

Screening Masses at High Temperature ...

- I** **Introduction**
- II** **... with quenched Wilson**
- III** **... with dynamical staggered**
- IV** **Summary**

based on work by
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I Introduction

aim at **analyzing existence and properties of hadronic excitations at $T > 0$**

\leadsto Correlator in momentum space defined at (boson) Matsubara frequencies $\omega_n = 2\pi T n$

$$\begin{aligned}\Delta(i\omega_n, \vec{p}) &= \oint \frac{dp_0}{2\pi i} \frac{\Delta(p_0, \vec{p})}{p_0 - i\omega_n} \\ &= \int \frac{dp_0}{2\pi} \frac{1}{p_0 - i\omega_n} \underbrace{\frac{1}{i} [\Delta(p_0 + i\epsilon, \vec{p}) - \Delta(p_0 - i\epsilon, \vec{p})]}_{\text{spectral density } \sigma(p_0, \vec{p})} + \text{Subtr.}\end{aligned}$$

because of limited physical extension $1/T$ in the temporal direction

\leadsto spatial correlator

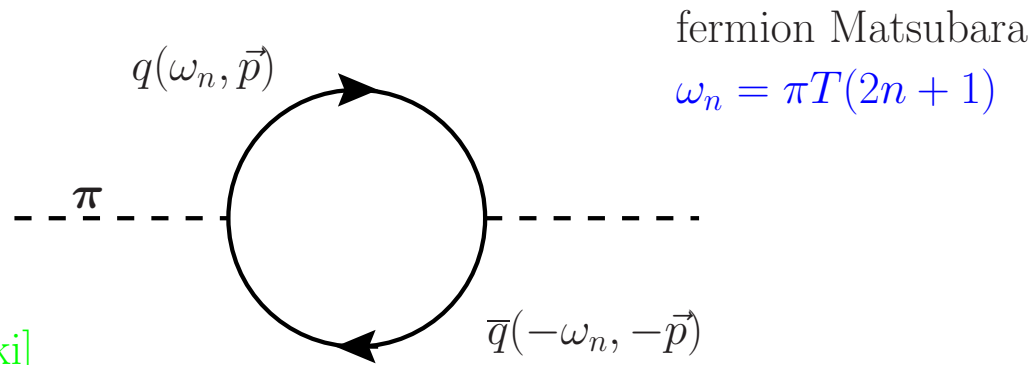
$$G^S(z) = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{ipz} \Delta(0, p) = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{ipz} \int_{-\infty}^{+\infty} dp_0 \frac{\sigma(p_0, p)}{p_0}$$

in particular

- symmetries
- comparison with free quark propagation

INTRODUCTION

At T large: expect **free quarks**



- free Continuum: [Friman, Florkowski]

$$G_\pi^S(z) = \frac{N_c T}{2\pi z^2 \sinh(2\pi T z)} [1 + 2\pi T z \coth(2\pi T z)]$$

define effective (z-dependent) screening mass

$$m_{\text{screen}}^{\text{eff}}(z) = -\frac{1}{G(z)} \frac{\partial G(z)}{\partial z} \simeq 2\pi T \left\{ 1 + \frac{1}{2\pi T z} + \dots \right\}$$

- lowest order correction: [Laine, Vepsäläinen]

$$m_{\text{screen}} = 2\pi T \left(1 + g^2 \times \left\{ \begin{array}{l} 0.022(N_F = 0) \\ 0.033(N_F = 3) \end{array} \right\} \right)$$

II. quenched Wilson

Numerical parameters

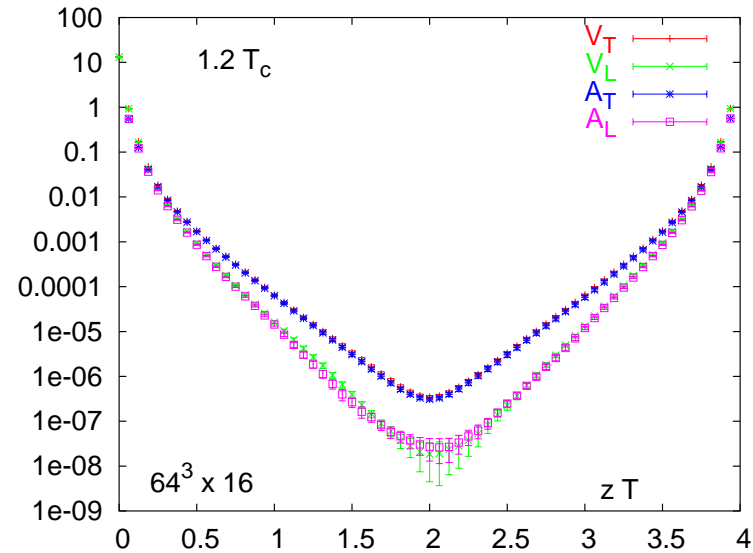
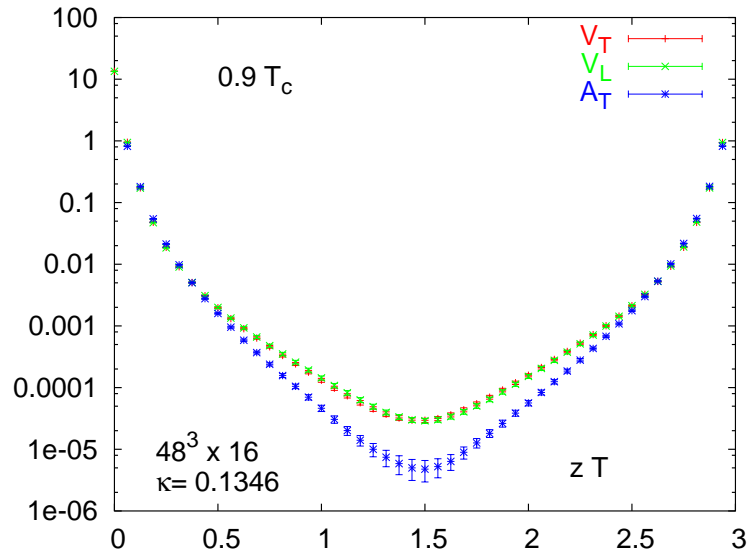
results based on

