The Conformal Window in $SU(3)$ Yang-Mills

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   - Flavor dependence
2. Setup and Methods
   - Program of study
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   - Looking forward: $N_f = 10$
   - Conclusion

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- "Lattice Study of the Conformal Window in QCD-like Theories" (Thomas Appelquist, George T. Fleming, EN.) PRL 100, 171607 (2008).

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- The second row defines the conformal window.
- The value of N_f^c and the nature of the transition are important to model builders.
- N_f^c is unknown - pert. theory breaks down near the bottom of the window. Need non-perturbative study!

Estimates of N_f^c

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- However, previous lattice investigation of the conformal window (Iwasaki et al, PRD 69: 014507, 2004) claims the result $6 < N_f^c < 7$.

Program of study

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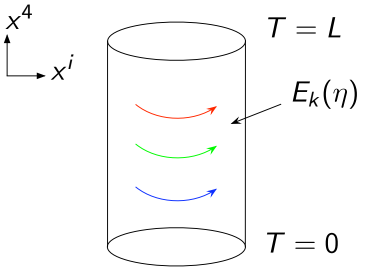
- SF boundary conditions lift fermionic zero modes to scale $1/L$ - simulate with $m = 0$ directly!
- Note: taking $m = 0$ further motivates the use of unrooted staggered fermions; trouble can arise if $m \to 0$ before $a \to 0$ (S. Sharpe, hep-lat/0610094.)

The Schrödinger Functional



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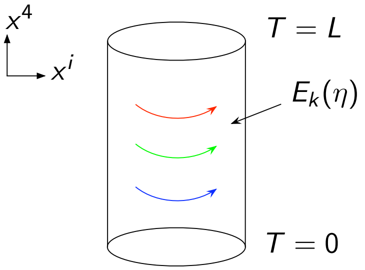

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Running coupling

The SF running coupling $\overline{g}^2(L)$ is defined to vary inversely with the response of the action to the strength η of the background field,

$$\frac{dS}{d\eta} = \left.\frac{k}{\overline{g}^2(L)}\right|_{\eta=0}.$$

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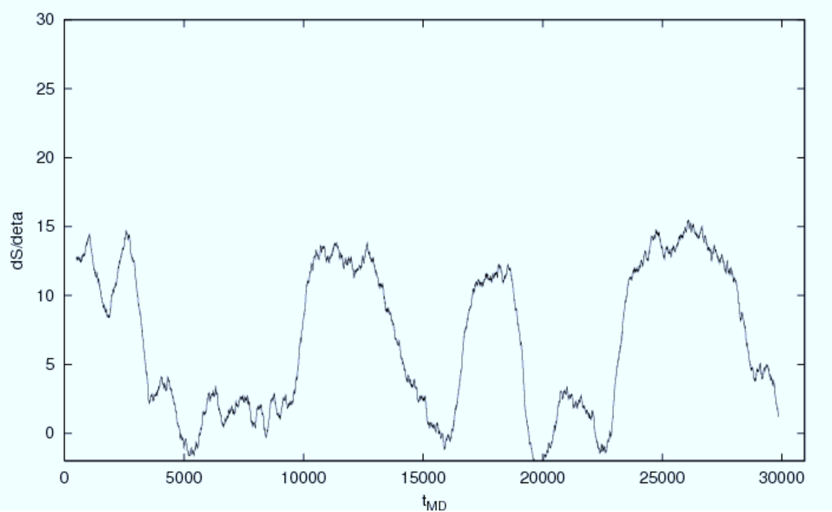
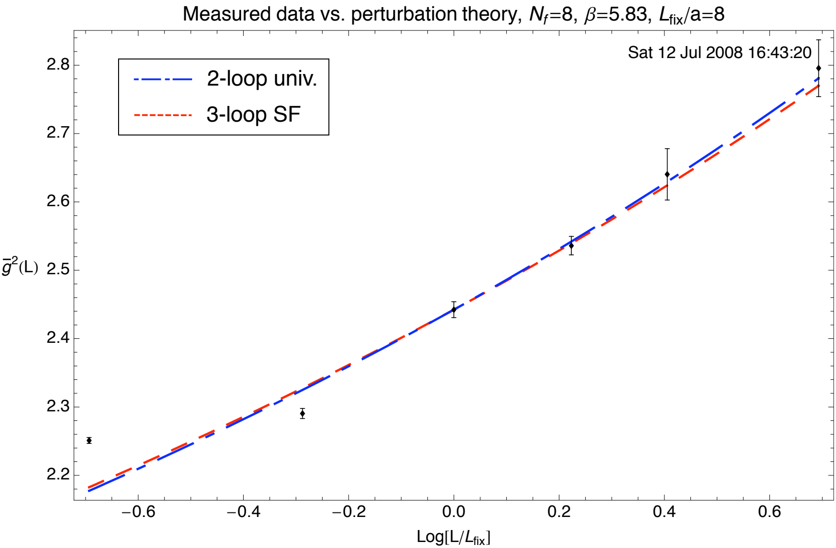
- Long autocorrelations; $\sim 20k - 80k$ MD trajectories are gathered at each (β, L) to accurately determine statistical error.

