# Physical Results from 2+1 Flavor Domain Wall QCD

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(for the RBC and UKQCD Collaborations)

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- Chiral Perturbation Theory
  - effective theory
  - expansion in light quark masses and momenta
  - application: light meson masses and decay constants, neutral kaon mixing (heavy-light meson, baryon $\chi$PT)

- Which quarks are considered as light? Convergence radius? NLO, NNLO, . . .
  - only up, down ( $m_{u,d} \approx$ 2 – 7 MeV): **SU(2)×SU(2)**
  - up, down, strange ( $m_s \approx$ 70 – 120 MeV): **SU(3)×SU(3)**

- lattice QCD & $\chi$PT
  - $\chi$PT tool for lattice simulations

    extrapolation to physical quark masses, partially quenching
  - lattice QCD can probe $\chi$PT (now)

    quark masses can be varied in lattice simulations
  - determine light quark masses, $f_\pi$, $f_K$, $B_K$, low-energy constants

    $B_K$: talks by Chris Kelly, Jan Wennekers

# $N_f = 2 + 1$ Domain Wall Fermion ensembles

- $N_f = 2 + 1$ ensembles with Iwasaki gauge action
  - $L^3 \times T \times L_s = 24^3 \times 64 \times 16$
  - preliminary: $32^3 \times 64 \times 16$, in preparation: $48^3 \times 64 \times 16$
- Rational Hybrid Monte Carlo algorithm [Clark, Kennedy]
- $24^3 \times 64 \times 16$ - ensemble
  - $\beta = 2.13$ (Iwasaki gauge action)
  - $a^{-1} = 1.72(3)$ GeV (from $\Omega^-$), $aL \approx 2.74$ fm
  - dynamical light quark mass: $m_l \in \{0.005, 0.01, 0.02, 0.03\}$
  - dynamical heavy quark mass: $m_h = 0.04$
  - lightest dynamical $m_\pi \approx 330$ MeV, $\widetilde{m}_l : \widetilde{m}_h = 1/5$
  - $m_{\text{res}} = 0.00315(02)$: $\widetilde{m}_X = m_X + m_{\text{res}}$
- valence quark mass (partial quenching): $m_{\text{val}} \in \{0.001, 0.005, 0.01, 0.02, 0.03, 0.04\}$

$$\Rightarrow \text{lightest valence: } m_\pi \approx 240\,\text{MeV}, \widetilde{m}_{\text{val}} \approx 1/10\, \widetilde{m}_h$$

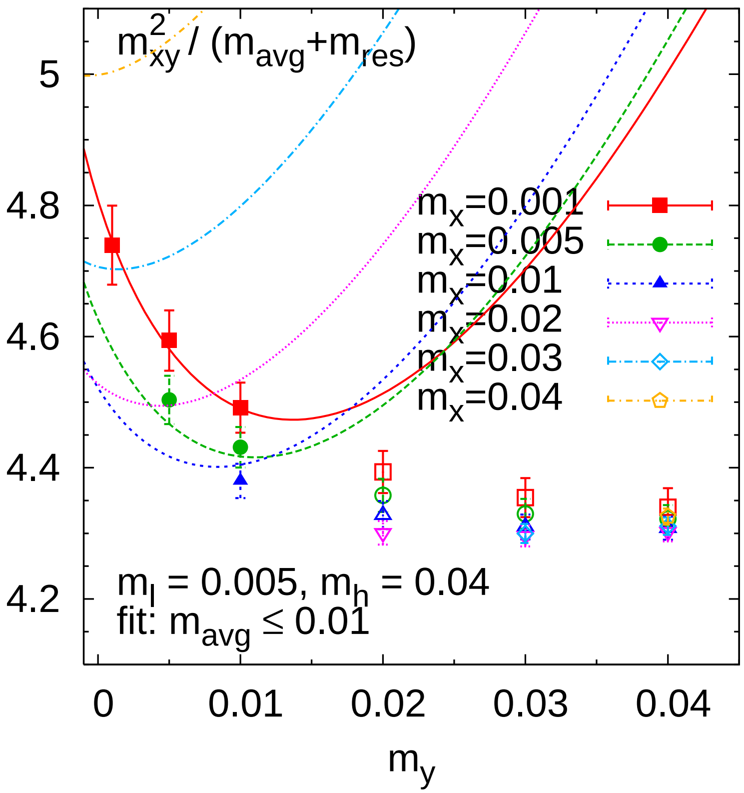
# SU(2) PQ$\chi$PT

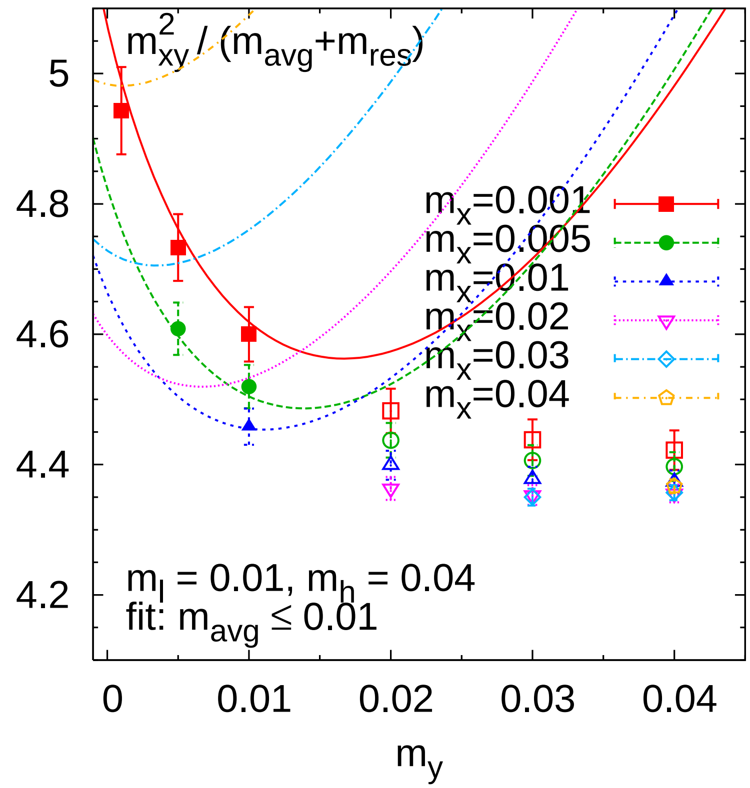
$$
\begin{aligned}
m_{xy}^2 &= \frac{\chi_x + \chi_y}{2}\Bigg\{1 + \frac{32}{f^2}(2L_6^{(2)} - L_4^{(2)})\chi_l + \frac{8}{f^2}(2L_8^{(2)} - L_5^{(2)})(\chi_x + \chi_y) \\
&\quad + \Big[\ldots \times \log(\chi_x), \log(\chi_y)\Big]\Bigg\} \\
f_{xy} &= f\left\{1 + \frac{16}{f^2}L_4^{(2)}\chi_l + \frac{4}{f^2}L_5^{(2)}(\chi_x + \chi_y)\right. \\
&\quad \left. + \Big[\ldots \times \log(\chi_x + \chi_l), \log(\chi_y + \chi_l), \log(\chi_x), \log(\chi_y)\Big]\right\} \\
\chi_X &= 2B\Big(m_X + m_{\mathrm{res}}\Big)
\end{aligned}
$$

$f$, $B$, $L_i^{(2)}$ depend on (background) $m_h$

# SU(2) PQ$\chi$PT I: meson masses

arXiv:0804.0473[hep-lat]

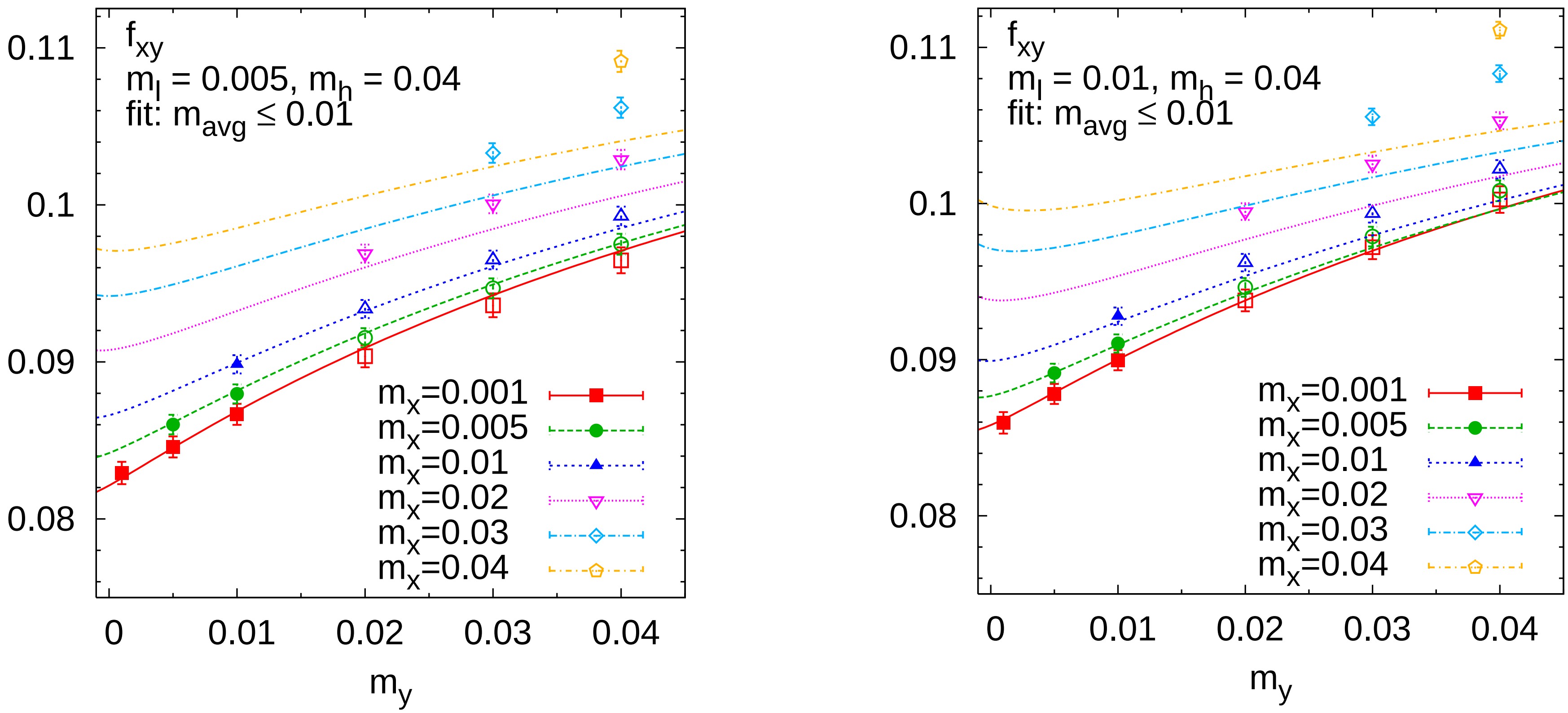



from $m_\pi = 135\,\text{MeV} \Rightarrow m_{\text{ud}} = 3.72(16)\text{MeV}$ (NPR)

fit range $m_{\text{avg}} \leq 0.01$, combined fits (only *filled symbols* inside fit range)

