D_s physics from fine lattices

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ALPHA Collaboration

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The D_s meson and lattice QCD

- f_{D_s} has well-known discrepancy between experiment and lattice simulations:

CLEO	268(8)(4) MeV	[arXiv:0806.3921]
HPQCD	241(3) MeV	[arXiv:0706.1726]

- New physics? Experimental issues? Discretisation effect?
- Here we present a progress report towards studying D_s physics on the large and fine lattices simulated by the CLS effort

The CLS coordinated lattice simulations effort

- CLS is a community effort to bring together the human and computer resources of several teams in Europe interested in lattice QCD
- Member teams: Berlin, CERN, DESY-Zeuthen, Madrid, Mainz, Rome, Valencia
- All CLS simulations use Lüscher's DD-HMC algorithm:
  - $N_f = 2$ Wilson QCD with non-perturbative $O(a)$ improvement
  - Domain decomposition separates block and inter-block MD forces
  - Use a Schwarz-preconditioned GCR solver on each block
  - Deflation to reduce the effort at smaller quark masses
  - Chronological solver to further accelerate MD dynamics

CLS ensembles used

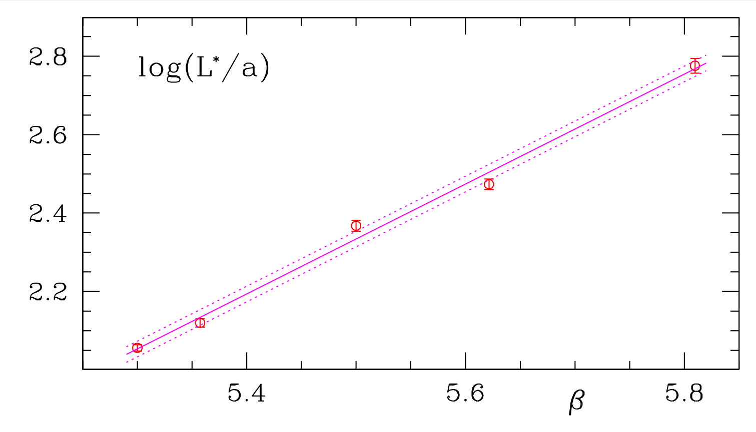
Name	Size	β	a	κ	MD τ
D1	48×24^3	5.3	0.08 fm	0.13550	2575
D2				0.13590	2565
D3				0.13610	2520
D4				0.13620	2505
D5				0.13625	2510
E1	64×32^3	5.3	0.08 fm	0.13550	2672
E2				0.13590	2512
E3				0.13605	2512
E4				0.13610	2497
E5				0.13615	2656
E6				0.13635	4960

CLS ensembles used

Name	Size	β	a	κ	MD τ
M1	64×32^3	5.5	0.06 fm	0.13620	4055
M2				0.13630	3772
M3				0.13640	2980
M4				0.13650	3790
M5				0.13660	2570
P1	96×48^3	5.7	0.04 fm	0.13620	1702
P2				0.13630	started
P3				0.13640	started
Q4	128×64^3	5.7	0.04 fm	0.13640	$1450+$
Q5				0.13650	$1120+$
Q6				0.136575	started

Setting the scale

- No unambiguous determination of scale via e.g. m_Ω available yet
- In [hep-lat/0610059], Del Debbio *et al.* determined scale at $\beta = 5.3$ via m_K as $a = 0.0784(10)$ fm
- We run this to other values of β via the scale L^* defined by Della Morte *et al.* in [arXiv:0710.1263] via $\bar{g}^2(L^*) = 5.5$ in the Schrödinger functional



- Specifically, we use their linear fit $\log(L^*/a) = 2.3338 + 1.4025(\beta - 5.5) \pm 0.02$
- Uncertainty about scale important source of error at $\beta > 5.3$

Measurements and Analysis

- Use $U(1)$ noise sources localised in time (6 sources per configuration)
- Measure $\langle PP\rangle$, $\langle PA_0\rangle$, $\langle A_0P\rangle$,$\langle A_0A_0\rangle$ correlators
- Used

  61 configurations of D2 (3 m_s, 2 m_c),

  28 configurations of E6 (3 m_s, 2 m_c),

  31 configurations of Q4 (2 m_s, 2 m_c)
- Fully correlated error analysis using the Jackknife method

