# Hadron spectrum of QCD with one quark flavor

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- Eur. Phys. J. C52, 305-314, 2007 (arXiv:0706.1131)
- PoS (LATTICE 2007) 135 (arXiv:0710.4454)

# Why $N_f = 1$ QCD?

Open questions:

- Solution of strong CP problem by $m_u = 0$?
- Spontaneous CP breaking for negative quark mass(es)?
- Relics of SUSY from an orientifold large $N_c$ equivalence?

Recent activity:

- Quark condensate [DeGrand, Liu and Schäfer (2006)]
- Finite temperature [Takaishi and Nakamura (2007)]
- Equivalence [Armoni, Lucini, Patella and Pica (2008)]

Related poster at the conference:

- K. Demmouche, "Simulation of SUSY Yang-Mills with light Wilson gluinos"

# $N_f = 1$ QCD

- Absence of chiral symmetry

$$U_V(1) \times U_A(1) \overset{\text{ABJ anomaly}}{\longrightarrow} U_V(1) \quad \text{(baryon number)}$$

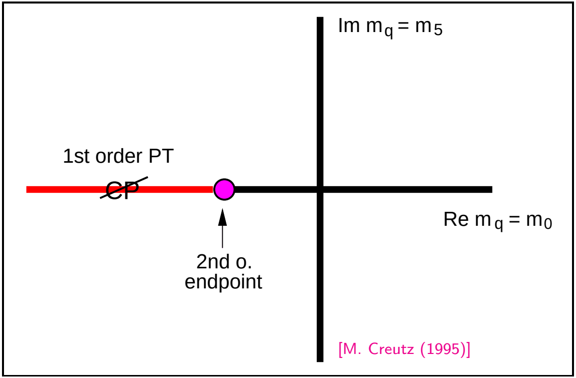
$$\partial_\mu A_\mu(x) = 2m_q\, P(x) + \overbrace{2N_f \frac{g^2}{32\pi^2} \tilde{F}F(x)}^{U_A(1)\text{-anomaly}}$$

- No new symmetry for $m_q = 0$!
- Scheme-dependent additive renormalization of $m_q$ possible

$\longrightarrow$ Massless limit physically not well defined? [M. Creutz (2004)]

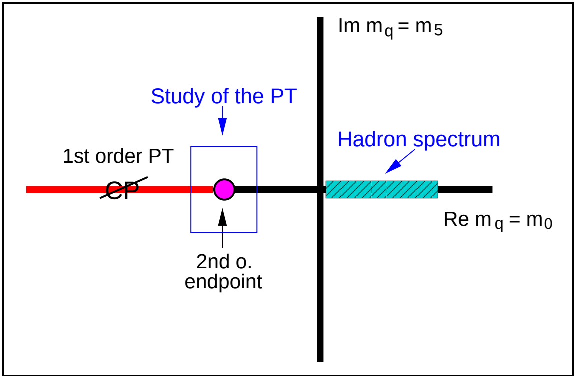
$\longrightarrow$ Technical: non-perturbative definition of the quark mass awkward
[(partial) solution to this problem proposed here]

# Expected T=0 phase diagram




- Mass term:
  $m_q\,\bar{\psi}_L\psi_R + m_q^*\,\bar{\psi}_R\psi_L =$
  $m_0\,\bar{\psi}\psi + im_5\,\bar{\psi}\gamma_5\psi$

# Expected T=0 phase diagram



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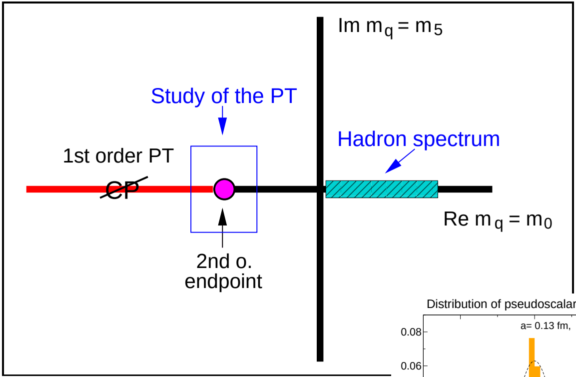
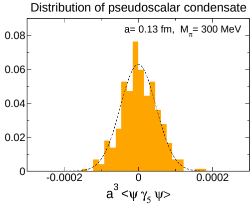


## Wilson Action

- Gauge sector: Tree-level Symanzik improved (tlSym)

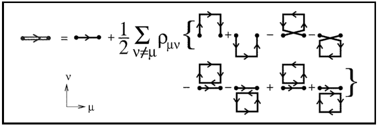
$$S_{glue} = \beta \sum_x \left( (1-8c_1) \square + c_1 \square\!\square \right)$$

$$c_1 = -\frac{1}{12}$$

- Fermion sector: Stout smeared fermion action


[© A.D.Kennedy (2004)]