# SU(2) PQ$\chi$PT II: decay constants

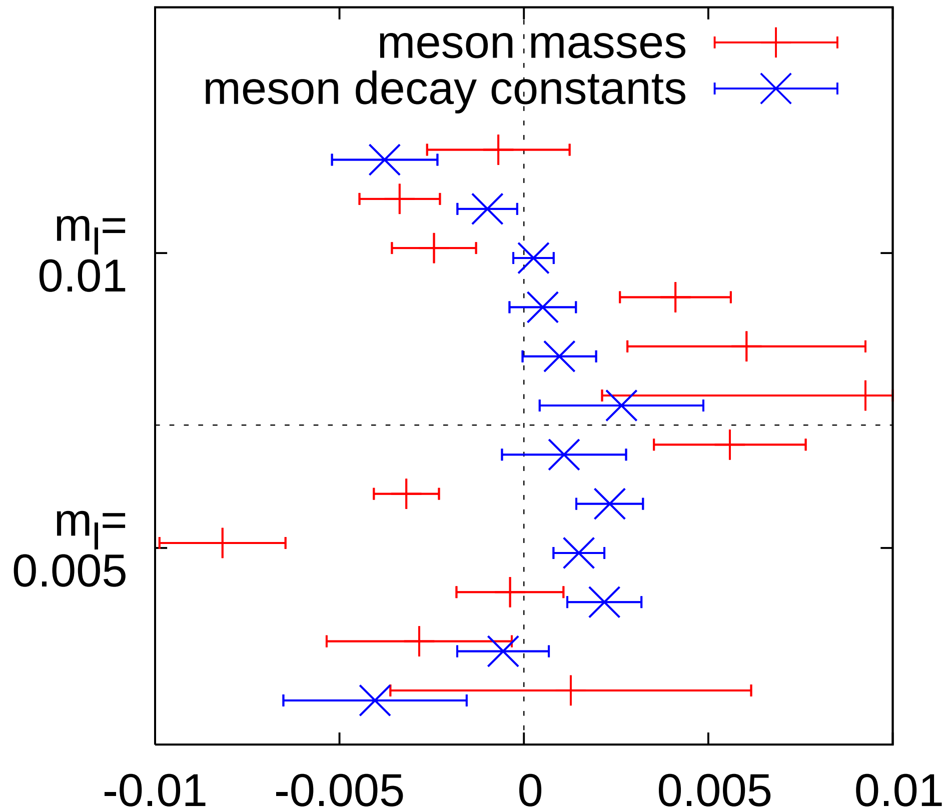



at $m_{ud}$: $f_\pi = 124.1(3.6)$ MeV (130.7)

fit range $m_{\text{avg}} \leq 0.01$, combined fits (only *filled symbols* inside fit range)

- uncorrelated $\chi^2/\text{d.o.f.} = 0.3$
- 2x45(blocked) configurations
- 2x12 data-points
- not fitting to an *exact* theory (NLO, NNLO, . . . )
- maximal deviation $\leq 1\%$
- acceptable for fit range $m_{\text{PS}} \leq 420\,\text{MeV}$

rel. error of fit [ (fit - value)/value ]

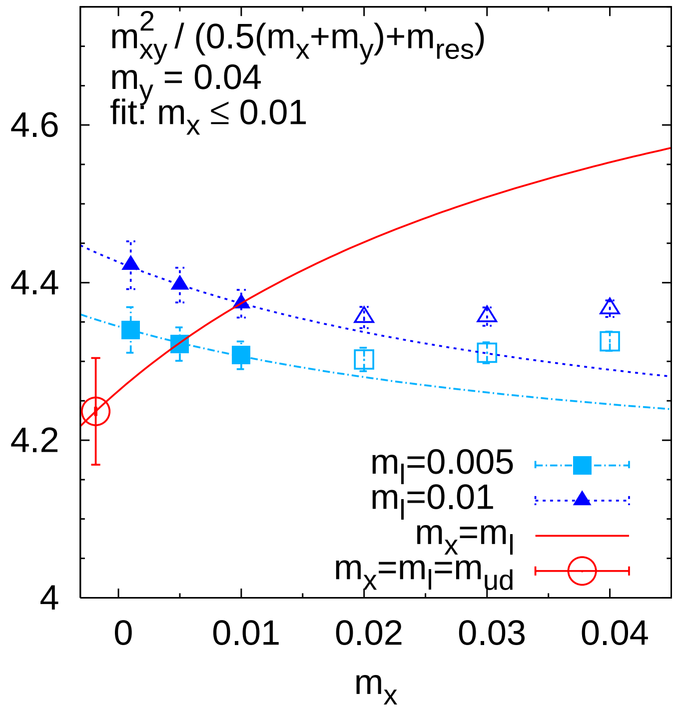


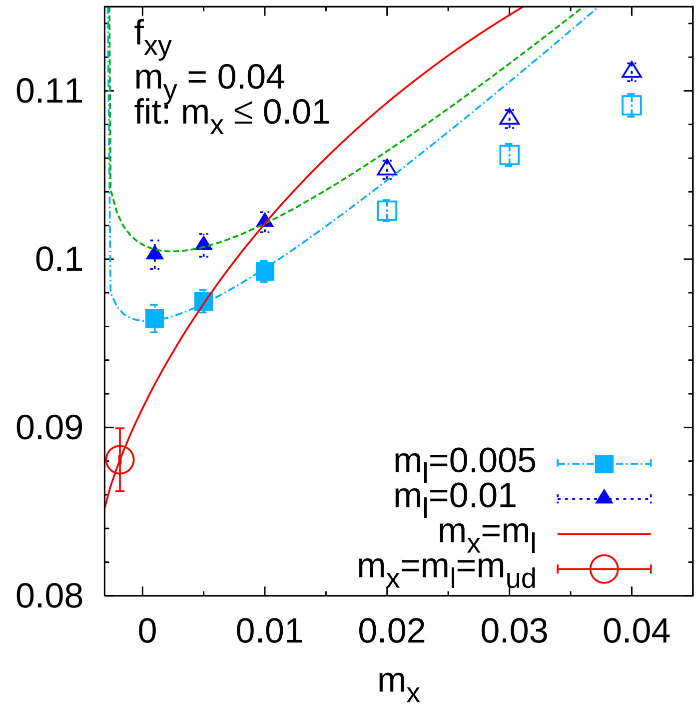

more on correlated vs. uncorrelated fits:
talk by Robert Mawhinney

$$m_{xh}^2 = B^{(K)}(m_h)\widetilde{m}_h\left\{ 1 + \frac{\lambda_1(m_h)}{f^2}\chi_l + \frac{\lambda_2(m_h)}{f^2}\chi_x \right\}$$

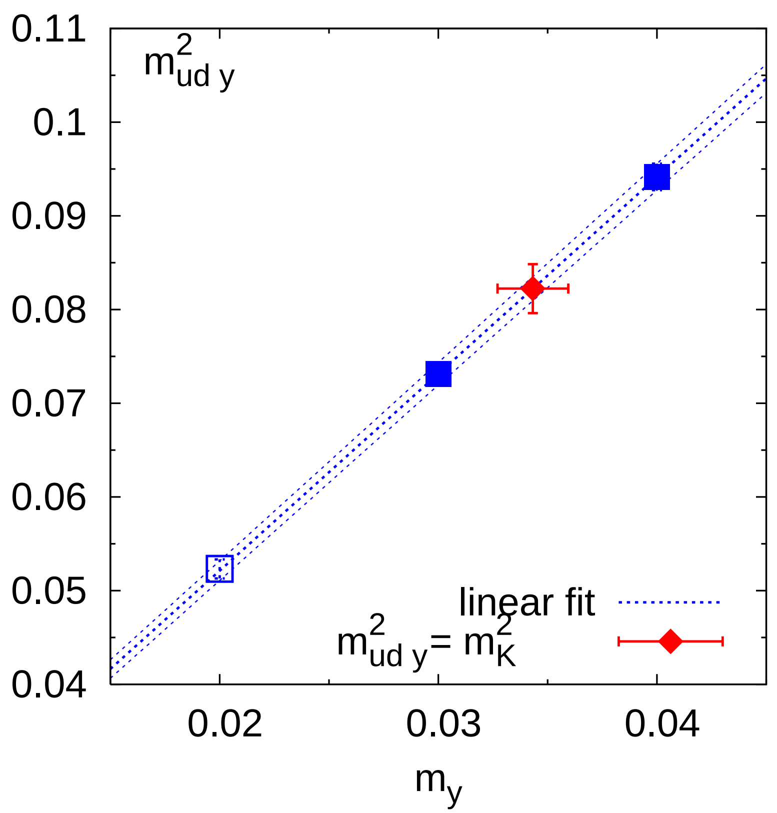
$$f_{xh} = f^{(K)}(m_h)\left\{1+\frac{\lambda_3(m_h)}{f^2}\chi_l+\frac{\lambda_4(m_h)}{f^2}\chi_x-\frac{1}{(4\pi f)^2}\left[\frac{\chi_x+\chi_l}{2}\log\frac{\chi_x+\chi_l}{2\Lambda^2}+\frac{\chi_l-2\chi_x}{4}\log\frac{\chi_x}{\Lambda^2}\right]\right\}$$

- $f$, $B$ from SU(2) $\times$ SU(2) fit, fixed, $m_{l,x} \leq 0.01$
- extrapolate to $m_{\rm ud}$

