\ell = 1, \ldots, L$ where

$$\frac{1}{L}\sum_\ell |\eta_\ell\rangle\langle\eta_\ell| = \overline{|\eta\rangle\langle\eta|}_L = \overline{|\eta\rangle\langle\eta|} = \mathbb{1} + \mathcal{O}(1/\sqrt{L}),$$
$$\overline{\langle\eta|} = \mathcal{O}(1/\sqrt{L}).$$

By solving

$$M|s_\ell\rangle = |\eta_\ell\rangle$$

for the $|s_\ell\rangle$ one can construct an unbiased estimate:

$$E(M^{-1}) = \overline{|s\rangle\langle\eta|}$$
$$= M^{-1} + M^{-1}\underbrace{\left(\overline{|\eta\rangle\langle\eta|} - \mathbb{1}\right)}_{\mathcal{O}(1/\sqrt{L})}$$

For tricks see also the posters of Christian Ehmann & Hagen.

Partitioning – We only set $|\eta_\ell\rangle \neq 0$ on one timeslice. This removes some of the (larger) off-diagonal noise elements $\overline{|\eta\rangle\langle\eta|} - \mathbb{1}$ and reduces the variance.

HPE – The first few terms of the hopping parameter expansion of $\mathrm{Tr}(\Gamma M^{-1}) = \mathrm{Tr}[\Gamma(\mathbb{1} - \kappa \not{D})^{-1}]$ vanish identically but still contribute to the noise. For the Wilson action, $\mathrm{Tr}(\Gamma M^{-1}) = \mathrm{Tr}(\Gamma \kappa^n \not{D}^n M^{-1})$, $n = 4, 8$, depending on Γ, where estimating the latter yields smaller errors.

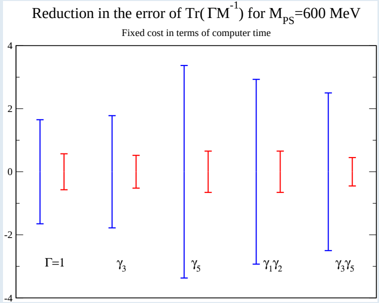
Eigenmodes – Calculate the N_{ev} lowest eigenvalues and eigenvectors of $\mathrm{Q} = \gamma_5 M$, $\mathrm{Q}^{-1} = \mathrm{Q}_\perp^{-1} + \sum_{i=1}^{N_{ev}} |u_i\rangle q_i^{-1} \langle u_i|$, and stochastically estimate the complement $\mathrm{Q}_\perp^{-1}$ (with deflation included for free).

TSM – Obtain approximate solutions $|s_{n_t,\ell}\rangle$ after n_t solver iterations (before convergence), and estimate the difference stochastically to obtain an unbiased estimate of M^{-1}:

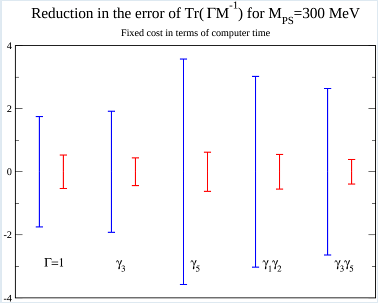
$$\mathrm{E}(M^{-1}) = \overline{|s_{n_t}\rangle\langle\eta|}_{L_1} + \overline{(|s\rangle - |s_{n_t}\rangle)\langle\eta|}_{L_2} \quad \text{with} \quad L_2 \ll L_1 \,.$$

Reduction of the stochastic error at fixed cost

Results for $\mathrm{Tr}(\Gamma M^{-1})$ on 1 configuration:



(a) Partitioning, HPE, TSM



(b) Partitioning, HPE, eigenmodes, TSM

- Significant gain for all Γs.
- Using different combinations of methods allows one to obtain similar gains at different quark masses.

Lattice details

Configurations: provided by the Wuppertal group

- $n_f \approx 2+1$ configurations using Symanzik improved gauge action and stout-link improved **staggered** fermion action
- $a \approx (1.55 \text{ GeV})^{-1} \approx 0.13$ fm, $L \approx 2$ fm, $m_s \approx$ physical, $m_{u,d} \approx 8\times$ physical, 980 configs.

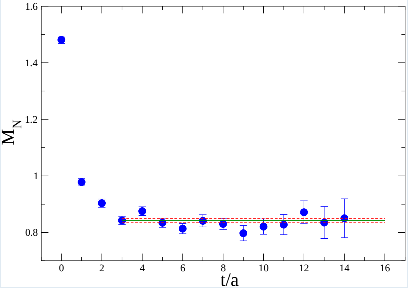
Propagators:

- Wilson action, $\kappa = 0.166$ ($M_{PS} \approx 600$ MeV), $\kappa = 0.1675$ ($M_{PS} \approx 450$ MeV) and $\kappa = 0.1684$ ($M_{PS} \approx 300$ MeV).
- Conjugate gradient algorithm with even-odd preconditioning [chroma] for stochastic propagators (with deflation at $\kappa = 0.1684$), BiCGStab for nonstochastic valence.

