The finite-temperature phase structure of lattice QCD with twisted-mass Wilson fermions

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Outline of the talk

1. Introduction and motivation
2. Anticipating the phase structure
3. Evidence for the Aoki phase
4. Closer look at the first order region
5. Search for Creutz' cone scenario
6. Summary and outlook

Some of our twisted-mass papers

- Twisted mass QCD at finite temperature, E.-M. I., M. Müller-Preussker, M. Petschlies, K. Jansen, M. P. Lombardo, O. Philipsen, L. Zeidlewicz, A. Sternbeck, PoS LATTICE2007:238 (2007) [arXiv:0710.0569 [hep-lat]]

- Probing the Aoki phase with $N_f = 2$ Wilson fermions at finite temperature, E.-M. I., W. Kerler, M. Müller-Preussker, A. Sternbeck, H. Stüben, [hep-lat/0511059]

- A numerical reinvestigation of the Aoki phase with $N_f = 2$ Wilson fermions at zero temperature, E.-M. I., W. Kerler, M. Müller-Preussker, A. Sternbeck, H. Stüben, Phys. Rev. D69:074511 (2004) [hep-lat/0309057]

1. Introduction and Motivation

- Lattice field theory exists in different discretizations
- General aim: optimization of the continuum and chiral limit
- Wilson fermions

  + locality realized

  + clear flavor assignment

  + competitive algorithms developed

  – chiral symmetry explicitely broken

  – subtle chiral behavior

  – complicated phase structure at $T = 0$ and finite T

  – slow approach to continuum

  + the latter can be cured

The goal of the tmfT Collaboration : taking advantage of twisted mass for QCD thermodynamics

One among three roads to improve the Wilson fermion action :

1. $O(a)$ improvement by clover term
2. chiral improvement by smearing
3. twisted mass improvement

What makes twisted mass attractive ?

- Prevents the occurrence of small eigenvalues of the Dirac operator
- This avoids "exceptional configurations".
- This allows to work at smaller quark masses.
- At maximal twist (with κ tuned to criticality) automatic $O(a)$ improvement is guaranteed.

Price: 3-dimensional phase diagram with complicated structure due to $O(a^2)$ parity and flavor violating effects

The gauge action :

$$S_G = \beta \sum_x \left[c_0 \sum_{\mu<\nu} \left(1 - \frac{1}{3} \text{Re Tr } U^{1\times1}_{x\mu\nu} \right) + c_1 \sum_{\mu\neq\nu} \left(1 - \frac{1}{3} \text{Re Tr } U^{1\times2}_{x\mu\nu} \right) \right]$$

tree-level Symanzik action with $\beta = 6/g_0^2$, $c_1 = -1/12$ and $c_0 = 1 - 8\ c_1$

In our previous Aoki phase studies : Wilson gauge action ($c_1 = 0$)

The fermion action :

$$S_F = a^4 \sum_x \left\{ \overline{\psi}(x) \ \left[\left(D[U] + m_0 \right) \mathbb{I}_{2\times2} + \text{i}\ \mu\ \tau_3\ \gamma_5 \right] \ \psi(x) \right\}$$

$$D[U] = \frac{1}{2} \left[\gamma_\mu \left(\nabla_\mu + \nabla^*_\mu \right) - a\ \nabla^*_\mu\ \nabla_\mu \right]$$

Wilson-Dirac fermion action with twisted-mass term for $N_f = 2$ light flavors (in the physical basis $\Psi = (u, d)$)

[Frezzotti, Grassi, Sint, Weisz 2001; Frezzotti, Rossi 2004]

Twisted mass - an irrelevant rotation in continuum, not on lattice

Phase diagram spanned by

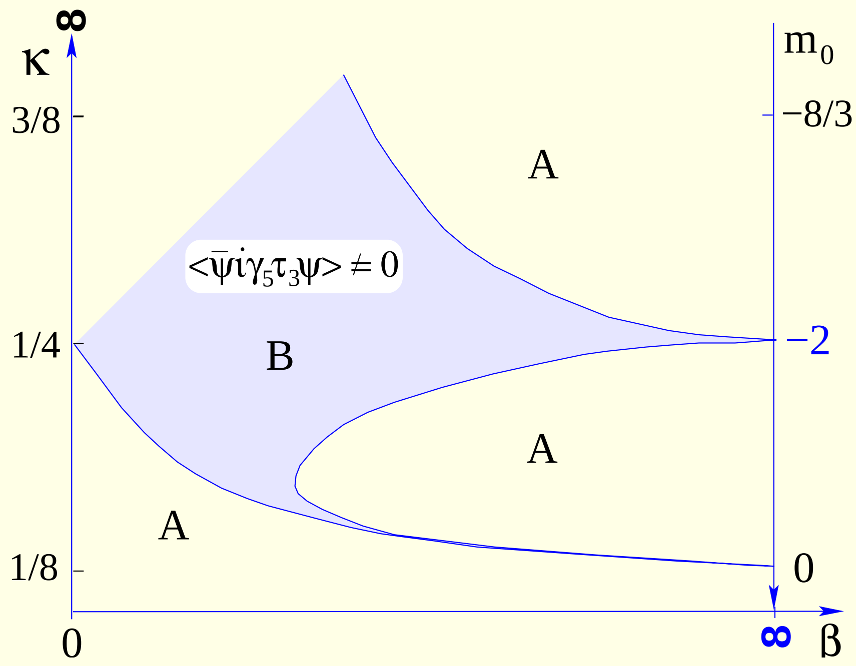
- inverse gauge coupling $\beta = 1/g_0^2$
- bare quark mass m_0, resp. hopping parameter $\kappa = \frac{1}{8+2\ a\ m_0}$
- twisted-mass μ, resp. polar mass $m_q = \sqrt{\left(\frac{1}{2\kappa} - \frac{1}{2\kappa_c}\right)^2 + \mu^2}$

