Dual quark condensate and dressed Polyakov loops

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Lattice 2008, William and Mary

with Erek Bilgici, Christian Hagen and Christof Gattringer

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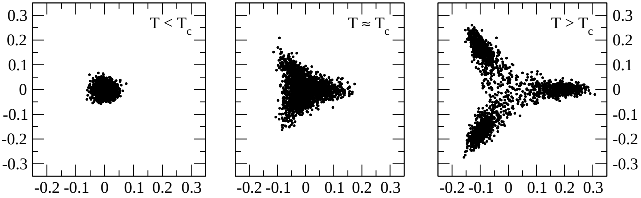
Motivation

QCD at finite temperature: confinement and chiral symmetry breaking

quenched $\sim$ Yang-Mills theory: same T_c

- Polyakov loop: $\mathcal{P}(\vec{x}) = \mathcal{P}\exp\left(i\int_0^\beta dx_0 A_0(x_0,\vec{x})\right), \quad \beta = 1/k_B T$

  $\mathrm{tr}_c\,\mathcal{P}$ in SU(3):

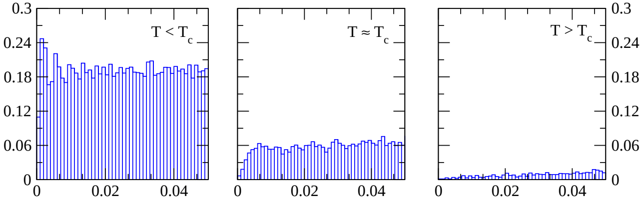


- order parameter for confinement:

  related to the free energy of a single quark

  confined phase: $\langle \mathrm{tr}_c\,\mathcal{P}\rangle = 0 \quad (F_{\mathrm{quark}} \to \infty)$

- spectral density $\rho(\lambda)$ of the Dirac operator (in background A_μ):



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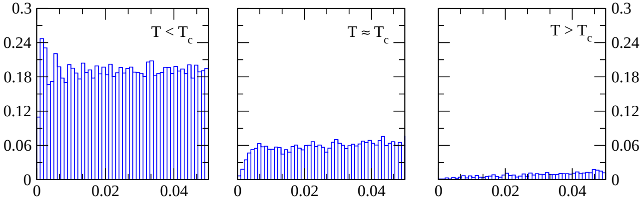

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Is there an underlying mechanism connecting the two?

does confinement leave a trace in the Dirac spectrum?

quarks should know that they are confined!

$\Rightarrow$ dressed Polyakov loops as a new order parameter

The idea

work on the lattice (regulator)

- Polyakov loop: $\mathcal{P}(x) \equiv \prod_{\tau=1}^{N_0} U_0(x_0+\tau,\vec{x})$

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$$D(x,y) \equiv \frac{1}{2a}\sum_{\mu}\eta_\mu(x)\big[U_\mu(x)\delta_{x+\hat{\mu},y} - h.c.\big] \quad \text{hopping by one link}$$

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- phase 'twisted' boundary conditions, as a tool: — Gattringer '06

$$\psi(x_0+\beta,\vec{x}) = z\,\psi(x_0,\vec{x}), \qquad z = e^{i\phi} \quad \text{imag. chem. potential}$$

realized by $U_0 \to zU_0$ at some time slice

$\Rightarrow$ Polyakov loops: $\mathcal{P} \to z\mathcal{P}$, trivial loops stay the same

- $\mathcal{P}$ itself turned out to be not suitable (UV dominated) FB et al. '06
- propagator: cf. Synatschke, Wipf, Wozar '07

$$\mathrm{tr}\,\frac{1}{m+D_\phi} = \frac{1}{m}\sum_{l=0}^{\infty}\frac{(-1)^l}{m^l}\,\mathrm{tr}(D_\phi)^l \qquad \ldots\text{all powers of } D_\phi$$

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- project onto particular winding q:

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{-i\phi q}$$

let's specify to a single winding $q = 1$ like the Polyakov loop:

A new observable

FB et al. '08

$$\tilde{\Sigma}_1 \equiv \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\phi} \frac{1}{V} \left\langle \mathrm{tr} \frac{1}{m + D_\phi} \right\rangle = \frac{1}{mV} \sum_{\substack{\text{loops} \\ \text{of length } l,\ \text{winding once}}} \frac{(\pm 1)}{(2am)^l} \left\langle \mathrm{tr}_c \prod_l U_\mu(x) \right\rangle$$

dual condensate

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- massless limit:

$$\lim_{m\to 0} \lim_{V\to\infty} \tilde{\Sigma}_1 = \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\phi} \rho(0)_\phi$$

dual chiral condensate
$\rho(0) \sim \langle \bar{\psi}\psi \rangle$
(integrated over phase bc.s)

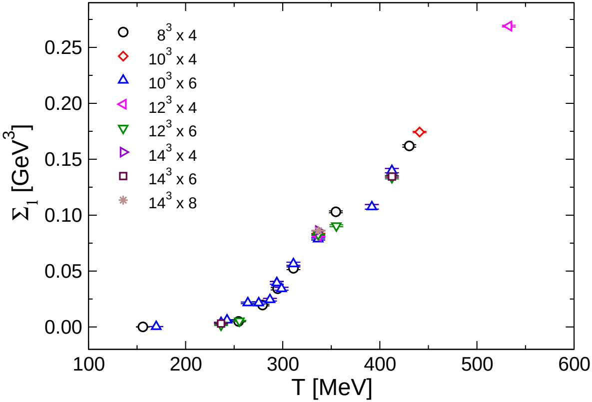
- massive limit:

$$\lim_{m\to\infty} \tilde{\Sigma}_1 \sim \langle \mathrm{tr}_c \mathcal{P} \rangle$$

thin Polyakov loop (shortest)
detours suppressed by $2am$

$\tilde{\Sigma}_1$ is an order parameter

numerical results (quenched):



