STRONG COUPLING CONSTANT AND FOUR QUARK CONDENSATES FROM VACUUM POLARIZATION FUNCTIONS WITH DYNAMICAL OVERLAP FERMIONS

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INTRODUCTION

- Calculation of strong coupling constant α_s

  Fundamental constant of QCD, and provides a high precision test of QCD.

  - ***Phenomenological determination (short distance physics)***
    - Deep-inelastic scattering,
    - tau decay (OPAL, ALEPH) , e^+-e^- annihilation.
    - Operator Product Expansion (OPE)

    . . . .

  - ***Lattice calculation***
    - (Heavy) hadron spectroscopy , [SESAM(1999), HP/UKQCD-Flab(2004)]
    - Heavy (static) quark potential, [HPQCD(2008)]
    - Schrödinger functional scheme, [ALPHA(2005)]

  . . . .

METHODOLOGY

- Matching the OPE with the lattice data of vector (V) and axial-vector (A) vacuum polarization functions in dynamical overlap fermion. See also [HPQCD(2008)]
- Exact chiral symmetry of overlap fermion
  - No additive renomalization terms (in chiral condensate)
  - No O(a) lattice artifacts due to the violation of chiral symmetry.
- Dynamical overlap fermion configurations Talk by S. Hashimoto
  - N_f=2, $16^3 \times 32$ lattice, a^{-1}=1.67 GeV, quark mass: $m_s/6$ ~ $m_s/2$
  - Non-perturbative renormalization factor
  - Topology is fixed in Q=0

VACUUM POLARIZATION FUNCTIONS

- Current correlator in the continuum

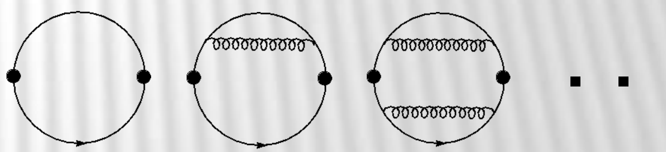
  Different spin (S=0, 1) components

$$i\int d^4x \langle T\{J_\mu(x), J_\nu^\dagger(0)\}\rangle e^{iqx} = -(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_J^{(1)}(q^2) + q_\mu q_\nu \Pi_J^{(0)}(q^2)$$

focus on $\Pi_J^{(0+1)}(Q^2) = \Pi_J^{(0)}(Q^2) + \Pi_J^{(1)}(Q^2)$, $Q^2 = -q^2 > 0$

- OPE

$$\begin{aligned}\Pi_J^{(0+1)}(Q^2) &= C_0(Q^2,\mu^2) + \frac{m^2 C_m^J(Q^2)}{Q^2} \\ &+ C_{\bar{q}q}^J(Q^2)\frac{\langle m\bar{q}q\rangle}{Q^4} \\ &+ C_{GG}(Q^2)\frac{\langle \alpha_s/\pi GG\rangle}{Q^4} \\ &+ \cdots\end{aligned}$$



C_0 and C_m are known at 4-loop, C_{qq} and C_{GG} are known at 3-loop.

CURRENT CORRELATOR ON THE LATTICE

- Local (non-conserved) current

$$V_\mu = Z_V \bar{q}\gamma_\mu\Big(1-\frac{D}{2m_0}\Big)q, \quad A_\mu = Z_A\bar{q}\gamma_\mu\gamma_5\Big(1-\frac{D}{2m_0}\Big)q$$

- Correlator

$$\int d^4x \langle T\{J_\mu(x), J_\nu(0)\}\rangle^{\mathrm{lat}} e^{iQx} = \delta_{\mu\nu}Q^2\Pi_J^{(1)}(Q) - Q_\mu Q_\nu \Pi_J^{(0+1)}(Q)$$

$$-B_0(Q)\delta_{\mu\nu} - \sum_{n=1} B_n(Q)Q_\mu^{2n}\delta_{\mu\nu} - \sum_{m,n=1} C_{mn}(Q)(Q_\mu^{2m+1}Q_\nu^{2n+1} + Q_\nu^{2m+1}Q_\mu^{2n+1})$$

  - 1st and 2nd term [Π_J]: Vacuum polarization function
  - 3rd term: [B_0]

    Same Lorentz structure as $\Pi_J^{(1)}$ and contains contact term which is divergent as $1/a^2$.
  - 4th and 5th term: [B_n, C_{mn}]