- quenched approximation
 - standard plaquette gauge action + non-perturbatively improved Wilson fermion action
 - \Rightarrow discretization errors at $\mathcal{O}(a^2)$
- temperatures: here mainly $1.5T_c$ and $3T_c$:
- quark masses: 0 (above T_c)
- statistics: $\mathcal{O}(100 - 200)$ configurations per parameter set
- channels analyzed: $PS, S, V_T, V_L, A_T, A_L$ flavor non-singlets
- isotropic lattices of various sizes:
 - discretization effects controlled by $aT = 1/N_\tau$ with $N_\tau = 8, 12, 16$
 - volume effects controlled by $LT = N_\sigma/N_\tau$ with aspect ratios $N_\sigma/N_\tau = 2, 3, 4, 8$
 - \Rightarrow lattice size up to $128^3 \times 16$

Symmetry restorations

- at $T \neq 0$, for spatial correlations: rotational $SO(3) \rightarrow SO(2) \times Z(2)$ [S.Gupta]

$$\Rightarrow V_T \neq V_L, A_T \neq A_L \text{ possible}$$



$$T < T_c: \quad V_T = V_L \neq A_T = A_L$$

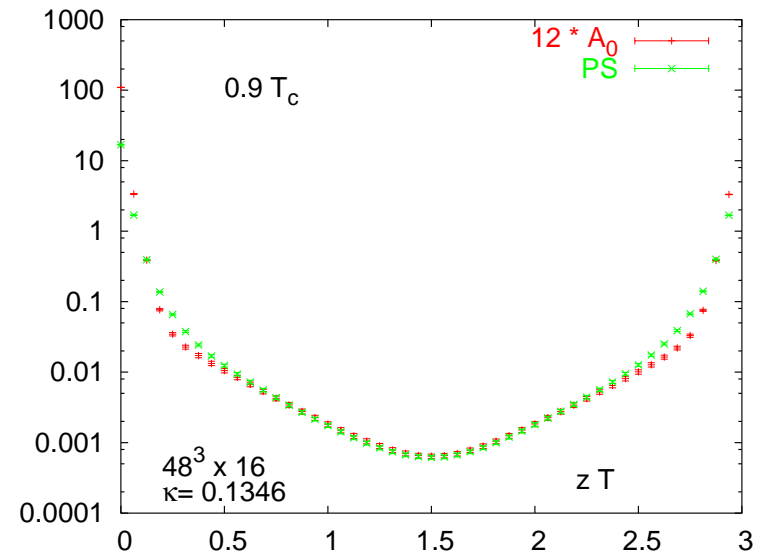
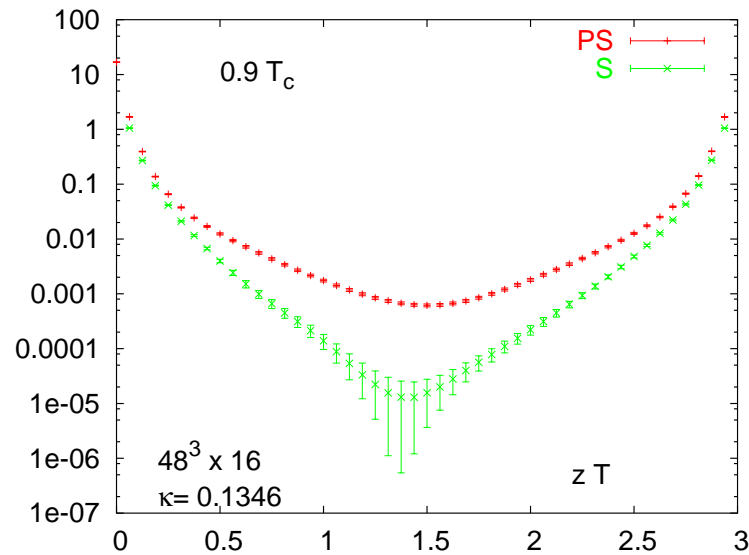
$$T > T_c: \quad V_T = A_T \neq V_L = A_L$$

