Fluctuation of Goldstone modes and the chiral transition in QCD†

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- Goldstone modes in 3 and 4 dimensions

  O(N) models

- (2+1)-flavor QCD below T_c

  $N_\tau = 4,\ 6$ and 8

- (2+1)-flavor QCD at T_c

  $N_\tau = 4$

- Conclusions

† This talk is based on preliminary numerical results obtained by the hotQCD and RBC-Bielefeld collaborations

2 (+1)-flavor QCD and O(N) spin models

physics of QCD at low energies as well as close to the chiral phase transition is described by effective, O(N) symmetric spin models

- $T = 0$: chiral symmetry breaking at $T = 0,\ m_q = 0$ as well as leading temperature and quark mass dependent corrections are related to universal properties of 4-dimensional, $O(4)$ symmetric spin models
- $T \simeq T_c$: chiral symmetry restoration at $T = T_c,\ m_q = 0$ as well as leading temperature and quark mass dependent corrections are related to universal properties of 3-dimensional, $O(4)$ symmetric spin models

R. Pisarski and F. Wilczek, PRD29 (1984) 338
K. Rajagopal and F. Wilczek, hep-ph/0011333
A. Pelissetto and E. Vicari, Phys. Rept 368 (2002) 549

Spontaneous Symmetry Breaking

$O(N)$ spin models in d-dimensions

- non-vanishing expectation value, Σ, of the scalar field, $\Phi_{||}$, parallel to the symmetry breaking field H
- $(N-1)$ transverse (Goldstone) modes give corrections for non-zero H (spin waves); controlled by Σ and the decay constant F for Goldstone modes

$$\Sigma_H = \Sigma_0 \left(1 - \frac{N-1}{32\pi^2} \frac{\Sigma_0 H}{F_0^4} \ln\left(\Sigma_0 H / F_0^2 \Lambda_\Sigma\right) + \mathcal{O}(H^2) \right) \quad , \quad d = 4$$

$$\Sigma_H = \Sigma_0 \left(1 + \frac{N-1}{8\pi} \frac{(\Sigma_0 H)^{1/2}}{F_0^3} + \mathcal{O}(H) \right) \quad , \quad d = 3$$

P. Hasenfratz and H, Leutwyler, NPB343, 241 (1990)

D.J. Wallace and R.K.P. Zia, PRB12, 5340 (1975)

Spontaneous Symmetry Breaking (cont.)

- (chiral) susceptibilities diverge below T_c for $H \to 0$

$$\chi_H = \frac{\mathrm{d}\Sigma_H}{\mathrm{d}H} \sim \langle \Phi_{||}^2 \rangle - \langle \Phi_{||} \rangle^2 \sim \begin{cases} H^{-1/2} & , \ d = 3 \\ -\ln H & , \ d = 4 \end{cases}$$

- divergence in the zero-field (chiral) limit

$$\chi_{H=0}(T) = \begin{cases} \infty & , \ T \le T_c \\ A(T - T_c)^{-\gamma} & , \ T > T_c \end{cases}$$

- divergence at T_c

$$\chi_H(T = T_c) = H^{1/\delta - 1} \ , \quad T = T_c$$

crit. exp. O(2) [O(4)]: $\gamma = 1.32$ $[1.45]$, $1 - 1/\delta = 0.79$ $[0.79]$

O(N) spin models in 3-dimensions

influence of Goldstone modes on spontaneous symmetry breaking below T_c and the consistency with critical behavior at T_c has been established in numerical simulations J. Engels and T. Mendes, NP B572 (2000) 289

- $T < T_c$: $\Sigma_H = c_0(T) + c_1(T) H^{1/2} \;\Rightarrow\; \chi_H = \partial\Sigma/\partial H \sim H^{-1/2}$
- $T \simeq T_c$: scaling functions, e.g.

$$\Sigma_H = H^{1/\delta} f_s(t/H^{1/\beta\delta}) \;,\; t = |T - T_c|/T_c$$
$$\Rightarrow \;\; H = 0: \;\Sigma \sim t^{\beta} \;\;,\; t = 0: \Sigma \sim H^{1/\delta}$$

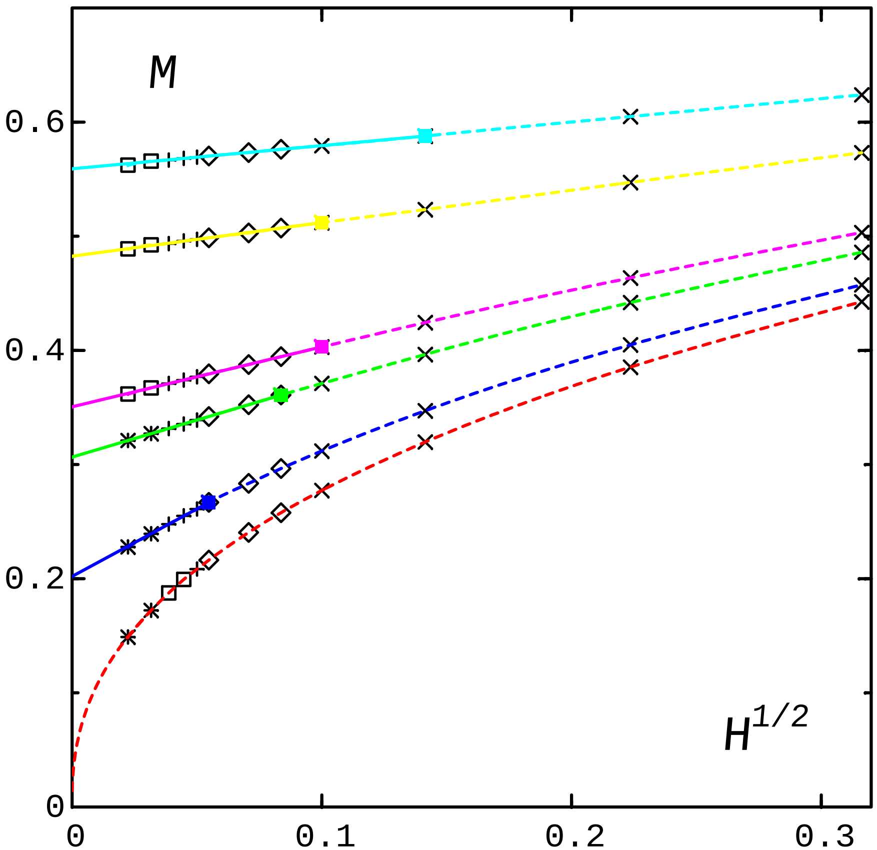
- magnetic equation of state incorporates both features

