Controlling Residual Mass in Domain Wall Fermion Simulations

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Motivation

- Perform DWF simulations at strong coupling and with topology change for large volume simulations and thermodynamic simulations near transition
- Achieve selected mres at smaller L_S or smaller mres at selected L_S
  - Measures chiral symmetry breaking due to overlap in 5'th dimension of chiral fermions nominally bound to walls
- Add weighting function to path integral to steer MD trajectories away from gauge configurations where 5D DWF transfer matrix has near unit eigenvalues
  - Unit eigenvalues of 5D DWF transfer matrix lead to relatively undamped propagation and overlap in 5'th dimension

Background

- 4D Wilson fermion, $D_W(N, m_0)$, zero modes lead to unit eigenvalues of 5D DWF transfer matrix
  - transfer matrix is a function of hermitian Wilson Dirac operator, $H_4(m_0)$
- Add a Wilson fermion to action

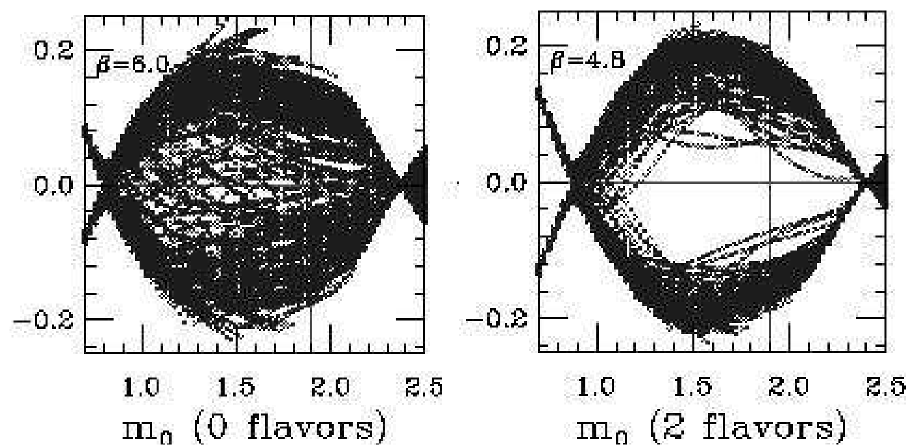
$$S_G(\beta, N) + S_{DWF}(N, L_S, m_0, m_f) + S_{PV}(N, L_S, m_0) + \mathbf{S_W}(\bar{\mathbf{X}}, \mathbf{X}; \mathbf{N}, \mathbf{m_0})$$

- Vranas, 01006v2, 0606014v2: adds Wilson fermions
  - Gap generated for $1 < m_0 < 2$

•Lowest eigenvalues of $H_4(m_0)$ vs m_0 ;

•Same physical lattice spacings;

•Quenched DWF sea fermions



Background (continued)

- Add a Wilson 'boson' with 'twisted' mass cancel unwanted 'ultraviolet' effects of Wilson fermion

$$S_W \Rightarrow S_W(\bar{X}_f, X_f, U; N, -m_0) - S_W(\bar{X}_b, X_b, U; N, -m_0 + \imath\epsilon_b\gamma_5))$$

- weighting term $\mathcal{W}$ in path integral becomes

$$\mathcal{W} = \det(H_W(m_0)H_W^\dagger(m_0)) \Rightarrow \mathcal{W} = \frac{\det(H_W(m_0)H_W^\dagger(m_0))}{\det(H_W(m_0)H_W^\dagger(m_0) + \epsilon_b^2)}$$

- Fukaya et al., 2006, Phys. Rev. D 094505
  - demonstrates clear effect on near 0 eigenvalues but little effect on other eigenvalues

Current approach

- Now add a 'twisted' mass to Wilson fermion different from 'twisted' mass of Wilson 'boson' in order to prevent complete suppression of 0 eigenvalues and allow topology change

$$S_W \Rightarrow S_W(\bar{X}_f, X_f, U; N, -m_0 + \imath\epsilon_f\gamma_5) - S_W(\bar{X}_b, X_b, U; N, -m_0 + \imath\epsilon_b\gamma_5))$$