    Violation of the Lorentz symmetry (lattice artifacts),

CURRENT CORRELATOR ON THE LATTICE

$$\int d^4x \langle T\{J_\mu(x), J_\nu(0)\}\rangle^{\mathrm{lat}} e^{iQx} = \delta_{\mu\nu} Q^2 \Pi_J^{(1)}(Q) - Q_\mu Q_\nu \Pi_J^{(0+1)}(Q)$$

$$-B_0(Q)\delta_{\mu\nu} - \sum_{n=1} B_n(Q) Q_\mu^{2n} \delta_{\mu\nu} - \sum_{m,n=1} C_{mn}(Q)(Q_\mu^{2m+1} Q_\nu^{2n+1} + Q_\nu^{2m+1} Q_\mu^{2n+1})$$

Our method:

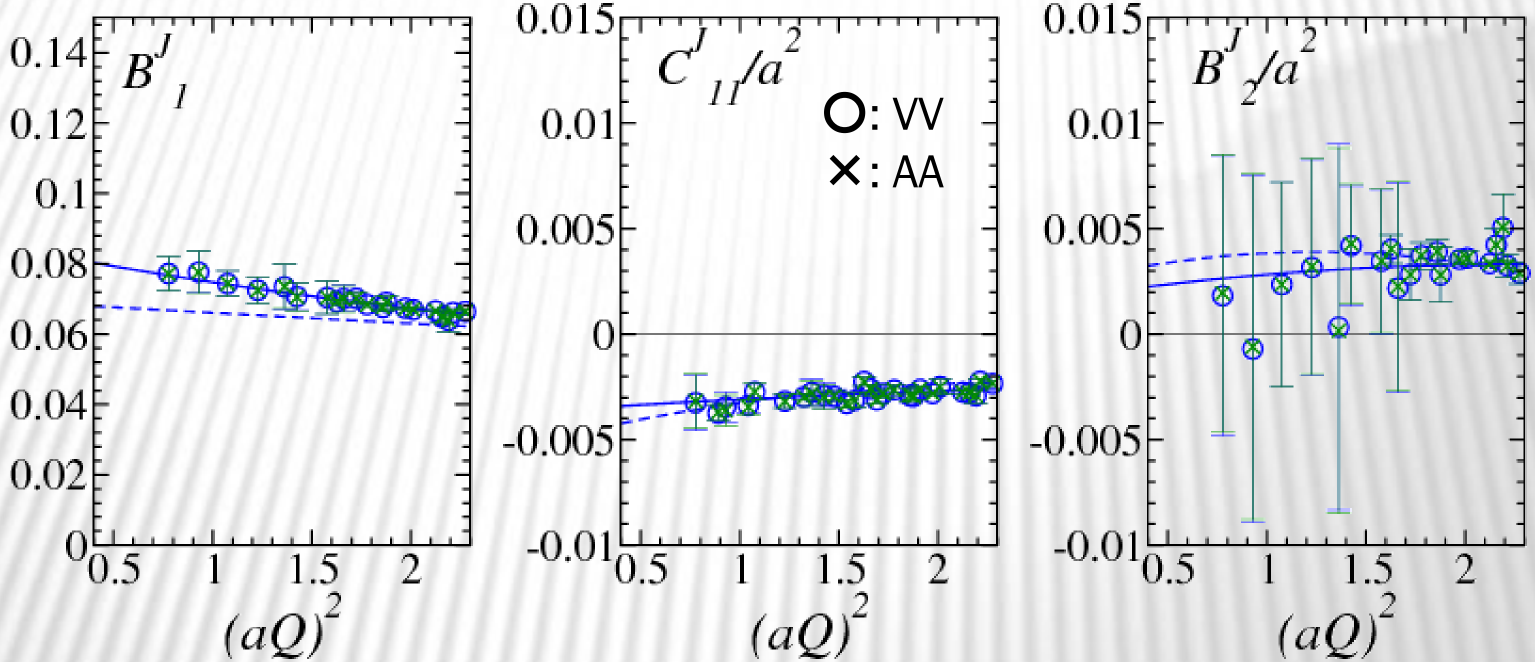
- Focus on $\Pi_J^{(0+1)}$, then $Q^2\Pi_J^{(1)}$ + B_0 can be ignored.
- Truncate the terms of $O(Q^6)$ and higher, we only consider $B_{1,2}$ and C_{11}
- Off-diagonal part (μ≠ν)

  extract $\Pi_J^{(0+1)}$ and C_{11}
- Diagonal part (μ=ν)

  extract $\Pi_J^{(0+1)}$ and $B_{1,2}$ using C_{11} from off-diagonal part.
- Comparison of $\Pi_J^{(0+1)}$ obtained from diagonal (μ=ν) and off-diagonal (μ≠ν) provides a good check of consistency.