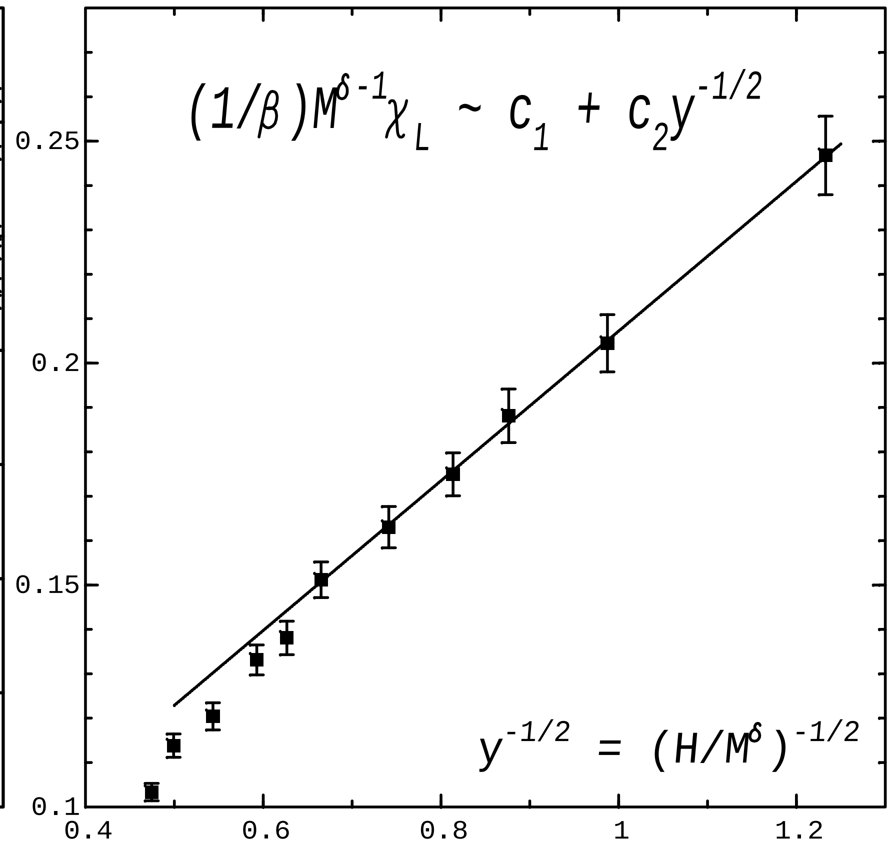
D.J. Wallace and R.K.P. Zia, PRB12, 5340 (1975)

J.Engels and T. Mendes, NP Proc.Suppl. 83, 700 (2000)

$$\frac{1}{\beta}\Sigma^{\delta-1}\chi_H = \tilde{c_0} + \tilde{c_1} y^{-1/2} \;\;,\; y = H/\Sigma^{\delta}$$

3-d, O(4) models close to T_c





J. Engels and T. Mendes 2000

- condensate shows $\sqrt{H}$ dependence;
  scaling sets in for smaller H closer to T_c
- magnetic equation of state reflects $O(4)$ scaling

Goldstone modes in adjoint QCD

QCD with 2-flavor, adjoint (staggered) fermions

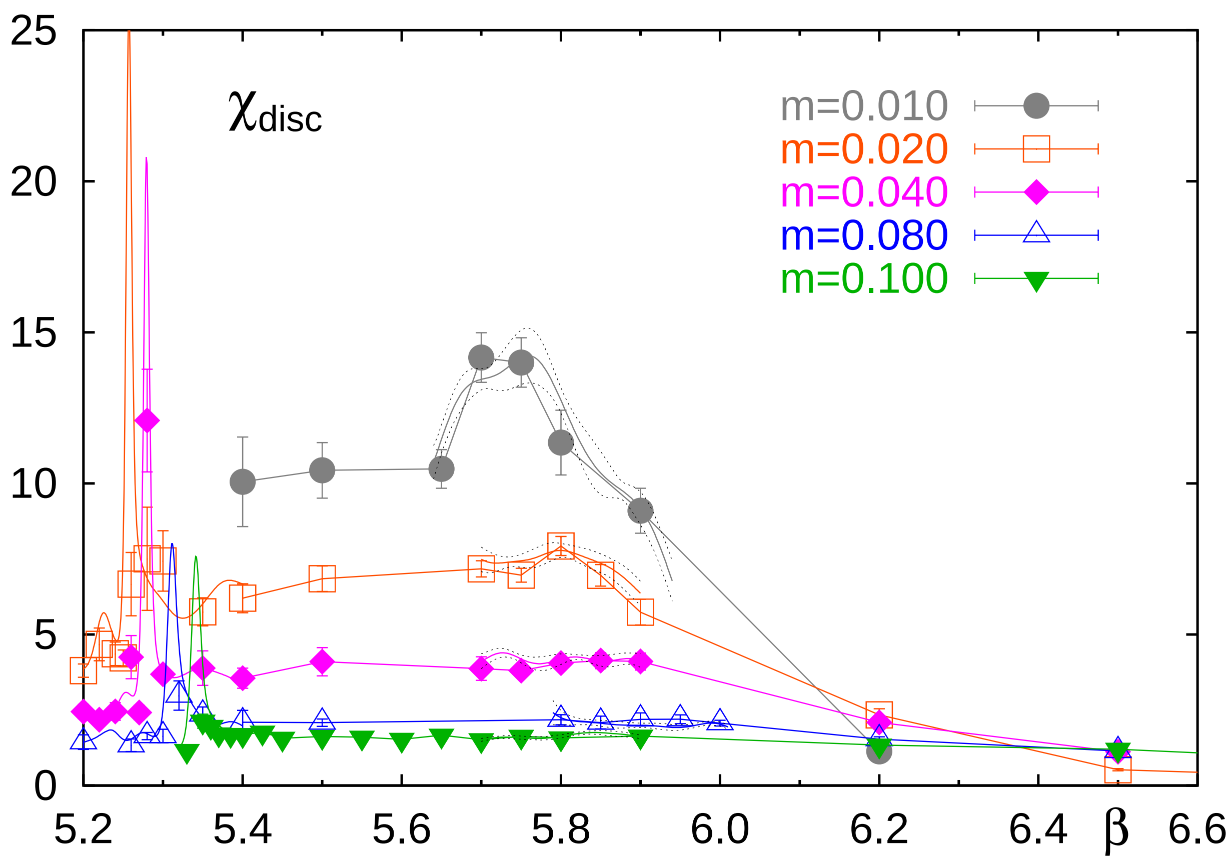
- QCD with fermions in the adjoint representation has two distinct phase transitions with $T_{deconf} < T_{chiral}$
- the intermediate deconfined phase shows chiral behavior as expected from 3-dimensional $O(N)$ models
- the disconnected part of the chiral susceptibility diverges with $1/\sqrt{m_a}$ for all $T_{deconf} < T < T_{chiral}$:

FK and M. Lütgemeier, NPB550, 449 (1999)

J, Engels, S. Holtmann and T. Schulze, NP B724, 357 (2005)

$$\begin{aligned} \chi_m &= \frac{\partial \langle \bar{\psi}\psi \rangle}{\partial m_q a} \equiv \chi_{disc} + \chi_{con} \\ \chi_{disc} &= \frac{1}{N_\sigma^3 N_\tau} \left(\left\langle (\mathrm{Tr} M^{-1})^2 \right\rangle - \left\langle \mathrm{Tr} M^{-1} \right\rangle^2 \right) \end{aligned} \quad (1)$$

2-flavor adjoint QCD, $N_\tau = 4$



- $T_{deconf} \le T \le T_{chiral}$: weak T-dependence
- Goldstone modes lead to $\sqrt{m}$ −terms in $\langle \bar{\psi}\psi \rangle$

  $\Rightarrow 1/\sqrt{m}$ singularity in χ_m
- $O(N)$ scaling at T_c barely visible; buildts up for small m_q only

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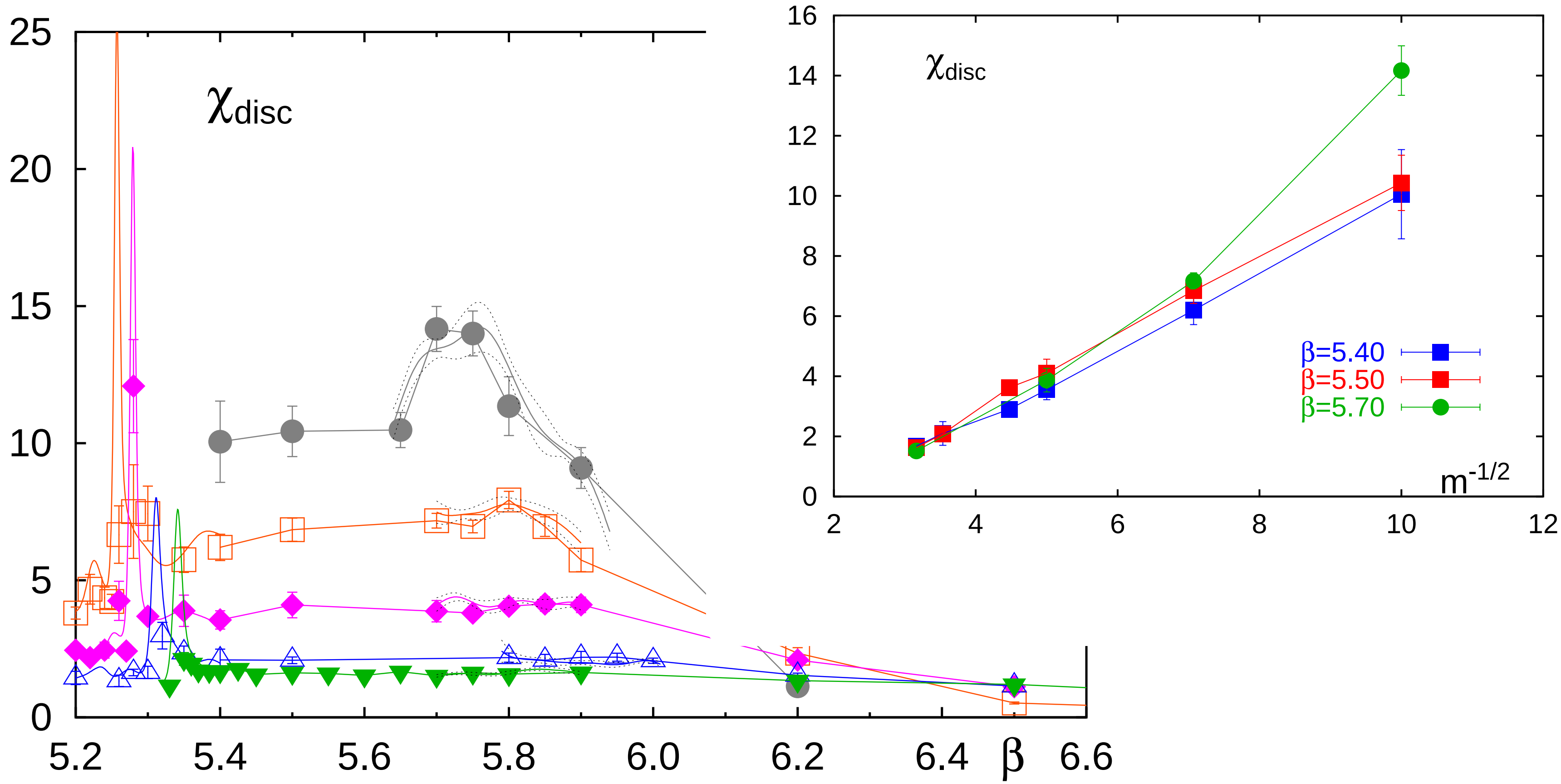