- weighting term now becomes

$$\mathcal{W} = \frac{\det(H_W(-m_0)H_W^\dagger(-m_0) + \epsilon_f^2)}{\det(H_W(-m_0)H_W^\dagger(-m_0) + \epsilon_b^2)} = \prod_i \frac{\lambda_i^2 + \epsilon_f^2}{\lambda_i^2 + \epsilon_b^2}$$

where λ_i are the eigenvalues of H_W

Current approach (continued)

- Little or no effect ($\mathcal{W} \simeq 1$) if
  - $0 < \epsilon_f \simeq \epsilon_b$ or
  - $0 < \epsilon_f < \epsilon_b \ll |\lambda_i|$
- Suppression of near 0 eigenvalues if
  - N ‘small’ eigenvalues $|\lambda_i| \simeq 0$; remaining eigenvalues ‘large’ $|\lambda_i| > \epsilon_b$; a low but non-zero weight

$$0 < \mathcal{W} = \mathcal{W}_{small}\mathcal{W}_{large} = \simeq \left(\frac{\epsilon_f}{\epsilon_b}\right)^N \prod_{large} \frac{\lambda_i^2 + \epsilon_f^2}{\lambda_i^2 + \epsilon_b^2} \simeq \left(\frac{\epsilon_f}{\epsilon_b}\right)^N \mathcal{W}_{large}$$

- Elimination of near 0 eigenvalues if
  - $\epsilon_f = 0$

Implementation in CPS (Columbia Physics System)

- Complication because CPS ‘preconditions’ Dirac operator into odd-even blocks

$$D_{2N\times 2N} = \begin{pmatrix} m_\epsilon I_{oo} & W_{eo} \\ W_{oe} & m_\epsilon I_{ee} \end{pmatrix} \qquad \begin{array}{c} m_\epsilon = \kappa^{-1}\gamma_5(\theta) \\ \gamma_5(\theta) = \cos\theta + \imath\sin\theta\gamma_5 \end{array}$$

  and Dirac determinant becomes

$$\det(D_{2N\times 2N}) = \det(M_{N\times N}) = \det[\kappa^{-2} I_{oo} - \gamma_5(-\theta) W_{eo} \gamma_5(-\theta) W_{oe}]$$

- Determinant ratio was represented as symmetrized quotient (alg_quotient integrator)

$$S = \phi^\dagger M_b (M_f^\dagger M_f)^{-1} M_b^\dagger \phi \quad \phi = M_b (M_b^\dagger M_b)^{-1} M_f^\dagger \eta \quad \chi = [(M_f^\dagger M_f)^{-1}] M_f^\dagger \eta$$

- The quotient force (derivative of action w.r.t. MD time)

$$\partial_t S = \chi^\dagger \partial_t [M_f^\dagger M_f] \chi + \phi^\dagger \partial_t [M_b] \chi + \chi^\dagger \partial_t [M_b^\dagger] \phi$$

Implementation in CPS (continued)

