The curvature of the QCD phase transition line

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Lattice 2008
Williamsburg, 18. July 2008.

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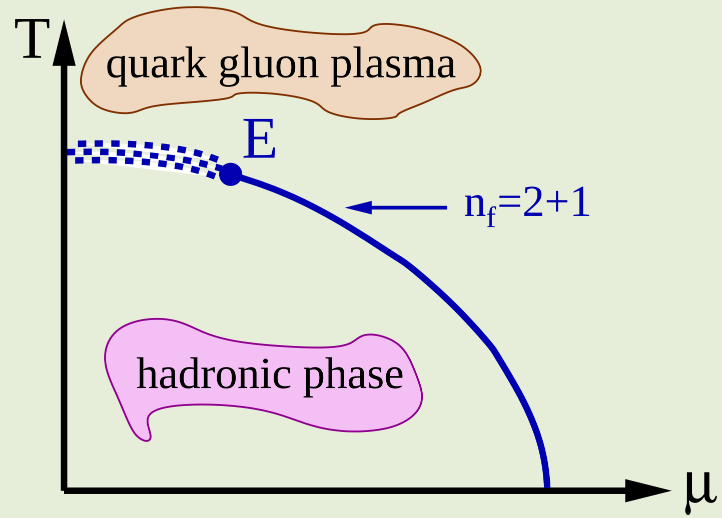
Introduction, motivation



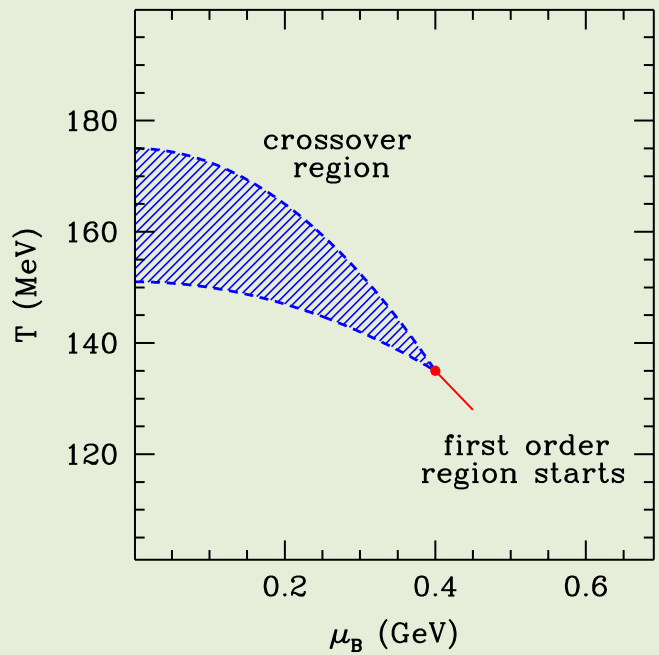
- Zero μ area is relevant for - the early Universe
  - high energy collisions
- Nature of QCD transition at zero μ is found to be a crossover [Y. Aoki, G.Endrődi, Z. Fodor, S.D. Katz, K.K. Szabó]
- Different observables give different values for T_c namely, $T_c(\chi_{\bar{\psi}\psi}) \approx 151$ MeV, $T_c(\chi_s) \approx 175$ MeV [Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabó]