Measurements and Analysis (II)

- Define effective masses as in [hep-lat/0701009] via

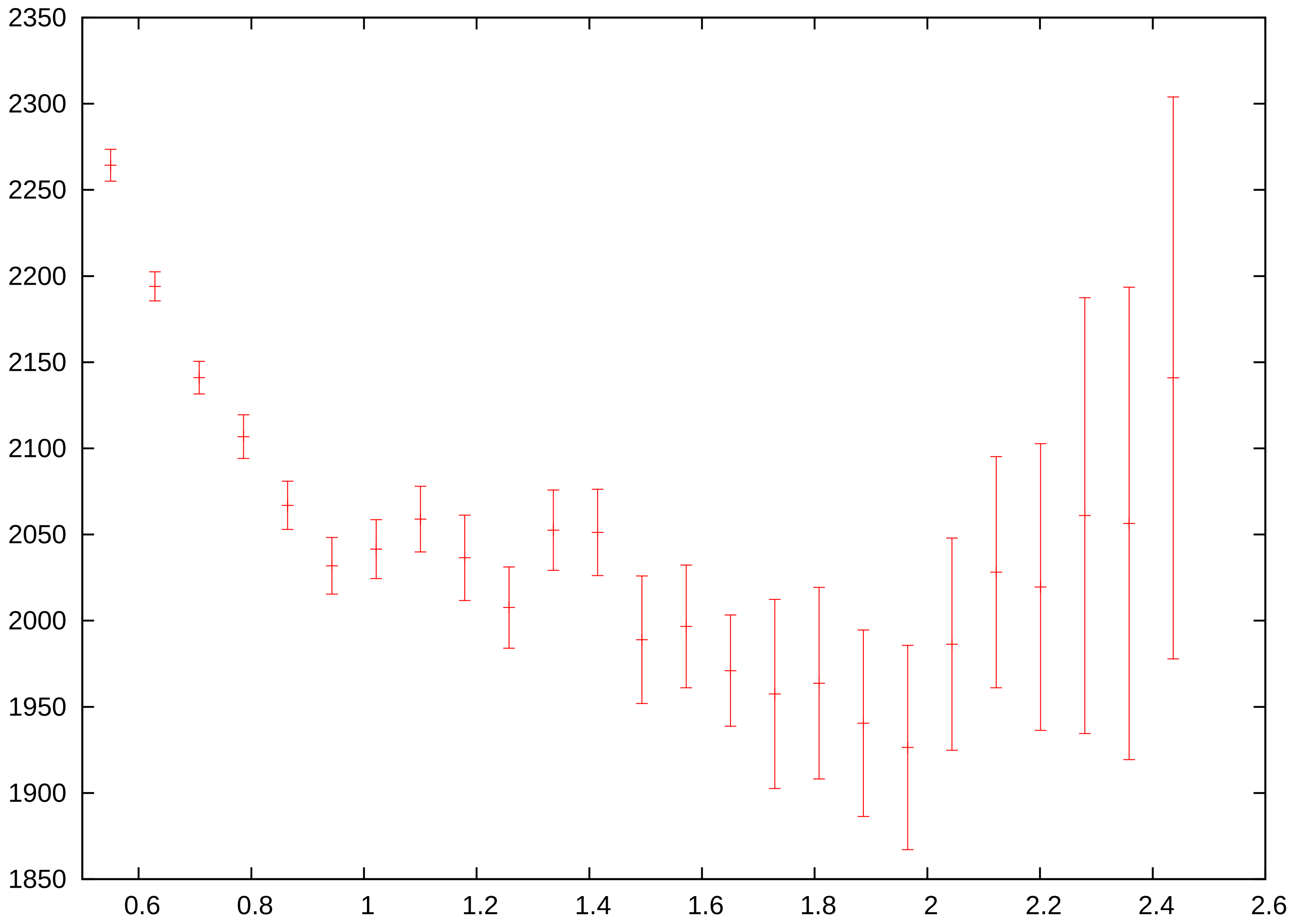
$$\frac{g(M_{eff}(x_0), x_0 - a)}{g(M_{eff}(x_0), x_0)} = \frac{C(x_0 - a)}{C(x_0)}$$