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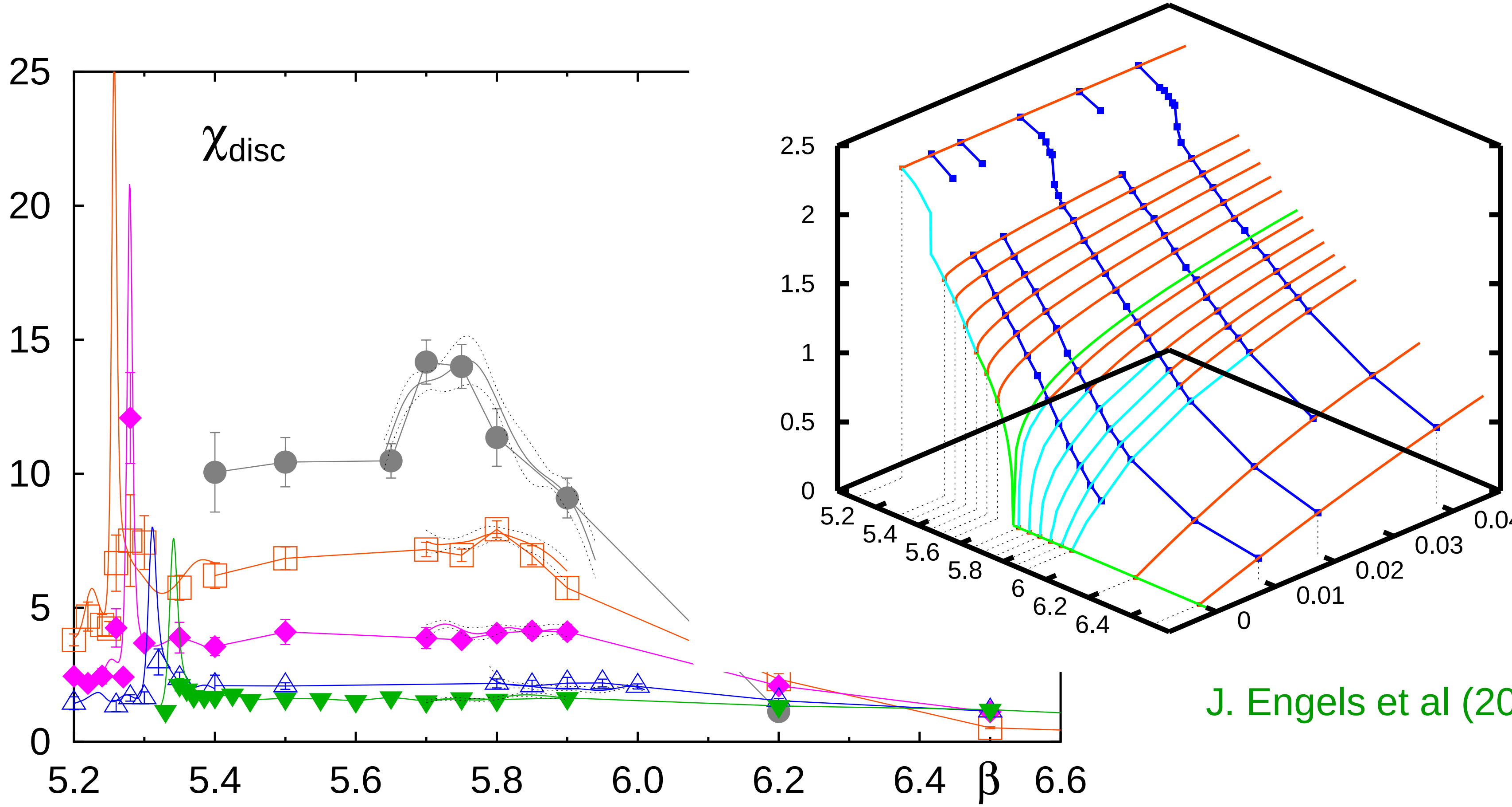

J. Engels et al (2005)

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Goldstone modes in (2+1)-flavor QCD

(RBC-Bielefeld and hotQCD Collaborations)

QCD with 2 light and a 'physical' strange quark mass;

staggered fermions, p4 and asqtad actions, RHMC simulations

- calculations have been performed on $N_\sigma^3 N_\tau$ lattices for $N_\tau = 4,\ 6$ and 8
- check volume dependence: $4 \le N_\sigma / N_\tau \le 8$
- at present, most detailed analysis for $N_\tau = 4$:

$$0.0125 \le m_l / m_s \le 0.4$$

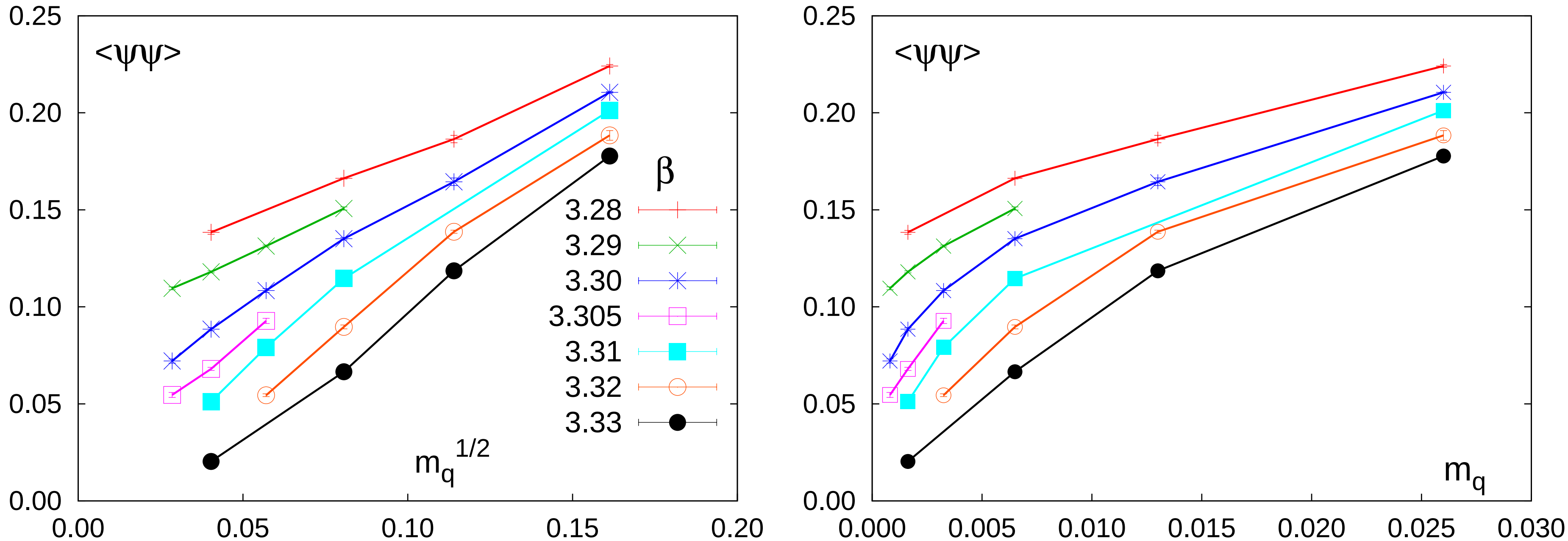
physical value: $m_l / m_s \simeq 0.05 \Rightarrow 70\ \mathrm{MeV} \le m_\pi \le 320\ \mathrm{MeV}$