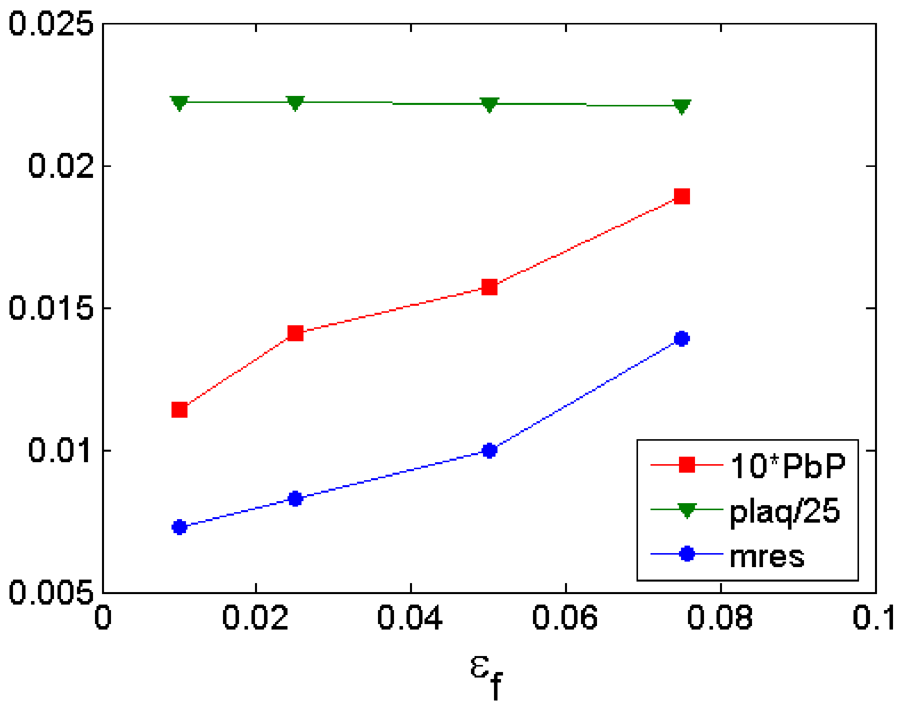
- CPS is object structured; actions and lattices are objects
- Add subclass of the Wilson fermion action class
  - pre/post multiply Dirac operator by $\gamma_5(\theta)$ (since $\gamma_5(\theta)$ does not commute with W)
  - Provide forces in new subclass that take proper account of non-commutativity of $\gamma_5(\theta)$
- Modify quotient 'integrator' class to be aware of new force type

Data - introduction

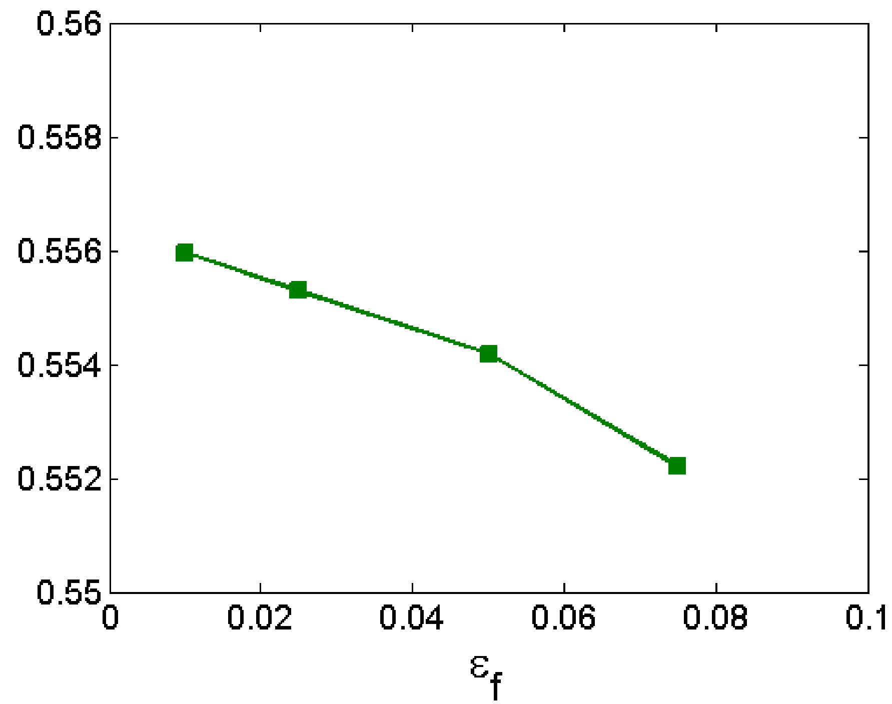
- Data without weighting factor ("old data")
  - QCDOC; Columbia (M. Cheng)
  - $16^3 \times 8 \times 32;\ m_0 = 1.8;\ m_l = 0.003;\ m_s = 0.037$; Iwasaki action; $\beta = 1.95, 2.00, 2.0375, 2.05, 2.08, 2.11, 2.14$
- Data with weighting factor ("new data")
  - 'New York Blue'; Brookhaven National Laboratory
  - $16^3 \times 8 \times 16, 32;\ m_0 = 1.8;\ m_l = 0.003;\ m_s = 0.037$; Iwasaki action; $\beta = 1.95, 2.00;\ \epsilon_b = 0.10;\ \epsilon_f = 0.01, 0.25, 0.05, 0.075$
- Initial experimentation with weighting factor
  - find optimum simulation parameters first

