Extracting ρ and Δ resonances from lattice simulations at small quark masses

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– QCDSF Collaboration –

With

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Outline

Preliminaries

Rho

Delta

Conclusions

Preliminaries

Action

$N_f = 2$

$$S = S_G + S_F$$

$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(x)\right)$$

$$\begin{aligned} S_F = \sum_x \Big\{ & \bar{\psi}(x)\psi(x) - \kappa\, \bar{\psi}(x) U_\mu^\dagger(x-\hat{\mu})[1+\gamma_\mu]\psi(x-\hat{\mu}) \\ & - \kappa\, \bar{\psi}(x) U_\mu(x)[1-\gamma_\mu]\psi(x+\hat{\mu}) - \frac{1}{2}\kappa\, c_{SW}\, g\, \bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \Big\} \end{aligned}$$

$\Uparrow$

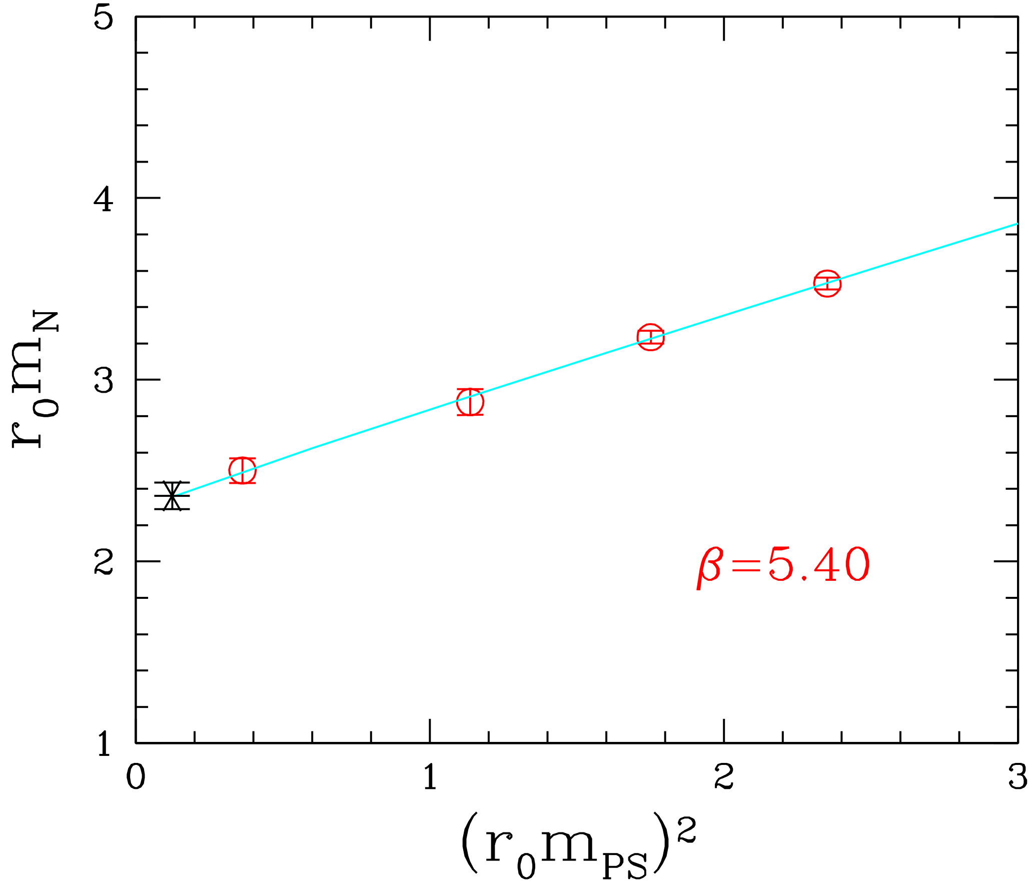
Nonperturbatively

Clover Fermions

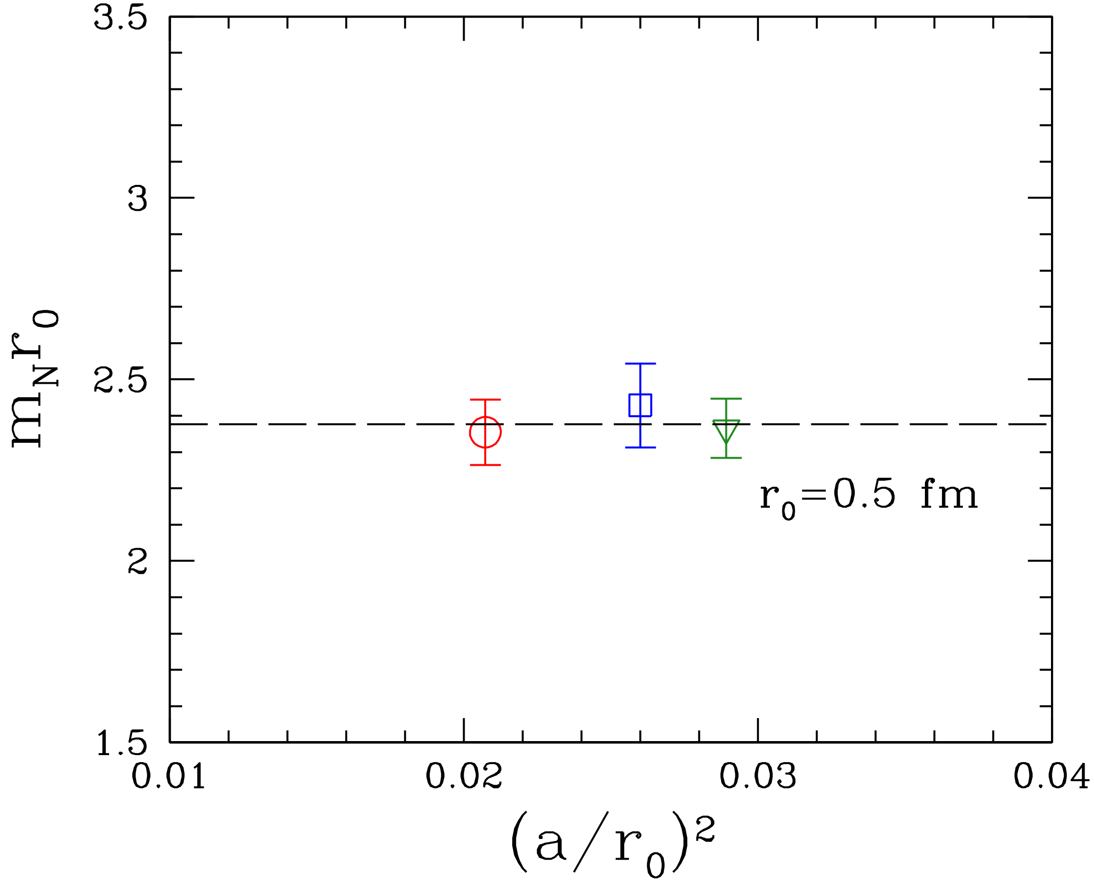
Data sample

β	κ_{sea}	Volume	a [fm]	m_{PS} [MeV]
5.25	0.13575	$24^3 \times 48$	0.084	599
5.25	0.13600	$24^3 \times 48$	0.084	431
5.29	0.13550	$24^3 \times 48$	0.080	808
5.29	0.13590	$24^3 \times 48$	0.080	593
5.29	0.13620	$24^3 \times 48$	0.080	390
5.29	0.13632	$32^3 \times 64$	0.080	264
5.29	0.13632	$40^3 \times 64$	0.080	255
5.40	0.13500	$24^3 \times 48$	0.072	808
5.40	0.13560	$24^3 \times 48$	0.072	772
5.40	0.13610	$24^3 \times 48$	0.072	611
5.40	0.13625	$24^3 \times 48$	0.072	527
5.40	0.13640	$24^3 \times 48$	0.072	425
5.40	0.13640	$32^3 \times 64$	0.072	425
5.40	0.13660	$32^3 \times 64$	0.072	240