NUMERICAL RESULTS: LATTICE ARTIFACTS

- Subtraction coefficences (lightest quark mass)



Solid line: fit function(polynomial), **Dashed line:** one-loop in lat. PT.

- Dominated by the perturbative contribution
- B_1 (in diagonal part), is much larger than others,
- These coefficients mostly cancel in V-A.

[arXive: 0806.4222] and Yamada's talk

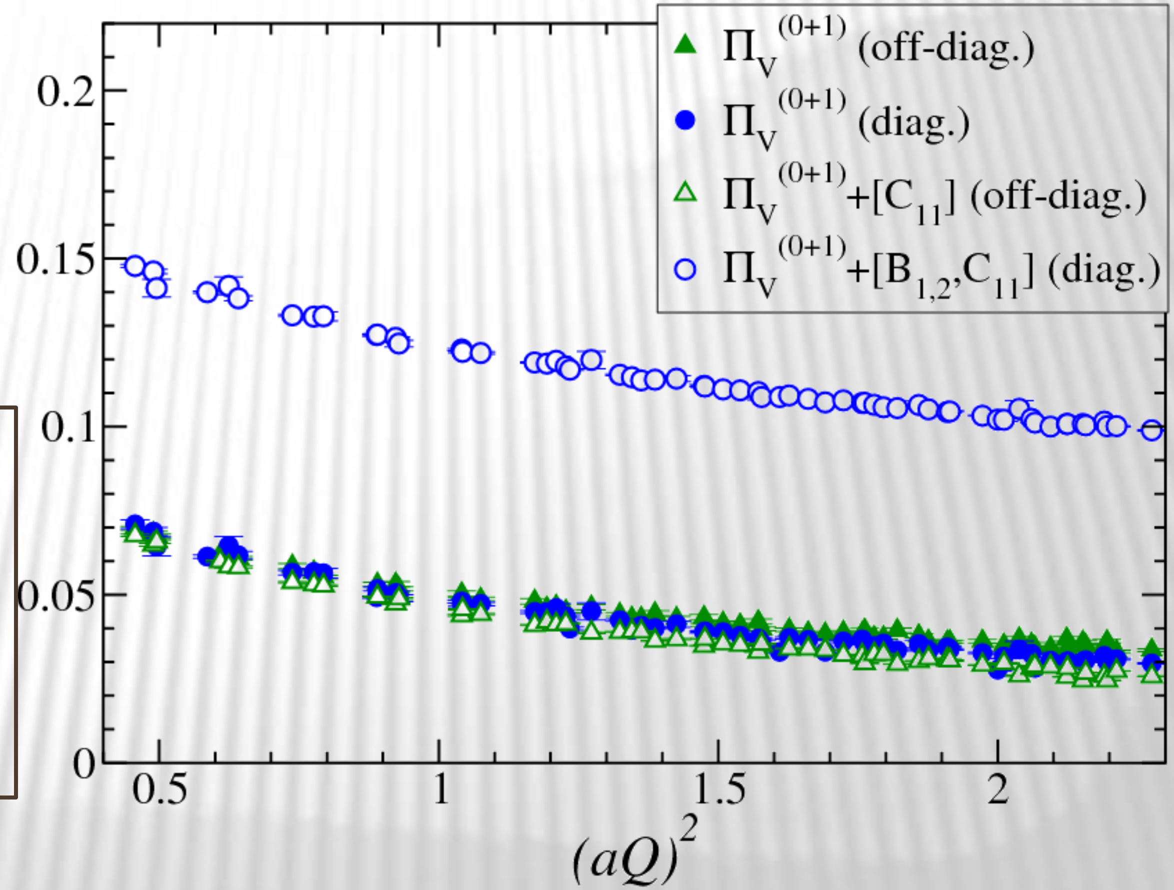
NUMERICAL RESULTS: SUBTRACTION

- $\Pi_V^{(0+1)}$ and subtraction factor

blue filled circle and green filled triangle reasonably agree in $(aQ)^2 < 1.4$

↓

- Higher order is small
- Truncation at $O(Q^6)$ is enough to reduce the Lorentz violating terms