$\Rightarrow$ find evidence for $1/\sqrt{m_l}$ divergence in χ_{disc} in the symmetry broken phase

$N_\tau = 4$: chiral condensate

(RBC-Bielefeld collaboration, in preparation)

$$\langle \bar\psi\psi \rangle = \frac{1}{N_\sigma^3 N_\tau} \frac{n_f}{4} \frac{\partial \ln Z}{\partial m_l a}$$



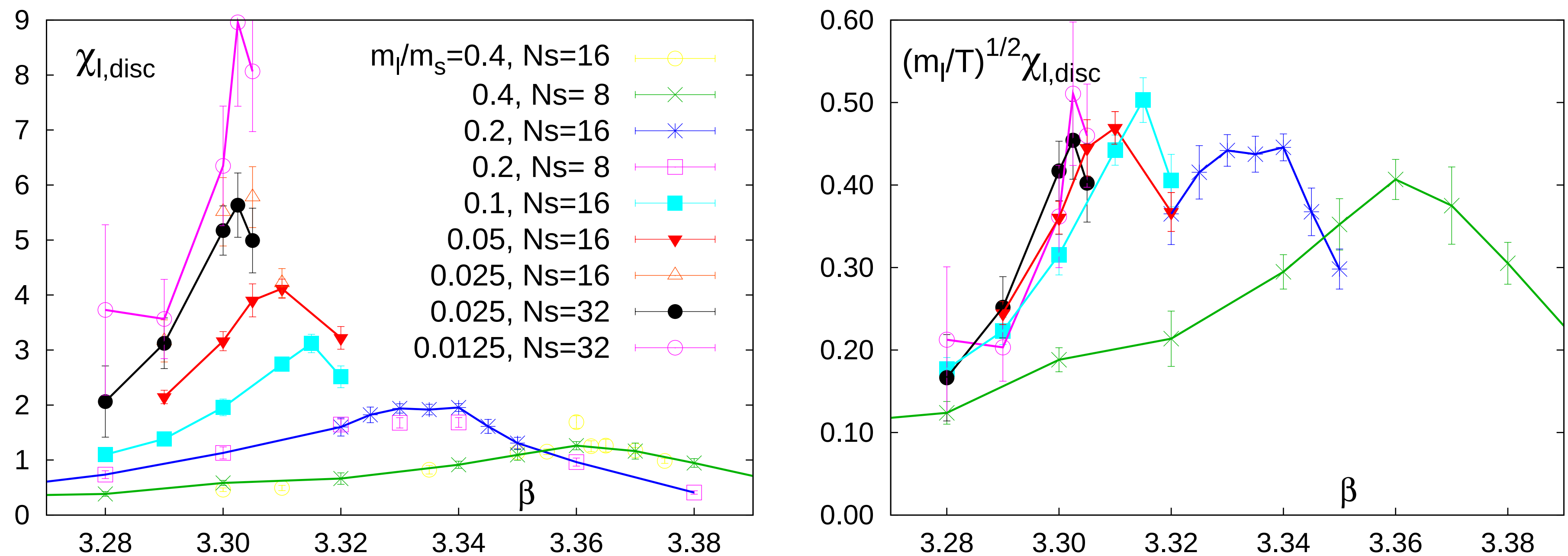
- evidence for $\sqrt{m_l}$ term in $\langle \bar\psi\psi \rangle$

for orientation: $\beta = 3.28$ $T \simeq 188$ MeV, $\beta = 3.30$ $T \simeq 196$ MeV

$N_\tau = 4$: $0.0125 \le m_l/m_s \le 0.4$

(RBC-Bielefeld collaboration, in preparation)

$$\chi_{disc} = \frac{1}{N_\sigma^3 N_\tau} \left(\frac{n_f}{4}\right)^2 \left(\left\langle (\mathrm{Tr} M^{-1})^2 \right\rangle - \left\langle \mathrm{Tr} M^{-1} \right\rangle^2 \right)$$



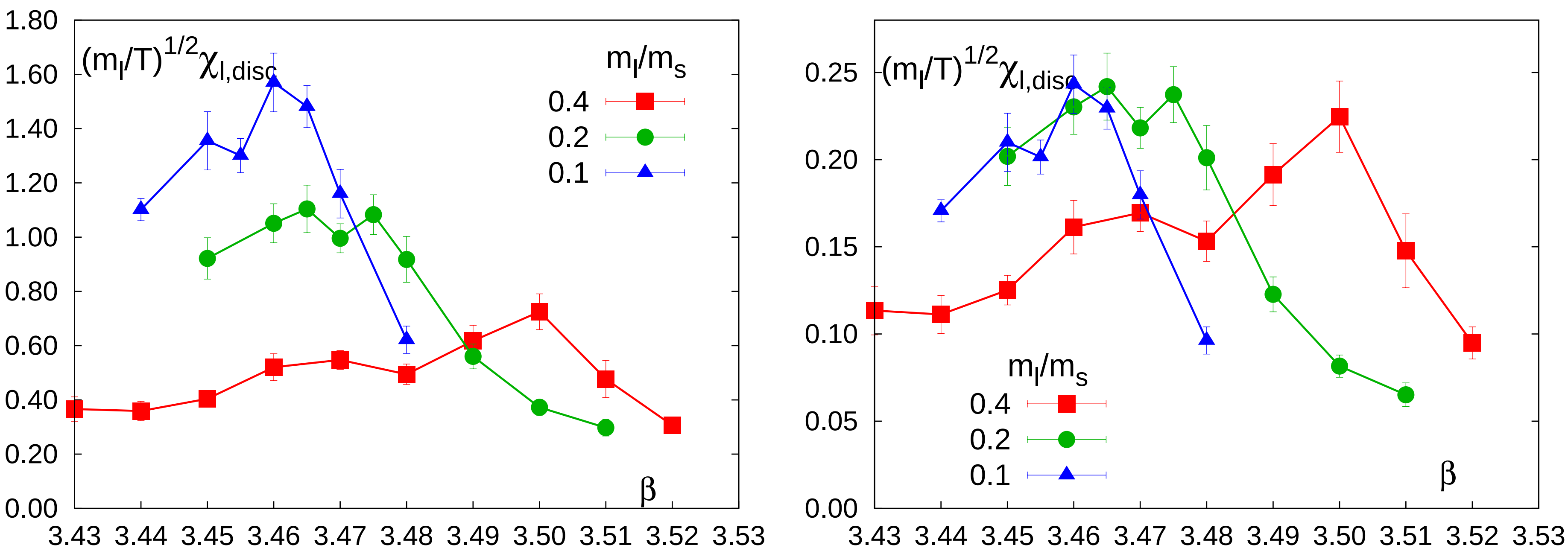
- evidence for $1/\sqrt{m_l}$ singularity in χ_{disc}

