Generalisations of the Ginsparg-Wilson relation and a remnant of supersymmetry on the lattice

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Lattice 2008
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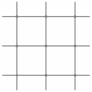
GB, F. Bruckmann, J. M. Pawlowski arXiv:0807.1110

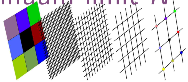
Introduction

continuum
path integral



lattice
N integrations

continuum limit $N \to \infty$

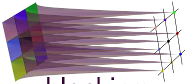


symmetry
$\{\gamma_5, \mathcal{D}\} = 0$

should generate
fine tuning

?
broken by the lattice?;
anomalies?; realisation?

Introduction



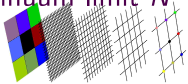
blocking

RG inspired

continuum
path integral

lattice
N integrations

continuum limit $N \to \infty$



symmetry

$\{\gamma_5, \mathcal{D}\} = 0$

imply

condition for generation

relation

$\{\gamma_5, \mathcal{D}\} = a\mathcal{D}\gamma_5\mathcal{D}$

defines a "symmetry"

$\{\gamma_{5,\mathrm{def}}, \mathcal{D}\} = 0$

$\gamma_{5,\mathrm{def}} = \gamma_5(\mathbb{1} - a\mathcal{D})$

The blocking transformation

- averaging of the continuum field $\varphi(x)$ around the lattice point $x_n = an$:

$$\Phi_n[\varphi] := \int dx\; f(x - x_n)\varphi(x)$$

- define a blocked lattice action $S[\phi]$ depending on lattice fields ϕ_n for a given continuum action $S_{\rm cl}[\varphi]$

$$e^{-S[\phi]} := \frac{1}{\mathcal{N}} \int d\varphi\; e^{-\frac{1}{2}(\phi-\Phi[\varphi])_n \alpha_{nm} (\phi-\Phi[\varphi])_m}\; e^{-S_{\rm cl}[\varphi]}$$

- simple interpretation if $f(x - x_n) \longrightarrow \delta(x - x_n)$ and $\alpha \longrightarrow \infty$ as $a \longrightarrow 0$ since $S \longrightarrow S_{\rm cl}$; more generally

$$\int d\phi\; e^{-S[\phi]+J\phi} = e^{\frac{1}{2}J\alpha^{-1}J} \int d\varphi\; e^{-S_{\rm cl}[\varphi]+J\Phi[\varphi]}$$

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A lattice symmetry

- continuum action is invariant under infinitesimal continuum symmetry transformations:
  $S_{\mathrm{cl}}[\varphi + \delta\varphi] = S_{\mathrm{cl}}[(1 + \varepsilon \tilde{M})^{ij} \varphi^j] = S_{\mathrm{cl}}[\varphi]$
- to translate the continuum symmetry transformations $\tilde{M}$ into naive lattice transformations M:

$$\Phi^i_n[\tilde{M}\varphi] = \int dx \; f_n(x) \; \tilde{M}^{ij} \varphi^j(x) = M^{ij}_{nm} \Phi^j_m[\varphi]$$

- can not be found for every $\tilde{M}$ and f $\hookrightarrow$ additional constraint
- naive lattice symmetry transformations: $(\delta\phi)^i_m = \varepsilon M^{ij}_{nm} \phi^j_m$
- naive invariance: $S[\phi + \delta\phi] = S[\phi]$

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Inherited symmetry of the blocked action

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- infinitesimal continuum transformation of φ; use additional constraint: $\Phi[\tilde{M}\varphi] = M\Phi[\varphi]$
- express $(\phi - \Phi)$ in terms of $\frac{\delta}{\delta\phi}$ and α^{-1}

$$M^{ij}_{nm}\phi^j_m \frac{\delta S}{\delta\phi^i_n} = (M\alpha^{-1})^{ij}_{nm}\left(\frac{\delta S}{\delta\phi^j_m}\frac{\delta S}{\delta\phi^i_n} - \frac{\delta^2 S}{\delta\phi^j_m\delta\phi^i_n}\right) + (\mathrm{STr} M - \mathrm{STr}\tilde{M})$$

$\mathrm{STr}\,\tilde{M}$ accounts for infinitesimal change of the measure $\rightarrow$ anomaly