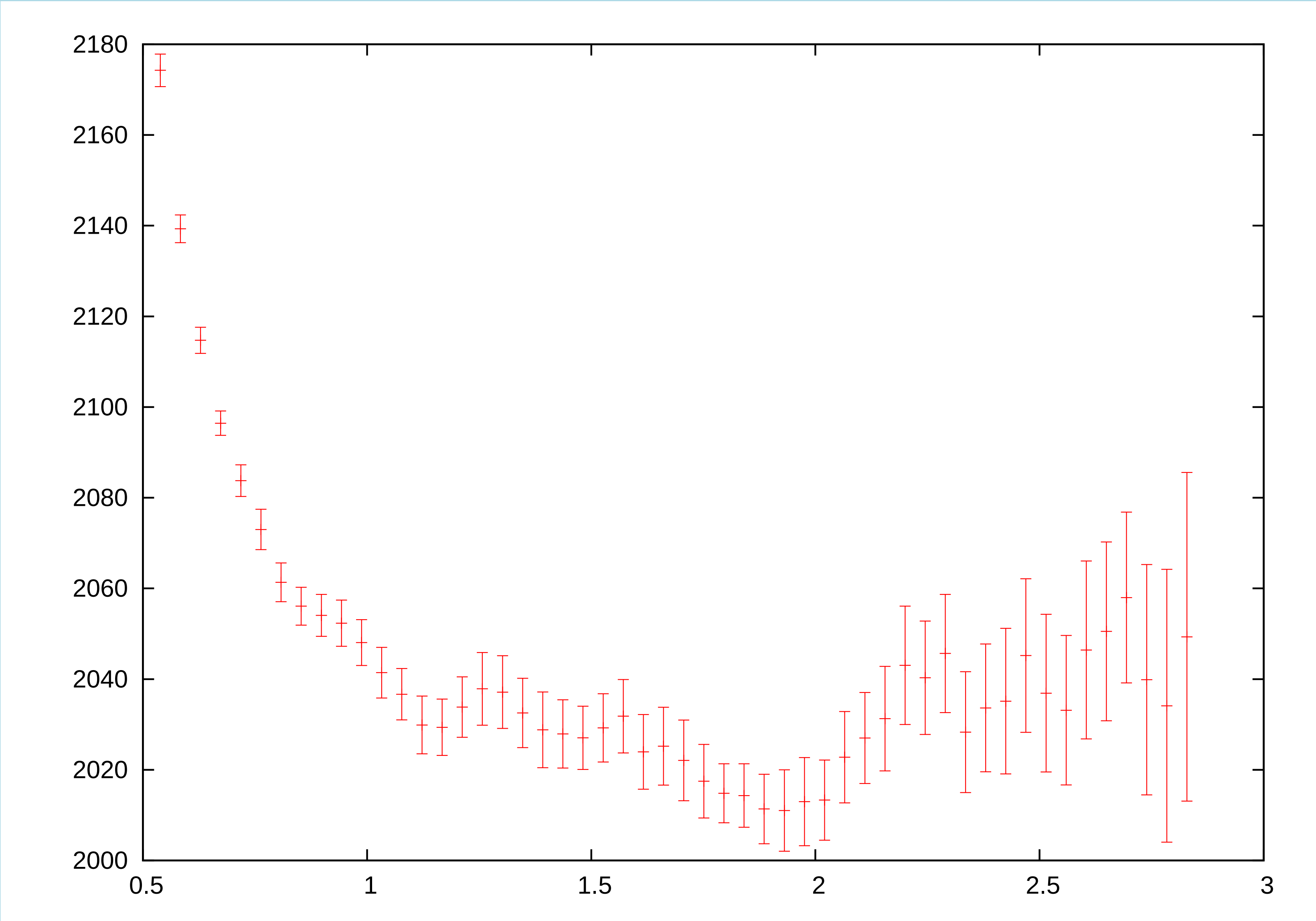
and effective matrix elements via

$$G_{eff}(x_0) = \sqrt{\frac{C(x_0) M_{eff}(x_0)}{g(M_{eff}(x_0), x_0)}}$$

where

$$g(M, x) = e^{-Mx} + e^{-M(T-x)}$$


2350
2300
2250
2200
2150
2100
2050
2000
1950
1900
1850
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6