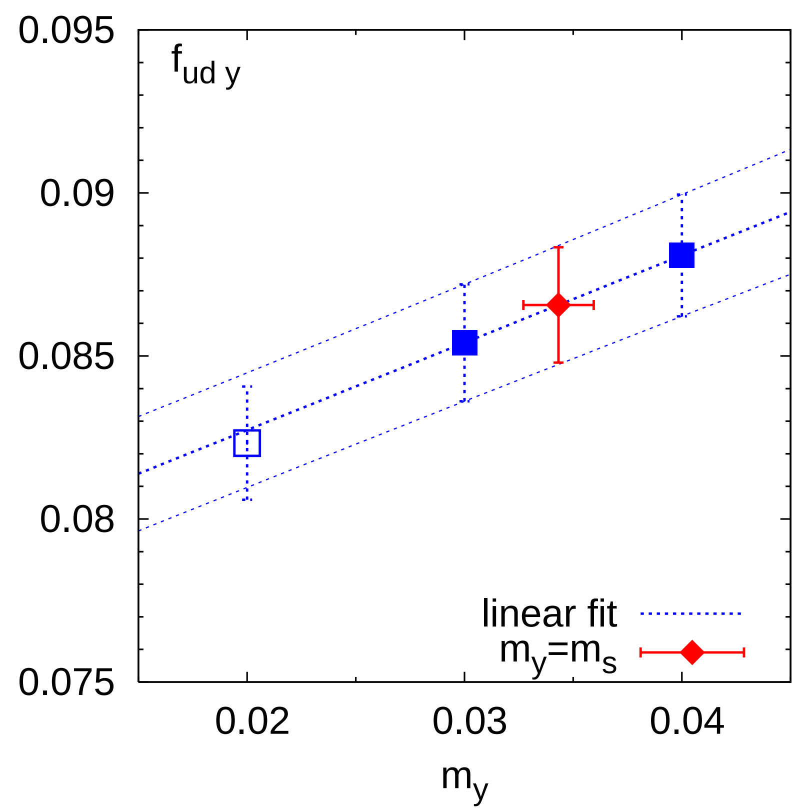


## $m_s$ from physical $m_K^2$

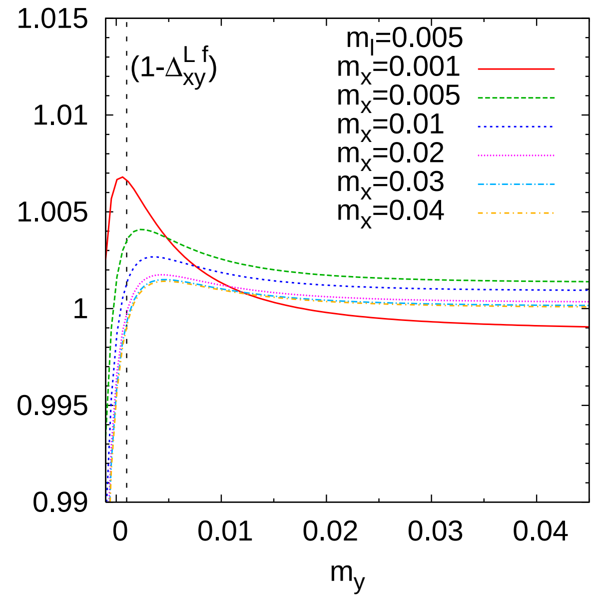



## interpolate $f_K$ to $m_s$

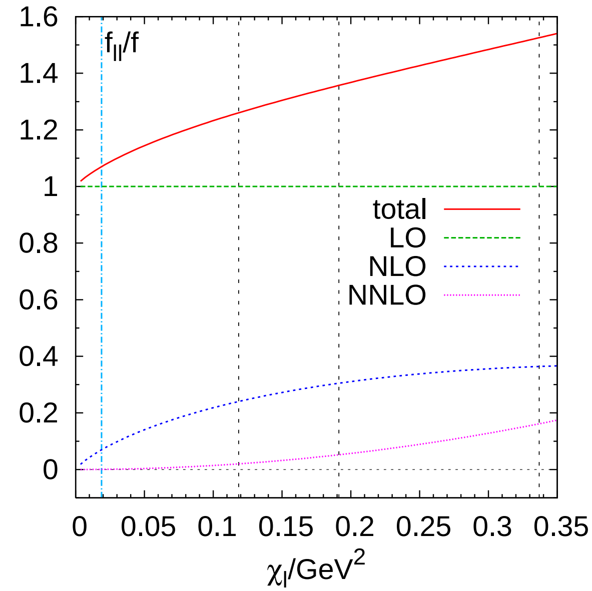



- $m_s = 107.3(4.4)$ MeV (NPR)
- $m_l : m_{\text{strange}} = 1 : 28.8(4)$
- $f_K = 149.6(3.6)$
  (159.8 MeV)
- $f_K/f_\pi = 1.205(18)$
  (1.223)

## Systematic Errors



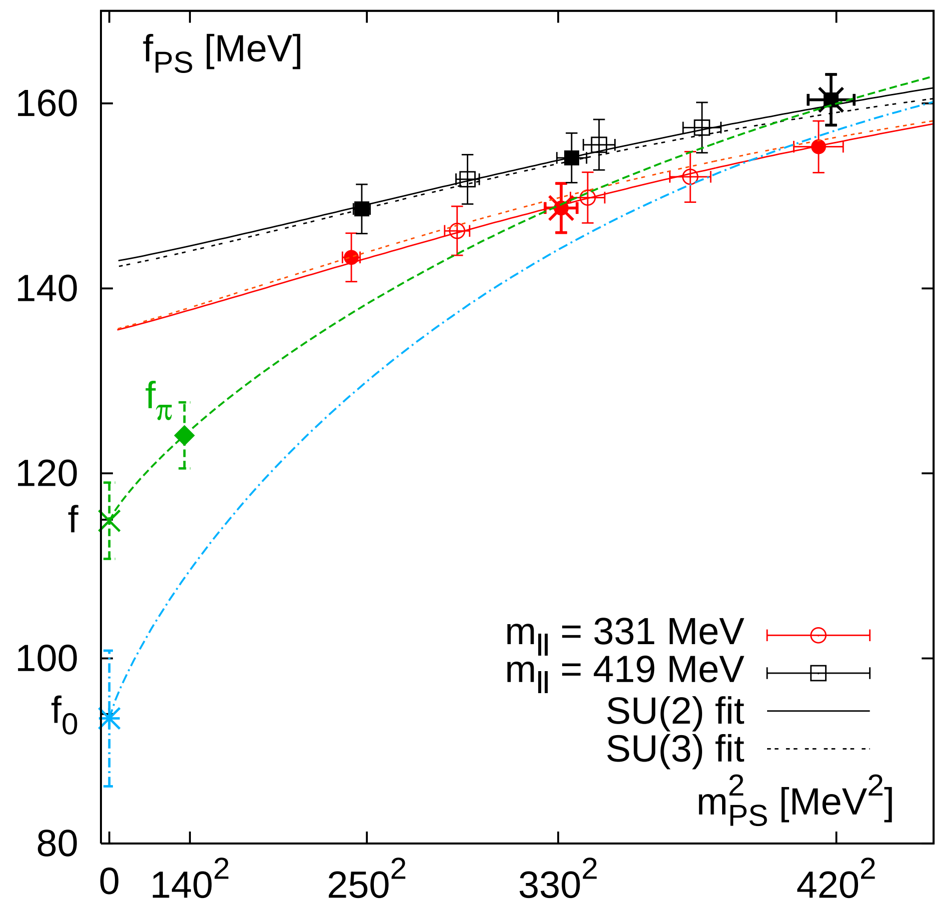

- finite volume effects
  - $*$ $(aL)^3 \approx (2.74\text{fm})^3$
  - $*$ FV-$\chi$PT: small effect
- continuum extrapolation
  - $*$ flat estimate $(a\Lambda_{\text{QCD}})^2 \approx 4\%$
  - $*$ data at 2nd $a$ preliminary, improve/justify error
- $\boldsymbol{m_s \neq m_h}$
  - $*$ 15% too large $m_h$
  - $*$ estimated effect by SU(3) $\longrightarrow$ SU(2) conversion
- $\chi$PT extrapolation
  - $*$ fit range $m_l \leq 0.02$
  - $*$ added analytic NNLO terms



$$
\begin{aligned}
f &= 114.8(4.1)_{\mathsf{stat}}(8.1)_{\mathsf{syst}}\ \mathsf{MeV}\,,\\
B^{\overline{\mathsf{MS}}}(2\ \mathsf{GeV}) &= 2.52(0.11)_{\mathsf{stat}}(0.23)_{\mathsf{ren}}(0.12)_{\mathsf{syst}}\ \mathsf{GeV}\,,\\
\Sigma^{\overline{\mathsf{MS}}}(2\ \mathsf{GeV}) &= \Big(255(8)_{\mathsf{stat}}(8)_{\mathsf{ren}}(13)_{\mathsf{syst}}\ \mathsf{MeV}\Big)^3\,,\\
\bar{l}_3 &= 3.13(0.33)_{\mathsf{stat}}(0.24)_{\mathsf{syst}}\,,\\
\bar{l}_4 &= 4.43(0.14)_{\mathsf{stat}}(0.77)_{\mathsf{syst}}\,,\\
f_\pi &= 124.1(3.6)_{\mathsf{stat}}(6.9)_{\mathsf{syst}}\ \mathsf{MeV}\,,\\
f_K &= 149.6(3.6)_{\mathsf{stat}}(6.3)_{\mathsf{syst}}\ \mathsf{MeV}\,,\\
f_K/f_\pi &= 1.205(0.018)_{\mathsf{stat}}(0.062)_{\mathsf{syst}}\,,\\
m_{ud} &= 3.72(.16)_{\mathsf{stat}}(.33)_{\mathsf{ren}}(.17)_{\mathsf{syst}}\,,\quad \text{(NPR)}\\
m_s &= 107.3(4.4)_{\mathsf{stat}}(9.7)_{\mathsf{ren}}(4.9)_{\mathsf{syst}}\,,\quad \text{(talk by Yasumichi Aoki)}\\
m_{ud}:m_s &= 1:28.8(0.4)_{\mathsf{stat}}(1.6)_{\mathsf{syst}}
\end{aligned}
$$

# SU(3) vs SU(2)

- SU(2) chiral limit better controlled
- SU(2) plus kaon: no need for SU(3)
- NLO SU(3) not sufficient for $\boldsymbol{m_s}$ . . .
- . . . but only one value of $m_s$ so far