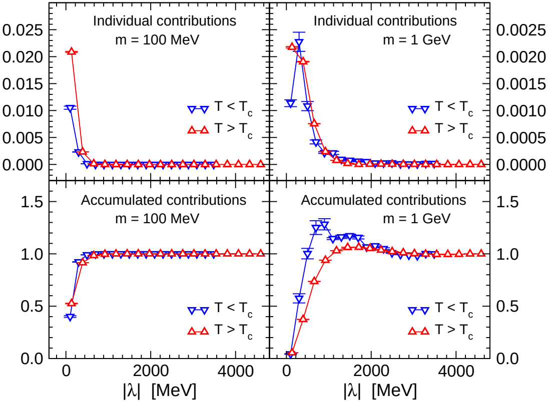
$\tilde{\Sigma}_1$ as a function of temperature for $m = 100 MeV$

Spectral representation

$$\tilde{\Sigma}_1 \equiv \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\phi} \frac{1}{V} \left\langle \mathrm{tr} \frac{1}{m + D_\phi} \right\rangle = \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\phi} \frac{1}{V} \left\langle \sum_i \frac{1}{m + \lambda_\phi^{(i)}} \right\rangle$$

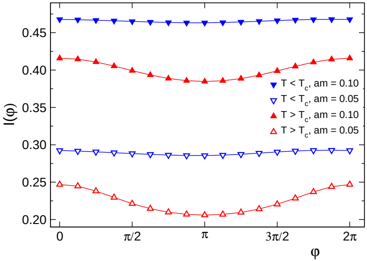
truncate the sum: IR dominance expected since λ in denominator!

confirmed by lattice data (if m not too large):



how is a vanishing/finite Polyakov loop built up by the eigenvalues?

respond differently to bc.s in confined and deconfined phase



$\frac{1}{V}\langle\sum_i \frac{1}{m+\lambda_\phi^{(i)}}\rangle$ as a function of ϕ for real $\mathcal{P}$

nonvanishing cos ϕ-part only in the deconfined phase $\Rightarrow \tilde{\Sigma}_1 \neq 0$

non-real $\mathcal{P}$: the plot is shifted by $\pm 2\pi/3$

$\Rightarrow$ periodicity $2\pi/3$, known from imag. μ Lombardo et al.

How about the chiral condensate?

remember:

$$\tilde{\Sigma}_1 \overset{m\to 0,V\to\infty}{\longrightarrow} \int_0^{2\pi} d\phi\, e^{-i\phi} \rho(0)_\phi = \int_0^{2\pi} d\phi\, e^{-i\phi} \langle\bar{\psi}\psi\rangle_\phi$$

- confined phase:
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  no: $\rho(0)_{\text{periodic}} \neq 0$ for real $\mathcal{P}$ Gattringer, Schaefer '03
  always one bc. where $\rho(0) \neq 0$

  $\langle\bar\psi\psi\rangle_\phi \sim \delta(\phi + \phi_{\mathcal{P}}) \quad \Rightarrow \quad$ nonvanishing $\tilde{\Sigma}_1$
  for all $T > T_c$

Center symmetry

the deconfinement transition of pure gauge theory can be described as spontaneous breaking of the center symmetry:

- the action is invariant under

$$U_0 \to z\, U_0 \quad \text{at some time slice,} \quad z \in \text{center}(SU(3))$$

- the Polyakov loop changes as

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- all functions of the form Synatschke, Wipf, Langfeld '08

$$\int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\phi} f(D_\phi)$$

transform this way, thus are order parameters for center symm.

Generalisation: Locally resolved Polyakov loops

so far: $\sum_x \mathcal{P}(x) \to$ eigenvalues $\lambda_\phi^{(i)}$

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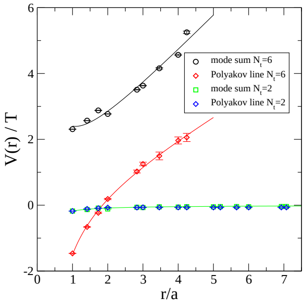
- static quark potential $V_{q\bar{q}}(|\vec{x}-\vec{y}|) \sim \ln\langle \mathrm{tr}\,\mathcal{P}(\vec{x})\,\mathrm{tr}\,\mathcal{P}(\vec{y})\rangle$

  SU(2): Synatschke, Wipf, Langfeld '08



$\Rightarrow$ string tension preserved by a truncated mode sum

mechanism not fully clear — Bilgici, Gattringer '08

Summary

the response of Dirac spectra to different temporal bc.s contains information about confinement

the dressed Polyakov loop $\tilde{\Sigma}_1$ is a novel deconfinement order param. that relates the dual chiral condensate to the thin Polyakov loop

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outlook:

- random matrix theory description of D_ϕ Bruckmann, Verbaarschot in progr.
- gauge group $G(2)$: no nontrivial center Gattringer, Maas in progr.
- full QCD and 4-fermi deformation (Sinclair): $T_{\chi sb} \neq T_{deconf}$
  how in the formalism?!