- at $T > T_c$, chiral symmetry restoration $SU_V(N_F) \rightarrow SU_L(N_F) \times SU_R(N_F)$

QUENCHED WILSON

Pseudoscalar - Scalar (connected: a_0/δ) sector

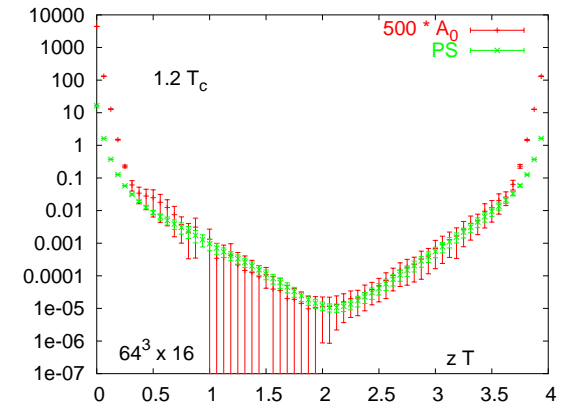
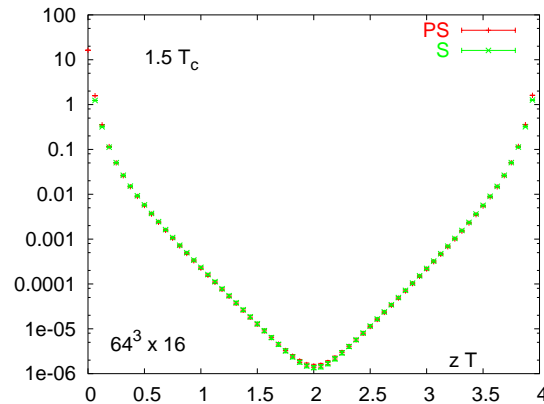
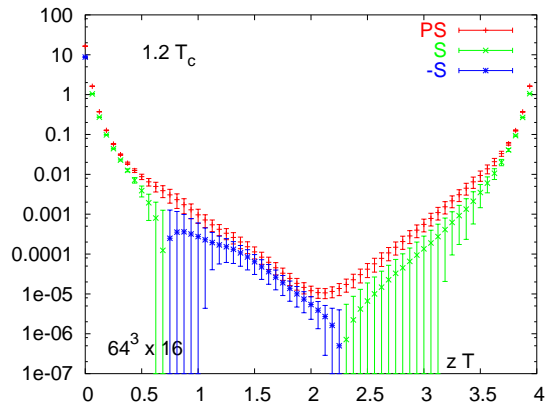
- below T_c



- pseudoscalar π much lighter than scalar a_0/δ
- aside: π couples to axialvector 0-component A_0 with relative strength $1/12$

QUENCHED WILSON

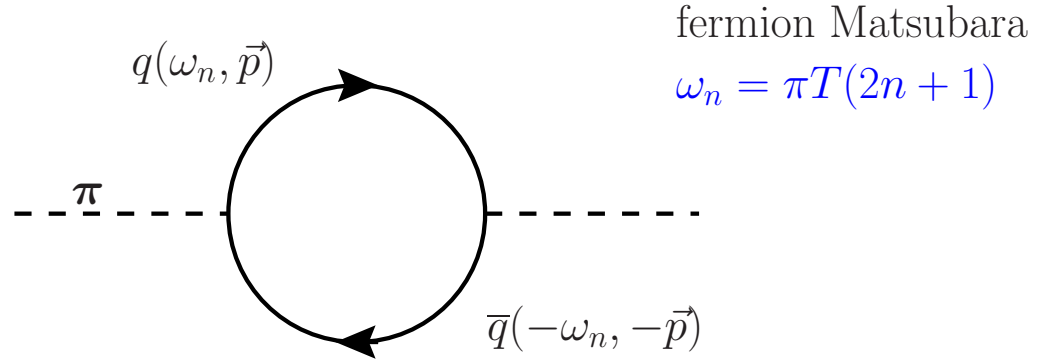
- at/above T_c
 - effective $U_A(1)$ restoration would predict $\pi - a_0/\delta$ degeneracy
 - not observed at $T = 1.25T_c$: (equivalent to saying that topologically non-trivial configurations survive up to (at least) $1.25T_c$)



– at $1.5T_c$: $m_{PS} = m_S$

– aside: A_0 couples to PS with much less strength $\simeq 1/500$

free lattice Wilson quarks



$$G_M^S(z) = \frac{1}{N_\sigma^2 N_\tau} \sum_{k_1, k_2, \omega_n} \frac{1}{(1+M)^2} \frac{1}{\sinh^2(EN_\sigma/2)} \left\{ b_M \cosh \left[2E \left(\frac{N_\sigma}{2} - z \right) \right] + d_M \right\}$$

where $M = \sum_{1,2,4} (1 - \cos(k_i))$

$$\cosh(E) = 1 + \frac{\sum_{1,2,4} \sin^2(k_i) + M^2}{2(1+M)}$$

	b_M	d_M
π	1	0
$\frac{1}{2}(\rho_1 + \rho_2)$	$1 - \frac{1}{2} \frac{\sin^2(k_1) + \sin^2(k_2)}{\sinh^2 E}$	$-\frac{1}{2} \frac{\sin^2(k_1) + \sin^2(k_2)}{\sinh^2 E}$
ρ_3	0	1
ρ_4	$1 - \frac{1}{2} \frac{\sin^2(k_4)}{\sinh^2 E}$	$-\frac{1}{2} \frac{\sin^2(k_4)}{\sinh^2 E}$