Time series of observable



$N_f = 12, \beta = 4.7, 16^3 \times 17$. Running average of 800 traj.

Data vs. perturbation theory



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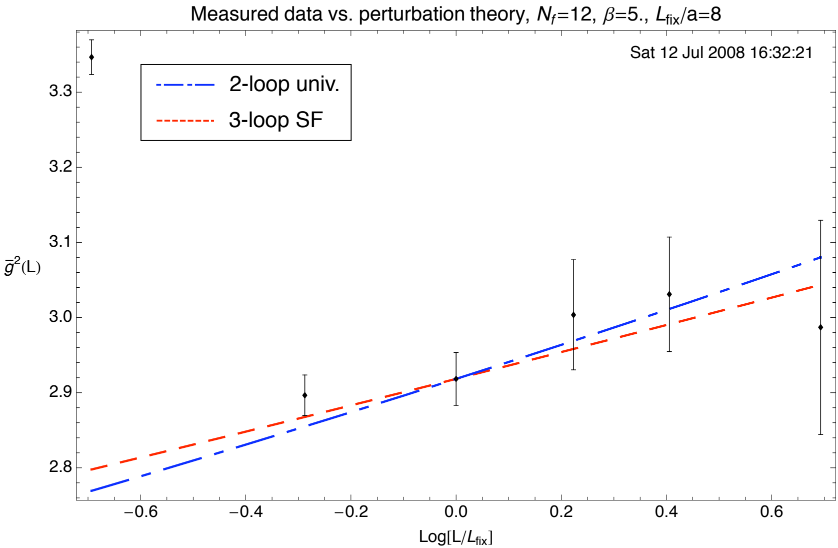

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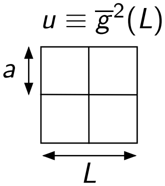
- We use the step scaling procedure to link together results of simulations at many different a. Measure in discrete steps: $\overline{g}^2(L) \to \overline{g}^2(2L) \to \ldots$
- Define the step-scaling function,

$$\Sigma(2, \overline{g}^2(L), a/L) \equiv \overline{g}^2(2L) + O(a/L)$$

The continuum limit $\sigma(2, u) \equiv \lim_{a \to 0} \Sigma(2, u, a/L)$ is basically a discretized version of the β-function.

Step scaling, visually



(see R. Sommer, hep-lat/0611020)

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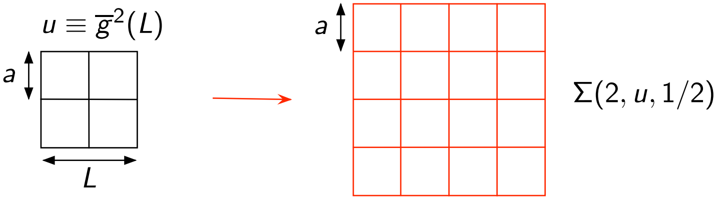