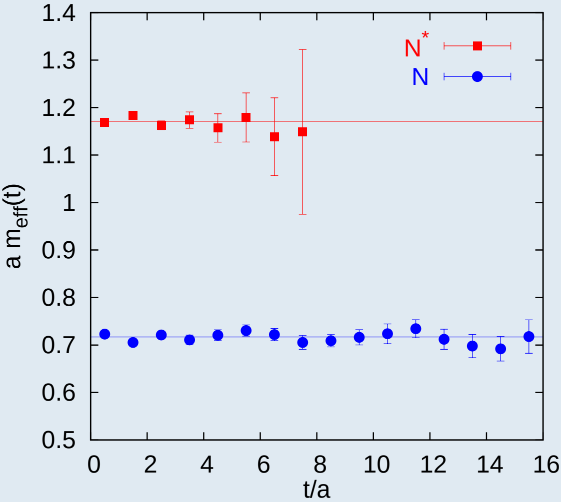
Smearing

Wuppertal smearing with APE smeared transporters.

$M_{PS} \approx 600$ MeV, i.e. $m \approx m_s$, 324 configurations:
nucleon effective mass (smeared-smeared).



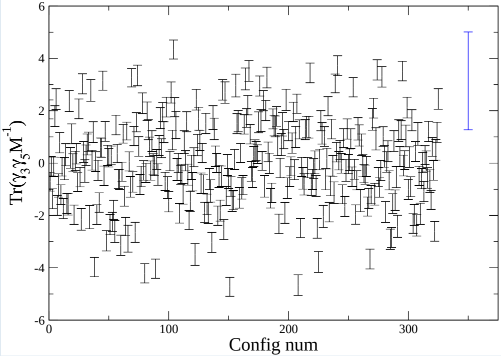
To avoid excited states we choose $t_0 = 0$ and $t = 3a \approx 0.38$ fm.
Better smearing is possible!



$n_F = 2$ Wilson, $a \approx 0.077$ fm, $m_\pi \approx 700$ MeV.

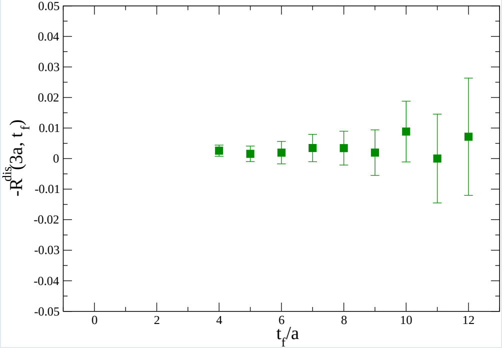
Disconnected loop: $\mathrm{Tr}(\gamma_3\gamma_5 M^{-1})$

- $\kappa_{loop} = 0.166 \approx \kappa_s$
- Partitioning: $t = 3a$
- TSM: $n_t = 90$, $L_1 \approx 2000$, $n_c \approx 480$, $L_2 \approx 100$.
- HPE: $(\kappa \not{D})^8$



$R^{dis}(3a, t_f) = \Delta q^{dis}$

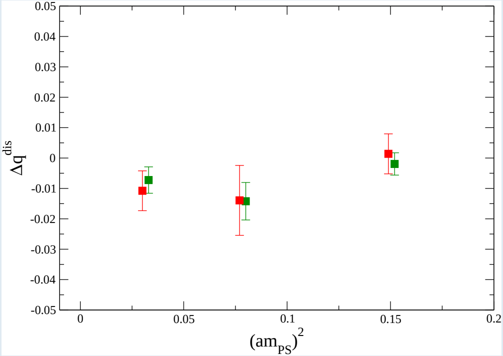
- $\kappa_{loop} = \kappa_{proton} = 0.166 \approx \kappa_s$: Δs for an *sss* proton.
- 324 configurations, $a \approx 0.13$ fm.
- The error is dominated by that of the two point function!



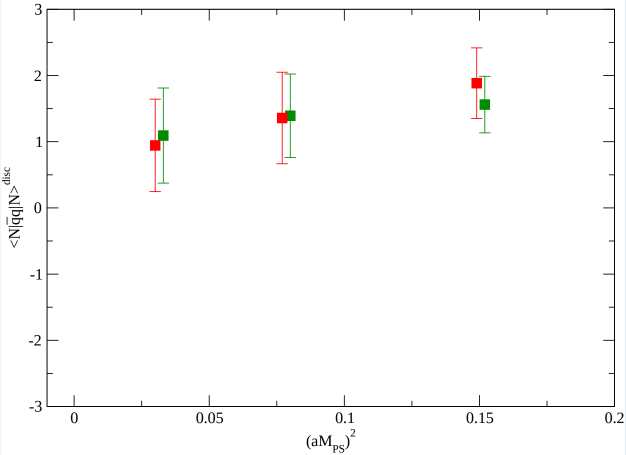
Variation of Δq^{dis} with the quark mass

Fix κ_N, i.e. the proton mass, vary κ_{loop}: $M_{PS,loop} \approx 300, 450, 600$ MeV.

- green points with $\kappa_N = 0.166 \approx \kappa_s$, **324 configs**.
- red points with $\kappa_N = 0.1675$ ($M_{PS} \approx 450$ MeV), **167 configs**.



Lowest moment of the disconnected scalar quark density



Outlook

Work in progress . . .

- Accurate calculations of isosinglet contributions to hadronic structure observables are now possible.
- Our results suggest a very small Δs in the proton. Higher twist?

Further analysis:

- Other disconnected contributions, e.g. to $\langle x \rangle$, G_M, G_E.

This was just a feasibility study on $16^3 \times 32$ volumes, using tiny computing resources (mostly desktop PCs).

Next step:

- Calculation with QCDSF on large $n_F = 2$ Clover and $n_F = 2 + 1$ SLiNC configurations.