PHYSICS TARGET

Analysis of two forms of Π_J using the OPE

- V+A

  Coupling constant α_s (Λ_{MS}) and gluon condensate

- V - A

  Four quark condensate, $a_6(\mu)$, $b_6(\mu)$:

$$a_6(\mu) = 2\Big[2\pi\langle\alpha_s O_8\rangle + A_8\langle\alpha_s^2 O_8\rangle + A_1\langle\alpha_s^2 O_1\rangle\Big]$$

$$b_6(\mu) = 2\Big[B_8\langle\alpha_s^2 O_8\rangle + B_1\langle\alpha_s^2 O_1\rangle\Big]$$

with

$$\langle O_1\rangle = \left\langle \bar{q}\gamma_\mu \frac{\tau^3}{2} q\bar{q}\gamma^\mu \frac{\tau^3}{2} q - \bar{q}\gamma_\mu\gamma_5 \frac{\tau^3}{2} q\bar{q}\gamma^\mu\gamma_5 \frac{\tau^3}{2} q \right\rangle$$

$$\langle O_8\rangle = \left\langle \bar{q}\gamma_\mu\lambda_a \frac{\tau^3}{2} q\bar{q}\gamma^\mu\lambda_a \frac{\tau^3}{2} q - \bar{q}\gamma_\mu\gamma_5\lambda_a \frac{\tau^3}{2} q\bar{q}\gamma_5\gamma^\mu\lambda_a \frac{\tau^3}{2} q \right\rangle$$

which corresponds to $K\rightarrow\pi\,\pi$ (I=2) matrix element.

[Donaghue(2000)]

ANALYSIS OF V+A

- For $\Pi_{V+A}^{(0+1)} = \Pi_V^{(0+1)} + \Pi_A^{(0+1)}$

$$\Pi_{V+A}^{(0+1)}|_{\mathrm{OPE}}(Q^2) = c + C_0(Q^2, \mu^2) + \frac{m^2 C_m^{V+A}(Q^2)}{Q^2} + C_{\bar{q}q}^{V+A}(Q^2)\frac{\langle m\bar{q}q\rangle}{Q^4} + C_{GG}(Q^2)\frac{\langle \alpha_s/\pi GG\rangle}{Q^4}$$