First example of an “unphysical” phase “pocket”

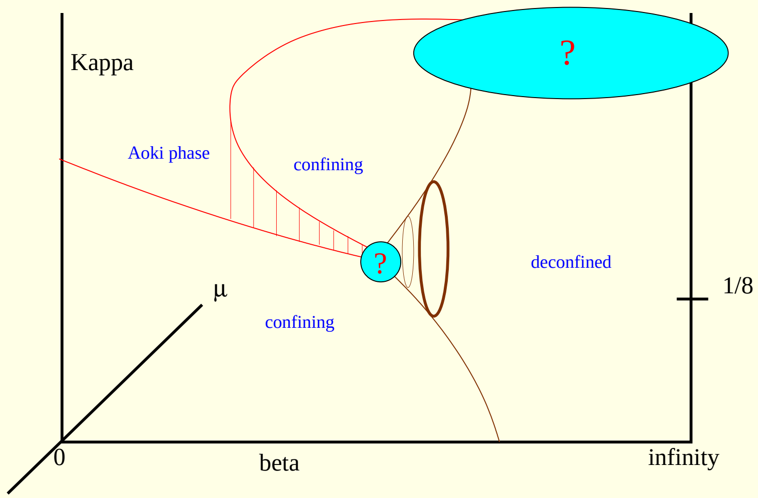
- $h = 2\ \kappa\ \mu$ – an external “magnetic field” $\Rightarrow$ induces spontaneous breaking of combined flavor-parity symmetry [Aoki 1984,1987] in some κ interval
  $\Rightarrow$ order parameter $= \langle \overline{\psi}\ \mathrm{i}\ \gamma_5\ \tau_3\ \psi \rangle \neq 0$
- no phase transition at $h \neq 0$ (cf. Ising model at $H \neq 0$)

2. Anticipating the phase structure

Aoki phase put into the full β-κ phase diagram



Aoki's conjecture [1984]: the Aoki phase (B) in the β-κ plane

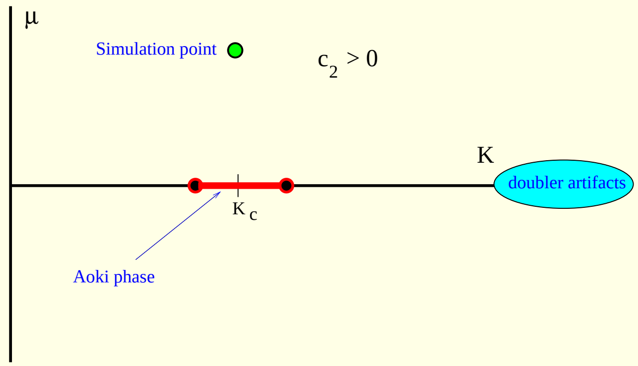


Connecting strong and weak coupling [Creutz, 2007]

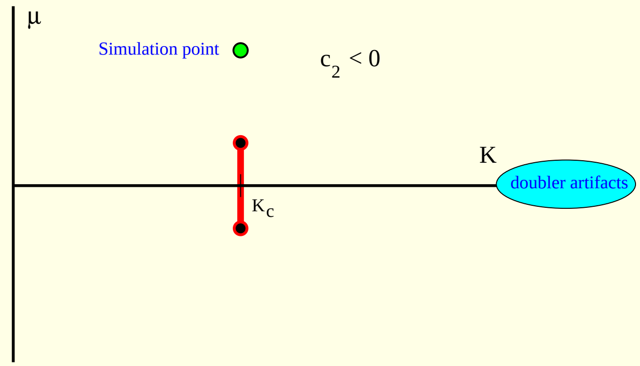
What follows after the Aoki phase before the confinement – deconfinement transition can be studied ?