Nucleon

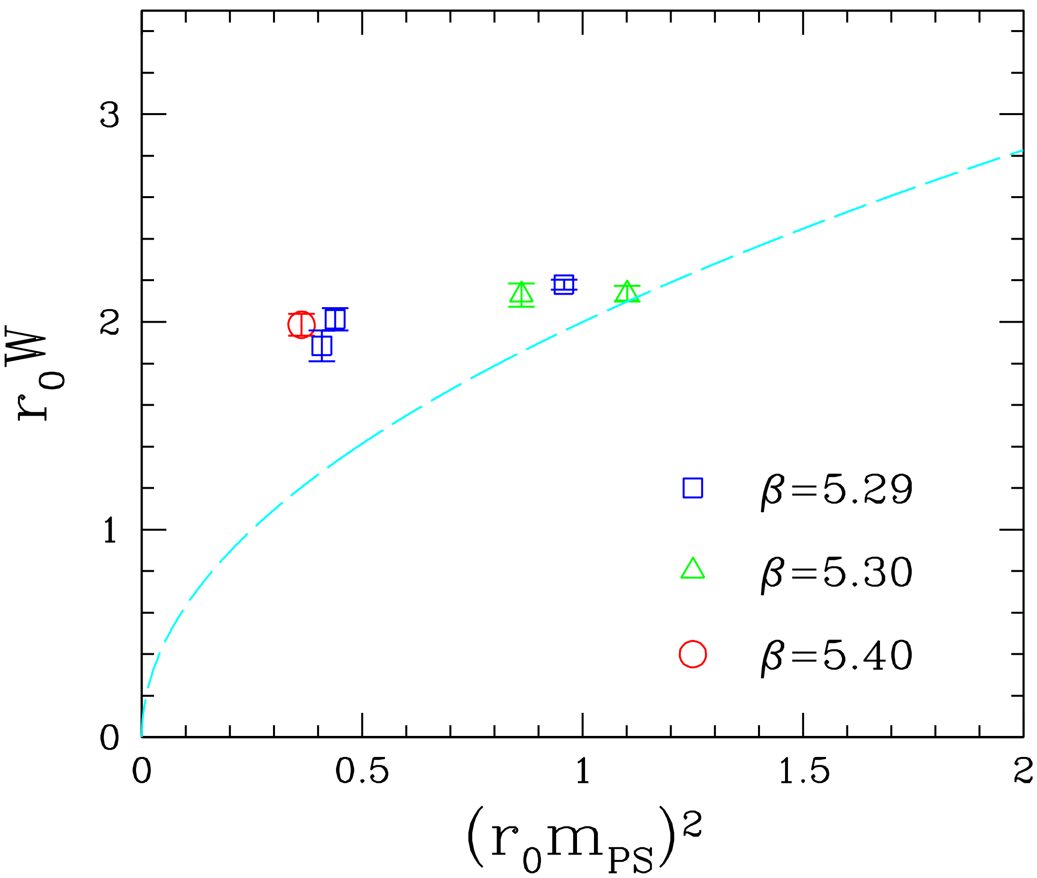


Scale & scaling



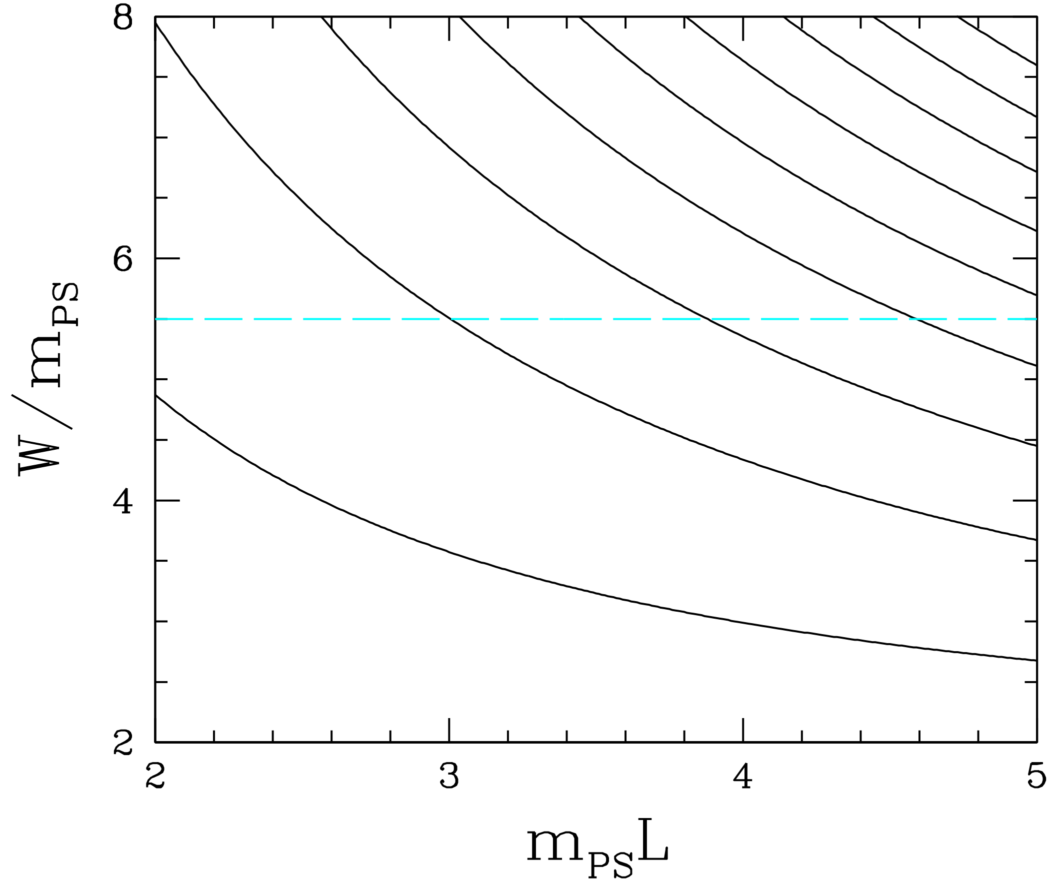
Rho

Energies $(W > 2m_{PS})$

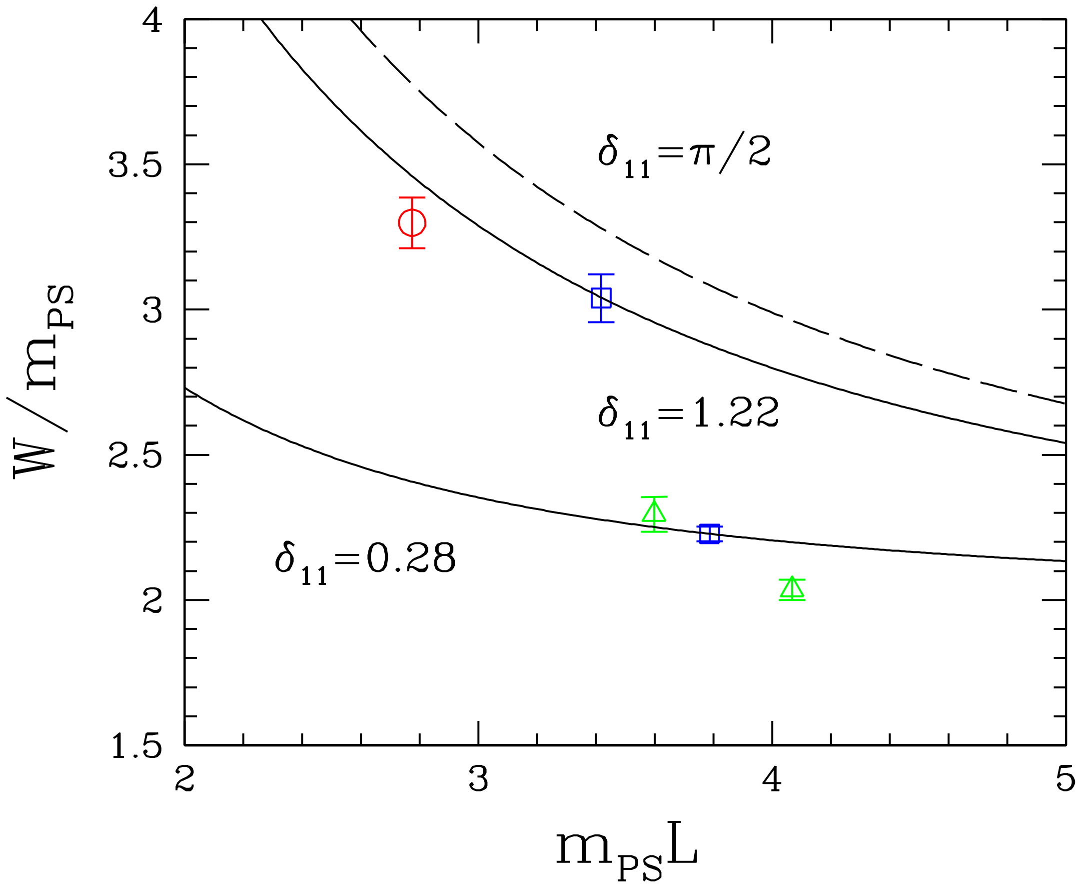


$$W = 2\sqrt{m_{PS}^2 + k^2}$$

Expected energy spectrum at physical point

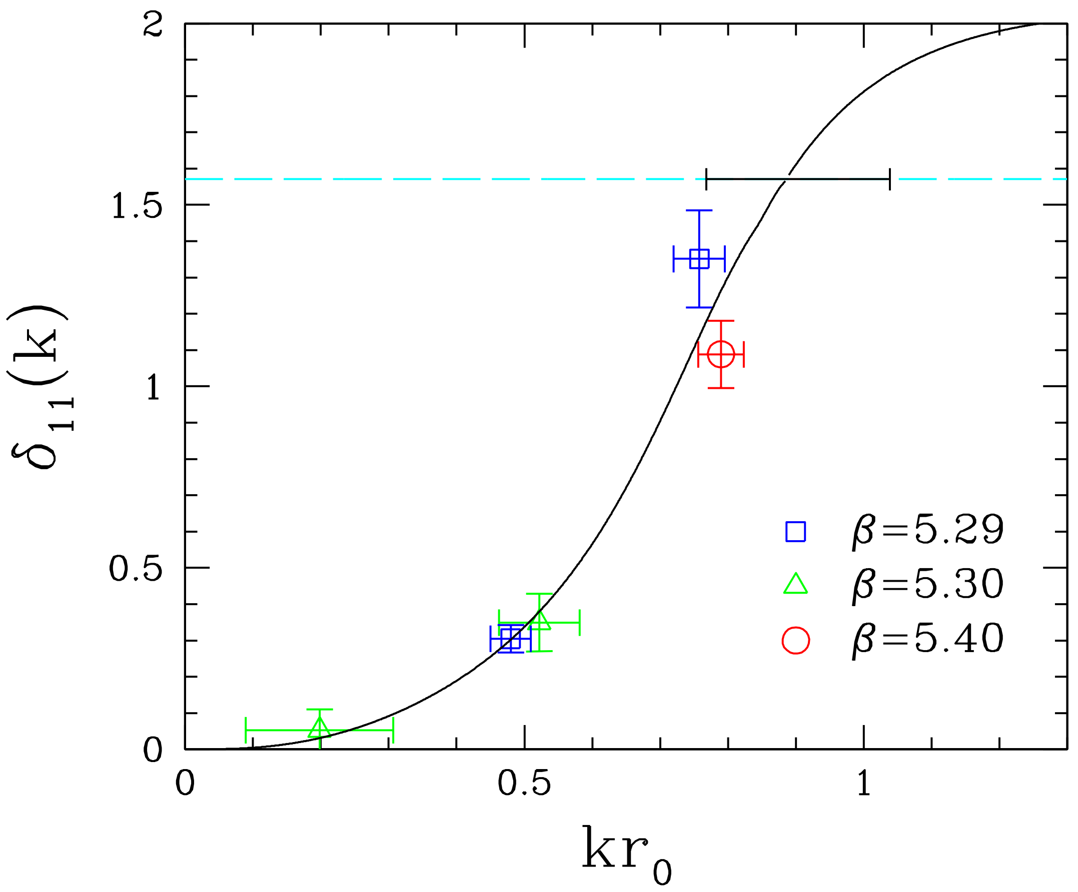


Energy spectrum