In the following, we used

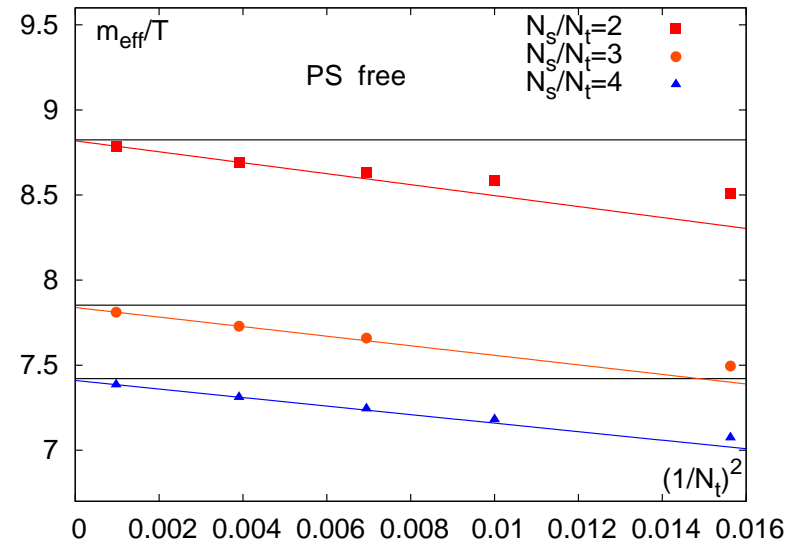
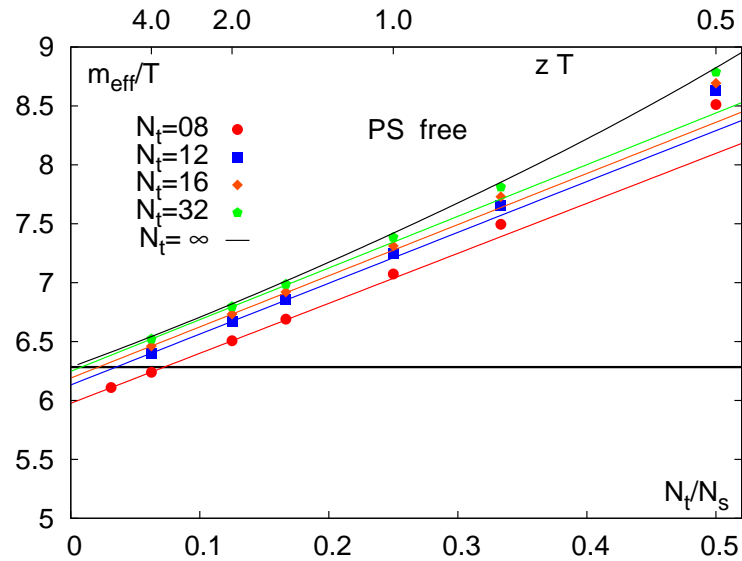
$$G_M^{S,subtr}(z) = G_M^S(z) - G_M^S(z = N_\sigma/2)$$

and solved for $m^{\text{eff}}(z)$

$$\frac{G_M^{S,subtr}(z)}{G_M^{S,subtr}(z+1)} = \frac{\sinh^2[(m^{\text{eff}}/2)(N_\sigma/2 - z)]}{\sinh^2[(m^{\text{eff}}/2)(N_\sigma/2 - z - 1)]}$$