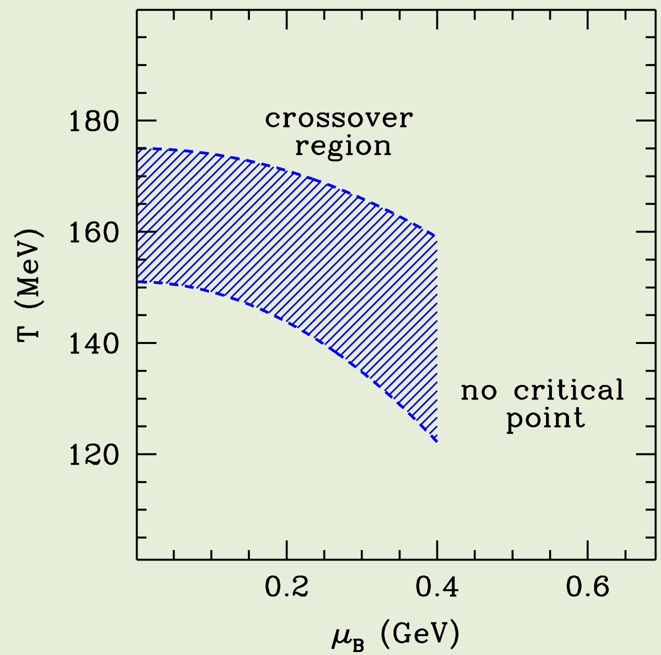
The role of the curvature

- Explore the $\mu \neq 0$ region of the phase diagram
- At $\mu \neq 0$ the fermion determinant is complex
  $\rightarrow$ importance sampling not possible
- Use Taylor-expansion in μ
  first term vanishes
  second term given by the curvature (κ)
- Aims:
  determine the curvature for different observables
  $\chi_{\bar{\psi}\psi}$, $\bar{\psi}\psi$; and for L, χ_s
- Comparison: no continuum limit results; the curvature is in the range of $\kappa = 0.003 \ldots 0.01$

  [Bielefeld-Swansea; Philipsen, de Forcrand; D'Elia, Lombardo; Fodor, Katz]

Scenarios





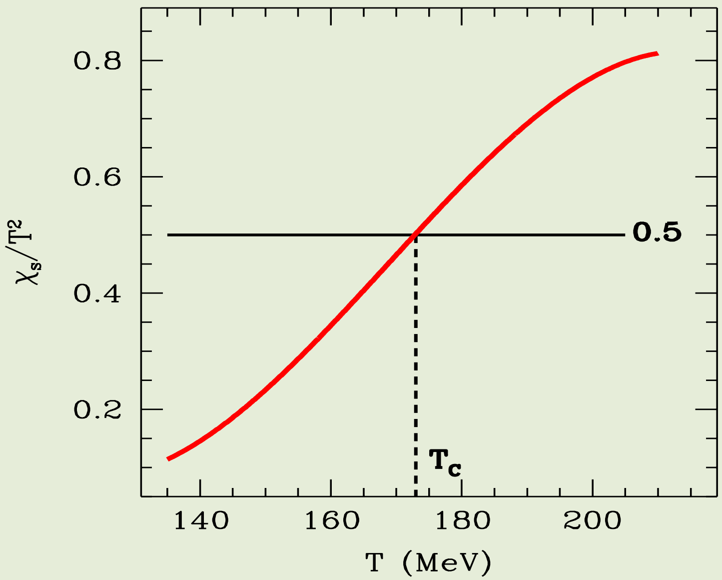
- Does the crossover region shrink or expand?
- The curvature can affect the existence of the critical endpoint
- Estimation: if $\mu_{crit} = 360$ MeV $\rightarrow \Delta\kappa \approx 0.02$
- $\mu \equiv \mu_B$

Definition of κ

- Equation of transition line is $T_c(\mu) = T_c\left(1 - \kappa\frac{\mu^2}{T_c^2}\right)$

$$\rightarrow \kappa = -T_c \left.\frac{\mathrm{d}T_c(\mu)}{\mathrm{d}\mu^2}\right|_{\mu=0}$$

- Let's assume we define T_c as $\mathcal{O} = 0.5$ (e.g. $\mathcal{O} = \chi_s/T^2$)
- For $\mathcal{O}(T,\mu^2)$: $\mathrm{d}\mathcal{O} = \frac{\partial\mathcal{O}}{\partial T}\cdot \mathrm{d}T + \frac{\partial\mathcal{O}}{\partial\mu^2}\cdot\mathrm{d}\mu^2$