Chiral effective action proposes the landscape of the phase diagram embedded in the β-κ-μ diagram.
(Sharpe, Singleton, Creutz)

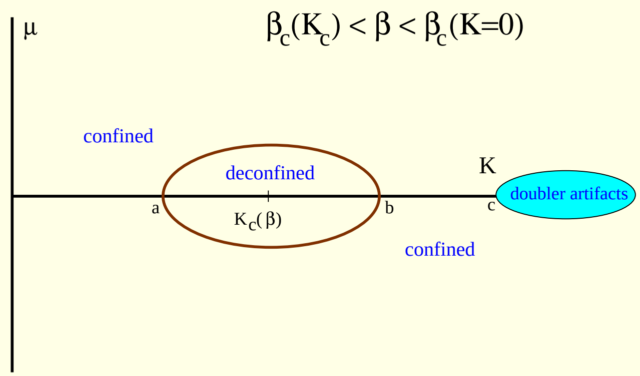
viewed in the κ-μ plane, going from low β to higher β



discontinuity in μ

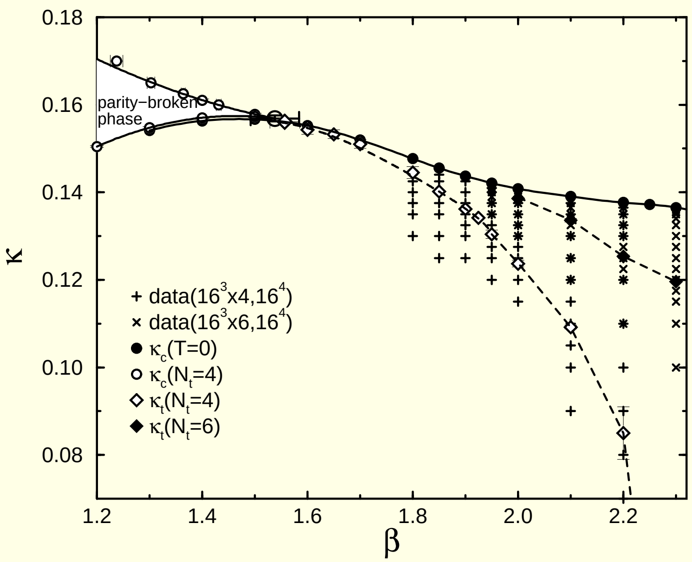


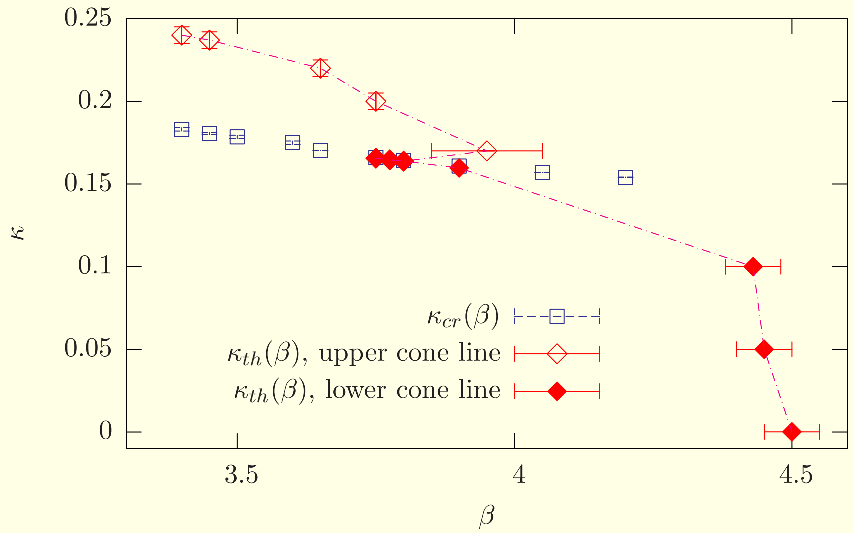
discontinuity in κ



deconfinement in a disk around the $\kappa_c(\beta)$ line

A closer study of the transition region towards higher β





The β-κ diagram for $16^3 \times N_t$ lattices with $N_t = 4, 6$ (here for Iwasaki gauge action and clover-improved Wilson fermion action [CP-PACS, 2001]) does not sufficiently resolve the "unphysical" phase structure.

The map of our simulation points on the $16^3 \times 8$ lattice, projected onto the β-κ plane from $0 \leq \mu < 0.007$, sketches the different transition lines (surfaces) under discussion. No transition has been found at $\beta > 4.5$.

We explore the phase structure using standard lattice variables :

– average plaquette $\Rightarrow$ indicator for bulk transitions

– average Polyakov loop $\Rightarrow$ thermal transition line

– chiral condensate : $\langle\overline{\psi}\psi\rangle$ interior of the "confinement" phase

– "pion norm" : $\sum_x \langle\overline{\psi}\psi(x)\ \overline{\psi}\psi(0)\rangle$ $\Rightarrow$ detects the chiral transition

– number N_{CG} of conjugate gradient iterations needed to invert the twisted-mass Wilson-Dirac operator $\Rightarrow$ sensitive to small eigenvalues, detects the chiral limit