2180
2160
2140
2120
2100
2080
2060
2040
2020
2000
0.5
1
1.5
2
2.5
3

Renormalisation and Improvement

- Define PCAC quark mass as

$$m_{PCAC}(x_0) = \frac{\frac{1}{2}(\partial_0 + \partial_0^*)C_{PA}(x_0) + c_A a \partial_0 \partial_0^* C_{PP}(x_0)}{C_{PP}(x_0)}$$

- Needs renormalisation and $O(a)$ improvement:

$$m(\mu) = Z_A Z_P^{-1}(\mu)(1 + \frac{1}{2}(b_A - b_P)(m_q a))m_{PCAC}$$

- Renormalised pseudoscalar decay constant defined as

$$F_{PS} = Z_A(1 + \frac{1}{2}b_A(m_q a))\frac{m_{PCAC}G_{PS,eff}}{M^2_{PS,eff}}$$

Renormalisation and Improvement (II)

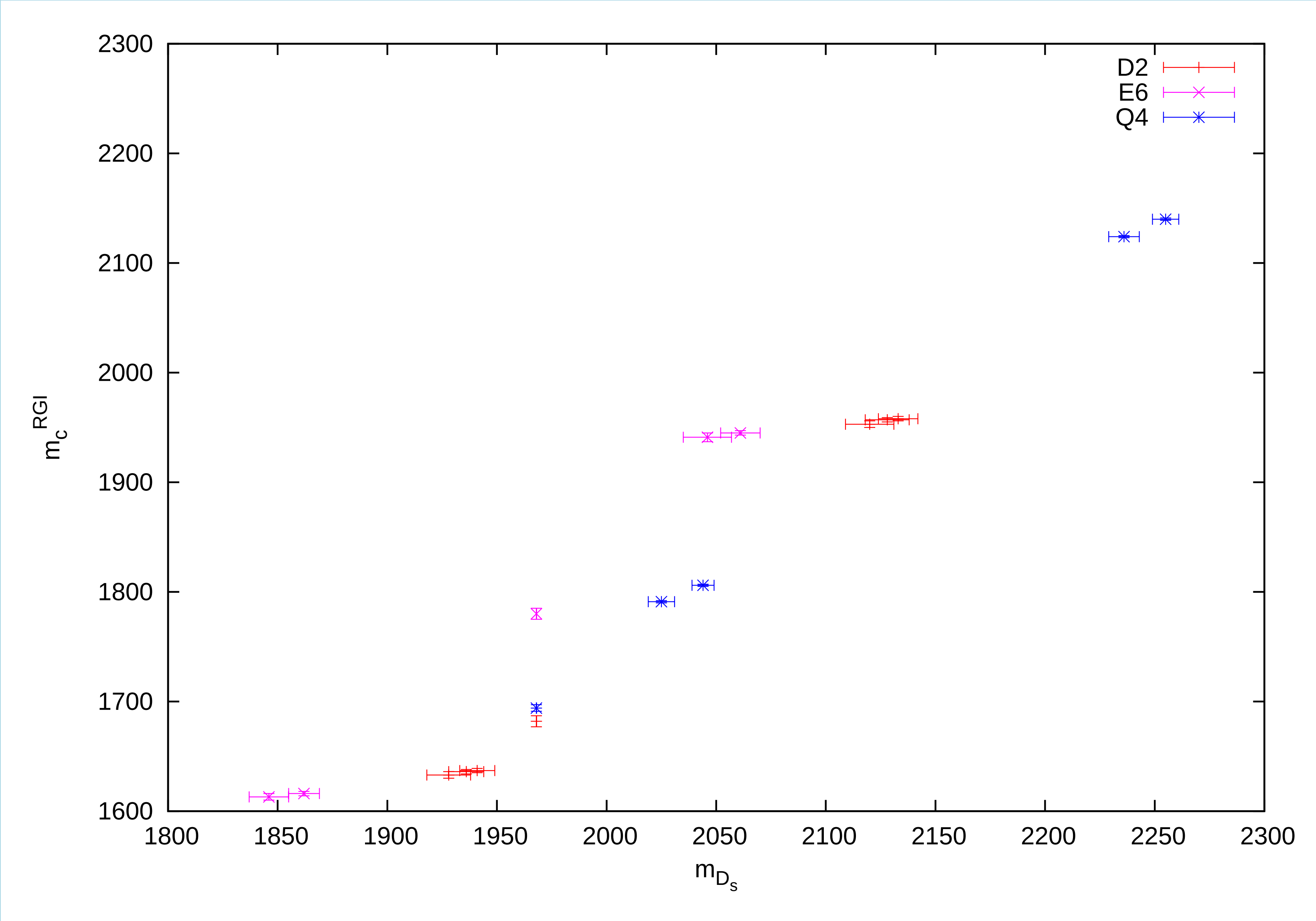
- Use non-perturbative renormalisation wherever possible:
  - c_A from Della Morte *et al.* [hep-lat/0503003]
  - Z_A from Della Morte *et al.* [arXiv:0807.1120]
  - Z_P from Della Morte *et al.* [hep-lat/0507035]
  - $b_A - b_P$ from Heitger *et al.* [private communication]
- Perturbative renormalisation (one-loop) used for b_A only
- Translate PCAC masses into RGI scheme via the Schrödinger functional as in Della Morte *et al.* [hep-lat/0507035]