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Symmetry relation for the lattice action

$$M^{ij}_{nm}\phi^j_m \frac{\delta S}{\delta \phi^i_n} = (M\alpha^{-1})^{ij}_{nm}\left(\frac{\delta S}{\delta \phi^j_m}\frac{\delta S}{\delta \phi^i_n} - \frac{\delta^2 S}{\delta \phi^j_m \delta \phi^i_n}\right) + (\mathrm{STr}M - \mathrm{STr}\tilde{M})$$

- α_S^{-1} drops out if $(\alpha_S^{-1}M)^T + M\alpha_S^{-1} = 0$ (supertransposed $\alpha = \alpha^T$)
  $\Rightarrow$ same relations for α^{-1} and $\alpha^{-1} + \alpha_S^{-1}$
- for a quadratic action, $S = \frac{1}{2}\phi^i_n K^{ij}_{nm}\phi^j_m$, the relation turns into $M^T K + (M^T K)^T = K^T\left[(M\alpha^{-1})^T + M\alpha^{-1}\right]K$ and can be rewritten as

$$M_{\mathrm{def}}{}^T K + K^T M_{\mathrm{def}} = 0; \quad M_{\mathrm{def}} = M(\mathbb{1} - \alpha^{-1}K)$$

- conditions for M_{def} to define a deformed symmetry
  1. M_{def} local
  2. M_{def} approaches continuum counterpart (excludes $M_{\mathrm{def}} = 0$)

  $\Rightarrow$ restricts possible choices of α and K

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Ginsparg-Wilson relation

$$\alpha_{nm} = \frac{1}{a}\delta_{nm}$$
$$\{\gamma_5, \mathcal{D}\} = a\mathcal{D}\gamma_5\mathcal{D}$$

- α_S^{-1} drops out if $(\alpha_S^{-1}M)^T + M\alpha_S^{-1} = 0$ (supertransposed $\alpha = \alpha^T$) ⇒ same relations for α^{-1} and $\alpha^{-1} + \alpha_S^{-1}$
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$$\boxed{M^T_{\mathrm{def}}K + K^T M_{\mathrm{def}} = 0; \quad M_{\mathrm{def}} = M(\mathbb{1} - \alpha^{-1}K)}$$

GW: $\{\gamma_{5,\mathrm{def}}, \mathcal{D}\} = 0$; $\gamma_{5,\mathrm{def}} = \gamma_5(\mathbb{1} - \alpha^{-1}\mathcal{D})$

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  1. M_{def} local
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  ⇒ restricts possible choices of α and K

GW: excludes Wilson fermions

Solution of the additional constraint for SUSY

$$\int dx\, f(x-an)\, \tilde{M}^{ij} \varphi^j(x) = M^{ij}_{nm} \Phi^j_m[\varphi] = M^{ij}_{nm} \int dx\, f(x-am)\, \varphi^j(x)$$

- trivial if $\tilde{M}^{ij}$ merely acts on multiplet index j; but for SUSY derivative operators appear in the continuum transformations
- must hold for all φ; in Fourier space

$$[\nabla(p_k) - ip_k] f(p_k) = 0$$

for $p_k = \frac{2\pi}{L} k$, $k \in \mathbb{Z}$ and $\nabla(p + \frac{2\pi}{a}) = \nabla(p)$

- solutions: nonlocal SLAC-derivative; otherwise effective cutoff below $\frac{2\pi}{a}$ is introduced by $f(p)$

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Setting for supersymmetric quantum mechanics

- transformations in the continuum, $\varphi^i(x) = (\chi(x), F(x), \psi(x), \bar{\psi}(x))$:

$$\delta\chi = -\bar{\varepsilon}\psi + \varepsilon\bar{\psi} \qquad \delta F = -\bar{\varepsilon}\partial\psi - \varepsilon\partial\bar{\psi}$$
$$\delta\psi = -\varepsilon\partial\chi - \varepsilon F \qquad \delta\bar{\psi} = \bar{\varepsilon}\partial\tilde{\varphi} - \bar{\varepsilon}F$$

- naive transformations on the lattice, $\phi^i_n = (\chi_n, F_n, \psi_n, \bar{\psi}_n)$:

$$\delta\begin{pmatrix}\chi\\ F\\ \psi\\ \bar{\psi}\end{pmatrix} = \begin{pmatrix} 0 & 0 & -\bar{\varepsilon} & \varepsilon\\ 0 & 0 & -\bar{\varepsilon}\nabla & -\varepsilon\nabla\\ -\varepsilon\nabla & -\varepsilon & 0 & 0\\ \bar{\varepsilon}\nabla & -\bar{\varepsilon} & 0 & 0\end{pmatrix}\begin{pmatrix}\varphi\\ F\\ \psi\\ \bar{\psi}\end{pmatrix} = (\varepsilon M + \bar{\varepsilon}\bar{M})\phi$$