Data I – raw data

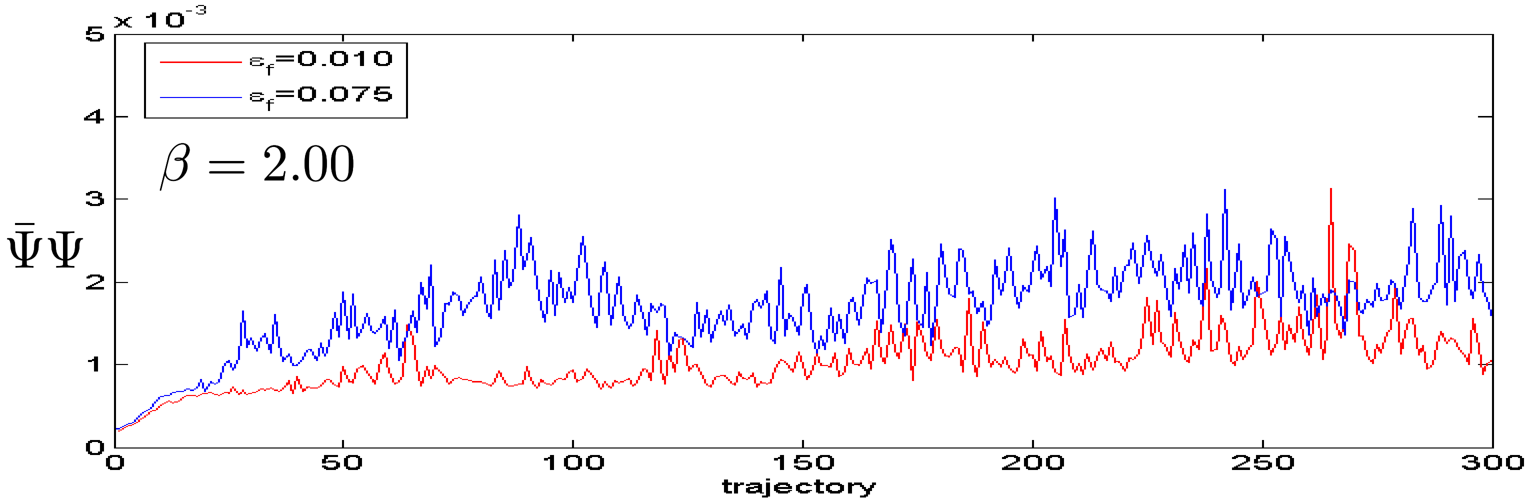
- Scaled $\bar{\Psi}\Psi$, plaquette, mres
- Lattice scale changing