CONTENT WARNING

All numbers presented on the following slides are highly preliminary and need to be checked.

D_s masses and m_c

m_{D_s}/MeV	$(m_c + m_s)$/MeV	$2m_s$/MeV	m_c/MeV	interpol.
2133(9)	2064(2)	211(1)	1958(2)	
2128(10)	2056(2)	199(1)	1957(2)	
2120(11)	2039(3)	172(1)	1953(3)	
				1682(5) MeV
1941(8)	1742(2)	211(1)	1637(2)	
1936(8)	1735(2)	199(1)	1636(2)	
1928(10)	1719(3)	172(1)	1633(3)	
2061(9)	2029(2)	168(1)	1945(2)	
2046(11)	2005(4)	128(1)	1941(4)	
				1780(5) MeV
1862(7)	1700(2)	168(1)	1616(2)	
1846(9)	1677(3)	128(1)	1613(3)	
2044(5)	1919(1)	226(1)	1806(1)	
2025(5)	1891(1)	200(1)	1791(1)	
				1694(3) MeV
2255(6)	2253(1)	226(1)	2140(1)	
2236(7)	2224(2)	200(1)	2124(1)	


D2
E6
Q4
2300
2200
2100
2000
1900
1800
1700
1600
1800
1850
1900
1950
2000
2050
2100
2150
2200
2250
2300
m_c^{RGI}
m_{D_s}