- along the $T_c(\mu)$ line $\mathrm{d}\mathcal{O} = 0$ by definition

$$\rightarrow \frac{\mathrm{d}T_c}{\mathrm{d}\mu^2} = -\left(\frac{\partial\mathcal{O}}{\partial\mu^2}\right) \Big/ \left(\frac{\partial\mathcal{O}}{\partial T}\right)$$

- For $\frac{\partial\mathcal{O}}{\partial\mu^2}$ we need to measure new operators

Technique

- Consider $\mathcal{Z} = \int \mathcal{D}U e^{-S_g(U)} \det M^{N_f/4}$

- $\frac{\partial \log \mathcal{Z}}{\partial \mu_{u,d}} = \langle n_{u,d} \rangle$; $\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_{u,d}^2} = \langle \chi_{u,d} \rangle$, where $(' \equiv \frac{\partial}{\partial \mu_{u,d}})$

$$n_{u,d} = \frac{N_f}{4} \mathrm{Tr} \left(M^{-1} M' \right) \text{ and}$$
$$\chi_{u,d} = n_{u,d}^2 + \frac{N_f}{4} \mathrm{Tr} \left(M^{-1} M'' - M^{-1} M' M^{-1} M' \right)$$

- Observables $\mathcal{O}$ that don't depend on $\mu_{u,d}$ (L, χ_s):

$$\frac{\partial \langle \mathcal{O} \rangle}{\partial (\mu_{u,d}^2)} = \frac{1}{2} \frac{\partial^2 \langle \mathcal{O} \rangle}{\partial \mu_{u,d}^2} = \langle \mathcal{O} \chi_{u,d} \rangle - \langle \mathcal{O} \rangle \langle \chi_{u,d} \rangle$$

- Observables $\mathcal{O}$ that depend on $\mu_{u,d}$ ($\bar{\psi}\psi$, $\chi_{\bar{\psi}\psi}$):

$$\frac{\partial \langle \mathcal{O} \rangle}{\partial (\mu_{u,d}^2)} = \frac{1}{2} \frac{\partial^2 \langle \mathcal{O} \rangle}{\partial \mu_{u,d}^2} = \langle \mathcal{O} \chi_{u,d} \rangle - \langle \mathcal{O} \rangle \langle \chi_{u,d} \rangle + \langle 2 \mathcal{O}' n_{u,d} + \mathcal{O}'' \rangle$$

Simulation setup

- Symanzik improved gauge and stout-link improved staggered fermionic lattice action
- Physical masses for $m_{u,d}$ and for m_s
- LCP determined by fixing m_K/f_K and m_K/m_π
- Scale set by f_K
- Lattice spacings used: $N_T = 4, 6, 8, 10$
- with aspect ratios $N_S/N_T = 4$ and 3
- Measurements carried out with 80 random vectors (balance between measurements and configuration production)

Observables I.

- Polyakov loop $L = \frac{1}{N_S^3} \sum_{\mathsf{x}} \mathrm{Tr} \prod_{\mathsf{t}=0}^{N_T - 1} U_4(x,t)$

  renormalization: $L_r = L \exp(V(r_0)/2T)$

- Strange susceptibility $\chi_s = \frac{T}{V} \frac{\partial^2 \log \mathcal{Z}}{\partial \mu_s^2}$

