

# Hadronic contribution to $g-2$ from twisted mass fermions

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## Muon g-2

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- muon anomalous magnetic moment

$$a_\mu = (g - 2)/2 = F_2(0) = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)$$

- experimental measurement at BNL\*

$$a_\mu^{\text{ex}} = 11659208.0(6.3) \times 10^{-10} \text{ [0.54 ppm]}$$

- standard model prediction† using  $e^+e^- \rightarrow \text{hadrons}$

$$a_\mu^{\text{th}} = 11659179.3(6.8) \times 10^{-10} \text{ [0.58 ppm]}$$

- discrepancy between theory and experiment

$$a_\mu^{\text{ex}} - a_\mu^{\text{th}} = 28.7(9.3) \times 10^{-10} \text{ [3.1 } \sigma \text{]}$$

- leading order hadronic (had) contribution dominates theory error

$$a_\mu^{\text{had}} = 692.1(5.6) \times 10^{-10} \text{ [60\% of theory error]}$$

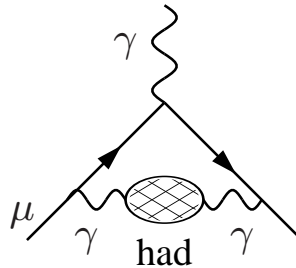
\*Muon G-2, PRD73:072003, 2006

†e.g. review by Jegerlehner, Acta.Phys.Polon.B38:3021, 2007

# Hadronic Vacuum Polarization

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- vacuum polarization by quarks or equivalently hadrons



- vacuum polarization tensor

$$\pi_{\mu\nu}(q^2) = \int d^4x e^{iq \cdot (x-y)} \langle J_\mu(x) J_\nu(y) \rangle = (q_\mu q_\nu - q^2 \delta_{\mu\nu}) \pi(q^2)$$

- leading order hadronic contribution\*

$$a_\mu^{\text{had}} = \alpha^2 \int_0^\infty \frac{dq^2}{q^2} w(q^2/m_\mu^2) (\pi(q^2) - \pi(0))$$

- $w(q^2/m_\mu^2)$  is maximum at  $q^2 = (\sqrt{5} - 2)m_\mu^2 \approx 0.003 \text{ GeV}^2$
- lowest momentum on lattice is  $q_{\text{min}}^2 = (2\pi/L)^2 \approx 0.05 \text{ GeV}^2$

\*Blum, PRL95:052001, 2003

# Twisted Fermions

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- continuum twisted QCD differs from QCD by a field transformation

$$\chi^{\text{tw}} = \exp(i\gamma_5\tau_3\theta)\chi^{\text{ph}} \quad \bar{\chi}^{\text{tw}} = \bar{\chi}^{\text{ph}} \exp(i\gamma_5\tau_3\theta)$$

- we use the maximally twisted Wilson action\*

$$S = \sum_x \bar{\chi}_x^{\text{tw}} [D_W(\kappa_c) + i\mu\gamma_5\tau_3] \chi_x^{\text{tw}}$$

- twisted quark mass  $\mu$  provides an infrared regulator:  $\det(D^\dagger D) \geq \mu^2$
- physical observables are accurate to  $\mathcal{O}(a^2)$  at maximal twist

\*e.g. review by Shindler, Phys.Rept.461:37, 2008

# Electromagnetic Currents

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- physical charge (and flavor) currents remain unchanged under twisting

$$Q\gamma_\mu = \exp(-i\gamma_5\tau_3\theta)Q\gamma_\mu\exp(-i\gamma_5\tau_3\theta) \quad \text{for } Q = 1, \tau_3$$

- we use the conserved vector current in the twisted basis

$$J_\mu^{\text{lc,ph}} \stackrel{a \rightarrow 0}{=} J_\mu^{\text{lc,tw}} \stackrel{a \rightarrow 0}{=} J_\mu^{\text{cc,tw}}$$

- conserved current has same point-split form as for Wilson fermions

$$J_{\mu x}^{\text{tw}} = \frac{1}{2} \left\{ \bar{\chi}_{x+\mu}^{\text{tw}}(r + \gamma_\mu) \chi_x^{\text{tw}} - \bar{\chi}_x^{\text{tw}}(r - \gamma_\mu) \chi_{x+\mu}^{\text{tw}} \right\}$$

- modified  $\gamma_5$  hermiticity,  $\gamma_5 D_u^\dagger \gamma_5 = D_d$ , requires twice the inversions

$$\gamma_\mu D_u^{-1}(x, y) \gamma_\nu D_u^{-1\dagger}(y, x) = \gamma_\mu D_u^{-1}(x, y) \gamma_\nu \gamma_5 D_d^{-1}(x, y) \gamma_5$$

# Flavor Breaking

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- $\gamma_5$  hermiticity,  $\gamma_5 D_u^\dagger \gamma_5 = D_d$ , relates  $u$  and  $d$  quark loops

$$\pi_{\mu\nu}^d(x, y) = \pi_{\mu\nu}^{u*}(x, y)$$

- explicit flavor symmetry breaking is removed by retaining only real part

$$\text{re}(\pi^d(q^2)) = \text{re}(\pi^u(q^2))$$

- this is true for each background gauge field
- real part accurate to  $\mathcal{O}(a^2)$  but imaginary part likely only  $\mathcal{O}(a)$
- implicit flavor breaking remains because the  $m_\rho^\pm \neq m_\rho^0$
- additionally, explicit parity and time-reversal breaking are eliminated