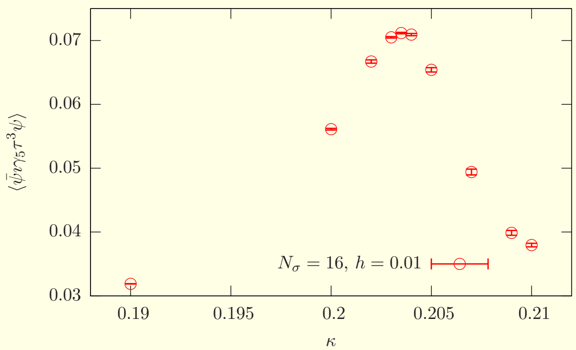
– parity-flavor breaking order parameter $\langle\overline{\psi}\ \mathrm{i}\ \gamma_5\ \tau_3\ \psi\rangle \rightarrow\neq 0$ in the double-limit $\lim_{h\to 0}\lim_{V\to\infty}$, exists only in the Aoki phase !

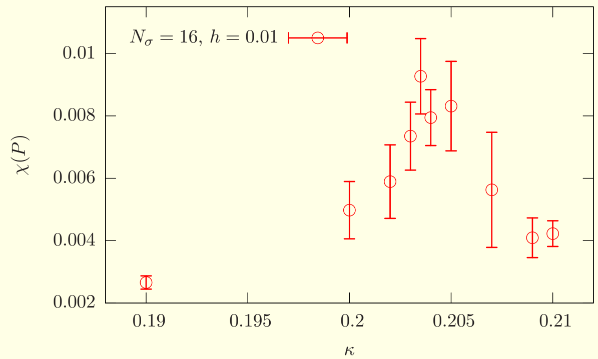
All simulations performed for $N_s^3 \times N_t = 16^3 \times 8$

Generalized HMC algorithm with even/odd preconditioning and Hasenbusch trick, in the multiple time-scale integration scheme.

3. Evidence for the Aoki phase

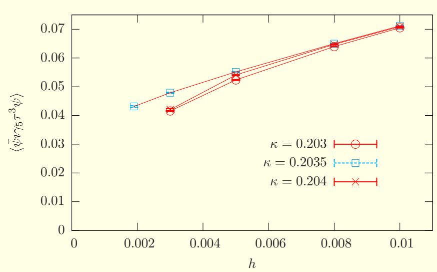
Studied anew for the tree-level Symanzik improved gauge action :

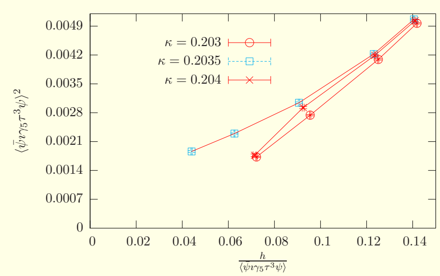


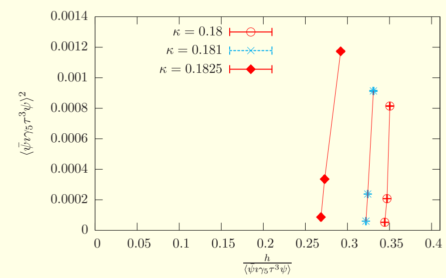


$\beta = 3.0$ order parameter

$\beta = 3.0$ plaquette susceptibility







Order parameter for $\beta = 3.0$ vs. $h \to 0$ for various κ

Fisher plots for various κ at $\beta = 3.0$

Fisher plots for various κ at $\beta = 3.4$

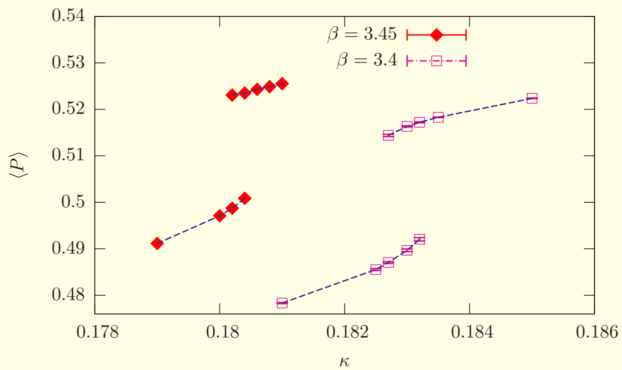
Conclusions concerning the Aoki phase

- $\beta = 3.0$: Aoki phase confirmed (Fisher plots are crucial !) for the new action (only for the available lattice size).

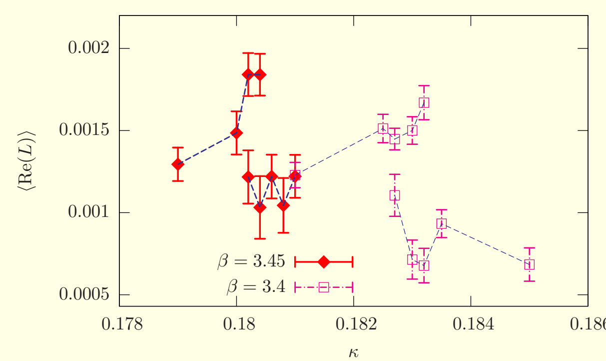
- $\beta = 3.4$: Only a "shadow" of the Aoki phase remains. The order parameter vanishes for $h \to 0$. Instead, first indications are found for metastability.

