Determination of Nucleon Excited States

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LHPC Baryon Spectroscopy

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- G.T. Fleming, Yale University
- K.J. Juge, University of the Pacific
- A. Lichtl, Brookhaven National Laboratory
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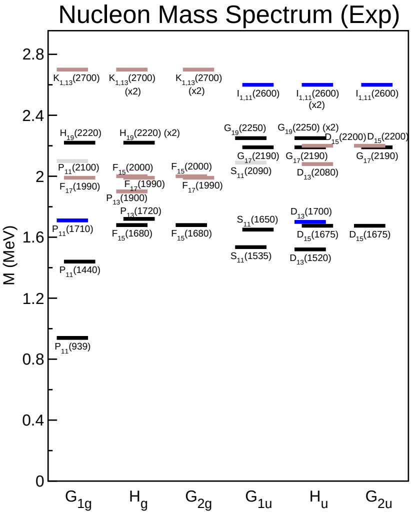
Outline

1. Intro and Previous Studies
2. Calculation Method
3. G_{1g} Results
4. G_{1u} Analysis
5. Conlusions and Outlook

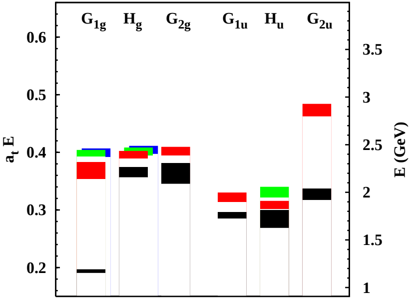
Introduction

- Lattice provides first principle calculations of spectrum
- Understand pattern of excited states, QCD
- Challenges: ordering of masses N' and N^* (Roper)



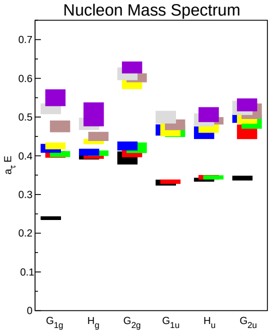
Quenched Results

Develop group theory, optimize operators, and refine analysis techniques.



Basak, et al. PRD76, 074504 (2007)

$16^3 \times 64$ and $24^3 \times 64$ lattices, $m_\pi = 490$ MeV



Lichtl, hep-lat/0609019

$12^3 \times 48$ lattice, $m_\pi = 700$ MeV

Calculation Overview

- Form a set of baryon operators $\{\bar{\mathcal{O}}_1, \bar{\mathcal{O}}_2, \ldots, \bar{\mathcal{O}}_n\}$
- Diagonalize matrix of correlators

$$\begin{aligned} C_{IJ}(t) &= \sum_{\vec{x}} \langle 0 | T \mathcal{O}_I(\vec{x}, t) \bar{\mathcal{O}}_J(0,0) | 0 \rangle \\ &= \sum_n c_n e^{-E_n t} \end{aligned}$$

- Principle correlator - diagonalize correlator matrix on each time slice, $\lambda_n(t) \propto e^{-E_n t}$
- Fixed coefficient - diagonalize at an early time slice, rotate each time slice into basis of eigenvectors. $C_{nn}(t) \propto e^{-E_n t}$

Group Theory

- Lattice breaks full rotational symmetry
- Construct operators that transform as irreducible representations of O_h^D: definite lattice spin and parity:

irreps	dim.
G_{1g}, G_{1u}	2
G_{2g}, G_{2u}	2
H_g, H_u	4

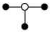
- Identify continuum spin via patterns of degenerate states in irreps:

J	irreps
1/2	G_1
3/2	H
5/2	H,G_2
7/2	G_1,G_2,H
9/2	G_1, $H(\times 2)$

Operators

Lichtl, hep-lat/0609019

- Gauge invariant displacements: capture radial and orbital structure of baryons:

Operator type	Displacement indices
 Single-Site	$i = j = k = 0$
 Singly-Displaced	$i = j = 0, k \neq 0$
 Doubly-Displaced-I	$i = 0, j = -k, k \neq 0$
 Doubly-Displaced-L	$i = 0, \|j\| \neq \|k\|, jk \neq 0$
 Triply-Displaced-T	$i = -j, \|j\| \neq \|k\|, jk \neq 0$