  no renormalization necessary

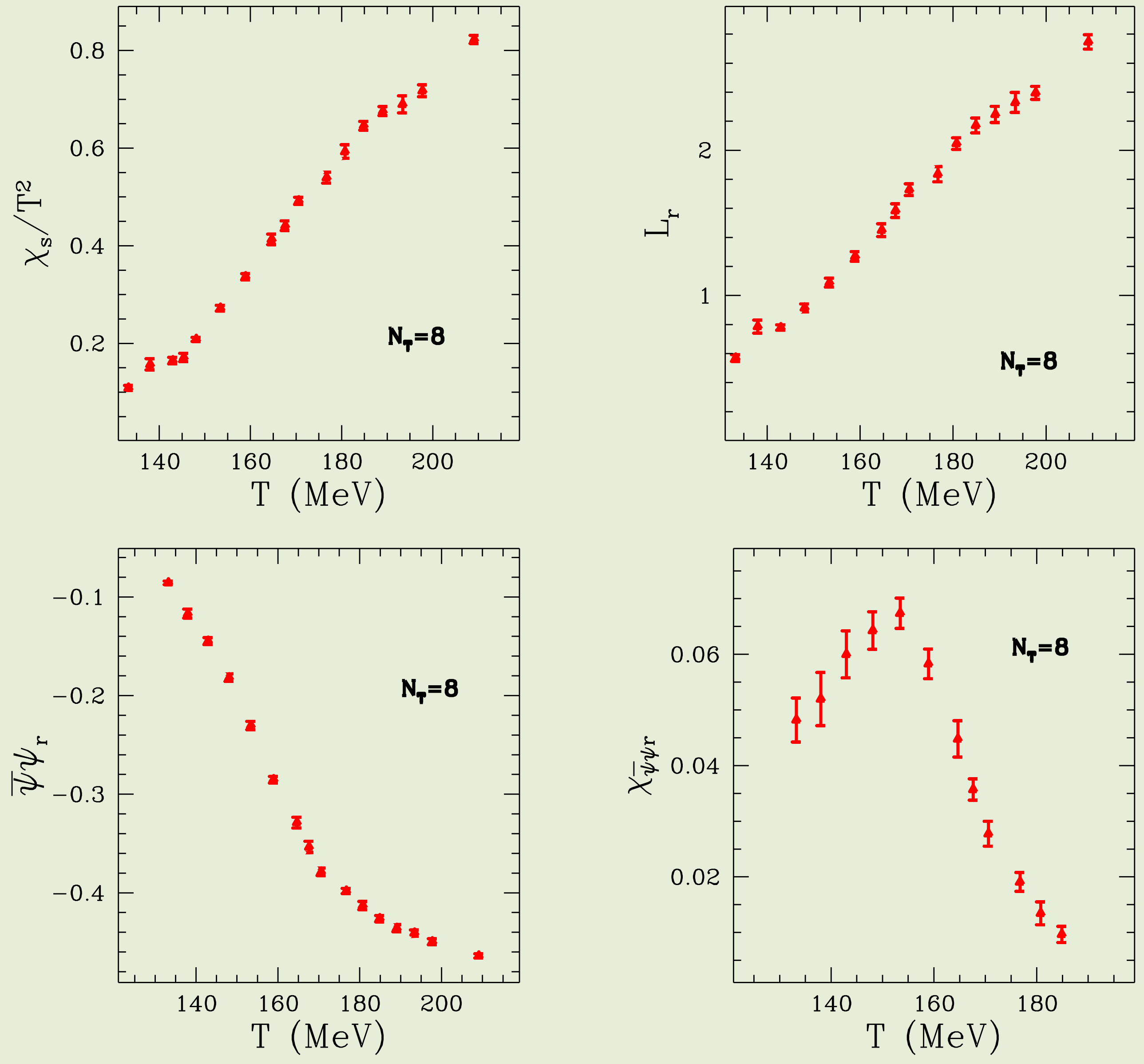
- Chiral condensate $\bar{\psi}\psi = \frac{T}{V} \frac{\partial \log \mathcal{Z}}{\partial m}$

  renormalization: $\bar{\psi}\psi_r = (\bar{\psi}\psi - \bar{\psi}\psi(T=0)) \cdot m \cdot \frac{1}{m_\pi^4}$