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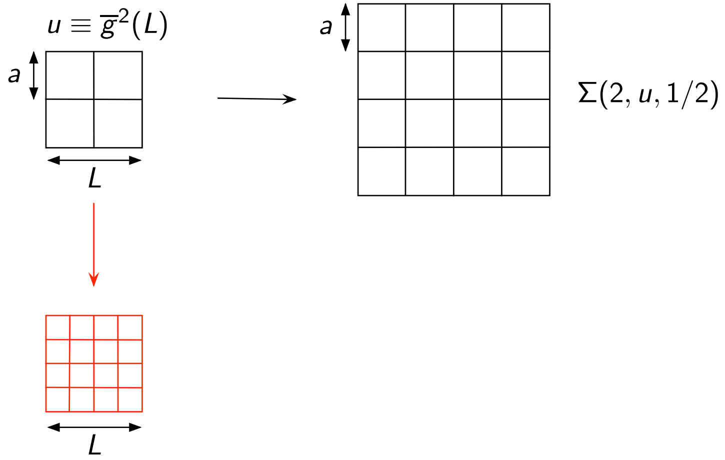

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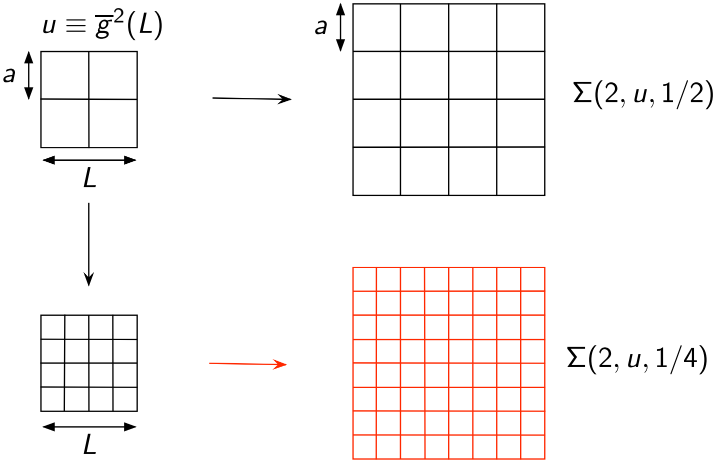

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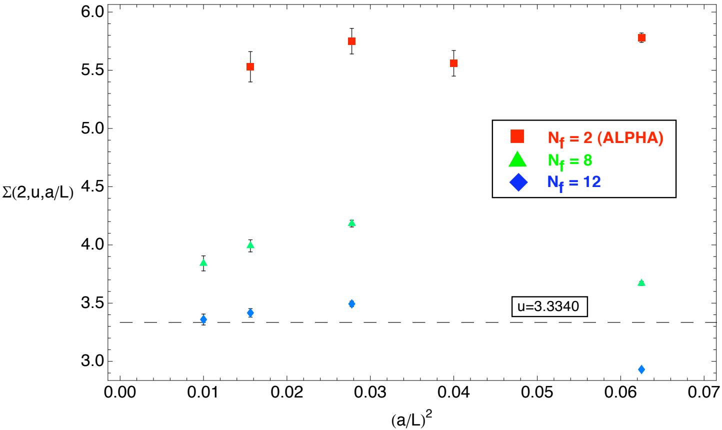
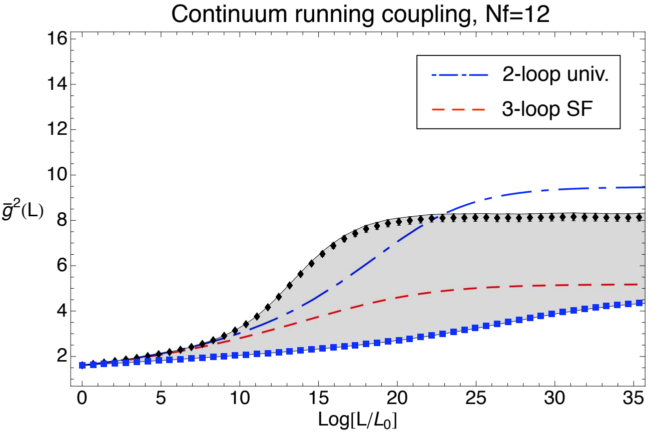
Data comparison with ALPHA



(Ref: Della Morte et. al. (ALPHA), hep-lat/0411025, NPB 713 (2005) p.378.)