4. Closer look at the first order region

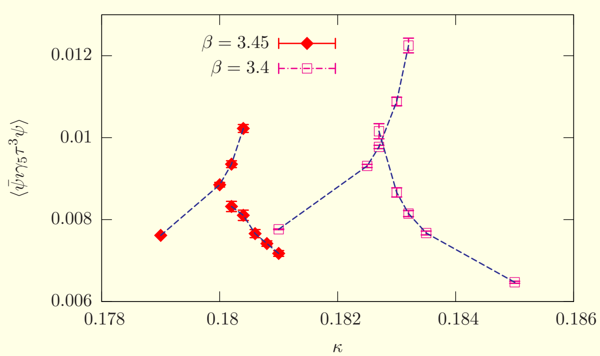
First look at the lower branch transition (low κ, metastability) as described by the effective action (also for $T = 0$)



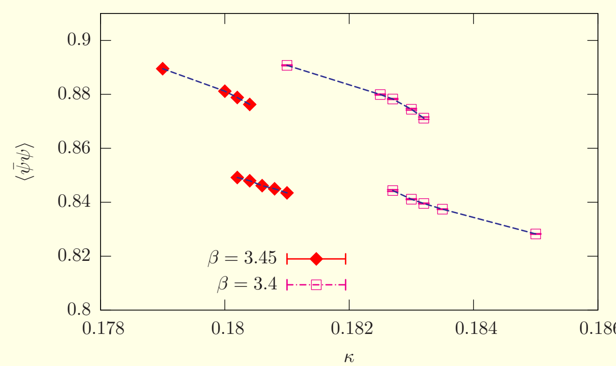
plaquette



real part of Polyakov loop



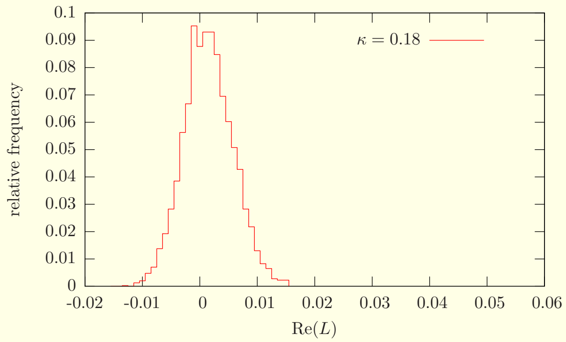
order parameter

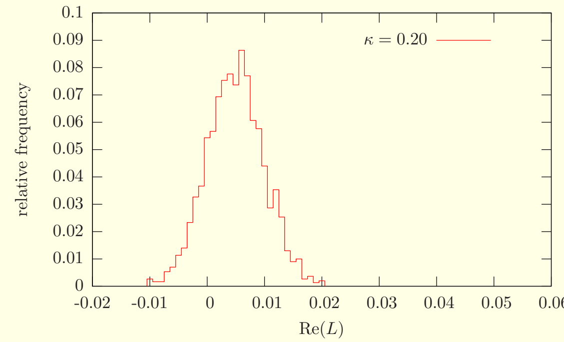


scalar condensate

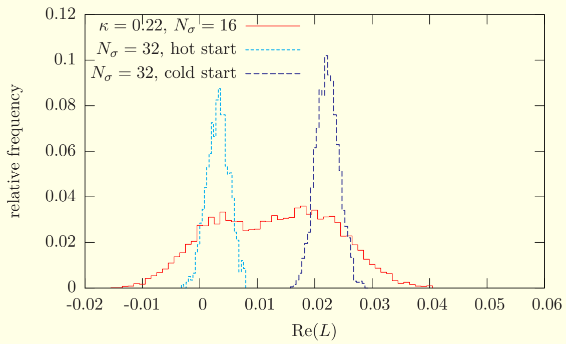
This is not a thermal transition: the Polyakov loop jumps down with increasing κ !

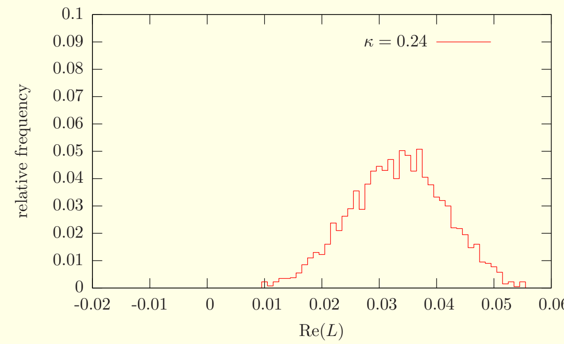
Next look at the upper branch transition (larger κ, also metastable). Histograms of the real part of the Polyakov loop:





$\kappa = 0.18$ $\kappa = 0.20$





$\kappa = 0.22$ $\kappa = 0.24$

This is a thermal transition. The Polyakov loop jumps up with increasing κ !