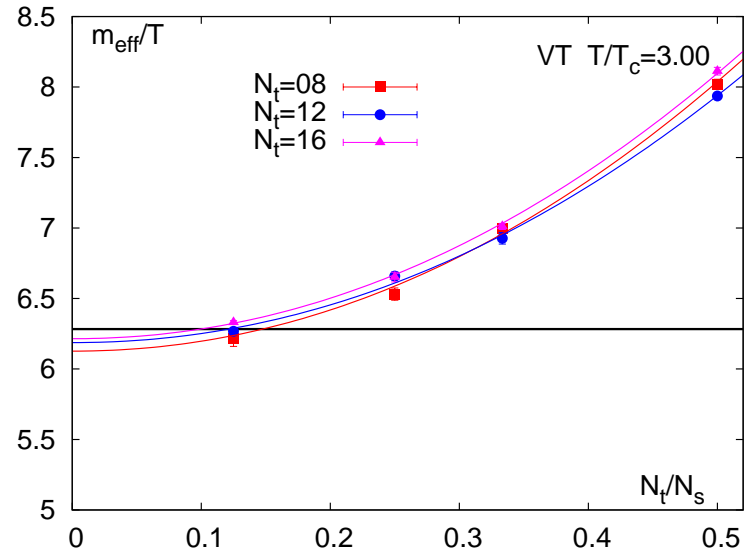
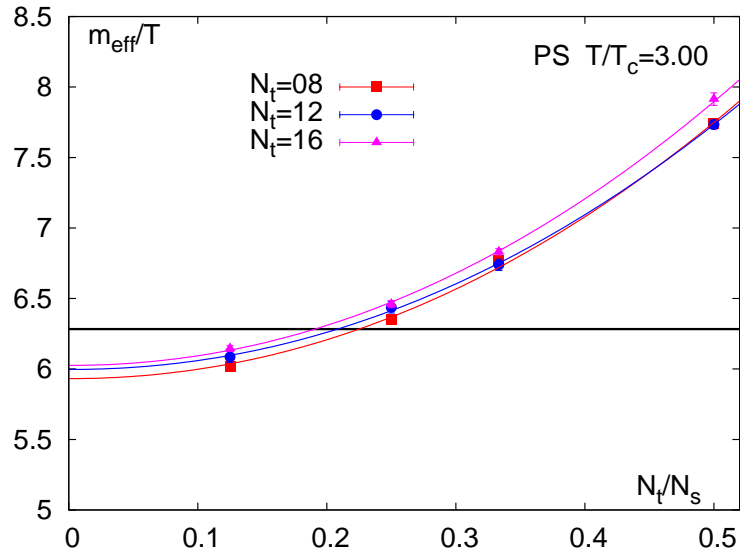
free Wilson results

in the following, to be definite, $z = N_\sigma/4 \rightarrow zT = \frac{1}{4} \times N_\sigma/N_\tau$



- finite volumes $LT = N_\sigma/N_\tau$ drive masses up
- finite lattice spacings $aT = 1/N_\tau$ drive them down
- leading behavior (free case) is $\sim 1/LT$
and $\sim a^2$

interacting case: thermodynamic limit



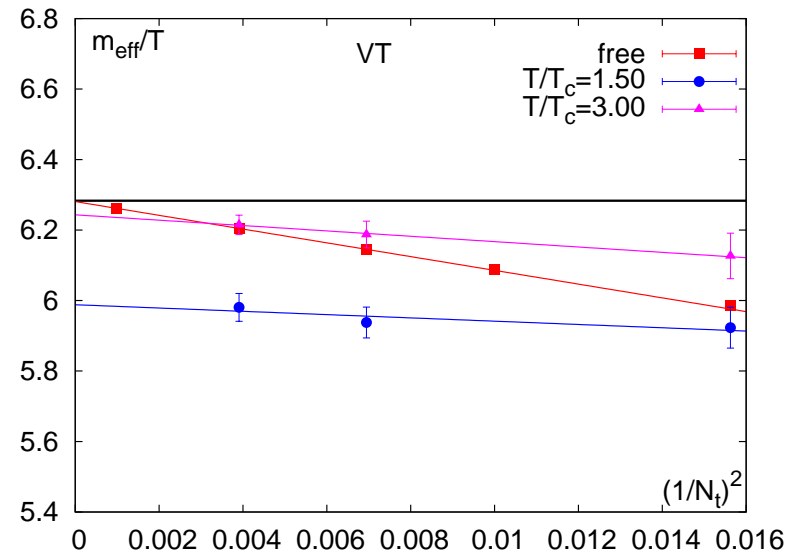
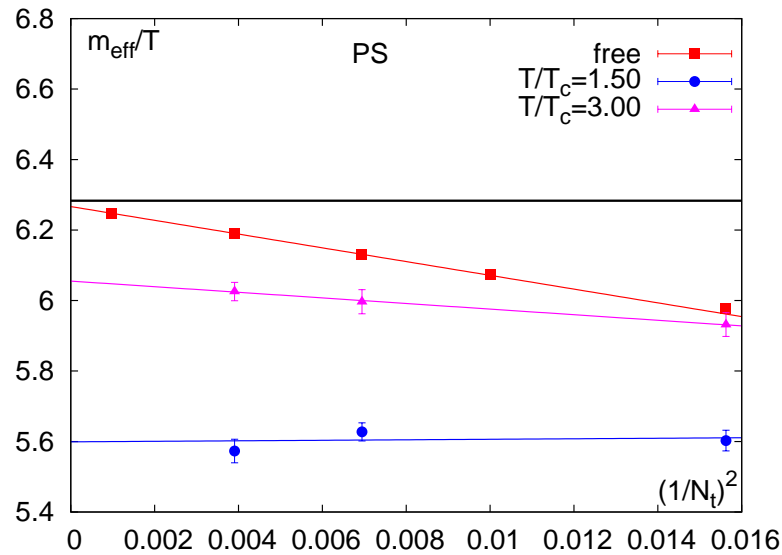
- finite volumes $LT = N_\sigma/N_\tau$ drive masses up