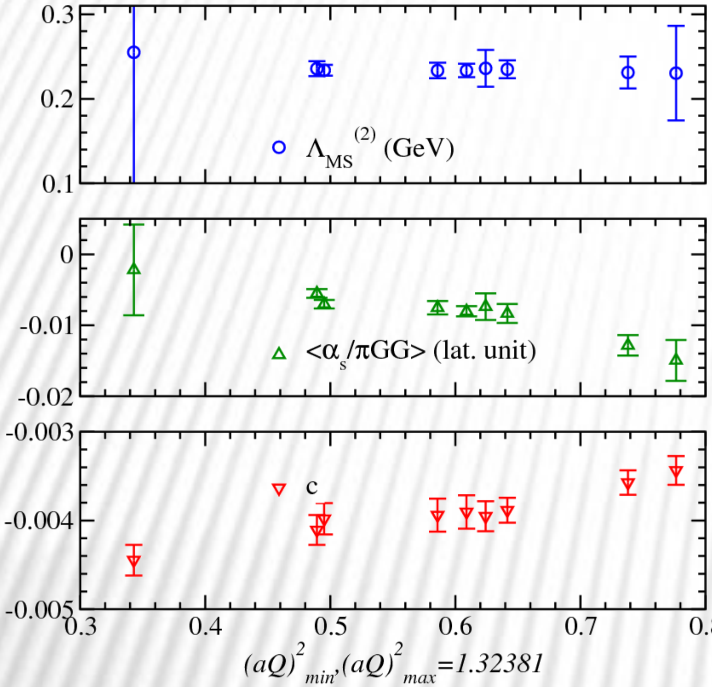
  - 3 free parameters

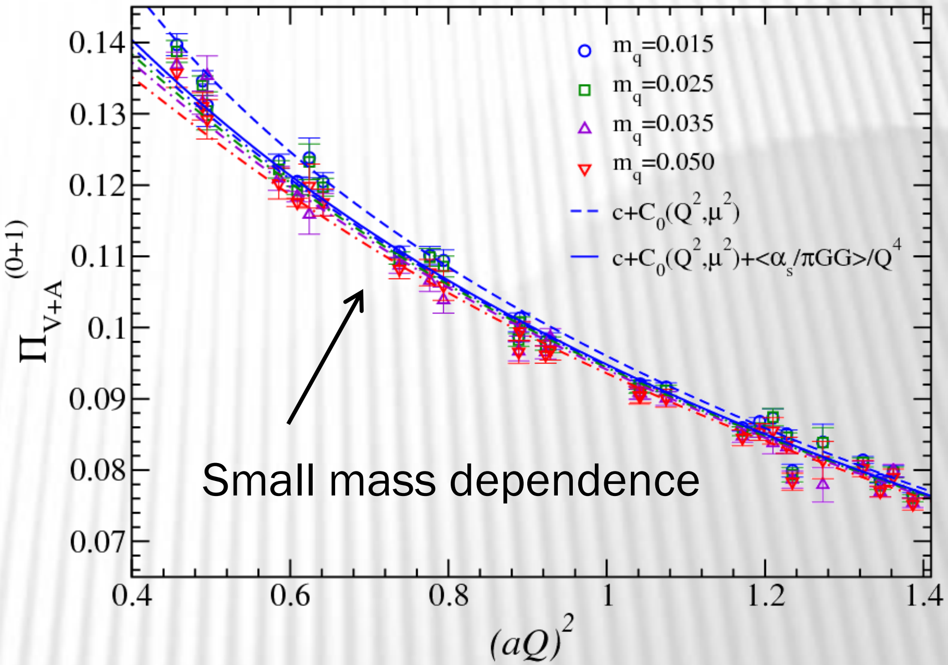
    α_s (Λ_{MS}) and gluon condensate $<\alpha_s/\pi GG>$,

    c: difference of renormalization scheme (lattice and dimensional regularization)
  - C_0, C_m, C_{qq}, C_{GG} from perturbation theory (3-loop)
  - Quark condensate is an input, $[0.251\ \mathrm{GeV}]^3$ [Fukaya(2007)]

    ↑ No additional renormalization necessary.
  - Mass dependence is controlled by the 4th term

NUMERICAL RESULTS: V+A





Fit range [0.58,1.3] statistical systematic

e.g. 0.250(16)(16) GeV

[ALPHA(2005)]

$$\Lambda^{(2)}_{\overline{MS}} = 0.234(9)(^{+16}_{-0})\,\mathrm{GeV} \quad \langle \alpha_s/\pi GG \rangle = -0.06 \sim 0.1\,\mathrm{GeV}^4$$

- Systematic error is estimated by replacing C_0 by lattice perturbation (one-loop)
- Gluon condensate has a large systematic error