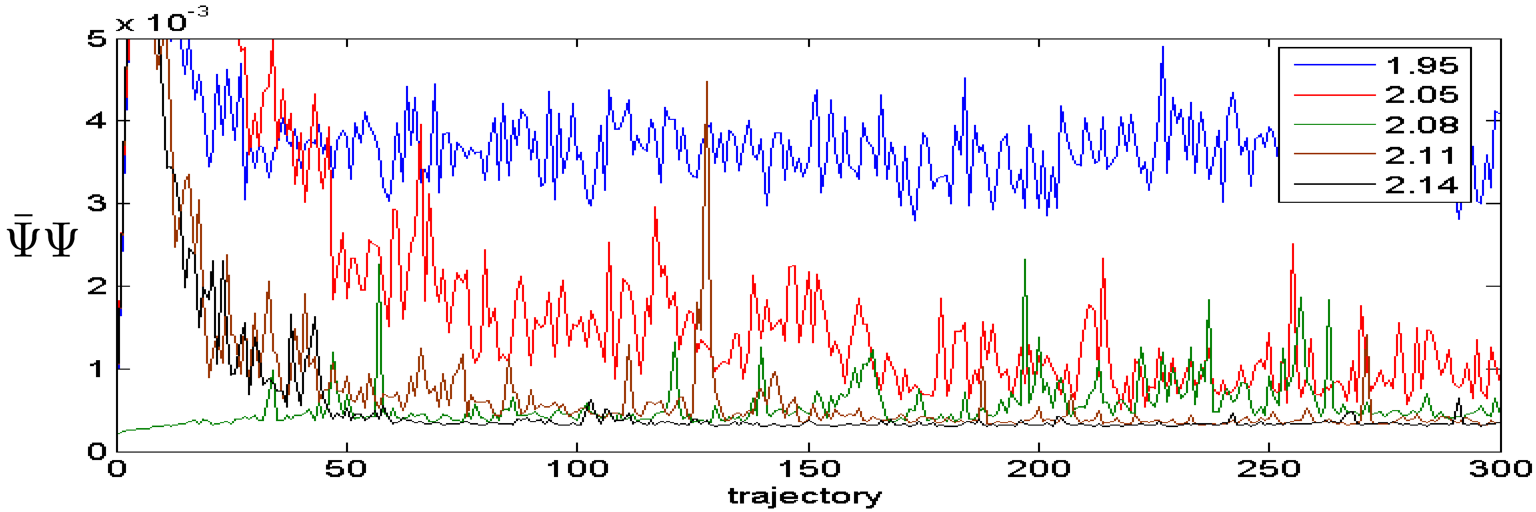


- Expanded plaquette
- Gauge fields smoother at higher ϵ_f



Data I (continued)