- assume power behavior

$$m/T = a + b \left(\frac{N_\tau}{N_\sigma} \right)^p$$

p	PS	VT
$1.5T_c$	2.22(10)	2.18(13)
$3.0T_c$	2.05(08)	2.05(10)

continuum limit



- also in interacting case: finite lattice spacings $aT = 1/N_\tau$ drive masses down $\sim a^2$ as expected
- slopes less than in free case
- T dependent deviations from free case
- up to $T = 3T_c \simeq 800\text{MeV}$ sizeable differences between π and ρ

III. Dynamical staggered

Simulation parameters

- $N_F = 2 + 1$: two degenerate u/d quarks + strange quark
- RHMC algorithm, exact to machine precision
 - polynomial approximation: 16/10 for light/strange quarks in molecular dynamics
20/16 for light/strange quarks in heatbath/ Metropolis
 - Sexton/Weingarten, Hasenbusch
- lattice sizes $16^3 \times 4$, $24^3 \times 6$, $32^3 \times 8$ (prelim.) ($T > 0$)
 $16^3 \times 32$, $24^3 \times 32$, $32^3 \times 32$, $24^2 \times 32 \times 48$ ($T = 0$, for scales and normalization)
- statistics $\mathcal{O}(10k - 60k)$ for $N_\tau = 4$, each $(\beta, \hat{m}_q, \hat{m}_s)$
 $\mathcal{O}(8k - 20k)$ for $N_\tau = 6$, each $(\beta, \hat{m}_q, \hat{m}_s)$
 $\mathcal{O}(5k - 20k)$ for $N_\tau = 8$, each $(\beta, \hat{m}_q, \hat{m}_s)$
 $\mathcal{O}(\geq 5k)$ for $T = 0$, each $(\beta, \hat{m}_q, \hat{m}_s)$
- along “line of constant physics” i.e. constant physical $m_K = 500\text{MeV}$, $m_\pi \simeq 220\text{MeV}$

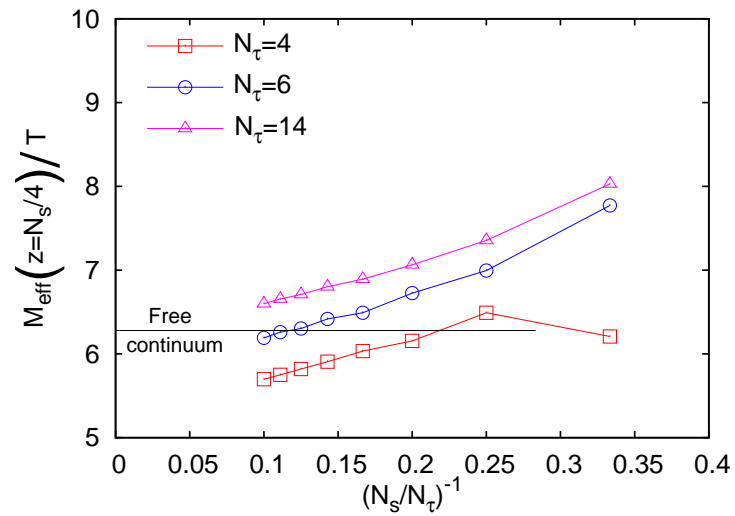
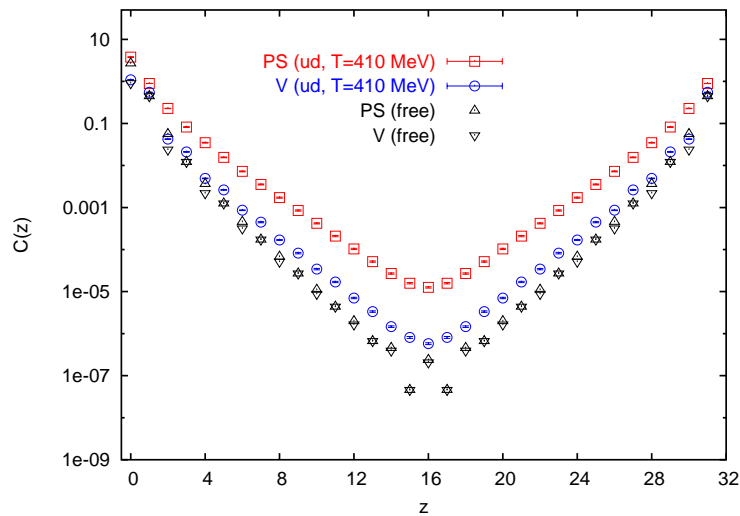
free lattice staggered fermions

$$G_M^S(z) = \frac{1}{N_\sigma^2 N_\tau} \sum_{k_1, k_2, k_4} \frac{1}{\cosh^2 E} \frac{1}{\sinh^2(EN_\sigma/2)} \left\{ b_M \cosh \left[2E \left(\frac{N_\sigma}{2} - z \right) \right] + d_M \right\}$$