## PRELIMINARY results from $32^3 \times 64 \times 16$ lattices PRELIMINARY

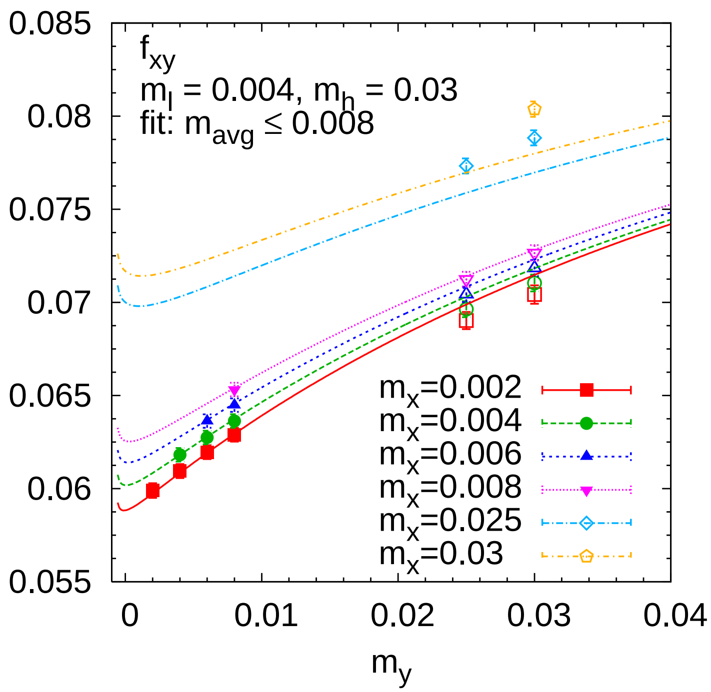
- Iwasaki-gauge action $\beta = 2.25$
- 3 dynamical light quark masses $m_l = 0.004, 0.006, 0.008,$ $m_s = 0.03$
- (dynamical) pion masses: $\approx 310,\ 365$ , $420$ MeV
- lightest PQ pion mass $\approx 235$ MeV ($m_x \in \{0.002, 0.004, 0.006, 0.008, 0.025, 0.03\}$)
- $m_{\text{res}} = 6.65(13) \cdot 10^{-4}$
- $a^{-1} = 2.42(4)$ GeV (from $r_0/a$, input: $r_0 = 0.47$ fm)
- $a \approx 0.08$ fm, $aL \approx 2.6$ fm
- FV-effects? $m_{ll}L \approx 4.1$–5.5, (smallest $m_{xx}L \approx 3.1$)

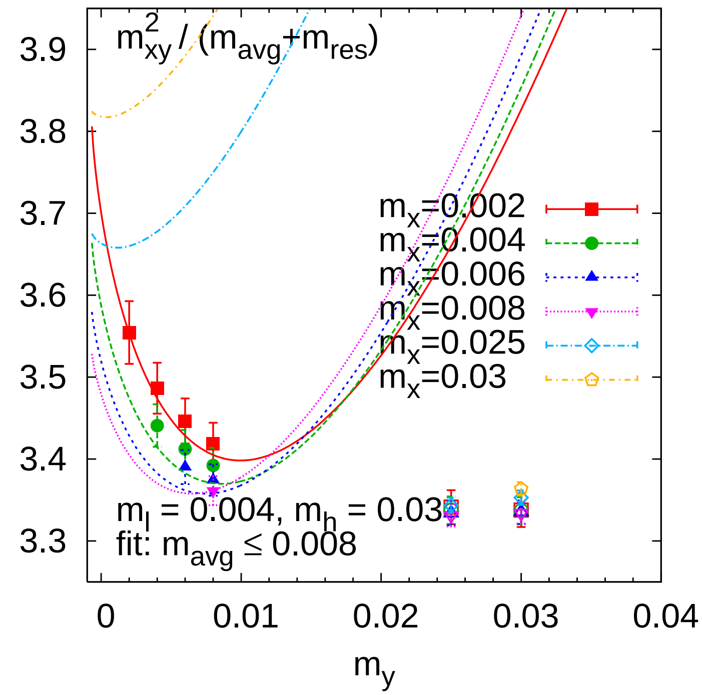
**. . . PRELIMINARY . . . NOT YET FULL STATISTICS . . . PRELIMINARY . . .**

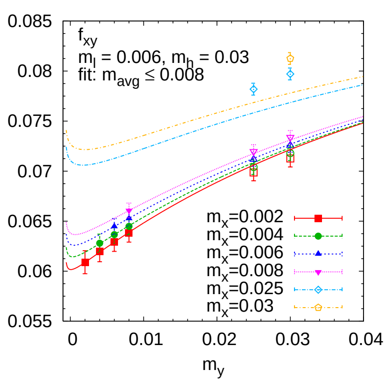
**PRELIMINARY** 

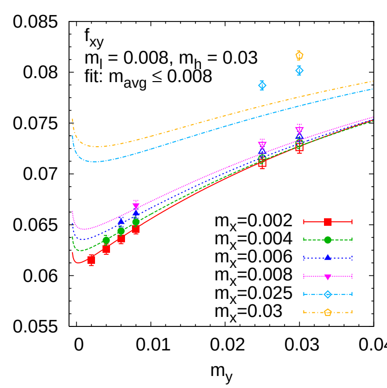
# SU(2)-PQChPT fits

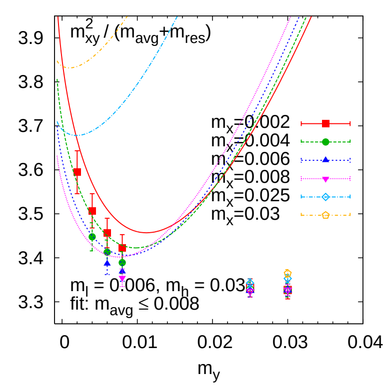
**PRELIMINARY**

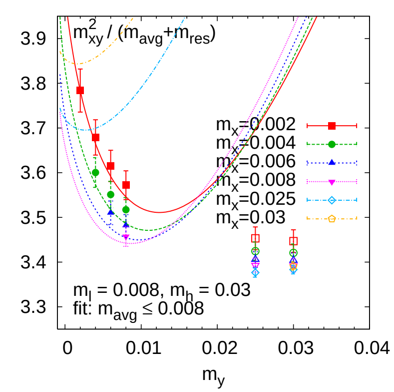


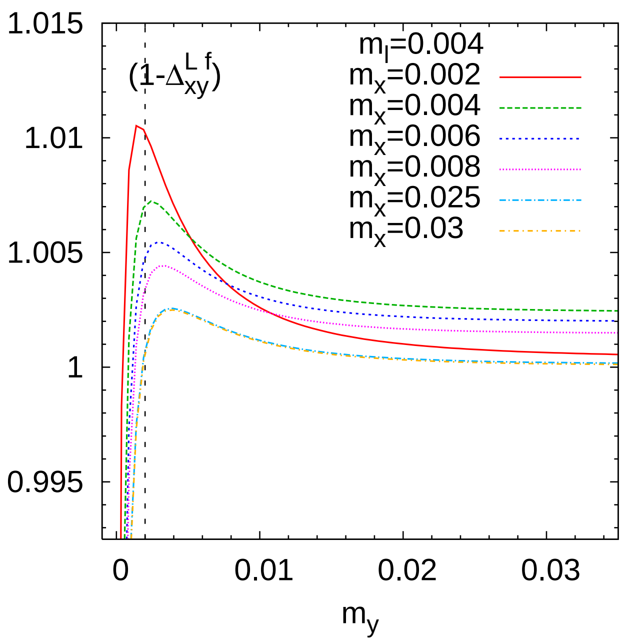



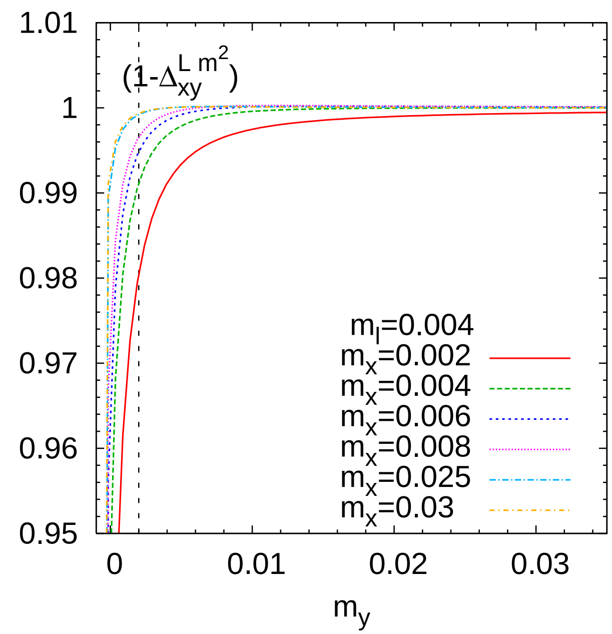




**. . . PRELIMINARY . . . NOT YET FULL STATISTICS . . . PRELIMINARY . . .**

PRELIMINARY **Finite volume correction $(2.6\,\mathrm{fm})^3$** PRELIMINARY




- correcting with SU(2) PQChPT for finite volume: [Gasser, Leutwyler]

for $m_{ll} \approx 310$ MeV: $R_m = 0.16(.01)\%, \quad -R_f = 0.65(.04)\%$

- compare with Colangelo, Dürr, Haefeli (resummed Lüscher formula):

$$R_m = 0.26(.07)\%, \quad -R_f = 0.64(.01)\%$$

**. . . PRELIMINARY . . . NOT YET FULL STATISTICS . . . PRELIMINARY . . .**

- SU(2)×SU(2) for kaon-sector more reliable
- estimates for LECs, light quark masses
- Outlook:
  - ∗ extending statistics on $24^3$ ensembles (330, 420 MeV)
  - ∗ enough data to compare with NNLO SU(3)×SU(3)
  - ∗ 2nd dynamical strange quark mass needed
  - ∗ continuum extrapolation with $a \approx 0.08$fm ongoing

computations were done on:
QCDOC Edinburgh, RBRC, USDOE
and BG/P at Argonne (ESP, INCITE)