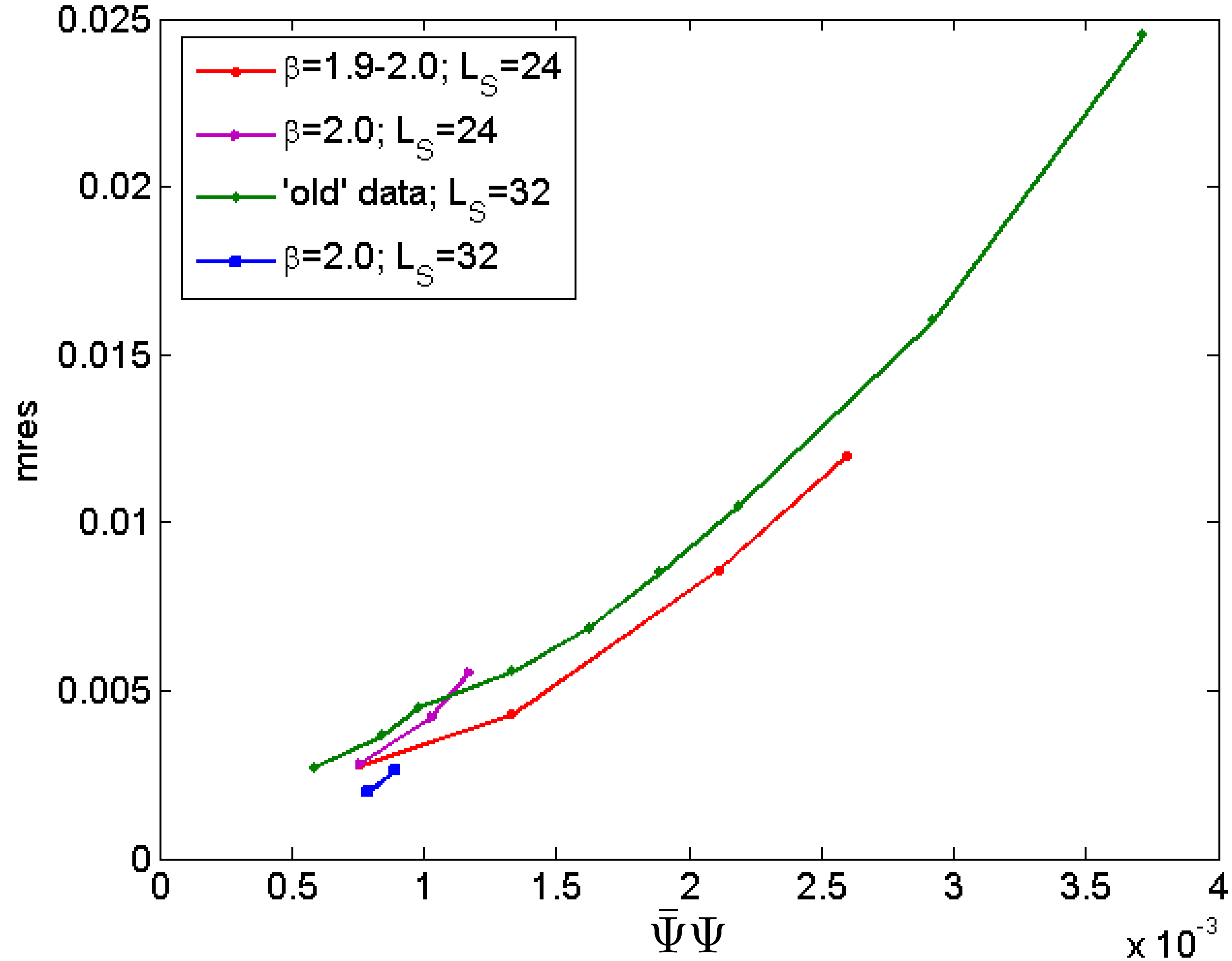


Data II

- Question: how much of change in mres, $\bar{\Psi}\Psi$ is due to lattice scale change and how much due to specific effects of suppressing 0 eigenvalues
- Obtain qualitative indications by comparing mres at equal $\bar{\Psi}\Psi$ values or at equal plaquette values
  - Assuming that $\bar{\Psi}\Psi$, plaquette reflect lattice scales to some degree
    - Note different L_S values
  - Scaled beta comparison

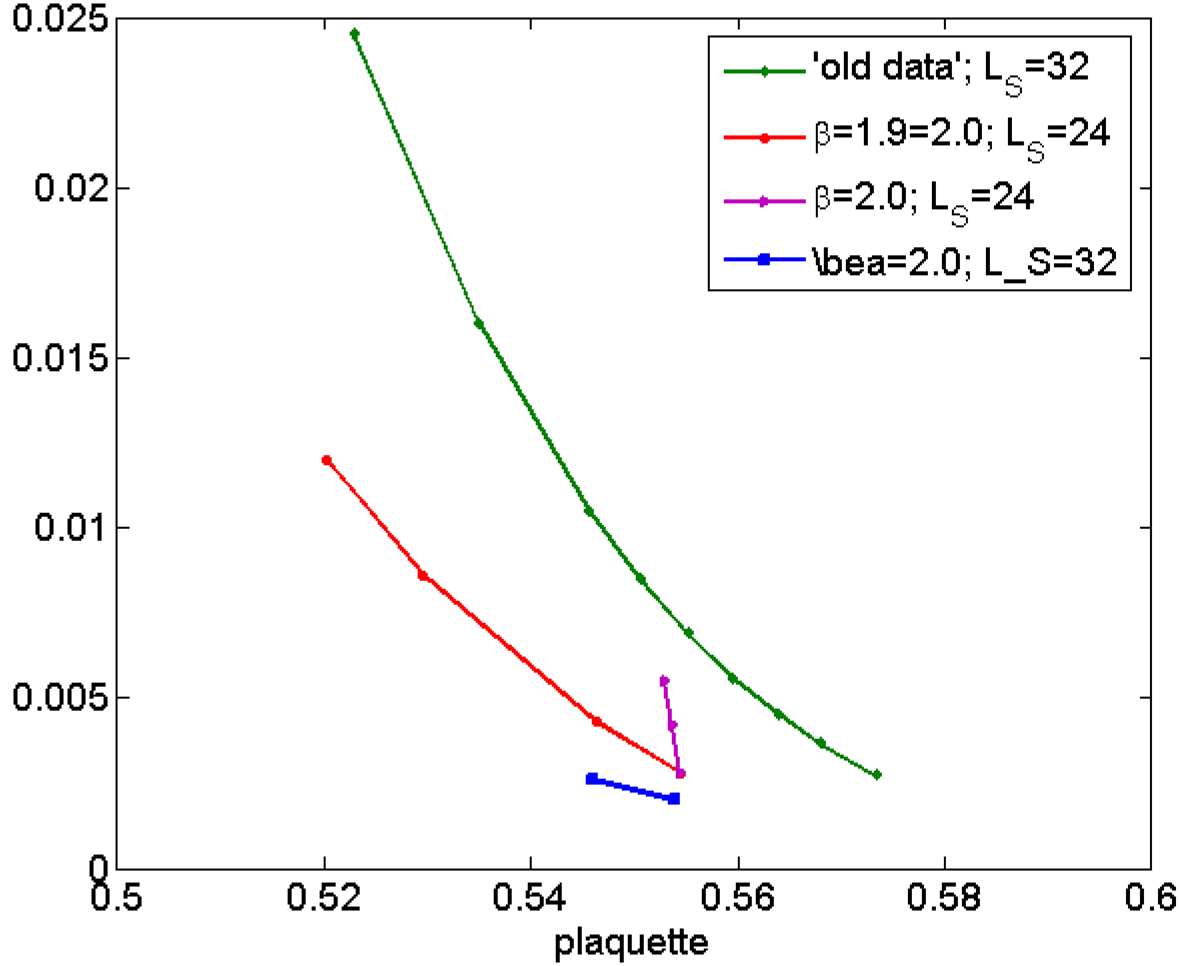
Data III – equal $\bar{\Psi}\Psi$ comparison