Results, $N_f = 8$ and 12



IR fixed point! First non-pert. evidence of an IRFP outside of SUSY.

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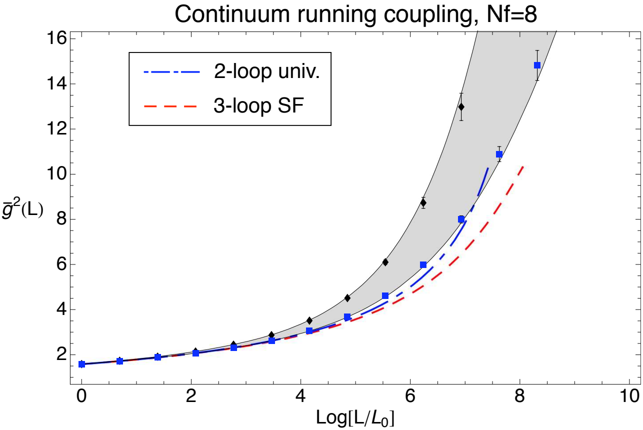

No evidence of a fixed point or inflection point! $8 < N_f^c < 12$.

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Wilson vs. staggered fermions

Wilson fermions are inherently more expensive than staggered, but we can offset this by making the continuum extrapolation easier:

- Use clover-improved fermion action, boundary improvement counterterms (2-loop perturbative values!)
- Simulate at odd L/a, more points in continuum extrapolation
- Use Chroma code package (with some modification.)
- Better algorithm: use rational HMC.

Conclusions

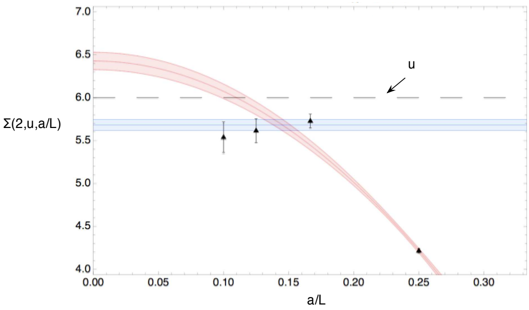
Summary

- We have constrained the lower boundary of the conformal window: $8 < N_f^c < 12$, in agreement with the ACS bound ($N_f^c \leq 12$) and contradicting Iwasaki et al ($6 < N_f^c < 7$.)
- We have provided the first non-perturbative evidence of an IR fixed point outside of supersymmetric theories.

Future work

- Continued simulations at 8 and 12 flavors, to reduce systematics.
- Study of running coupling at $N_f = 10$ (underway now.)
- Study of running coupling in QED3.
- $T = 0$ simulation at $N_f = 8$, to verify the presence of chiral symmetry breaking.
- Simulation at other N_c, other fermion reps.

Continuum extrapolation



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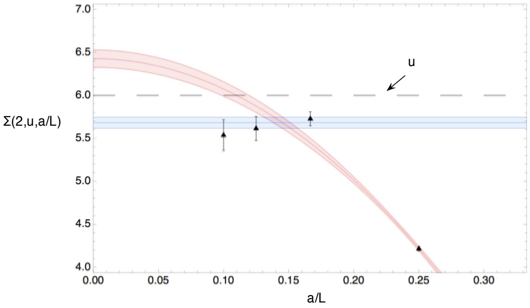

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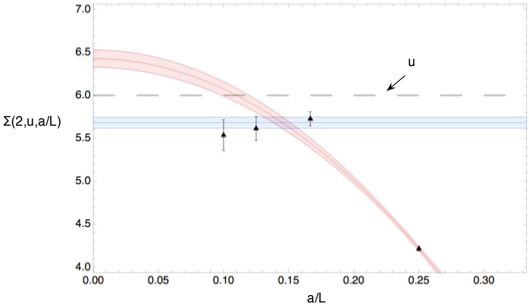

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Any reasonable continuum extrapolation should be bounded by the two methods shown above, so we take them to define a systematic error band. Other, more complex extrapolations yield intermediate results.