# BACKUP

$$\mathcal{L}_{\pi\pi} = \frac{f^2}{8}\mathrm{Tr}\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger + \frac{f^2 B}{4}\mathrm{Tr}\left(M^\dagger\Sigma + M\Sigma^\dagger\right)$$

$$\mathcal{L}_{\pi K} = D_\mu K^\dagger D^\mu K - M_K^2 K^\dagger K$$

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \qquad \Sigma = \xi^2 = \exp\frac{\mathrm{i}}{f}\begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}$$

$$\Sigma \to L\Sigma R^\dagger, \qquad \xi \to L\xi U^\dagger = U\xi R^\dagger,$$

$$K \to UK, \qquad D_\mu K \to U D_\mu K$$

$$D_\mu K = \partial_\mu K + V_\mu K, \qquad V_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)$$

$$m_{xh}^2 = B^{(K)}(m_h)\widetilde{m}_h\left\{1 + \frac{\lambda_1(m_h)}{f^2}\chi_l + \frac{\lambda_2(m_h)}{f^2}\chi_x\right\}$$

$$f_{xh} = f^{(K)}(m_h)\left\{1 + \frac{\lambda_3(m_h)}{f^2}\chi_l + \frac{\lambda_4(m_h)}{f^2}\chi_x - \frac{1}{(4\pi f)^2}\left[\frac{\chi_x+\chi_l}{2}\log\frac{\chi_x+\chi_l}{2\Lambda_\chi^2} + \frac{\chi_l - 2\chi_x}{4}\log\frac{\chi_x}{\Lambda_\chi^2}\right]\right\}$$

# Systematic Errors

- finite volume $(aL)^3 \approx (2.74\,\mathsf{fm})^3$
- (missing) continuum extrapolation $a \approx 0.11\,\mathsf{fm}$
- $\chi$PT - extrapolation
- dynamical heavy quark $\boldsymbol{m_h} \neq \boldsymbol{m_s}$ physical strange quark mass

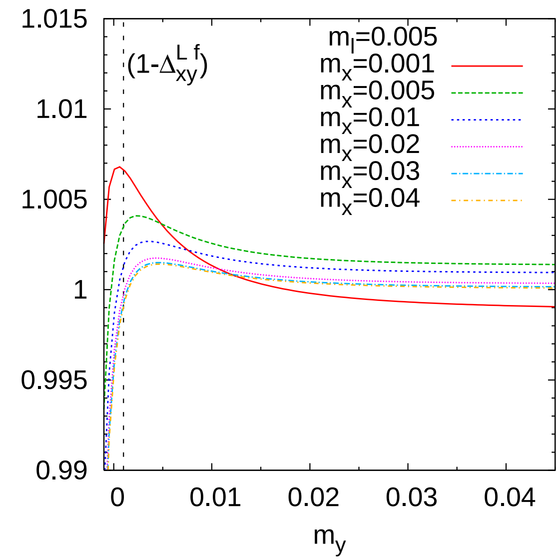
# Systematic Errors I: Finite Volume

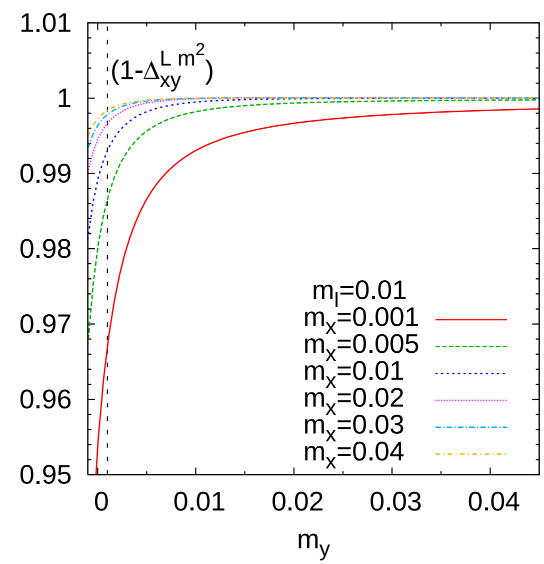
- using FV-corrected PQ$\chi$PT formulae

  Gasser, Leutwyler

- assign difference in measured quantity as syst. error (typical size $\approx 1-2\%$, larger effect (6%) on $\bar{l}_3$)
- compare at unitary points with "resummed Lüscher-formula" of Colangelo, Dürr, Haefeli

| $m_\pi$ [MeV] | $R_m$ [%] PQ$\chi$PT | $R_m$ [%] CDH | $-R_f$ [%] PQ$\chi$PT | $-R_f$ [%] CDH |
|---|---|---|---|---|
| 331 | 0.09(.01) | 0.13(.04) | 0.36(.03) | 0.32(.00) |
| 419 | 0.03(.00) | 0.04(.01) | 0.10(.01) | 0.09(.00) |



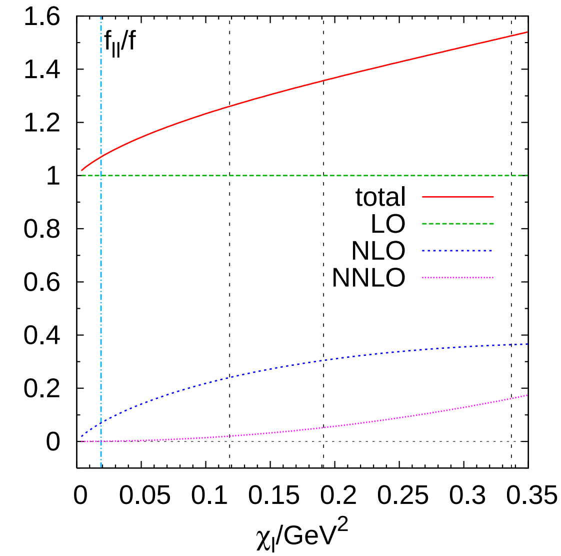



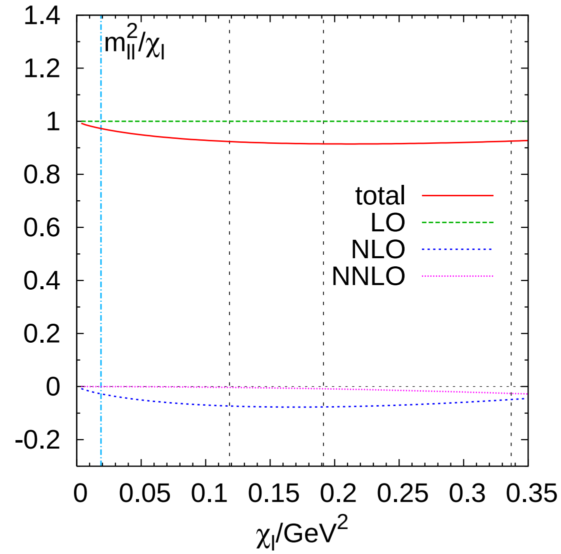
## Systematic Errors II: $\chi$PT extrapolation

- influence of NNLO?
- 4 possible NNLO terms, incl. first 3

$$(\chi_x + \chi_y)^2, \ (\chi_x - \chi_y)^2, \ (\chi_x + \chi_y)\chi_l, \ \chi_l^2$$

- extended fit range to $m_{\text{avg}} \leq 0.02$
- good convergence behaviour
- assigned (doubled) difference as syst. error
  ( typical size $\approx 2 - 4$ % (but large (17%) effect on $\bar{l}_4$)



# Systematic Errors III:

## Continuum Extrapolation

- currently only one lattice spacing available
- assign flat $(a\Lambda_{\mathrm{QCD}})^2 \approx$ 4% error
- better (smaller?) estimate from $32^3$-data, $a \approx 0.08\,\mathrm{fm}$

$$\boldsymbol{m_s \neq m_h}$$

- 15% too large $m_h$
- estimated effect by conversion SU(3) $\longrightarrow$ SU(2)
  (typical size $< 1\%$, except 2% for $m_s$)

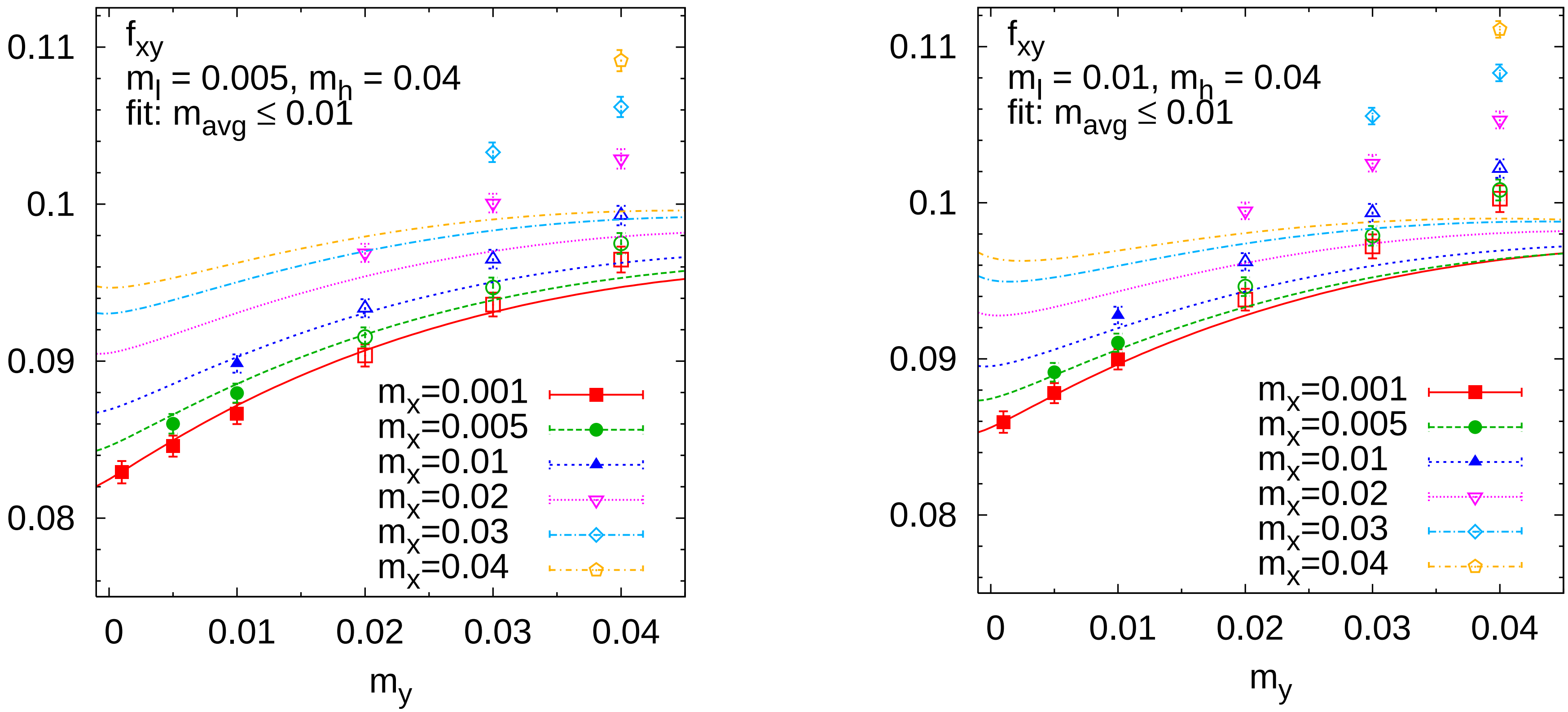
# SU(3) $\chi$PT for meson masses and decay constants