- ‘new’ data at $L_S = 24$ is equal to or better than ‘old’ data at at $L_S = 32$



Data IV – equal plaquette comparison

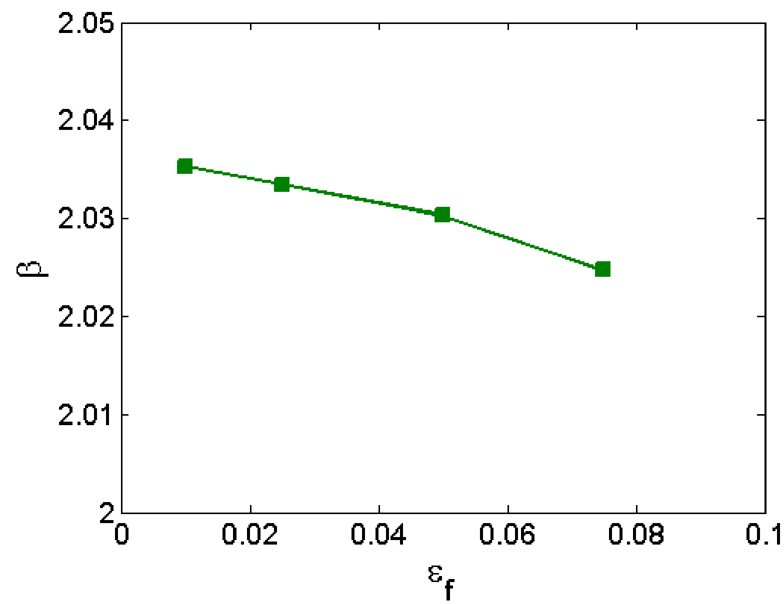
- ‘new’ data at $L_S = 24$ is better than ‘old’ data at $L_S = 32$



Data V – scaled β comparison

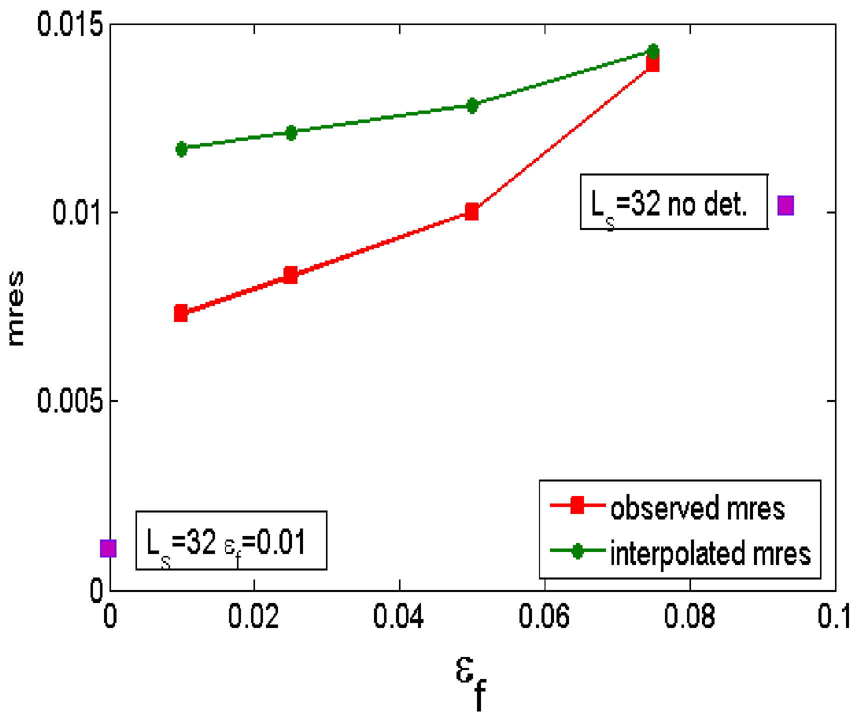
- determine 'effective' beta for each ϵ_f using relation of plaquette and beta from old data
- interpolate mres & $\bar{\Psi}\Psi$ to 'effective' beta using relations of mres & $\bar{\Psi}\Psi$ and beta from old data
- scale for visibility
- admittedly ad hoc