$$\frac{W}{m_{PS}} \approx 2\sqrt{1+\frac{4\pi\,\delta_{11}}{(m_{PS}L)^2}}$$

Phase shift



$$n\pi - \delta_{11}(k) = \arctan\left(-\frac{\pi^{3/2}\,q}{Z_{00}(1;q^2)}\right)\,, \quad q = \frac{k\,L}{2\pi}$$

Lüscher

Fit

$$\tan\delta_{11}(k) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{W(m^2_\rho - W^2)}$$

Breit–Wigner

Results

$r_0\,k_\rho = 0.90(13)$

$$r_0\,m_\rho = 2\sqrt{(r_0k_\rho)^2 + (r_0m_\pi)^2} = 1.94(28)$$

$r_0 = 0.50\,\mathrm{fm}$

$g_{\rho\pi\pi} = 7(2)$

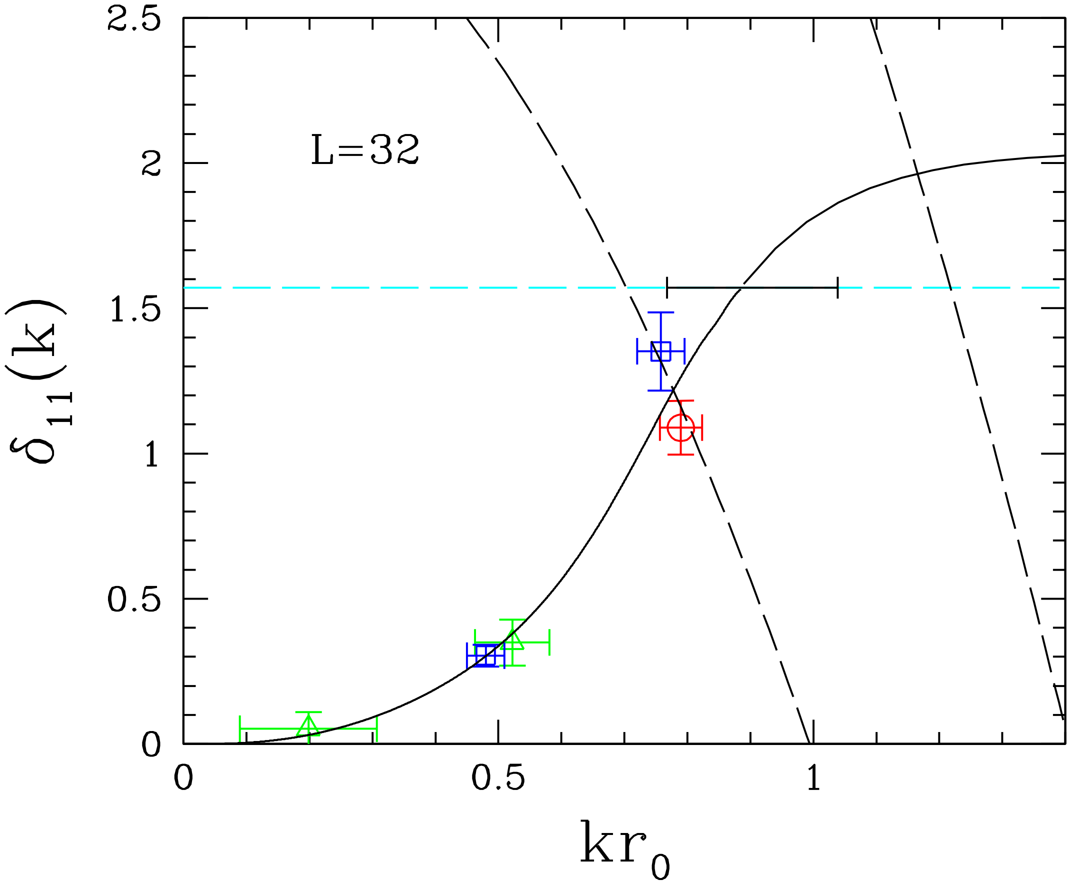
$$r_0\,\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{(r_0k_\rho)^3}{(r_0m_\rho)^2} = 0.50^{+0.32}_{-0.25}$$