- measure $m_{\rm PS}$, $f_{\rm PS}$ at different sea $(m_l, m_h)$ and valence $(m_x, m_y)$ quark masses
- SU(3) × SU(3) PQ$\chi$PT (NLO):

$$
\begin{aligned}
m_{xy}^2 &= \frac{\chi_x+\chi_y}{2}\Bigg\{1+\frac{48}{f_0^2}(2L_6^{(3)}-L_4^{(3)})\bar\chi+\frac{8}{f_0^2}(2L_8^{(3)}-L_5^{(3)})(\chi_x+\chi_y) \\
&\quad +\Big[\ldots\times\log(\chi_x),\log(\chi_y),\log(\chi_\eta)\Big]\Bigg\} \\
f_{xy} &= f_0\Bigg\{1+\frac{24}{f_0^2}L_4^{(3)}\bar\chi+\frac{4}{f_0^2}L_5^{(3)}(\chi_x+\chi_y)\;+\;\Big[\ldots\times\log(\chi_x+\chi_l),\log(\chi_x+\chi_h), \\
&\quad \log(\chi_y+\chi_l),\log(\chi_y+\chi_h),\log(\chi_x),\log(\chi_y),\log(\chi_\eta)\Big]\Bigg\}
\end{aligned}
$$

- $\chi_X = 2B_0(m_X+m_{\rm res})$, $\bar\chi = 2B_0\left(\frac{2m_l+m_h}{3}+m_{\rm res}\right)$

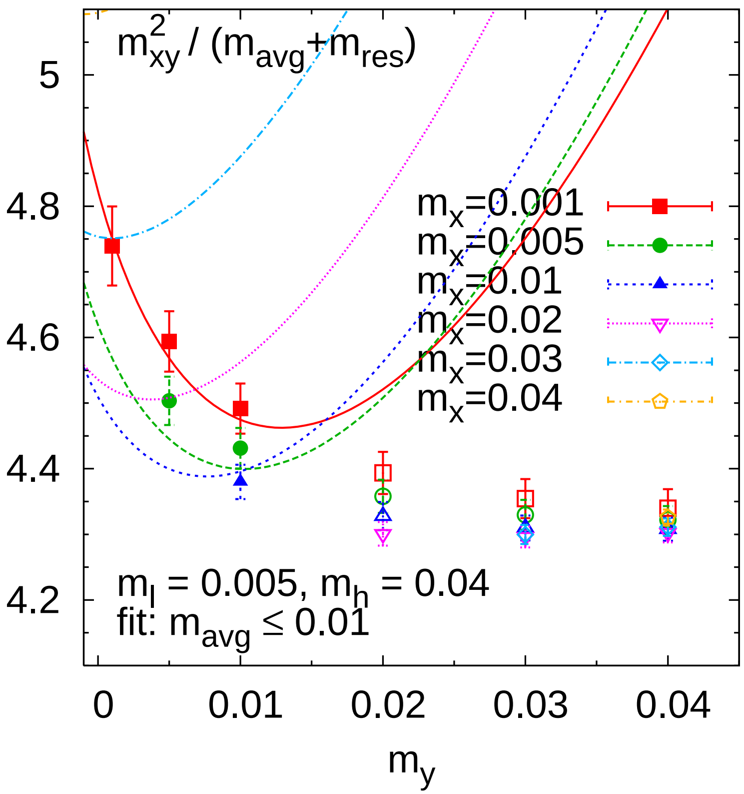
# SU(3) PQ$\chi$PT I: decay constants $f_{xy}$

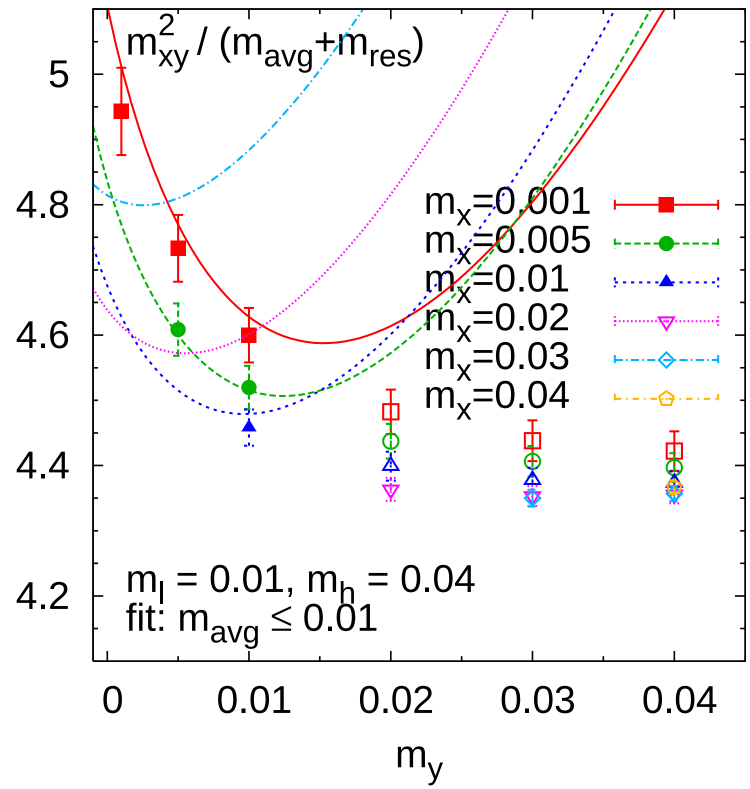



fit range $m_{\text{avg}} \leq 0.01$ corresponds to (deg.) PS-mass of 420 MeV
(only *filled symbols* inside fit range)