Smeared-Link in Dirac operator:




[Morningstar and Peardon (2004)]

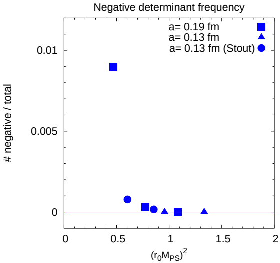
- One step of smearing, $\rho_{\mu\nu} = 0.15$

# Algorithm

- Polynomial Hybrid Monte Carlo (2-step)

  [Montvay and Scholz (2005)]

  - Sequence of PHMC trajectories + Metropolis accept-reject
  - Measurement correction
  - Determinant sign (see following)



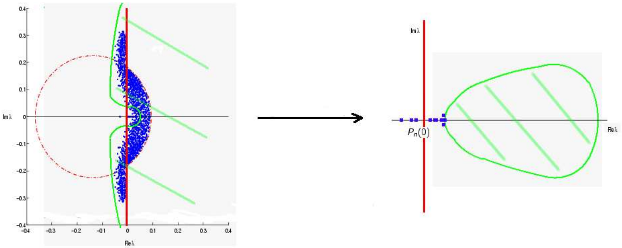
# Dirac operator determinant sign

- $\mathrm{sgn}(\det(D_W)) = \mathrm{sgn}(\prod_i \lambda_i) = \mathrm{sgn}(\prod_{i \in \mathbf{R}} \lambda_i) \implies$ look at $\lambda_i \in \mathbf{R}^-$

Real eigenvalues computation:

- Arnoldi algorithm
- Performance enhancement through polynomial preconditioning
  $D_W \to P_n(D_W)$ (compute eigenvales of $P_n(D_W)$)

  1. power preconditioning $D_W \to P_n(D_W) = (\sigma \mathbf{1} - D_W)^n$ [Neff (2001)]
  2. iterated version $D_W \to P_{2n}(D_W) = (\sigma \mathbf{1} - P_n(D_W)/\mathrm{const})^n$
  3. Faber polynomials (optimization) [Heuveline and Sadkane (1996); Driscoll(1996)]



# Hadron spectrum

Hadrons: $\bar{q}q$: $O^{++}$ $(\sigma)$, $O^{-+}$ $(\eta)$; $qqq$: $3/2^{+}$ $(\Delta^{++})$

Glueballs?

## Theoretical predictions (massless quark)

- t'Hooft large $N_c$ limit: Witten-Veneziano formula

$$m_\eta^2 = \frac{4N_f}{f_\eta^2}\chi_t$$

[Witten; Veneziano (1979)]

- Orientifold large $N_c$ equivalence with SUSY Yang-Mills:

$$\frac{m_\eta}{m_\sigma} = \frac{N_c-2}{N_c} + O\left(\frac{1}{N_c}, \frac{1}{N_c^2}\right)$$

[Armoni, Shifman and Veneziano (2003); Armoni and Imeroni (2005)]

Necessary and sufficient condition for validity: Unbroken charge-conjugation symmetry

[Ünsal and Yaffe (2006)]

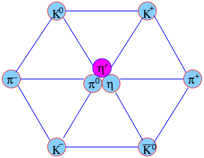
# Partially quenched extension

Add in the theory $N_V$ valence quarks: $(q;\, q_V^{(1)} \ldots, q_V^{(N_V)})$

$$SU(N_V+1|N_V)_L \otimes SU(N_V+1|N_V)_R \xrightarrow{\text{SSB}} \underbrace{SU(N_V+1|N_V)}_{\text{graded flavor symmetry}}$$

$m_{val} = m_{sea}$: $SU(N_V+1)$ "flavor symmetry"

$N_V = 2$: SU(3)




PCAC relation: $\partial_\mu \underbrace{(\bar{q}\,\frac{\lambda^a}{2}\gamma_\mu\gamma_5 q)}_{A^a_\mu(x)} \;=\; 2\, m_q^{PCAC}\; \underbrace{\bar{q}\,\frac{\lambda^a}{2}\gamma_5 q}_{P^a(x)}$

Interesting application: continuum limit (?) of rooted staggered QCD.