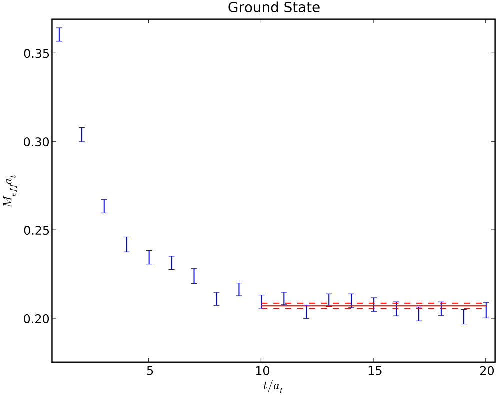
- Pruning: Choose 16 operators in each channel for use with the variational method: low noise, linear independence
- Smearing: Enhances coupling to low lying states - Gaussian quark smearing with stout smeared links.

Lattice Info

- Anisotropic lattices - finer temporal spacing for better measurement of excited states
- 860 configurations: $24^3 \times 64$ $N_f = 2$ Wilson with $m_\pi = 360$ MeV, $a_s = 0.13$ fm, $a_s/a_t = 3$,
- Ratio of spatial and temporal Wilson loops to measure gauge anisotropy, relativistic energy dispersion relation to measure fermion anisotropy
- r_0 to set scale

G_{1g} Preliminary Results

Ma_t
0.2048(25)
0.3967(94)
0.4079(94)
0.4237(69)



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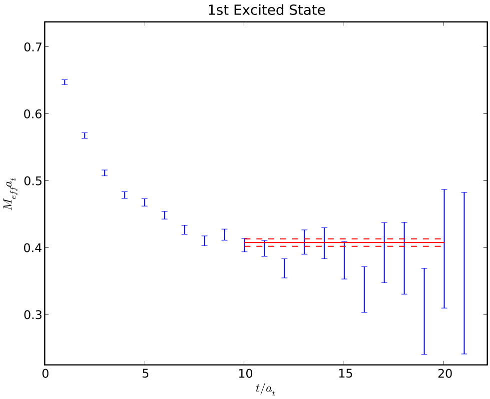

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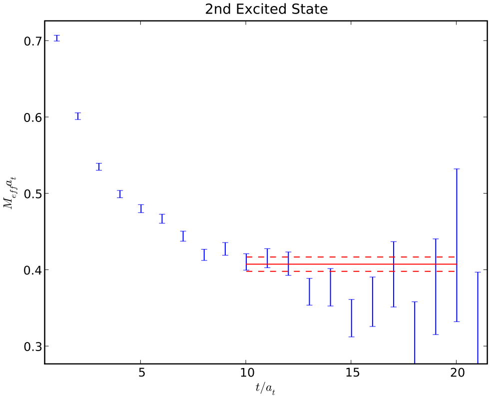

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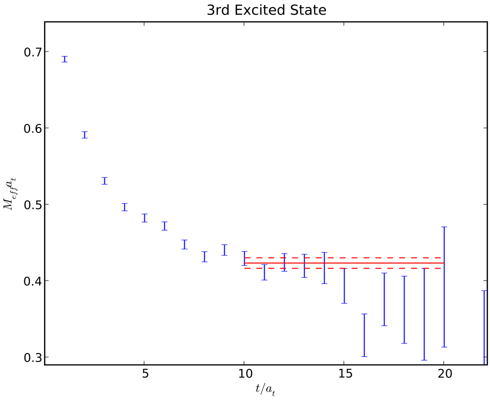
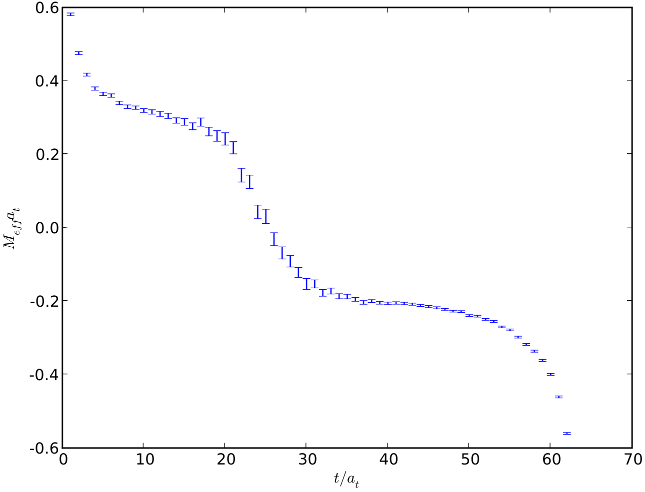