$\Downarrow$

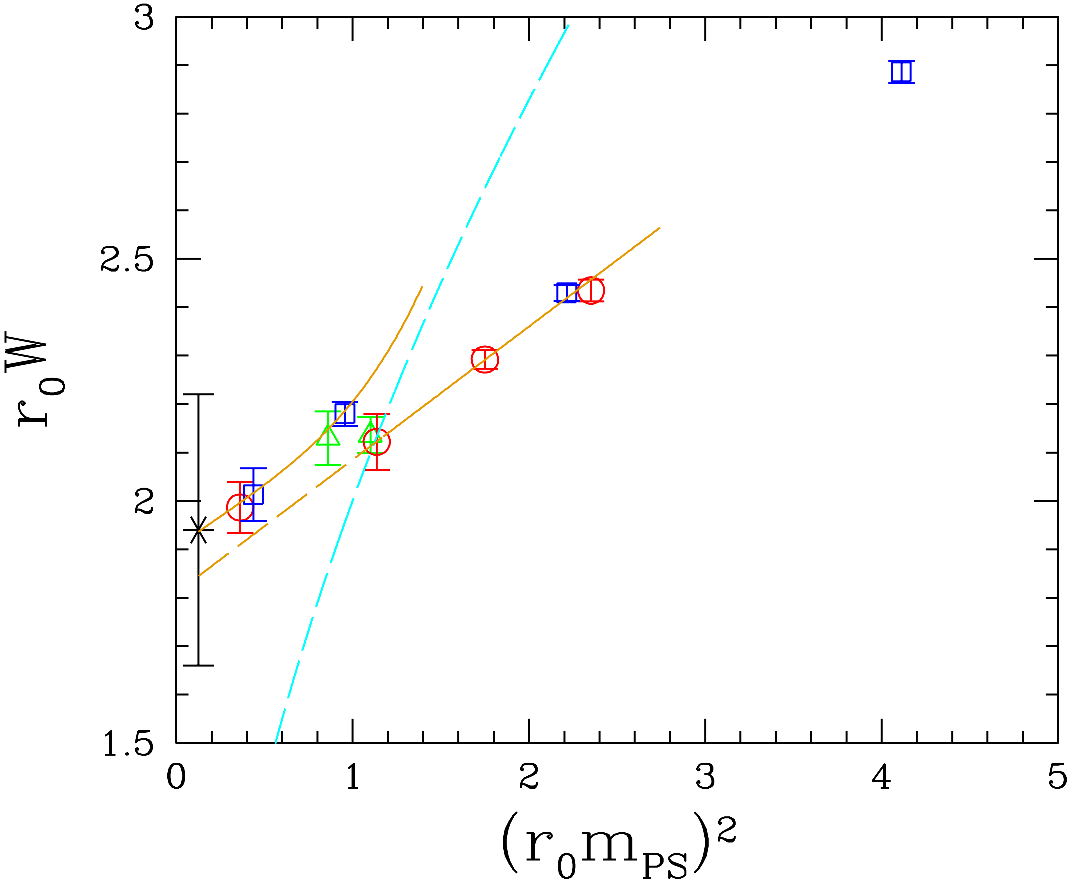
$m_\rho = 770(111)$ MeV

$\Gamma_\rho = 200^{+130}_{-100}$ MeV

Higher level(s)