The histogram at $\kappa = 0.22$ is supplemented with hot and cold starts on a 32^4 lattice.

Conclusions concerning the first order region

We observe two branches :

A lower transition surface (finite in μ) of a first order transition and (so far seen at $\mu = 0$) an upper branch of the transition which is thermal.

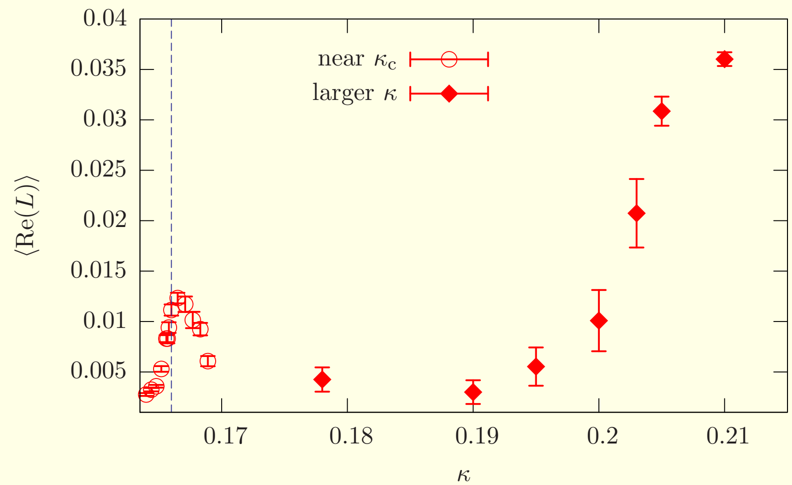
- Metastable states are observed at $\beta = 3.4$ and $\beta = 3.45$ in the region $\kappa = 0.18...0.184$ at small μ as predicted by the effective action (a remnant of the $T = 0$ transition).
- The upper branch is a thermal transition at $\kappa > \kappa_c(\beta)$. As an example, for $\beta = 3.6$, a first-order transition at $\kappa \approx 0.22$ has been shown (originating from the first doubler branch).

5. Search for Creutz' cone scenario

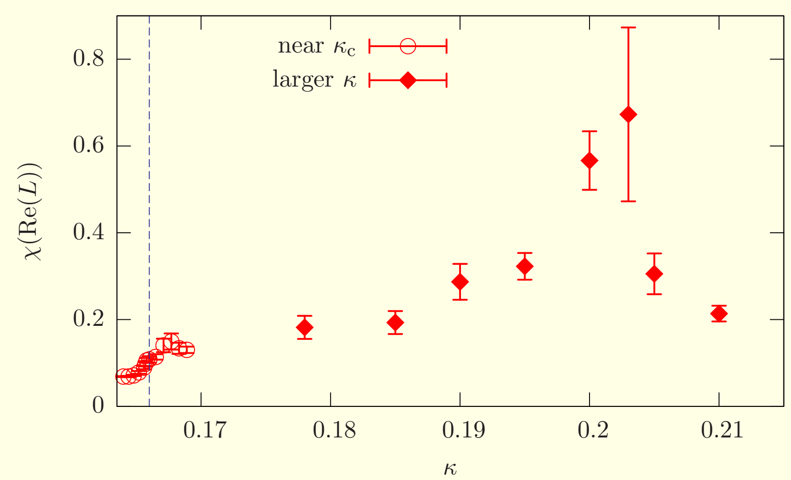
While the transitions come closer, the lower branch - before it ends - is enclosed by a cone around the $\kappa_c(\beta)$ line, that is opening towards large β.

Most useful so far : κ scan at $\mu \neq 0$ for several β values $\beta = 3.75$, 3.775 and 3.8

Example : a rough scan at $\beta = 3.75$ and $\mu = 0.005$



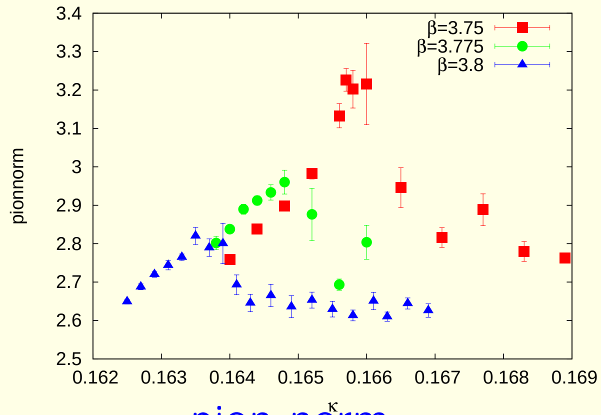
The real part of the Polyakov loop



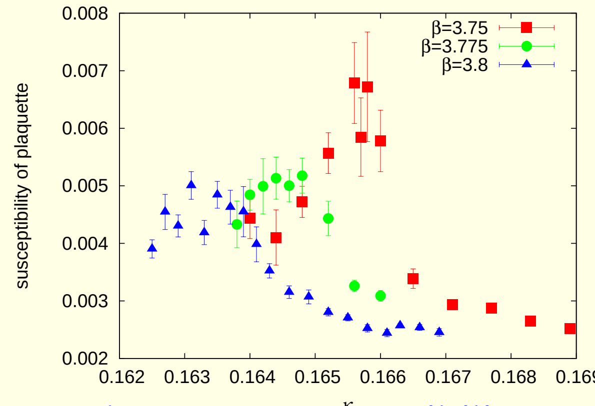
The susceptibility of the Polyakov loop

The lower peak (left) actually splits into two !