  for orientation: $\beta = 3.28$ $T \simeq 188$ MeV, $\beta = 3.30$ $T \simeq 196$ MeV

- scaling for $m_l/m_s \lesssim 0.1$ all the way to the (pseudo-)critical temperature
- scaling sets in for smaller quark masses closer to T_c (similar to 3-d, O(N) models)

$N_\tau = 6$: $0.1 \le m_l/m_s \le 0.4$

(RBC-Bielefeld collaboration)



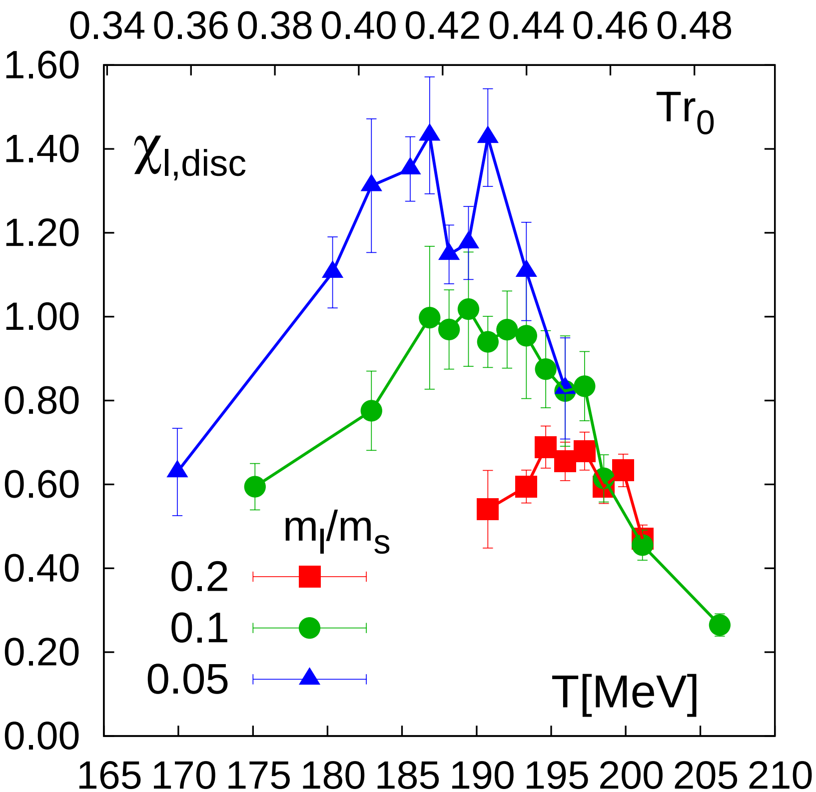
- evidence for $1/\sqrt{m_l}$ singularity in $N_\tau = 6$ data has been observed and commented upon already in
  M. Cheng et al. (RBC-Bielefeld), PRD74 (2006) 054507

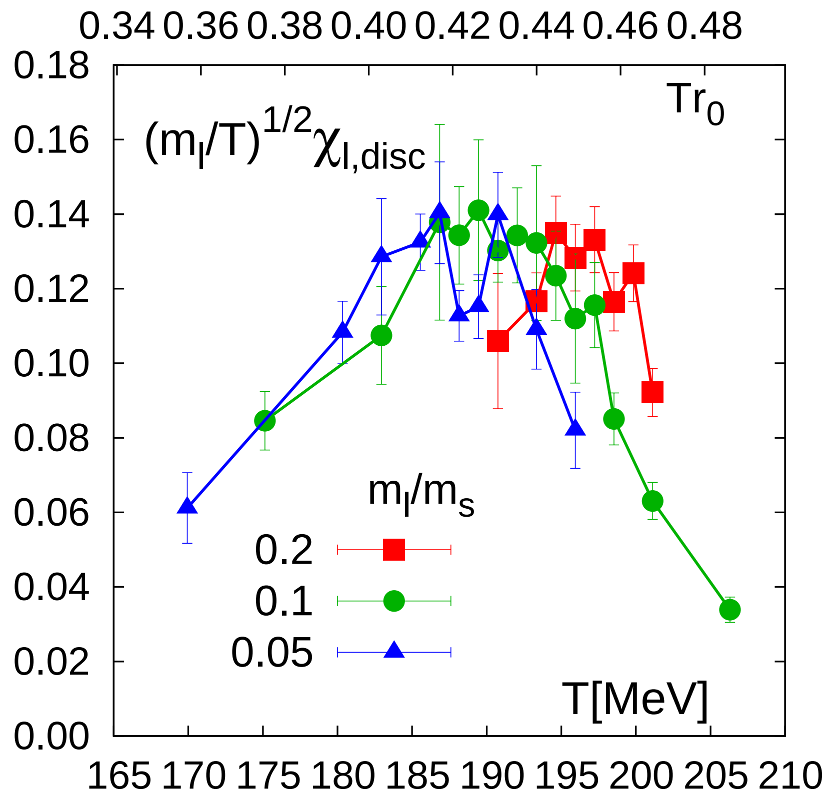
  scaling for $m_l/m_s \lesssim 0.2$,

  for orientation: $\beta = 3.52$ $T \simeq 185$ MeV, $\beta = 3.54$ $T \simeq 196$ MeV