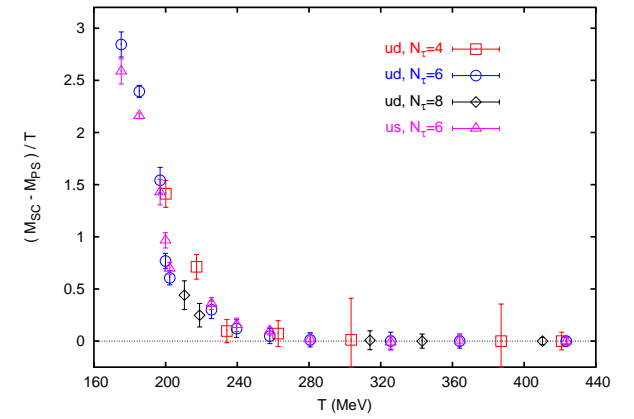
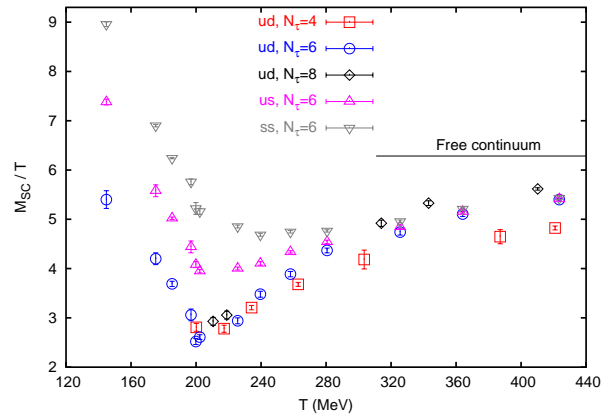
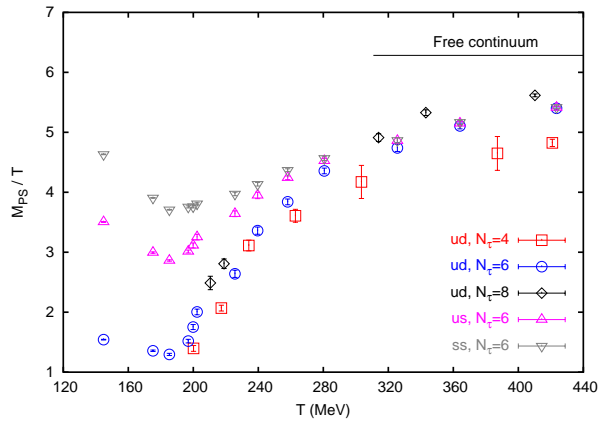
where $\sinh^2 E = m^2 + \sum_{1,2,4} \sin^2 k_i$

	b^{odd}	d^{odd}	b^{even}	d^{even}
P	1	-1	1	+1
$\frac{1}{2}(V_1 + V_2)$	1	-1	$\frac{m^2 + \sin^2 k_4}{\sinh^2(E)}$	$\frac{m^2 + \sin^2 k_4}{\sinh^2(E)}$
S	1	-1	$\frac{2m^2}{\sinh^2(E)} - 1$	$\frac{2m^2}{\sinh^2(E)} - 1$
$\frac{1}{2}(A_1 + A_2)$	1	-1	$\frac{m^2 - \sin^2 k_4}{\sinh^2(E)}$	$\frac{m^2 - \sin^2 k_4}{\sinh^2(E)}$

standard staggered; similar for p4

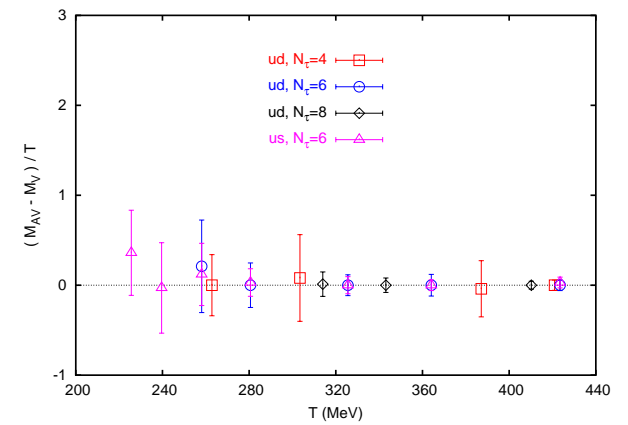
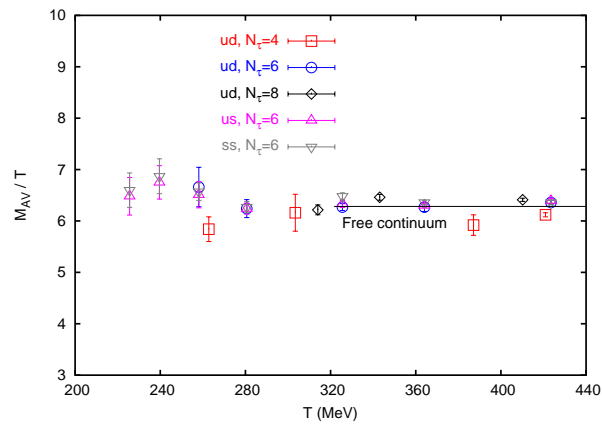
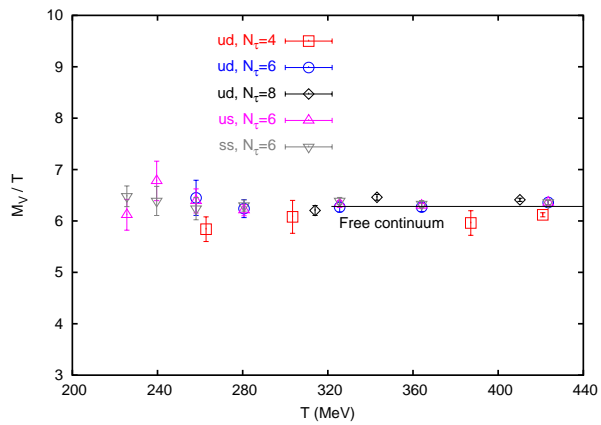