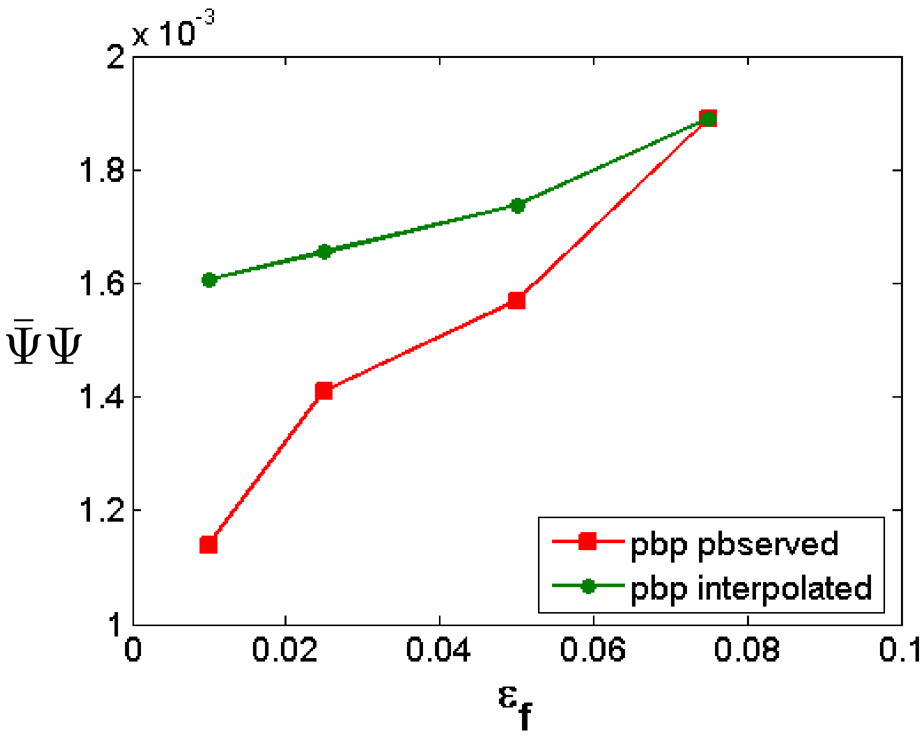
- 'effective' beta vs ϵ_f
- $\Delta\beta \sim 0.01/2.00$



Data VI (continued)

- observed mres vs. mres at 'effective' beta
- observed $\bar{\Psi}\Psi$ vs. $\bar{\Psi}\Psi$ at 'effective' beta





Conclusions

- Implemented weighting factor with ratio of ‘twisted’ mass Wilson fermions
  - Characterization on going
    - current emphasis on finding optimum simulation parameters
- Achieved 2 fold reduction of mres (most recent data)
  - Further improvements expected
- Future directions

$$\left\{ \frac{\det(H_W(-m_0)H_W^\dagger(-m_0) + \epsilon_f^2)}{\det(H_W(-m_0)H_W^\dagger(-m_0) + \epsilon_b^2)} \right\}^\gamma$$