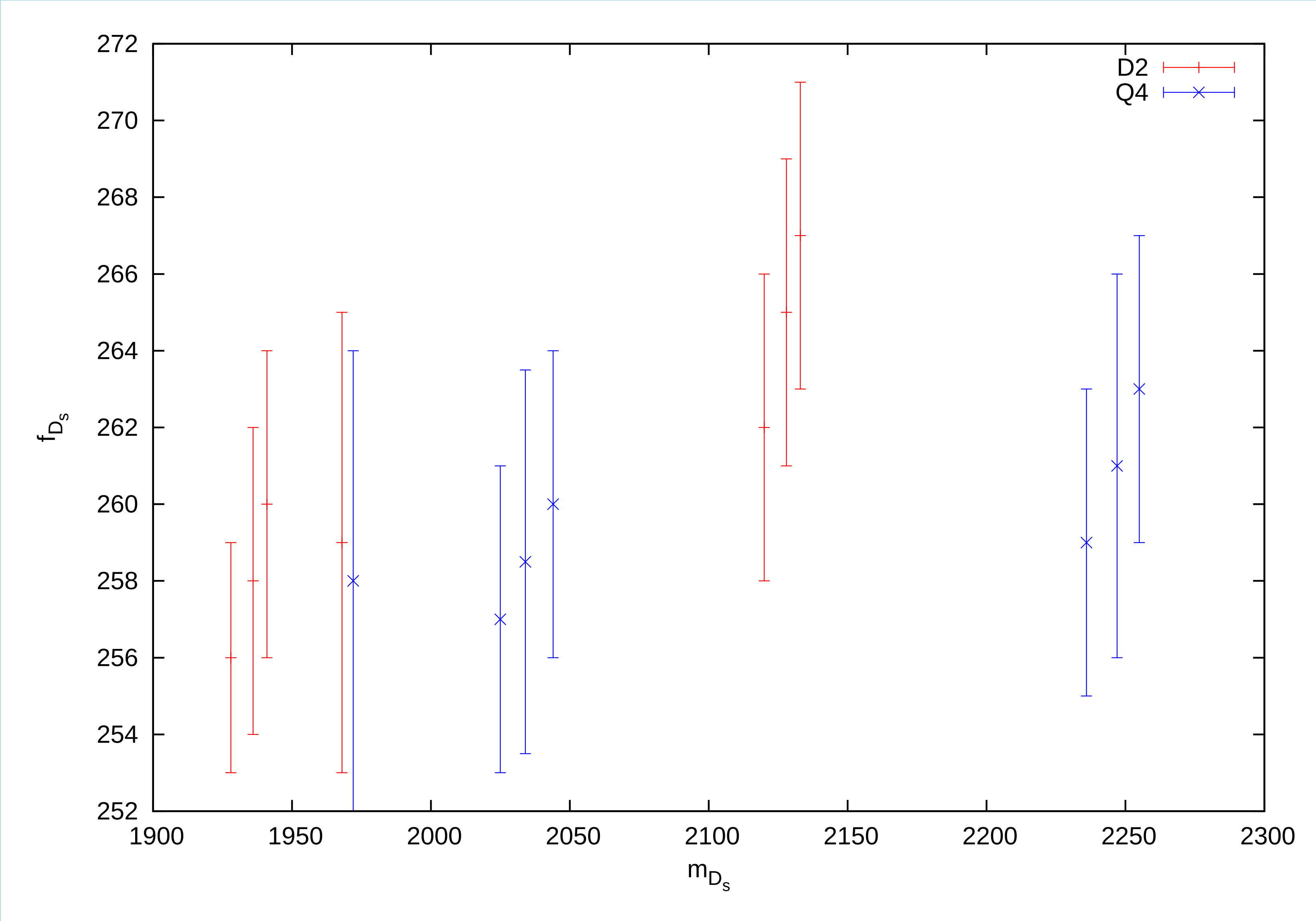
D_s masses and m_c (II)

m_{η_c}/MeV	$2m_c$/MeV	m_c/MeV	interpol.
3275(2)	4423(1)	2212(1)	
2930(2)	3608(1)	1804(1)	1863(3) MeV
3247(1)	4399(1)	2120(1)	
2889(2)	3562(1)	1781(1)	1867(3) MeV
3139(1)	3737(1)	1867(1)	
3527(1)	4485(1)	2243(1)	1685(5) MeV

- Extract RGI mass m_c from PCAC masses
- m_c from D_s is weakly a-dependent, but has significant sea-quark mass-dependence
- m_c from η_c is weakly sea-quark mass-dependent, but strongly a-dependent