Pseudoscalar - Scalar (connected a_0/δ) sector



- all results at fixed $N_\sigma/N_\tau = 4$
- π and a_0 do not become degenerate up to $T \simeq 240$ MeV

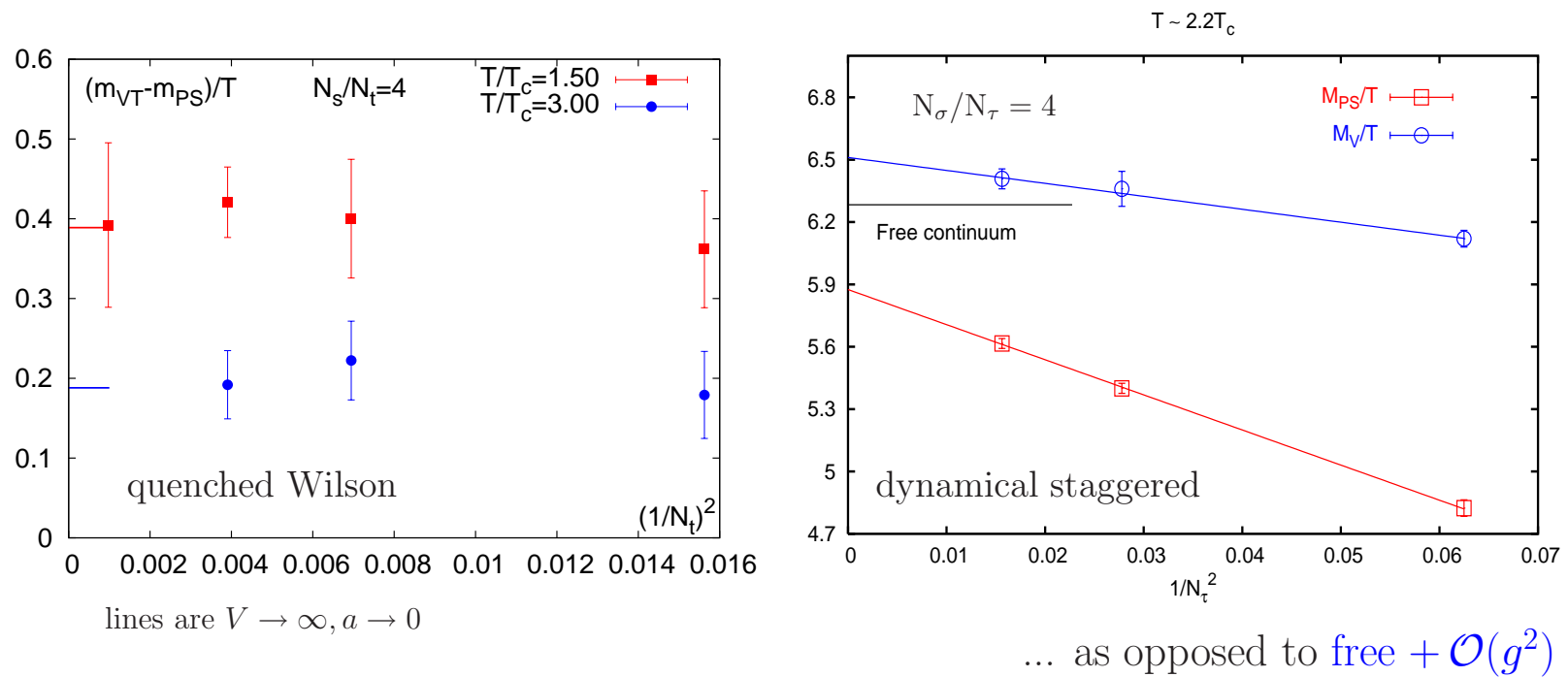
(transverse) Vector - Axialvector channel



- all results at fixed $N_\sigma / N_\tau = 4$: coincidence with 2π is presumably accidental
- V_T and A_T appear degenerate even in the us channel at $T > T_c$

IV Summary

- π and a_0 do not become degenerate at T_c neither in quenched Wilson nor in 2 + 1 dynamical staggered
- up to $T \simeq$ few times T_c : π and ρ do not become degenerate ...



- remains to be done: $V \rightarrow \infty$ for (dynamical) staggered