[Bernard, Golterman, Shamir and Sharpe (2008)]

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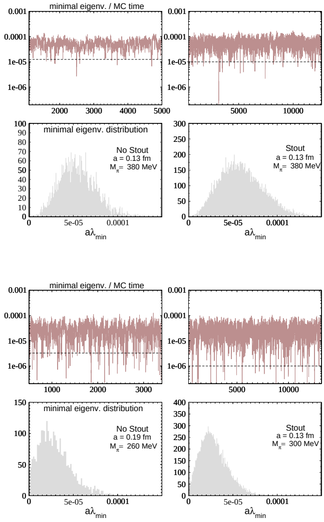
# Runs

| $L/a$ | Link | $\beta$ | $\kappa$ | $a$(fm) | $L$(fm) | $r_0/a$ | $M_\pi$(MeV) |
|---|---|---|---|---|---|---|---|
| 12 | Thin | 3.8 | 0.1700 | 0.19 | 2.1 | 2.66 | 404 |
| 12 | Thin | 3.8 | 0.1705 | 0.19 | 2.1 | 2.67 | 342 |
| 12 | Thin | 3.8 | 0.1710 | 0.19 | 2.1 | 2.69 | 260 |
| 16 | Thin | 4.0 | 0.1600 | 0.13 | 2.1 | 3.56 | 621 |
| 16 | Thin | 4.0 | 0.1610 | 0.13 | 2.1 | 3.61 | 484 |
| 16 | Thin | 4.0 | 0.1615 | 0.13 | 2.1 | 3.73 | 394 |
| 16 | Stout | 4.0 | 0.1440 | 0.13 | 2.1 | 3.74 | 374 |
| 16 | Stout | 4.0 | 0.1443 | 0.13 | 2.1 | 3.83 | 303 |
| 24 | Stout | 4.0 | 0.1443 | 0.13 | 3.1 | | 292 |

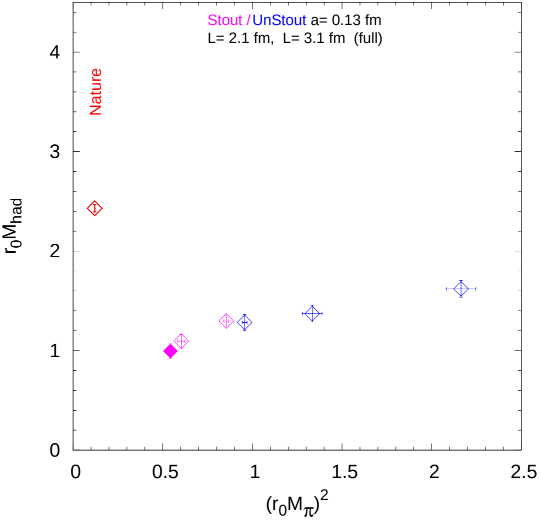
Lattice scale fixed by the Sommer scale $r_0$ (QCD units)

# Minimal eigenvalue: Stout vs. Thin link




- Squared Hermitian fermion matrix

# Hadron spectrum

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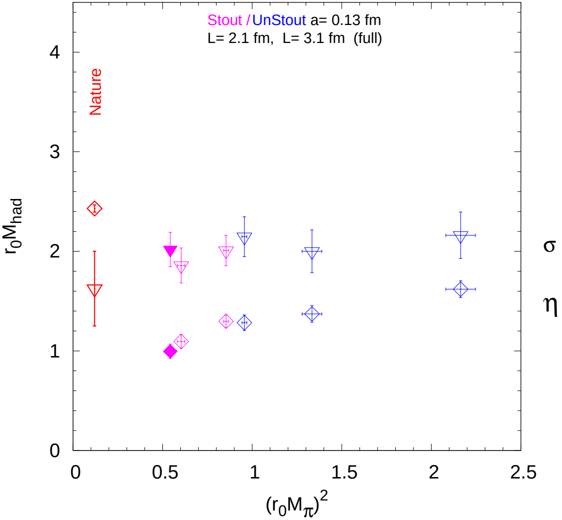

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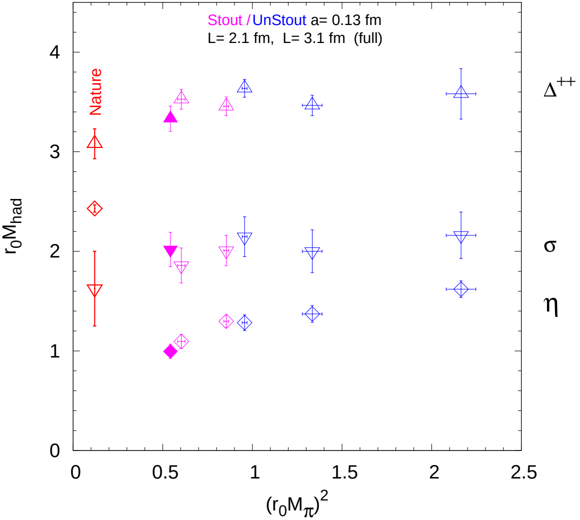