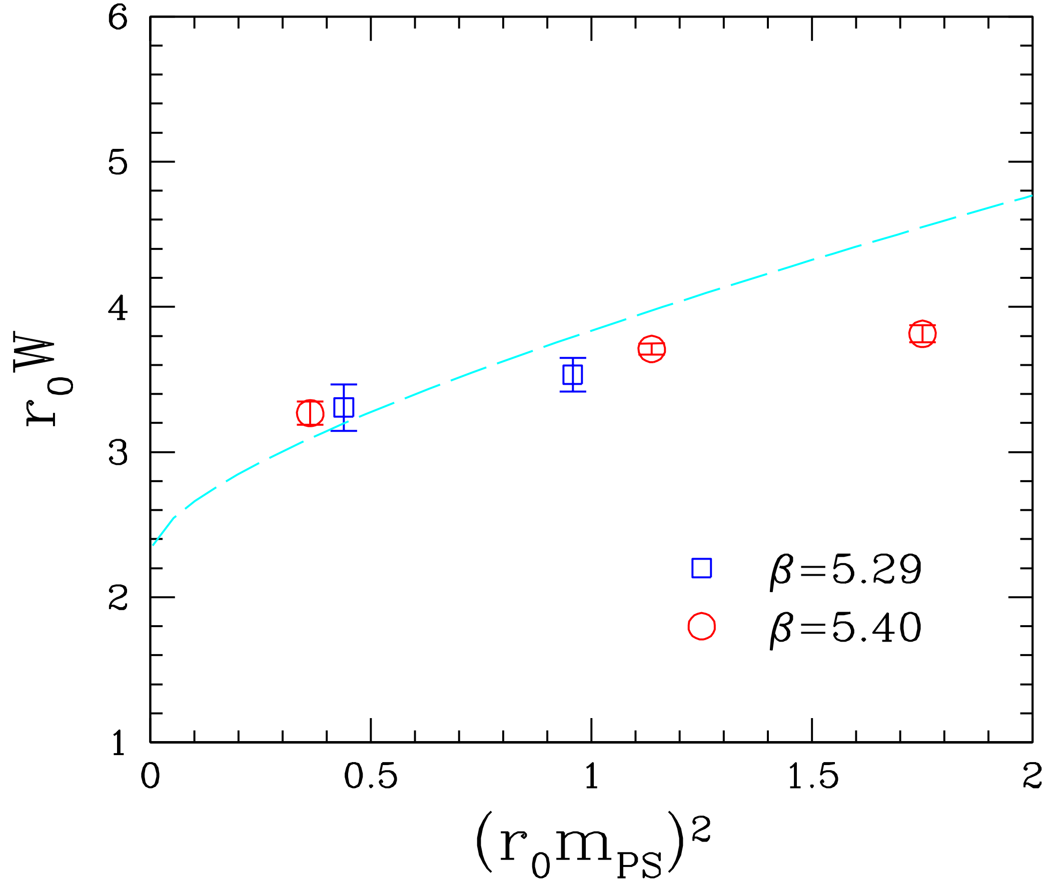


Kink ?