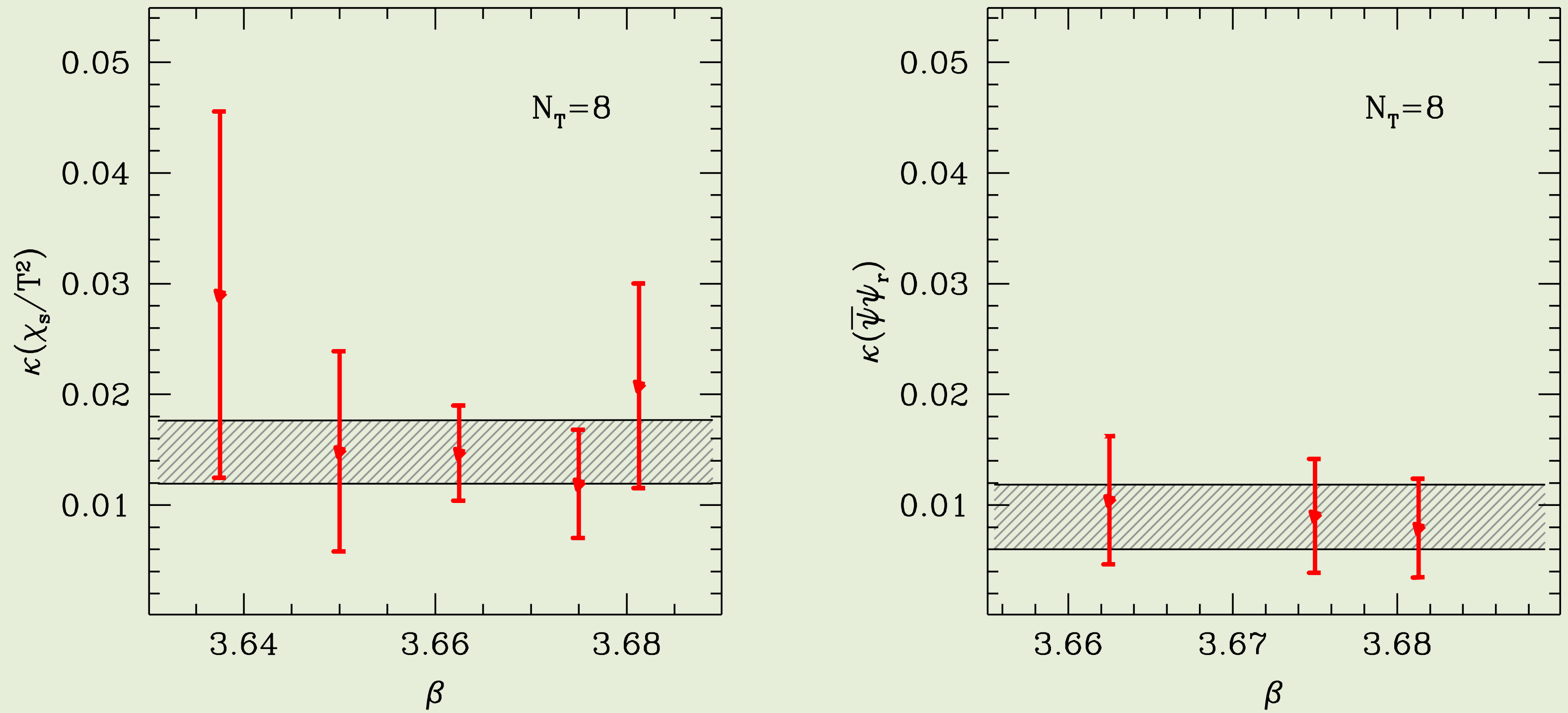
- Chiral susceptibility $\chi_{\bar{\psi}\psi} = \frac{T}{V} \frac{\partial^2 \log \mathcal{Z}}{\partial m^2}$

  renormalization: $\chi_{\bar{\psi}\psi r} = (\chi_{\bar{\psi}\psi} - \chi_{\bar{\psi}\psi}(T=0)) \cdot m^2 \cdot \frac{1}{T^4}$

Observables II.