combined fits: 2 light sea quark masses, $f_{xy}$ **and** $m_{xy}^2$

# SU(3) PQ$\chi$PT II: meson masses $m_{xy}^2$




$$\frac{m_{xy}^2}{(\widetilde{m}_x + \widetilde{m_y})/2}$$

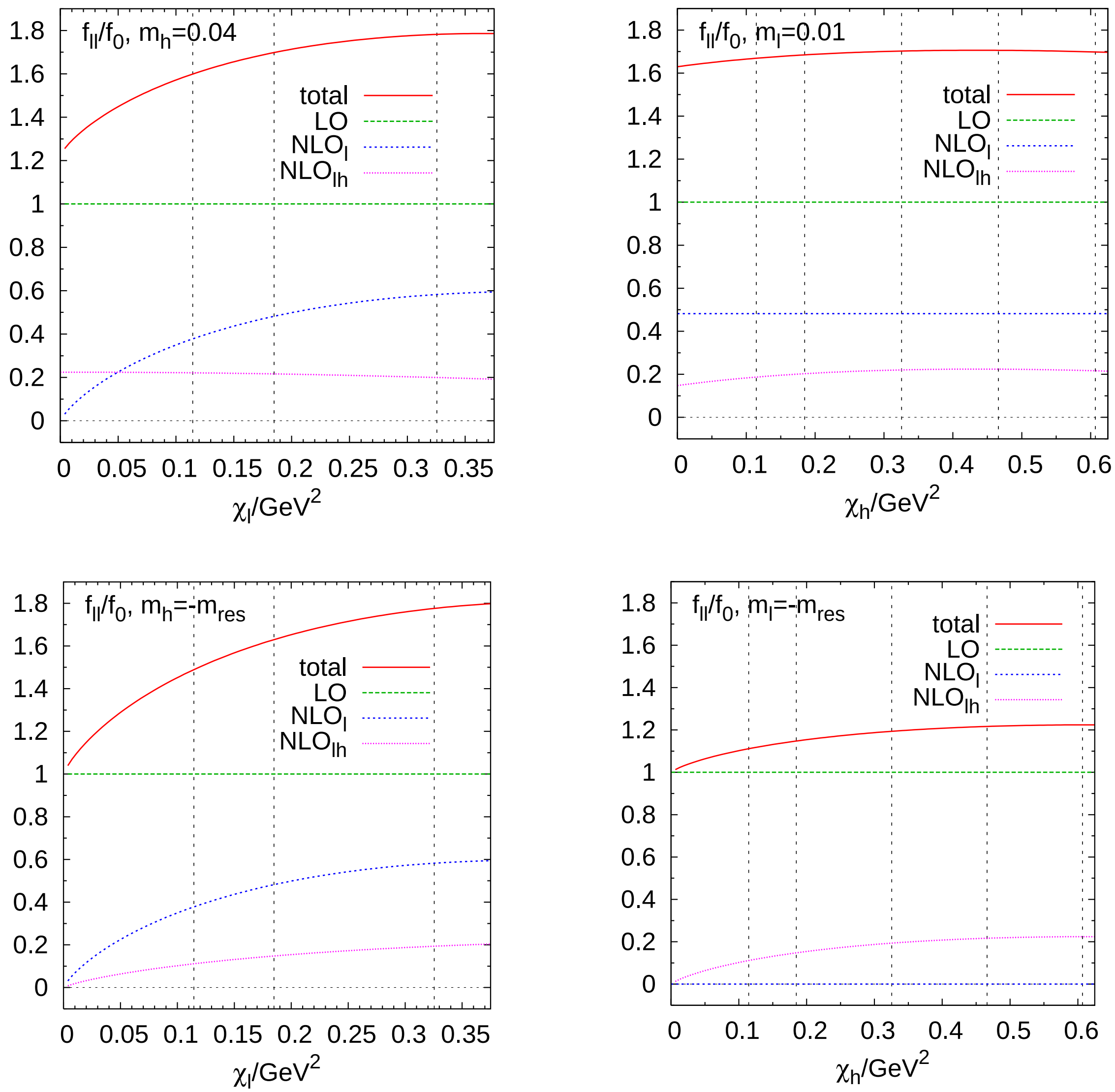
# SU(3) low-energy constants

| | $L_4^{(3)}$ | $L_5^{(3)}$ | $L_6^{(3)}$ | $L_8^{(3)}$ | $(2L_8^{(3)} - L_5^{(3)})$ | $(2L_6^{(3)} - L_4^{(3)})$ |
|---|---|---|---|---|---|---|
| | 1.4(0.8)(–) | 8.7(1.0)(–) | 0.7(0.6)(–) | 5.6(0.4)(–) | 2.4(0.4)(–) | 0.0(0.4)(–) |
| Bijnens (pheno) | | | | | | |
| NLO | $\equiv 0$ | 14.6 | $\equiv 0$ | 10.0 | 5.4 | $\equiv 0$ |
| NNLO | $\equiv 0$ | 9.7(1.1) | $\equiv 0$ | 6.0(1.8) | 2.3 | $\equiv 0$ |
| MILC, 2+1 stagg. fermions, (LO, NLO, anal. NNLO + NNNLO) | | | | | | |
| 2004 | -0.7(3.0)(3.0) | 10.9(3.0)(3.0) | – | – | 1.6(1.0)(2.0) | 4.4(2.0)(4.0) |
| 2007 | 1.3(3.0)$(^{+3.0}_{-1.0})$ | 13.9(2.0)$(^{+2.0}_{-1.0})$ | 2.4(2.0)$(^{+2.0}_{-1.0})$ | 7.8(1.0)(1.0) | 2.6(1.0)(1.0) | 3.4(1.0)$(^{+2.0}_{-3.0})$ |

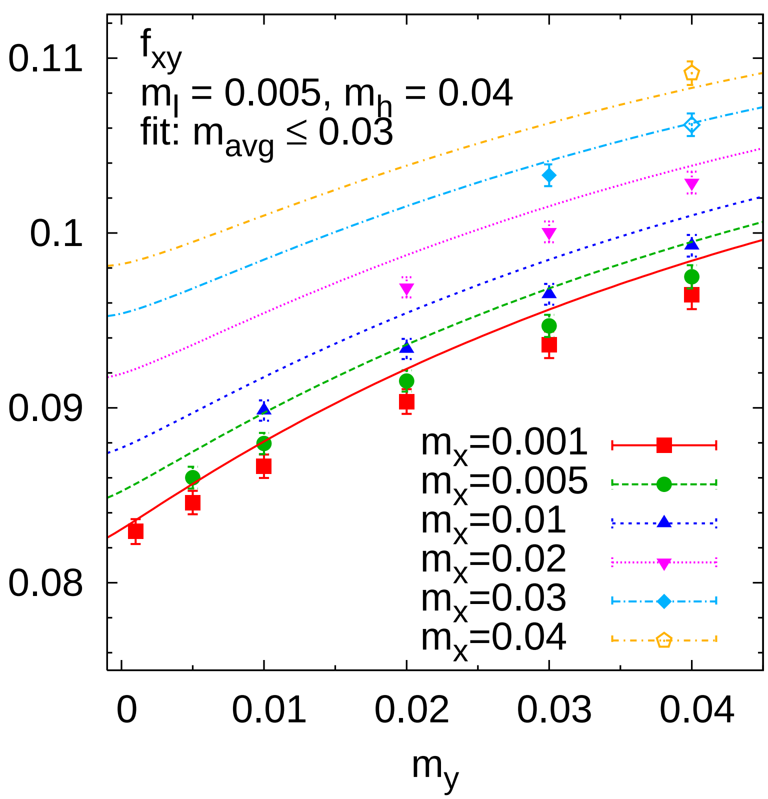
LEC's in $10^{-4}$ at $\Lambda_\chi = 770$ MeV

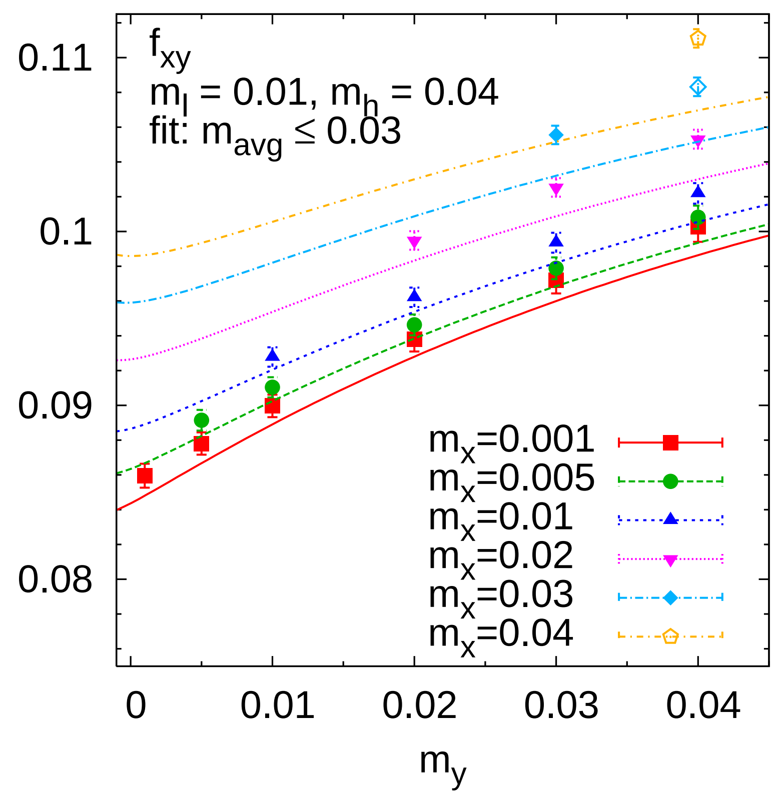
fit range sufficient to extract LEC's, but did not include the strange quark mass

# Convergence?

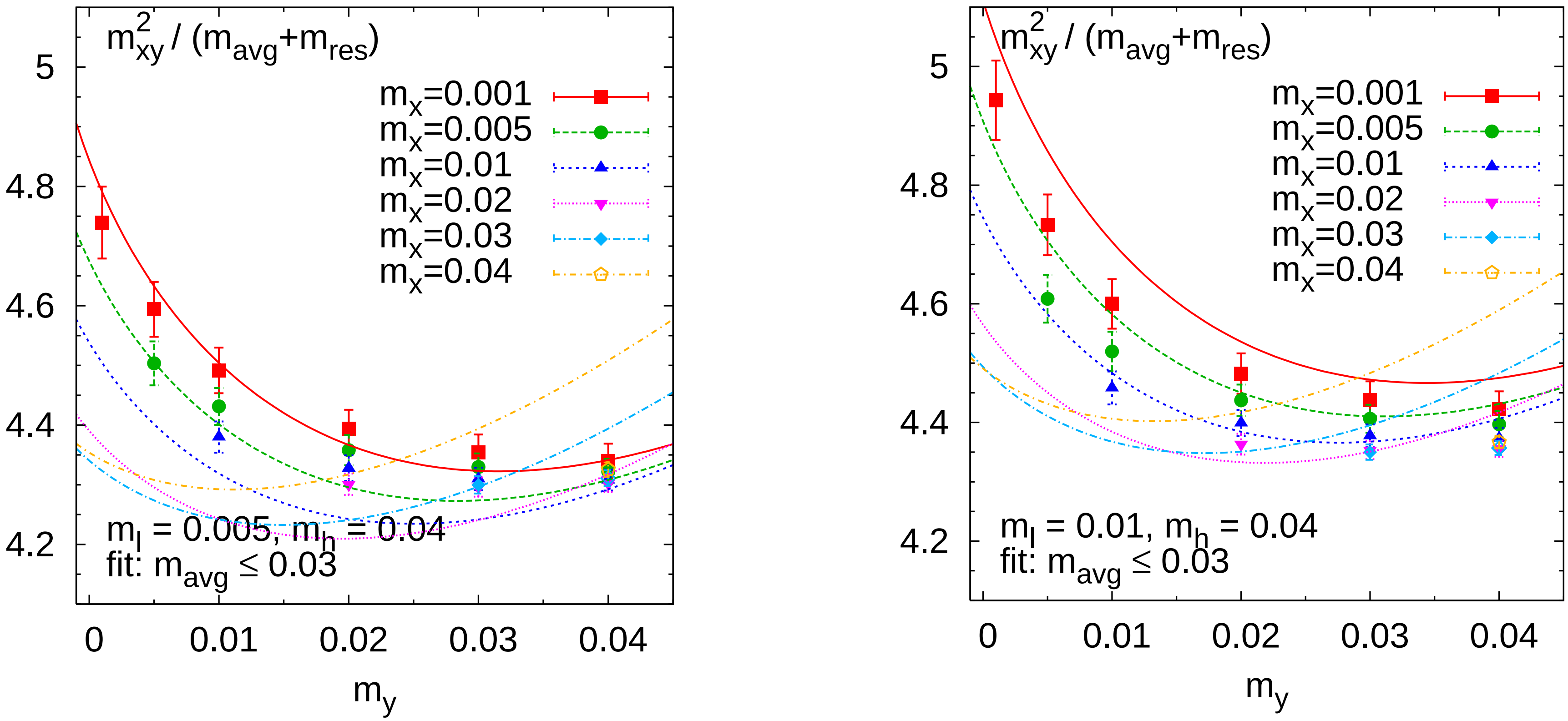


# SU(3) PQ$\chi$PT, fit range $m_{\text{avg}} \leq 0.03$




(combined) NLO-fits not working up to $m_s$


$m^2_{xy}$ / ($m_{avg}$+$m_{res}$)
$m_x$=0.001
$m_x$=0.005
$m_x$=0.01
$m_x$=0.02
$m_x$=0.03
$m_x$=0.04
$m_l$ = 0.005, $m_h$ = 0.04
fit: $m_{avg} \leq 0.03$
$m_l$ = 0.01, $m_h$ = 0.04
fit: $m_{avg} \leq 0.03$
$m_y$

- NLO-fits not working up to the strange quark mass
  ($m_x = 0.001, m_y = 0.04 \;\Rightarrow\; m_{xy} \;\approx\; 554\,$MeV)

- including NNLO-terms

  * complete formulae available Bijnens et al.,
    work in progress, will try to apply with $32^3$ data