∇ solution of additional constraint (SLAC-derivative)

Setting for supersymmetric quantum mechanics

- invariant quadratic action in the continuum:

$$S_{\rm cl} = \int dx \left[\frac{1}{2}(\partial_x\chi) + \bar\psi\partial_x\psi - \frac{1}{2}F^2 + \bar\psi W'(\chi)\psi - FW(\chi)\right]$$
$$= \int dx \left[\frac{1}{2}(\partial_x\chi) + \bar\psi\partial_x\psi - \frac{1}{2}F^2 + m\bar\psi\psi - mF\chi\right]$$

- ansatz for the lattice action $S = \frac{1}{2}\phi K\phi$:

$$\frac{K_{ij}}{a} = \begin{pmatrix} -\Box_{nm} & -m_{b,nm} & 0 & 0 \\ -m_{b,nm} & -I_{nm} & 0 & 0 \\ 0 & 0 & 0 & (\hat\nabla - m_f)_{nm} \\ 0 & 0 & (\hat\nabla + m_f)_{nm} & 0 \end{pmatrix}$$

I, $\Box$, m_b, m_f symmetric; $\hat\nabla$ antisymmetric
translation invariance: all circulant matrices ($\rightarrow$ commute)

Solutions for a quadratic action

- solve $M_{\mathrm{def}}^T K + K^T M_{\mathrm{def}} = 0$ with $M_{\mathrm{def}} = M(\mathbb{1} - \alpha^{-1} K)$
- diagonal blocking matrix (as for overlap: $\alpha \sim \delta_{nm}$) leads to nonlocal action (use freedom to choose α_S^{-1} to reduce matrix elements)

$$a(\alpha^{-1})_{nm} = \begin{pmatrix} a_2 & 0 & 0 & 0 \\ 0 & a_0 & 0 & 0 \\ 0 & 0 & 0 & -a_1 \\ 0 & 0 & a_1 & 0 \end{pmatrix} \delta_{nm}; \quad \begin{array}{l} \hat{\nabla} + m_f = \frac{\nabla + m_b}{1 + a_0 + a_1 mb + (a_1 + a_2 mb)\nabla} \\ -\Box + m_b^2 = \frac{-\nabla^2 + m_b^2}{1 + a_0 - a_2 \nabla^2} \\ I = \mathbb{1} \end{array}$$

- local solutions like $\hat{\nabla}$ symmetric derivative, $\Box = \hat{\nabla}^2$, $I = \mathbb{1}$, and $m_b = m_f = m + m_w$ generically lead to nonlocal α^{-1}
- demand M_{def} and K

$$M_{\mathrm{def}} = \begin{pmatrix} 0 & 0 & 0 & I \\ 0 & 0 & 0 & -I\nabla \\ -\nabla & -I\nabla & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \quad \begin{array}{l} \hat{\nabla} = I\nabla \\ I \to 1,\ I\nabla \to \partial_x \text{ cont. limit} \\ I \text{ and } I\nabla \text{ must be local} \end{array}$$

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Beyond the quadratic action

- final goal: construct a supersymmetric local interacting lattice action
- the given relation extends beyond the quadratic case
- it connects different orders of the field $\longrightarrow$ generically nonpolynomial solutions
- not unexpected since blocked action is comparable to the effective action
- under special conditions a truncation can be achieved

Conclusions and outlook

- symmetry of a continuum action implies the fulfilment of certain relations for the lattice action which ensure a symmetric continuum limit and define deformed lattice symmetry operators
- requirement: definition of a naive lattice transformation by the “averaged” continuum symmetry transformation (additional constraint) $\hookrightarrow$ SLAC-derivative for SUSY
- severe restriction: M_{def} and the action must be local; can be fulfilled under special conditions
- although the relation couples different orders of the fields, even for interacting theories a polynomial solution can be achieved
- from the GW point of view: more careful investigations of the conditions for lattice SUSY is needed: compare with other symmetries; use the knowledge from ERG studies for interacting case; generalise the setup