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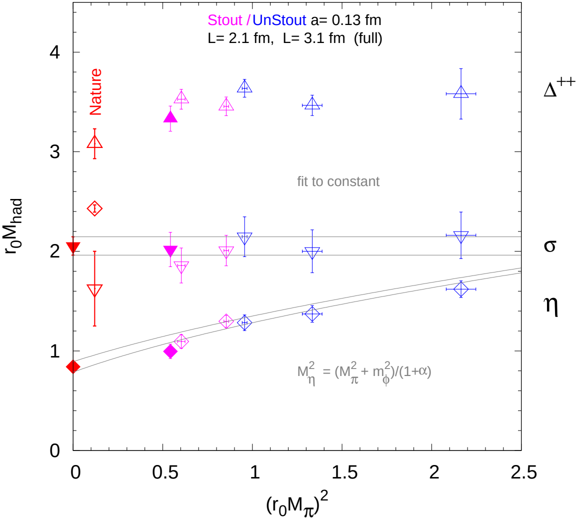

# Massless quark extrapolation

LO ChPT for the singlet: $M_\eta^2 = \dfrac{M_\pi^2 + m_\phi^2}{1+\alpha} \longrightarrow M_\eta(m_q = 0) = \dfrac{m_\phi}{\sqrt{1+\alpha}}$

[Bernard and Golterman (1993); Sharpe and Shoresh (2001)]

$m_q \longrightarrow 0$ extrapolated masses:

$r_0 m_\sigma = 2.05(9)$ [810(35)MeV]

$r_0 m_\eta = 0.84(5)$ [330(20)MeV] [SU(2)SYM: $m_\eta \simeq 490$MeV, poster K. Demmouche]

$\dfrac{m_\eta}{m_\sigma} = 0.410(32)(25)$

Orientifold planar equivalence with SYM :

$$N_c = 3: \quad \frac{m_\eta}{m_\sigma} = \frac{1}{3}[1+\Delta], \quad \Delta = O(\frac{1}{N_c}, \frac{1}{N_c^2})$$

[Armoni, Shifman and Veneziano (2003); Armoni and Imeroni (2005)]

We find: $\Delta = 0.23(12)$

Cf. with quark condensate: $\frac{\langle\bar{\psi}\psi\rangle}{\langle\lambda\lambda\rangle} = \frac{1}{3}[1+\Delta'] \quad \Delta' = -0.096(8) \quad (N_f = 0)$

[Armoni, Lucini, Patella and Pica (2008)]

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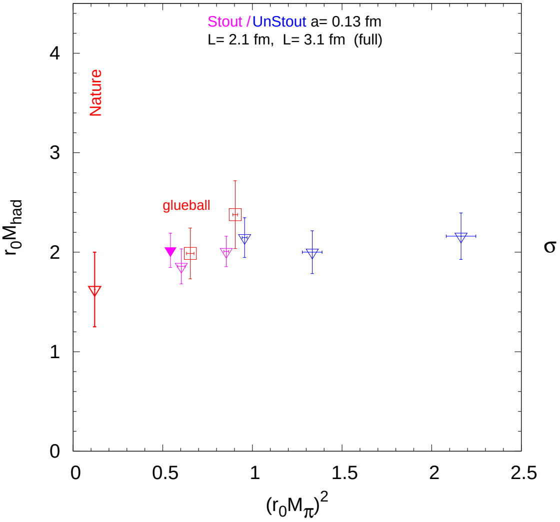
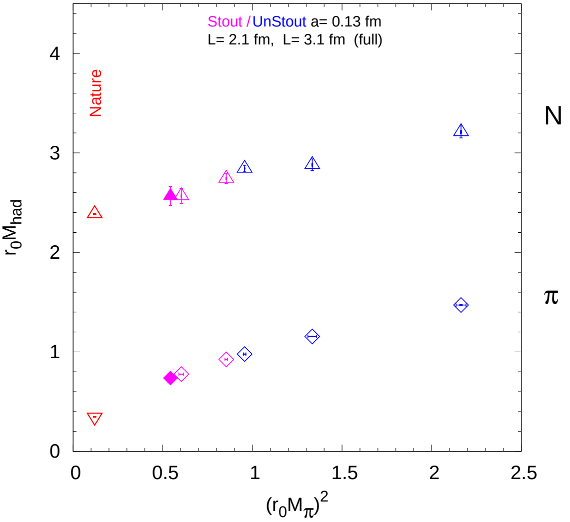
# Glueballs?




Better signal with Stout runs

# PQ hadron spectrum



# Perspectives

Hadron spectrum:

- more quark masses with Stout quarks
  $\rightarrow$ chiral fits in "pion" sector
- continue second larger volume
- second smaller lattice spacing

Phase structure:

- Explore negative quark masses on small lattices: fast determinant sign computation

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Thank you!