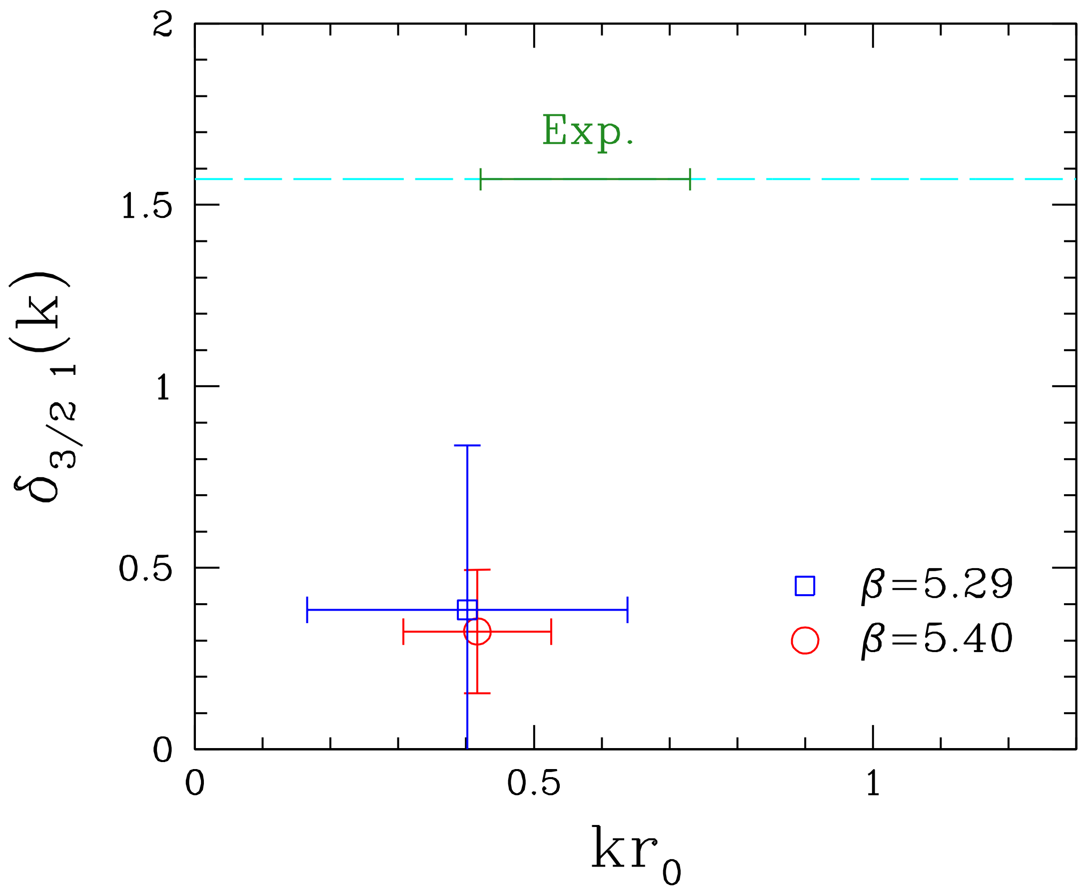
Delta

Energies



$$W = \sqrt{m_{PS}^2 + k^2} + \sqrt{m_N^2 + k^2}$$

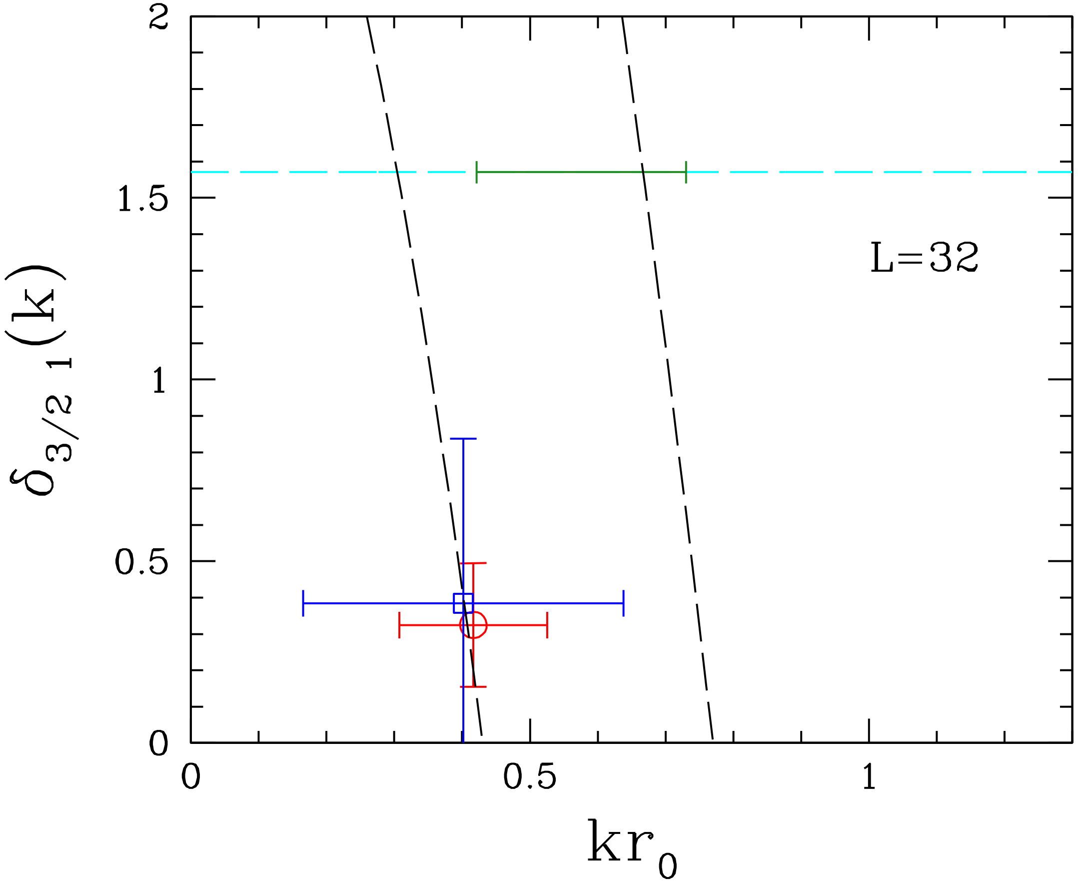
Phase shift



$$n\pi - \delta_{3/2\,1}(k) = \arctan\left(-\frac{\pi^{3/2}\,q}{Z_{00}(1;q^2)}\right)\;, \quad q = \frac{k\,L}{2\pi}$$

Bernard, Lage, Meissner & Rusetsky

Higher level(s)



Conclusions

- Simulations of full QCD on large lattices and at pion masses of $O(250\,\mathrm{MeV})$, with $m_\rho > 2m_\pi$ and $m_\Delta > m_N + m_\pi$, are in progress now

- Following Lüscher, this enabled us to compute the elastic $\pi\pi$ and πN scattering amplitudes from the respective particle spectrum in the finite volume

- First results on the mass and width of the ρ meson are encouraging and demonstrate the practicability of the method

- To be able to extract mass and width of the Δ resonance, one needs slightly smaller quark masses and $L = O(4\,\mathrm{fm})$ lattices