  * just include analytic NNLO-terms
    $(\chi_x + \chi_y)^2$, $(\chi_x - \chi_y)^2$, $\overline{\chi}^2$, $\overline{\chi}(\chi_x + \chi_y)$, $\overline{\chi^2}$
    * still right behaviour in light quark mass region?? non-analytic terms???
    * limited number of data points (sea quark mass)

- chiral symmetry only for up- and down-quarks: SU(2) $\times$ SU(2)

# SU(2) low-energy constants

- convert $B_0$, $f_0$, LECs from SU(3)-case to SU(2)
  (1-loop matching in Gasser, Leutwyler, 1985, 2-loop: Gasser et al. 2006)
- compare SU(2)-LECs at scale $m_\pi = 139$ MeV: $\bar{l}_{3,4}$

| | $B$ | $f$ | $\bar{l}_3$ | $\bar{l}_4$ |
|---|---|---|---|---|
| SU(2)×SU(2) | 2.414(61) | 0.0665(21) | 3.13(33) | 4.43(14) |
| SU(3)×SU(3) | 2.457(78) | 0.0661(18) | 2.87(28) | 4.10(05) |
| MILC ($N_f = 2 + 1$) | | | 0.6(1.2) | 3.9(5) |
| MILC, pure NLO | | | 2.85(07) | – |
| ETMC ($N_f = 2$) | | | 3.44(08)(35) | 4.61(04)(11) |
| CERN ($N_f = 2$) | | | 3.0(5) | – |
| phenom. | | | 2.9(2.4) | 4.4(0.2) |

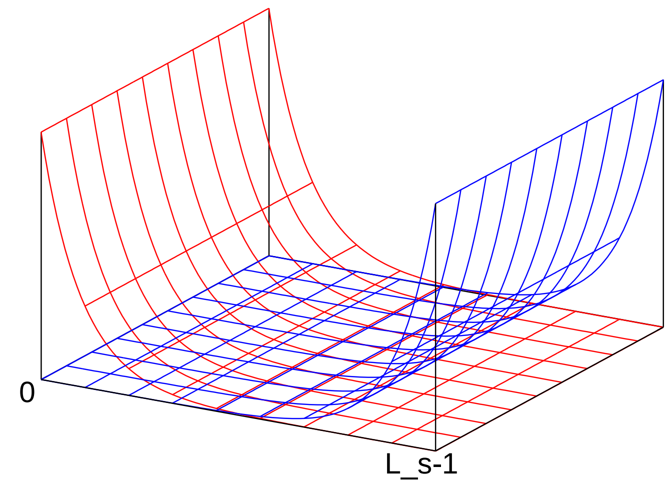
# physical quark masses

| | $N_f$ | fermion | $m_{ud}$/MeV | $m_s$/MeV<br>$m_s/m_{ud}$ |
|---|---|---|---|---|
| using non-perturbative renormalization | | | | |
| | **2+1** | **DWF** | $\mathbf{3.72(.16)_{stat}(.33)_{ren}(.17)_{syst}}$ | $\mathbf{107.3(4.4)_{stat}(9.7)_{ren}(4.9)_{syst}}$<br>$\mathbf{28.8(0.4)_{stat}(1.6)_{syst}}$ |
| RBC | 2 | DWF | $4.25(.23)_{stat}(.26)_{ren}$ | $119.5(5.6)_{stat}(7.4)_{ren}$<br>$28.10(.38)_{stat}$ |
| ETMC | 2 | TM-Wilson | $3.85(.12)_{stat}(.40)_{syst}$ | $105(3)_{stat}(9)_{syst}$<br>$27.3(0.3)_{stat}(1.2)_{syst}$ |
| QCDSF | 2 | impr. Wilson | $4.08(.23)_{stat}(.19)_{syst}(.23)_{scale}$ | $111(6)_{stat}(4)_{syst}(6)_{scale}$<br>$27.2(3.2)$ |
| using perturbative renormalization | | | | |
| MILC | $2+1$ | stagg. | $3.2(0)_{stat}(.1)_{ren}(.2)_{EM}(0)_{cont}$ | $88(0)_{stat}(3)_{ren}(4)_{EM}(0)_{cont}$<br>$27.2(.1)_{stat}(.3)_{syst}$ |
| PACS-CS | $2+1$ | impr. Wilson | $2.3(1.1)$ | $69.1(2.5)$<br>$30(?)$ |
| JLQCD | $2+1$ | impr. Wilson | $3.54(^{+.64}_{-.35})_{total}$ | $91.1(^{+14.6}_{-6.2})_{total}$<br>$25.7(?)$ |

($\overline{\text{MS}}$-scheme at $\mu = 2$ GeV)

# (Why) Domain Wall fermions

- different lattice fermions
  - ∗ Wilson fermions and improved versions
  - ∗ staggered fermions
  - ∗ domain wall fermions (DWF)
  - ∗ overlap-fermions
- DWF
  - ∗ fermion fields have a 5th dimension of extent $L_s$
  - ∗ *left* and *right* handed fermions on slice $0$ and $L_s - 1$
  - ∗ propagation through 5th dimension:
    residual chiral symmetry breaking ($m_{\text{res}}$)
  - chiral symmetry breaking under control
  - reduces (wrong chirality) operator mixing ($B_K$)
  - non-perturbative renormalization (quark masses, $B_K$)



## short intermission: CKM-matrix elements

$$\frac{|V_{us}|}{|V_{ud}|} = 0.2387(4)\,\frac{f_\pi}{f_K}\sqrt{\frac{\Gamma(K\to\mu\nu(\gamma))}{\Gamma(\pi\to\mu\nu(\gamma))}}$$

BLUCHER, MARCIANO, PDG '06

with $\frac{\Gamma(K\to\mu\nu(\gamma))}{\Gamma(\pi\to\mu\nu(\gamma))} = 1.3383(46)$ and our result for $f_k/f_\pi$ we obtain:

$$\begin{aligned}
\frac{|V_{us}|}{|V_{ud}|} &= 0.2292(034)_{\text{stat}}(138)_{\text{syst}}(005)_{\text{other}}\\
|V_{us}| &= 0.2232(033)_{\text{stat}}(115)_{\text{syst}}(005)_{\text{other}}\\
|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 &= 0.9980(15)_{\text{stat}}(51)_{\text{syst}}(06)_{\text{other}} = 0.9980(54)_{\text{total}}
\end{aligned}$$

(using $V_{ud} = 0.97377(27)$, $\beta$-decay, $|V_{ub}| = 0$)

## non-perturbative renormalization (Rome-Southampton)

- renormalization done at $16^3 \times 32 \times 16$ lattices (same gauge action, lattice spacing)

- match bare lattice operators to RI/MOM non-perturbatively

- perturbative matching to $\overline{\text{MS}}$ at 2 GeV

- Domain Wall Fermions:
  - $*$ control of chiral symmetry breaking
  - $*$ $\mathcal{O}(a)$-improved
  - $\longrightarrow$ operator mixing reduced
  - $\longrightarrow$ (partially) conserved axial and vector currents

- publication in preparation by RBC- and UKQCD-Collaborations

- here we are interested in $Z_m = 1/Z_S$

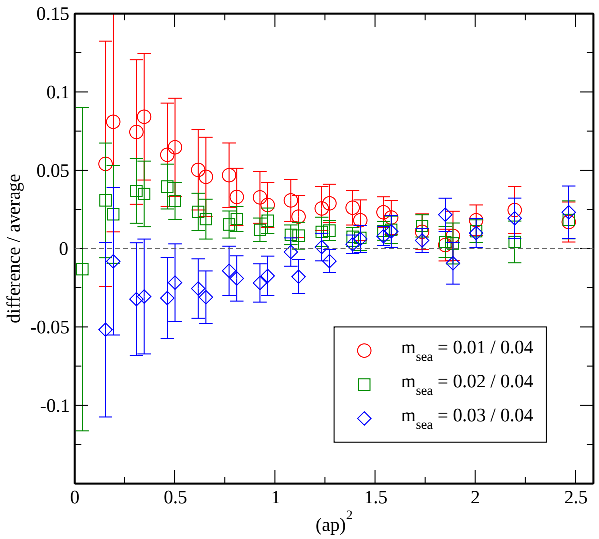
- renormalized amp. vertex functions $\Lambda_i^{\text{ren}} = Z_i/Z_q\,\Lambda_i = 1,\ i \in \{S, P, V, A, T\}$

$$Z_m^{\text{RI}} = \underbrace{\frac{Z_q}{Z_S}(p)}_{\Lambda_S}\ \underbrace{\frac{Z_A}{Z_q}(p)}_{\Lambda_A}\ \underbrace{\frac{1}{Z_A}}_{\text{hadronic ME}}$$

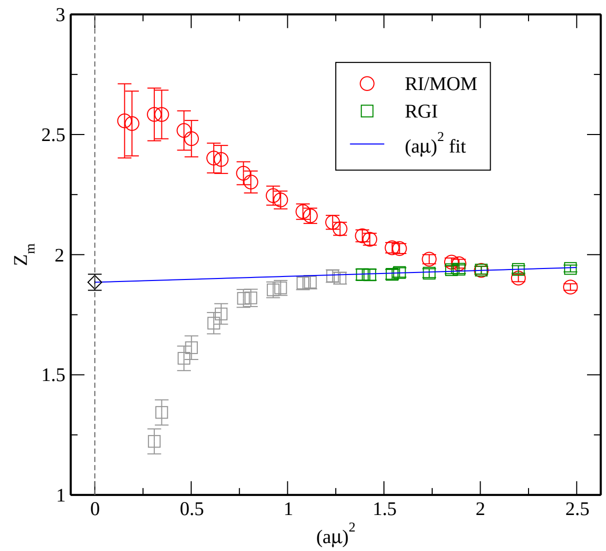
- four loop RG-running $Z_m^{\text{RGI}} = \frac{c(\alpha_S(\mu_0)/\pi)}{c(\alpha_S(\mu)/\pi)} Z_m^{\text{RI}}(\mu)$ [Chetyrkin et al., 2000]
- three loop matching RI/MOM to $\overline{\text{MS}}$

$$\boldsymbol{Z_m^{\overline{\text{MS}}}(2\ \text{GeV}) = 1.575(28)(15)(83)}$$

(error: statistical, $\Lambda_A \leftrightarrow \frac{1}{2}(\Lambda_A + \Lambda_V)$, linear vs. quadratic chiral extrap.)




comparing $\frac{1}{12}\frac{\partial \text{Tr} S^{-1}}{\partial m_v}$ and $\Lambda_S$




dividing out the running factor