G_{1u} Channel

- Baryon creation operators of parity P create backward propagating baryons of parity $-P$
- Short temporal dimension: interference between forward and backward states
- G_{1u} channel: backward propagating G_{1g} ground state has a lower energy

G_{1u} Channel

G_{1u} ground state effective mass:



Filtering

Filter out the backwards propagating state prior to diagonalization

$$C(t) = \sum_n c_n e^{-E_n t} + b e^{-E_0'(T-t)}$$

$$E_0' \int_t^{t_1} dt' C(t') = \sum_n \frac{E_0'}{E_n} c_n \left(e^{-E_n t} - e^{-E_n t_1}\right) - b \left(e^{-E_0'(T-t)} + e^{-E_0'(T-t_1)}\right)$$

$$\begin{aligned} C_{filt}(t, t_1) &= C(t) - C(t_1) + (1 - e^{-E_0'}) \sum_{j=t+1}^{t_1} C(j) \\ &= \sum_n c_n \left[1 + \frac{1 - e^{-E_0'}}{e^{E_n} - 1}\right] \left(e^{-E_n t} - e^{-E_n t_1}\right) \\ &= \sum_n c_n' \left(e^{-E_n t} - e^{-E_n t_1}\right) \end{aligned}$$

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Filtering

- The filtered correlators look like:

$$\sum_n c'_n e^{-E_n t} + K$$

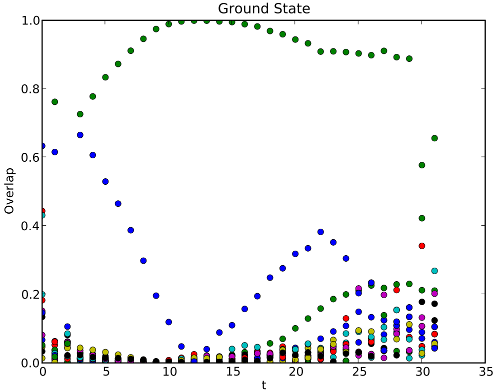
- Diagonalize
- Fit to $Ae^{-Et} + K$
- Compute the effective mass:

$$M_{eff} = \log\left(\frac{C(t) - K}{C(t+1) - K}\right)$$

Does the filter matter?

Does filtering change the diagonalization?

Look at the overlap between filtered and unfiltered eigenvectors



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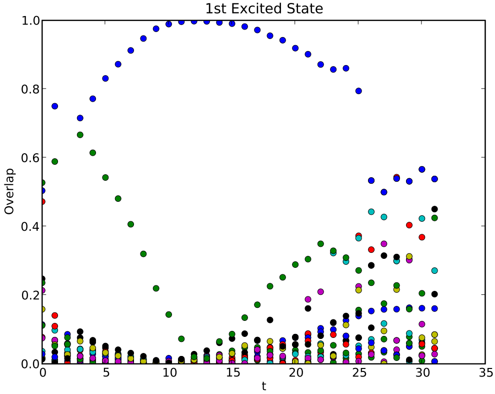

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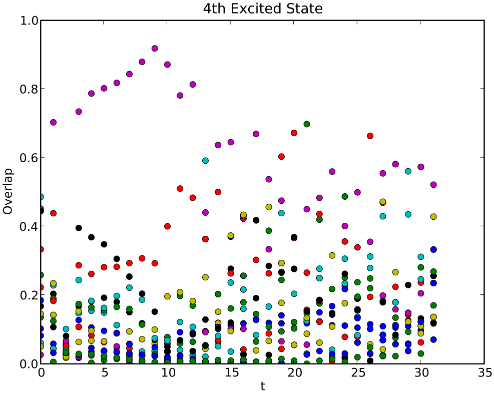

Filtering may be needed for higher excited states.