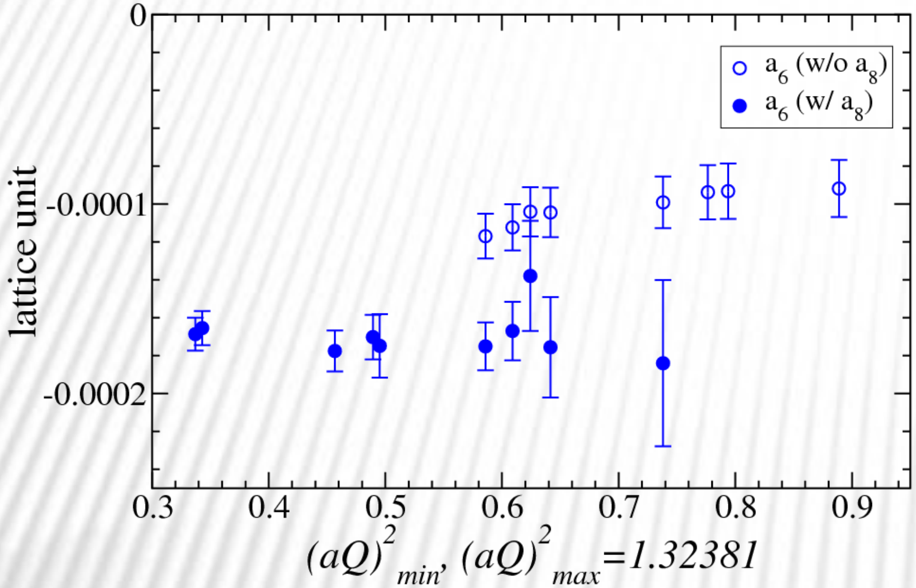
ANALYSIS OF V — A

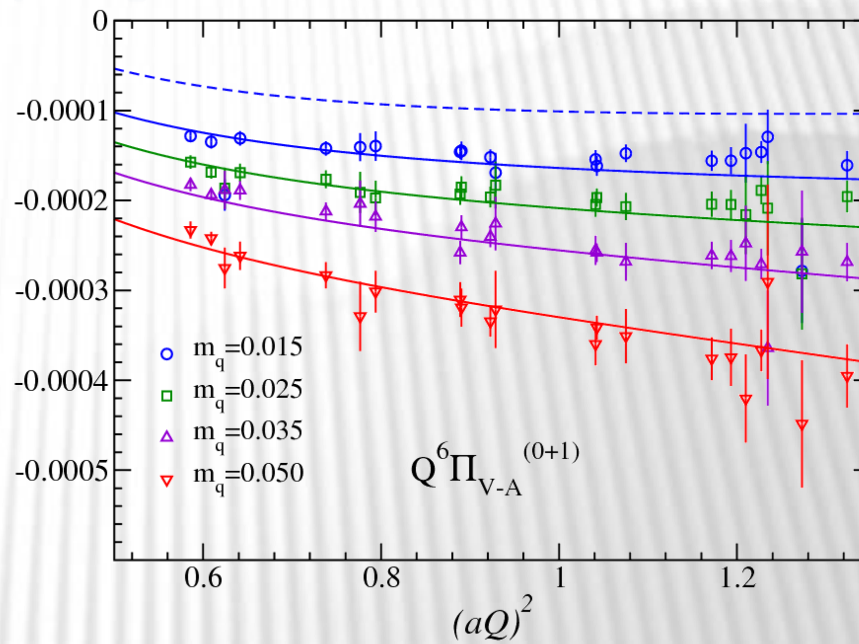
- For $\Pi_{V\text{-}A}^{(0+1)} = \Pi_V^{(0+1)} - \Pi_A^{(0+1)}$

$$\Pi_{V-A}^{(0+1)}|_{\mathrm{OPE}}(Q^2) = \frac{m^2 C_m^{V-A}(Q^2)}{Q^2} + C_{\bar{q}q}^{V-A}(Q^2)\frac{\langle m\bar{q}q\rangle}{Q^4} + \left[a_6(\mu) + b_6(\mu)\ln(Q^2/\mu^2) + m_q c_6\right]\frac{1}{Q^6} + \frac{a_8}{Q^8}$$

  - In the chiral limit, 1st and 2nd terms go to zero.
    ⇒ start from the dimension-six term (leading)
  - $C_m^{V-A}(Q^2)$, $C_{\bar{q}q}^{V-A}(Q^2)$ ~ $O(\alpha_s)$
    ⇒ sub-dominant
  - Fit with or without the a_8 term in order to estimate the truncation effect
  - Exact chiral symmetry of overlap fermion is important to remove additional operator mixing.

NUMERICAL RESULTS: V—A





Fit range [0.58,1.3]

$$a_6 = -0.0038(3)(^{+16}_{-0})\,\mathrm{GeV}^6, b_6 = +0.0017 \sim -0.0008\,\mathrm{GeV}^6$$

- Systematic error is determined by the comparison with and without the a_8 term.
- Phenomenological estimate: a_6= -0.003~-0.009 GeV^6 , b_6~0.03a_6

SUMMARY

- Calculation of strong coupling α_s and four-quark condensate a_6 from vacuum polarization function $\Pi_{V\pm A}$ by matching with OPE.
- Dynamical overlap fermions (N_f=2)
- For V+A
  - Subtract Lorentz violating terms $B_{1,2}$, C_{11}
  - We obtain $\Lambda^{(2)}_{\overline{MS}} = 0.234(9)(^{+16}_{-0}), \langle \alpha_s/\pi GG \rangle = -0.06 \sim 0.1\,\mathrm{GeV}^4$
- For V—A
  - Four-quark condensate is leading term in the chiral limit
  - We obtain $a_6 = -0.0038(3)(^{+16}_{-0})\,\mathrm{GeV}^6, b_6 = +0.0017 \sim -0.0008\,\mathrm{GeV}^6$
  - Good agreement with phenomenological estimation
- On-going project on N_f=2+1 configurations