$N_\tau = 8$: $0.05 \le m_l/m_s \le 0.2$

p4-data: (RBC-Bielefeld and hotQCD collaborations)

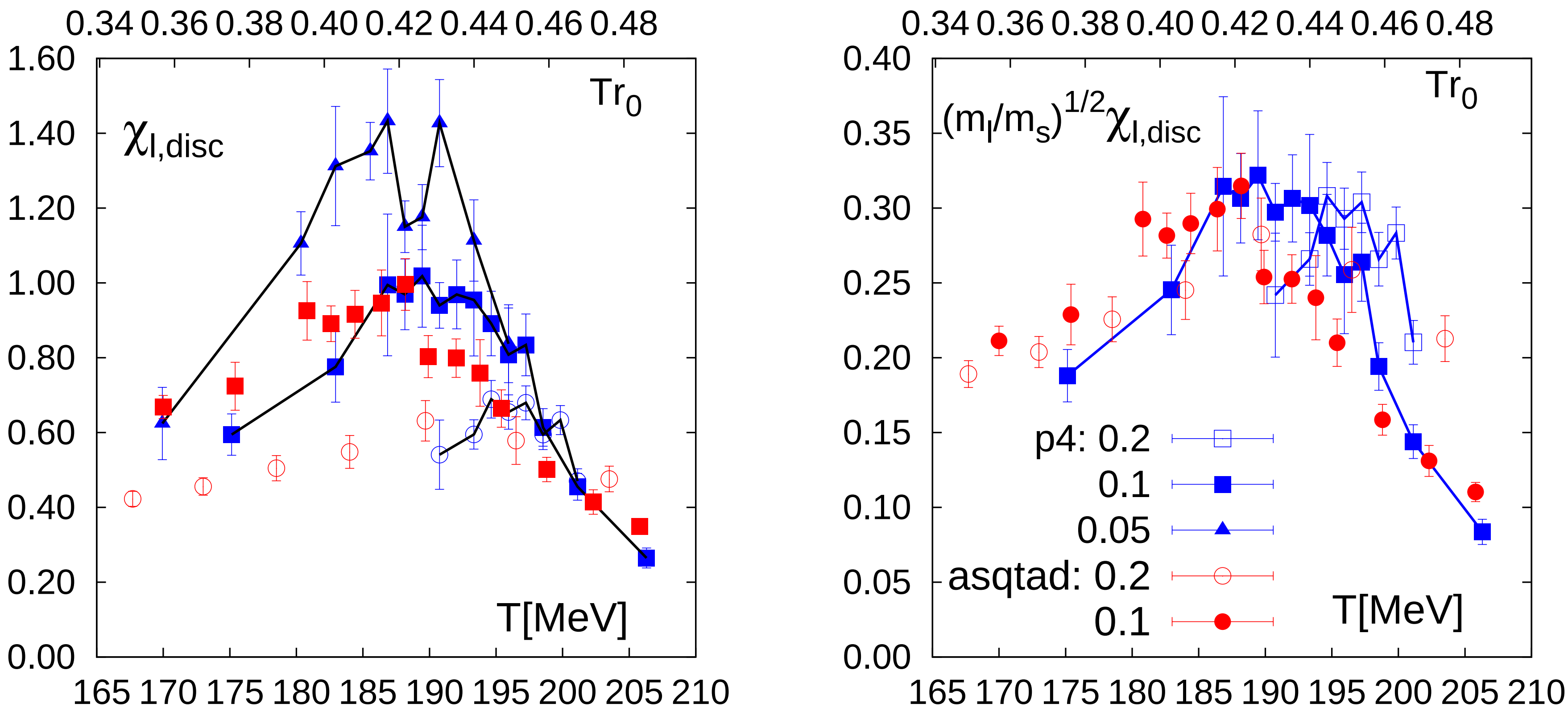




- evidence for $1/\sqrt{m_l}$ singularity in a wide temperature range
- steep edge at high temperature is approximately quark mass independent (as expected: $\langle\bar{\psi}\psi\rangle \sim c(T)m_l \;\Rightarrow\; \chi_m \sim c(T)$)

$N_\tau = 8$: p4 and asqtad

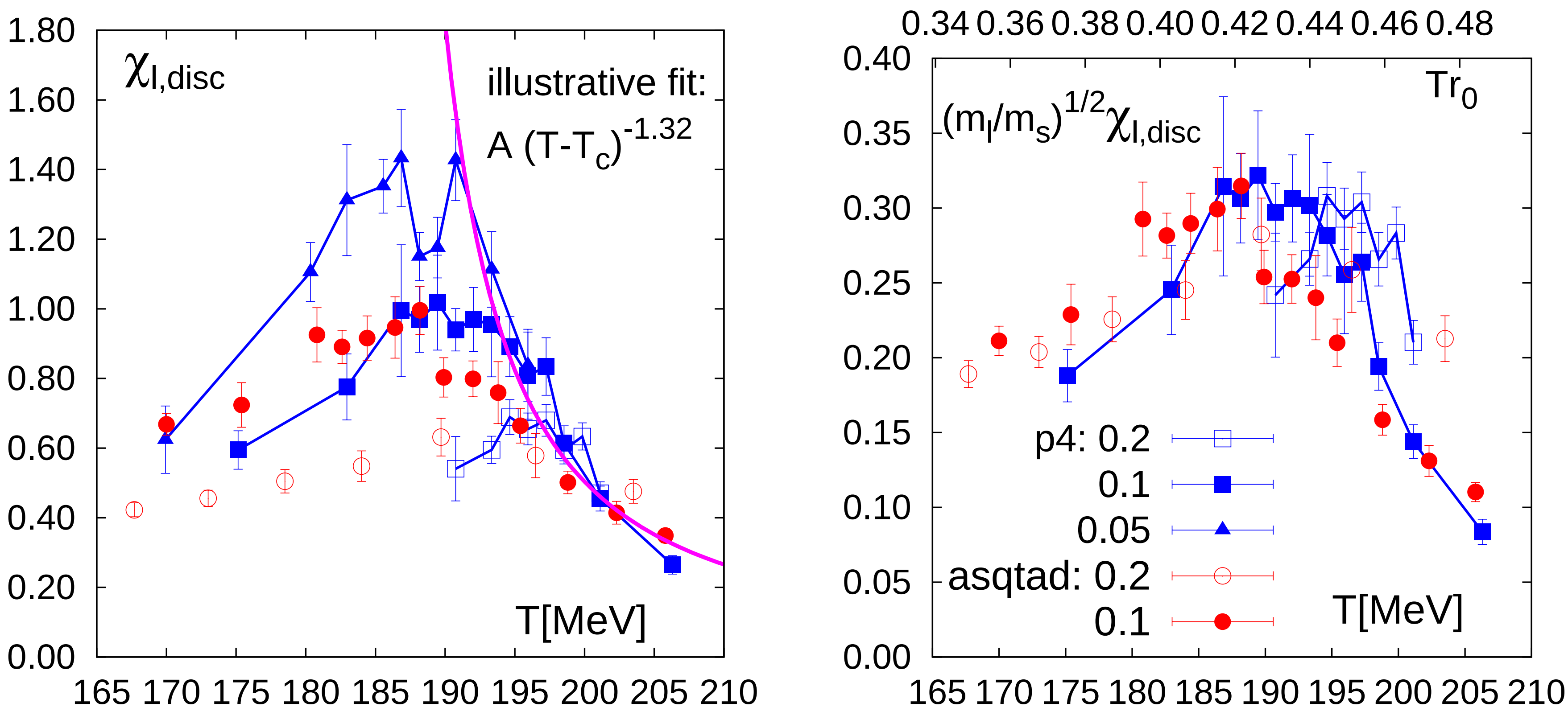
hotQCD and RBC-Bielefeld collaborations, preliminary