G_{1u} Preliminary Results

Ma_t
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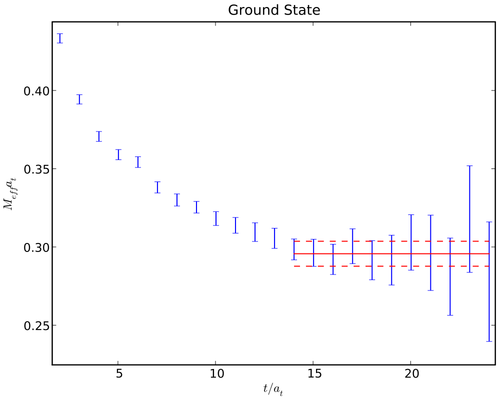
0.3232(92)
0.5024(128)
0.5314(72)
0.5316(109)



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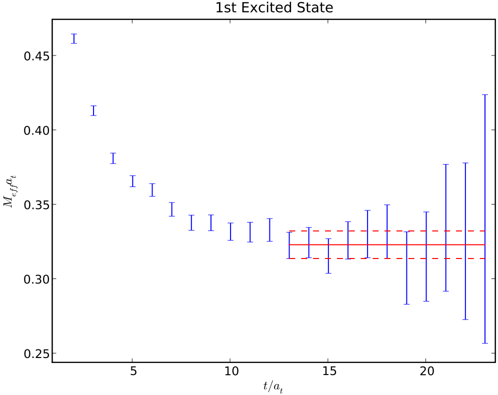

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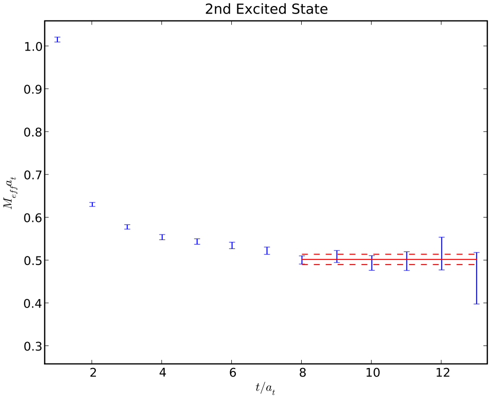

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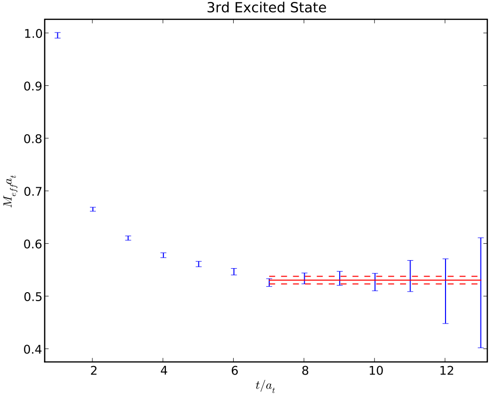

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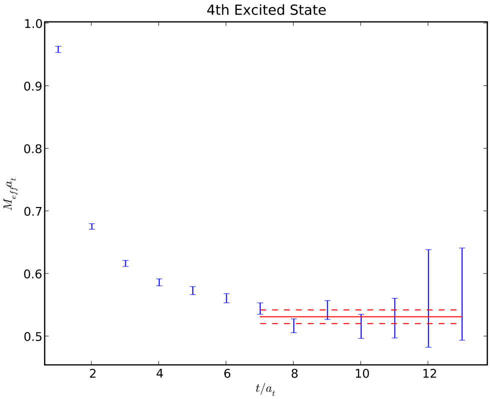
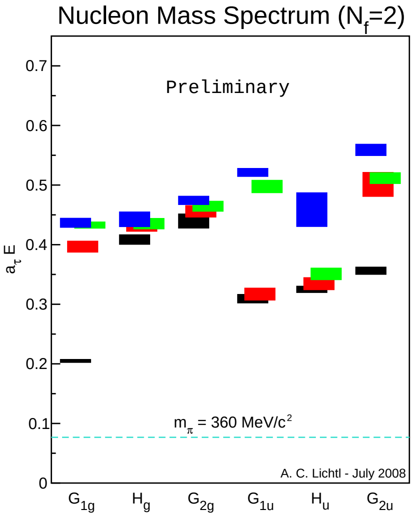


$N_f = 2$ Nucleon Spectrum



Outlook

- Analyze G_2 and H irreps with the filter
- Refine fitting and filtering - evaluate systematics
- Δ spectrum
- Other lattices - different volumes and pion masses

More filtering

Before the filter:

$$e^{-Ht} + e^{-\bar{H}(T-t)}$$

After the filter:

$$e^{-Ht} + Ce^{-Ht_1}$$

Eigenvalues:

$$e^{-Et} + Ce^{-Et_1}$$