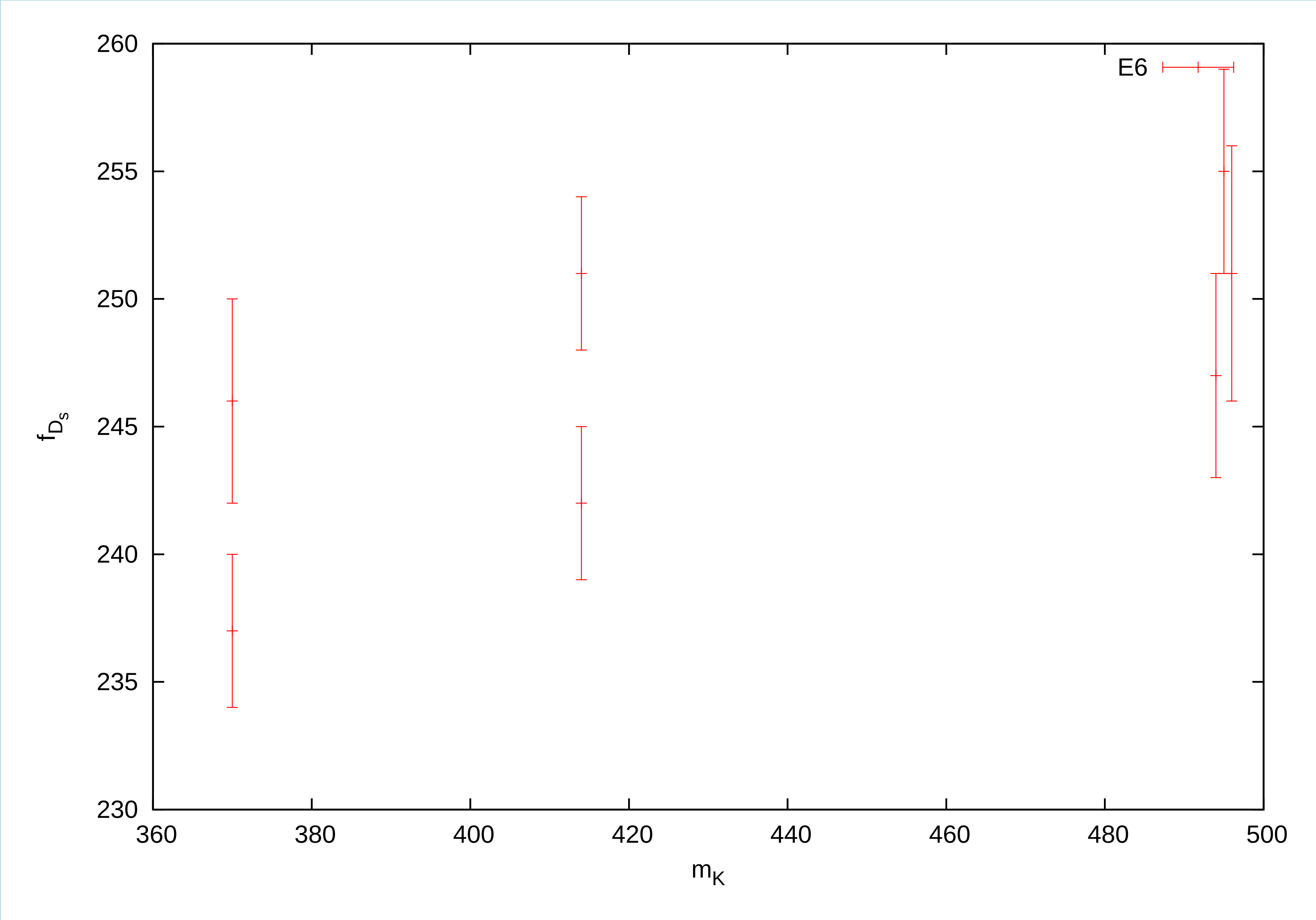
Matrix elements

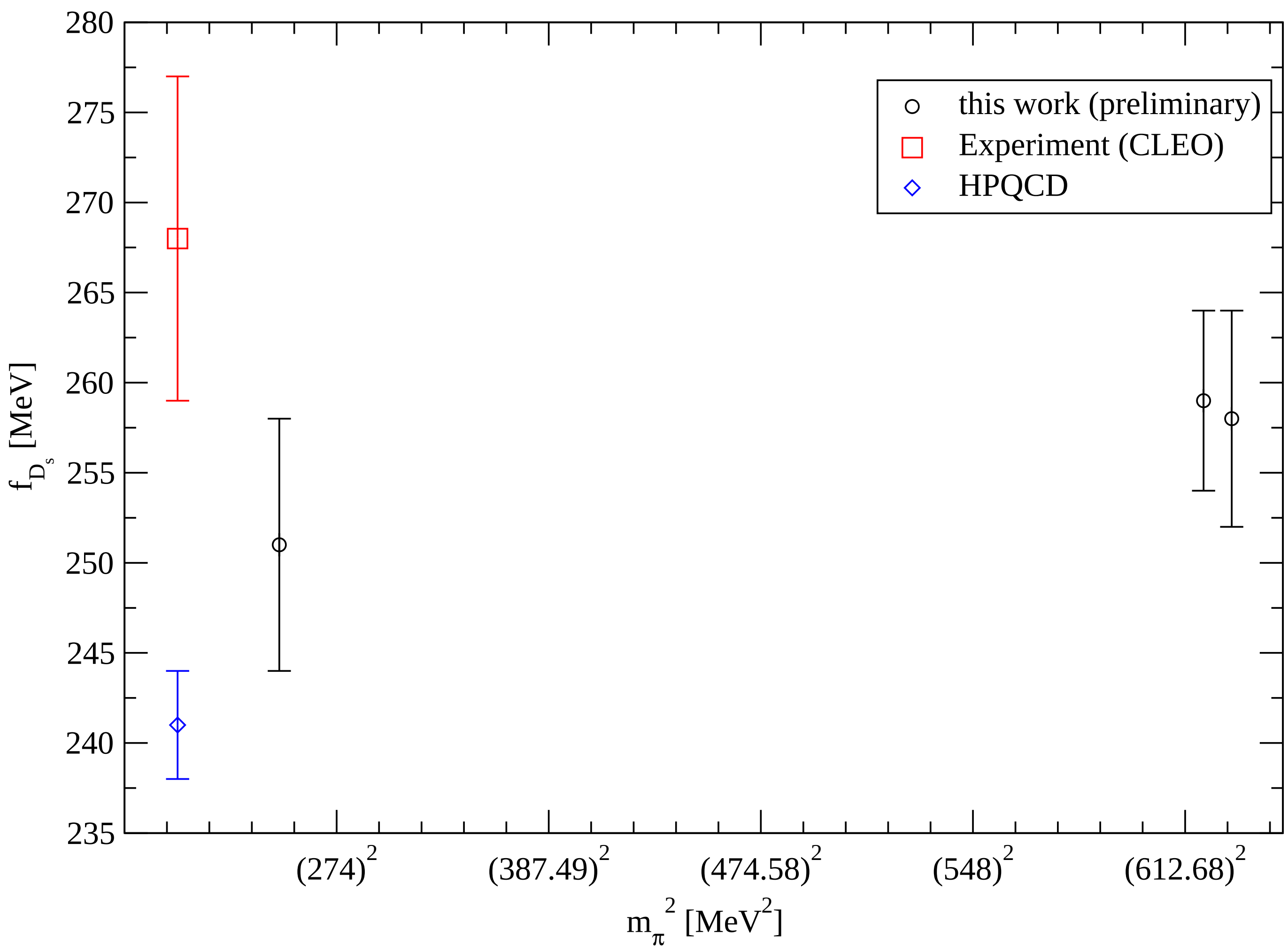
- Define reference point $m_\pi = m_K = 618$ MeV, $m_{D_s} = 1968$ MeV, to compare different β values
- This point is realised directly on the D2 ensemble, where $f_\pi^{ref} = 174(5)(2)$ MeV, $f_D^{ref} = 259(4)(4)$ MeV
- On the Q4 ensemble, need to interpolate in m_π to $f_\pi^{ref} = 187(3)(3)(2)$ MeV and $f_D^{ref} = 258(3)(4)(3)$ MeV
- Lattice spacing effects are $O(10\%)$ in f_π^{ref}, but small in f_D^{ref}


272
270
268
266
264
262
260
258
256
254
252
1900
1950
2000
2050
2100
2150
2200
2250
2300
f_{D_s}
m_{D_s}
D2
Q4

Matrix elements (II)

- To approach the physical point, take our lightest pion mass $m_\pi = 234(10)(3)$ MeV on the E6 ensemble
- Extrapolate from $m_K = 369(7)(5), 414(7)(5)$ MeV to $m_K = 495$ MeV
- Find $f_{D_s} = 255(3)(3)(3), 247(3)(3)(3)$ MeV at $m_{D_s} = 2068(8)(26)(3)$, $1870(7)(24)(3)$ MeV
- Interpolate linearly to $f_{D_s} = 251(3)(3)(5)(?)$ at $m_{D_s} = 1968$ MeV


E6
260
255
250
245
240
235
230
360
380
400
420
440
460
480
500
f_{D_s}
m_K



280
275
270
265
260
255
250
245
240
235
f_{D_s} [MeV]
this work (preliminary)
Experiment (CLEO)
HPQCD
$(274)^2$
$(387.49)^2$
$(474.58)^2$
$(548)^2$
$(612.68)^2$
m_π^2 [MeV^2]

Summary

- CLS is now simulating very large and fine $N_f = 2$ lattices
- Lattice spacings as small as $a = 0.04$ fm have become accessible
- Fully relativistic charm quarks are feasible
- Lighter sea quarks are being simulated
- Cutoff effects small and under control
- More precise scale determination is a priority
- With better statistics and more data points to come, expect accurate determination of f_{D_s} in the near future