- p4 and asqtad calculation lead to similar quark mass dependence

$N_\tau = 8$: p4 and asqtad

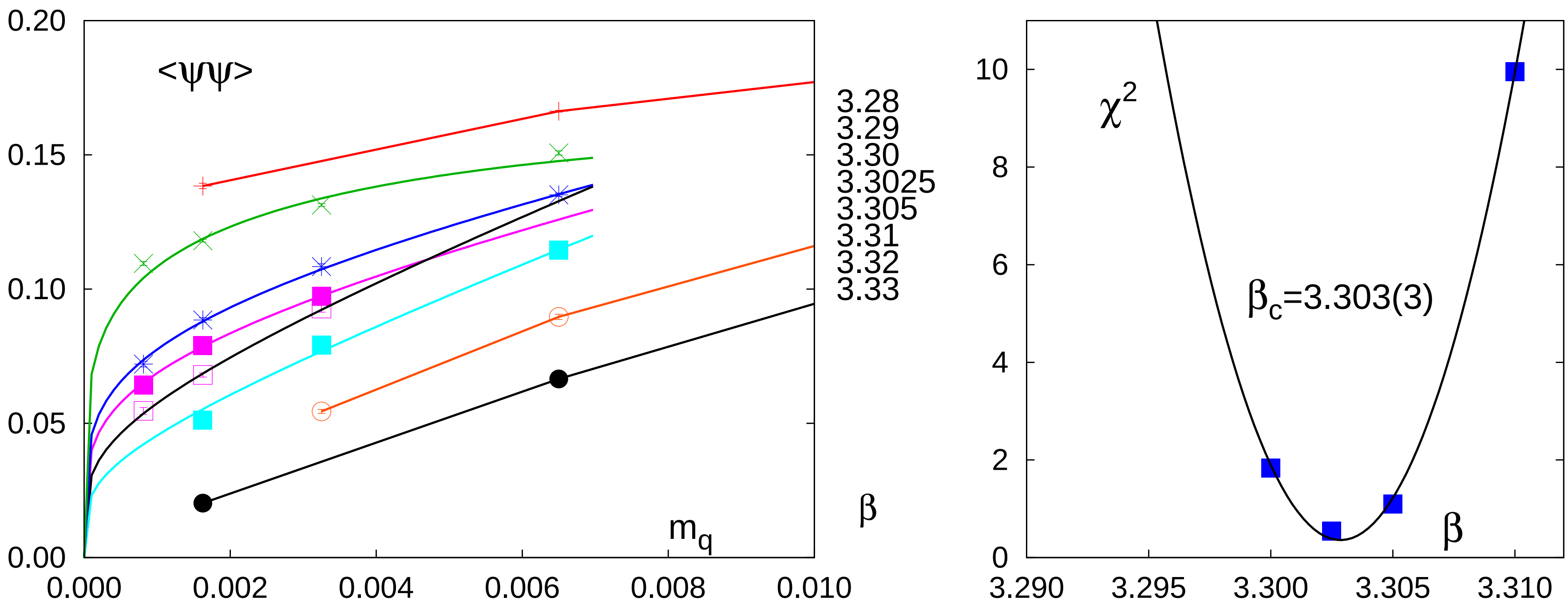
hotQCD and RBC-Bielefeld collaborations, preliminary



- p4 and asqtad calculation lead to similar quark mass dependence
- the rapid drop at large temperature is consistent with the expected O(2) [O(4)] scaling; however no 'critical behavior' of peak heights
- to firmly establish these features requires a more thorough analysis of the (m_l, T)-dependence of $\chi_{l,disc}$

$N_\tau = 4$: O(2) [O(4)] scaling at β_c

determining β_c using the χ^2-method: J. Engels et al., PLB298 (1993) 154



fit: $\langle\bar{\psi}\psi\rangle = a m_l^{1/\delta} + b m_l \ , \ 1/\delta = 0.21$

minimize $\chi^2 \ \Rightarrow \ \beta_c$; in agreement with earlier determination:

M. Cheng et al, PRD74 (2006) 054507

error on β_c: $\chi^2 \le 2 \ \Rightarrow$ narrow scaling window $\Delta T \simeq \pm 1.5$ MeV

evidence for $O(N)$ scaling at T_c for $m_l \lesssim m_{phys}$

Conclusions

- Goldstone modes control properties of the chiral condensate and its susceptibility in the confined phase

  3-d, O(N) scaling close to T_c

- (2+1)-flavor QCD

  the chiral susceptibility diverges like $1/\sqrt{m_l}$ for $T \lesssim T_c$;
  this scaling sets in ready in the regime of physical quark mass values

- in order to disentangle the thermal critical behavior of the chiral condensate and its susceptibility from the singular behavior induced by Goldstone modes a detailed analysis of the temperature and quark mass dependence is necessary