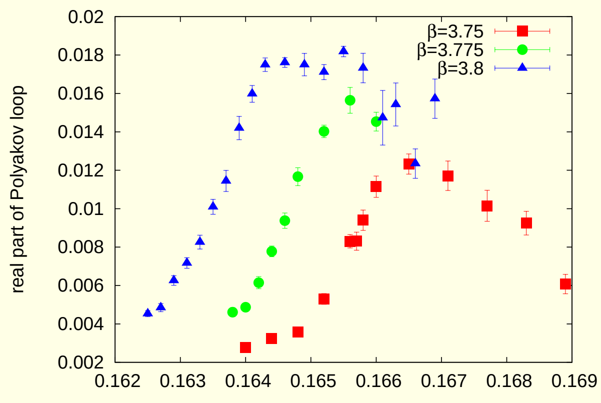
Zooming in the lower transition region for $\beta = 3.75$, 3.775 and 3.8



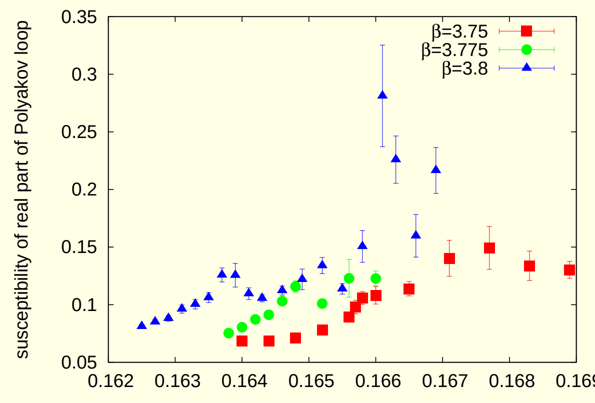
pion norm



plaquette susceptibility

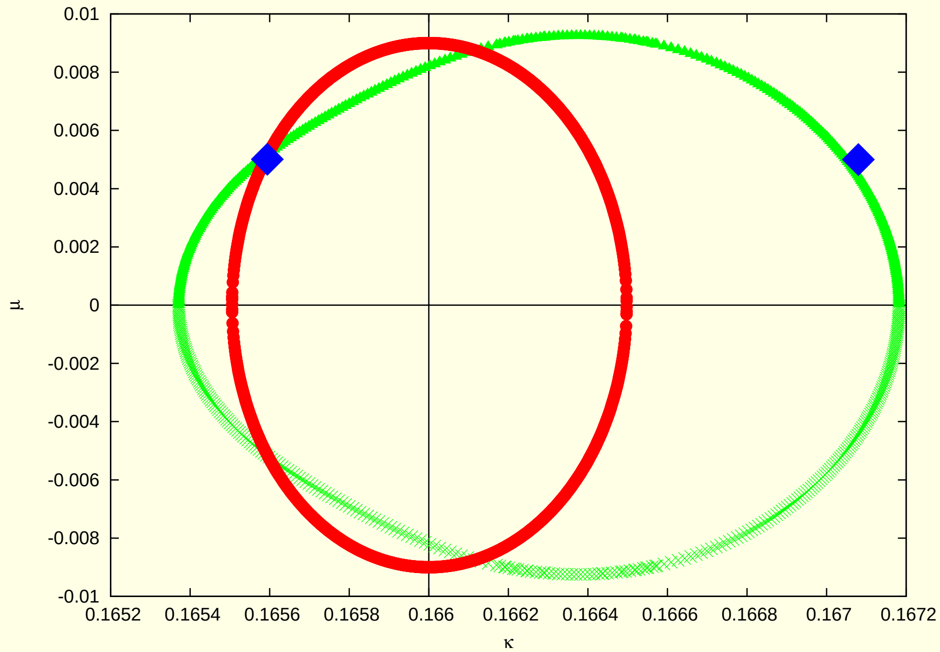


real part of Polyakov loop



susceptibility of Polyakov loop

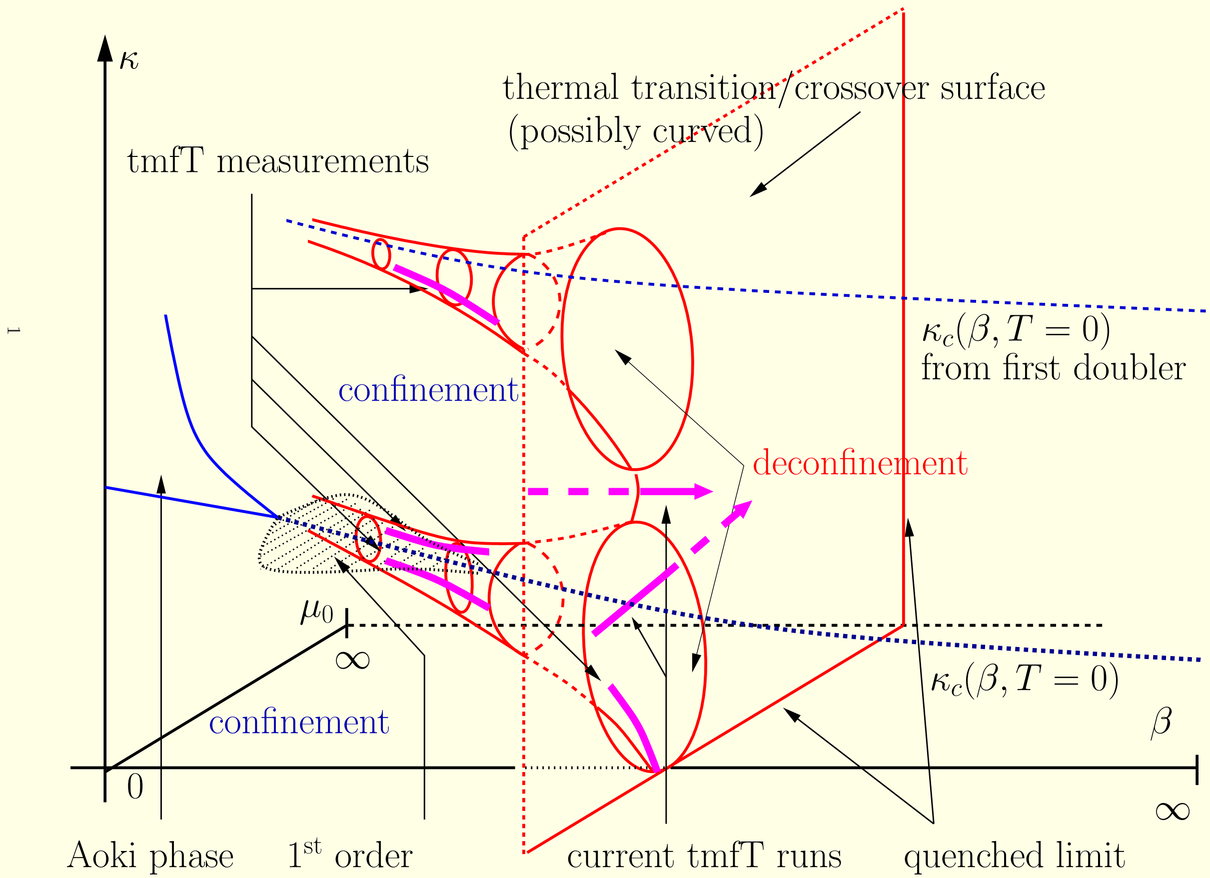
- The Polyakov loop susceptibility resolves two transitions.
- The Polyakov loop rises from both sides.
- The pion norm (and the plaquette susceptibility) have peaks at the lower (in κ) of the two transitions.
- The higher β, the more convincing becomes the two-transition picture.
- The transition bends down in κ with increasing β.
- The tip of the cone seems to be at $\beta \lesssim 3.75$
- At higher β, the bottom of the cone probably connects to the transition line coming from the β-axis ($\kappa = 0$, $m_q = \infty$, quenched limit).
- The upper deconfining transition, at $\kappa \approx 0.2$, is deconfining from below and related to the physics of the doubler.



A cut through the cone close to the tip: The schematic (ellipsoidal) transition line (red) centered at $\kappa_C(\beta, \mu = 0, T = 0)$ is compared with the prediction of χPT (green) with the two κ's located by actual simulations at $\beta = 3.75$ and $\mu = 0.005$.

6. Summary and outlook

In summary, our perspective view :



- The phase structure at $T \neq 0$ in the β-κ-μ phase space, for the preferred tree-level Symanzik gauge / twisted-mass Wilson fermion system and closer to the continuum (now with $N_t = 8$), has become clearer.
- We explored the vicinity of the β-κ plane at $0 \leq \mu < 0.007$.
- Now, for larger N_t, the different regimes are better separated from each other. The tip of the Aoki phase and the following first order surface are well localized.
- Higher in β, the $\kappa_c(\beta, \mu = 0, T = 0)$ line becomes the center of a cone-like surface enclosing the deconfined phase.
- The tip of the cone is at $\beta \lesssim 3.75$. The "transition circle" is partially seen.
- In the region of the opening cone more simulational work must be invested.
- Our final aim : determination of T_c and the equation of state close to the chiral limit, taking advantage of the automatic $O(a)$ improvement for twisted-mass fermions.