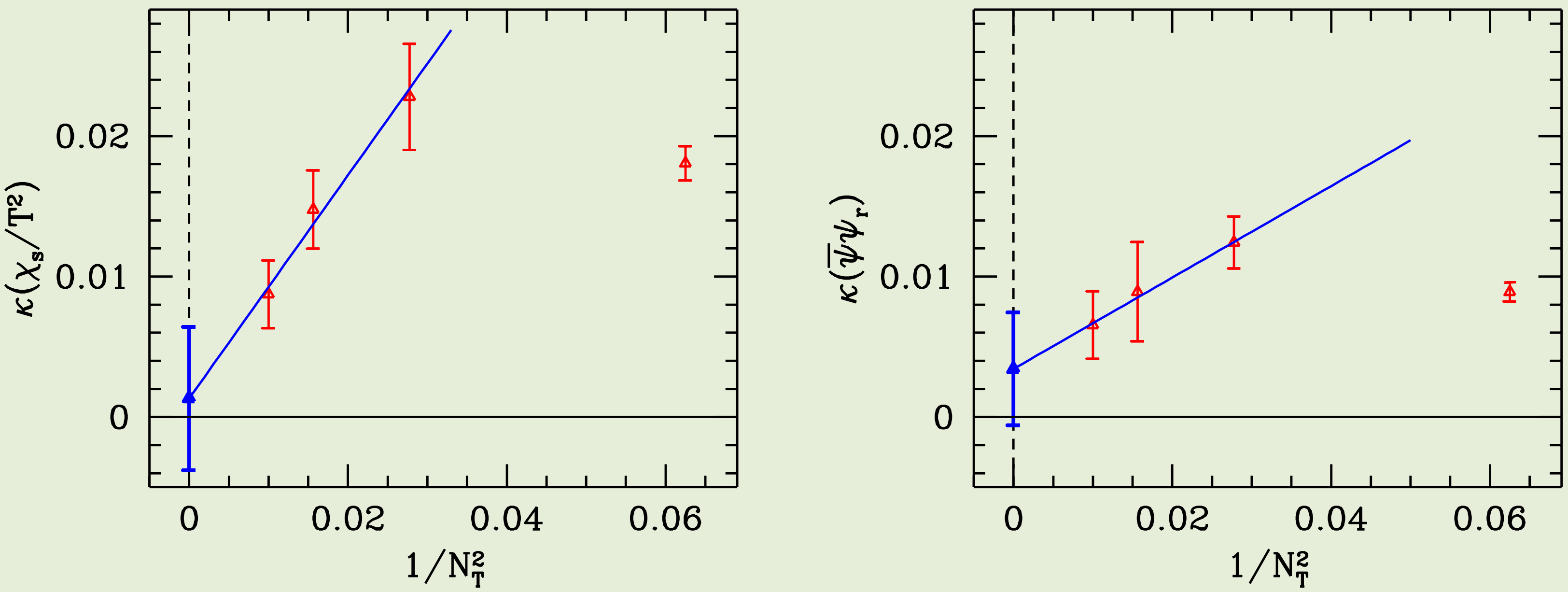


Results I.



- Independent measurements at different β values

Results II.



- Continuum extrapolated results are preliminary
- Notice large errors
  $\rightarrow$ for $N_T = 6, 8, 10$ more statistics is needed
  $\kappa(\chi_{\bar\psi\psi r}) = -0.0018(34)$, $\kappa(\bar\psi\psi_r) = 0.0034(40)$
  $\kappa(L_r) = -0.0095(93)$, $\kappa(\chi_s/T^2) = 0.0013(51)$

Summary

- Although $\kappa(\bar{\psi}\psi_r) < \kappa(\chi_s/T^2)$ for every N_T, they become consistent in the continuum limit
- For that reason we cannot make an estimate about the location of the critical endpoint
- We need to increase statistics to achieve a reliable continuum limit