# Lattice Calculation

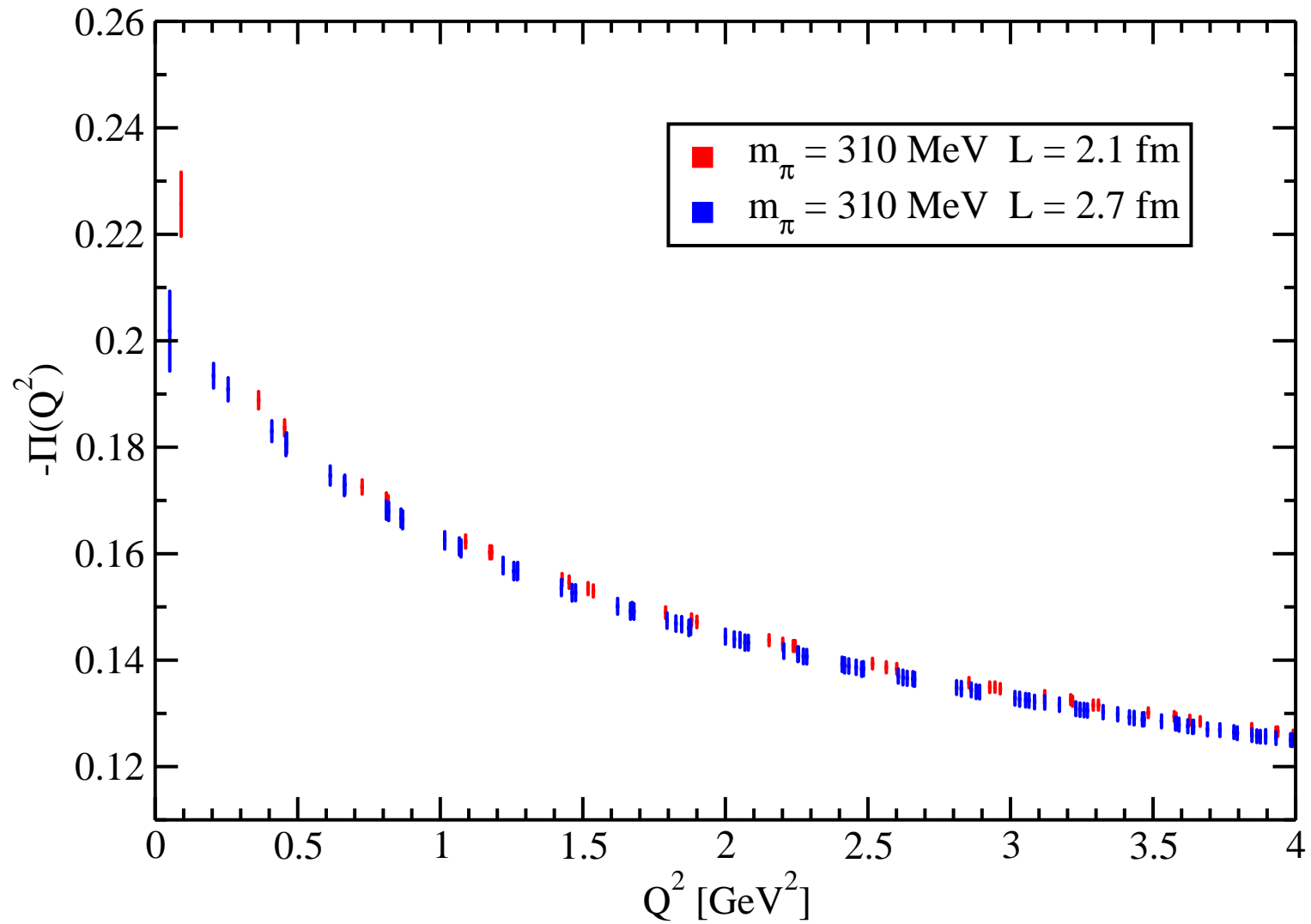
- $N_F = 2$  maximally twisted mass fermions from ETMC\*

$\beta$	$a\mu$	$V/a^4$	$a$	$L$	$m_\pi$	$m_\pi L$	$N_{\text{traj}}$
			fm	fm	MeV		
$m_\pi$ dependence							
3.9	0.0100	$24^3 \times 48$	0.086	2.1	480	5.0	120
3.9	0.0085	$24^3 \times 48$	0.086	2.1	450	4.7	207
3.9	0.0064	$24^3 \times 48$	0.086	2.1	390	4.1	139
3.9	0.0040	$24^3 \times 48$	0.086	2.1	310	3.3	178
3.9	0.0030	$32^3 \times 64$	0.086	2.7	270	3.7	101
$V$ dependence							
3.9	0.0040	$32^3 \times 64$	0.086	2.7	310	4.3	124
$a$ dependence							
4.05	0.0030	$32^3 \times 64$	0.067	2.1	310	3.3	104

- we calculate with degenerate u, d and s

\*Jansen and Urbach, PoS LAT2006:203, 2006

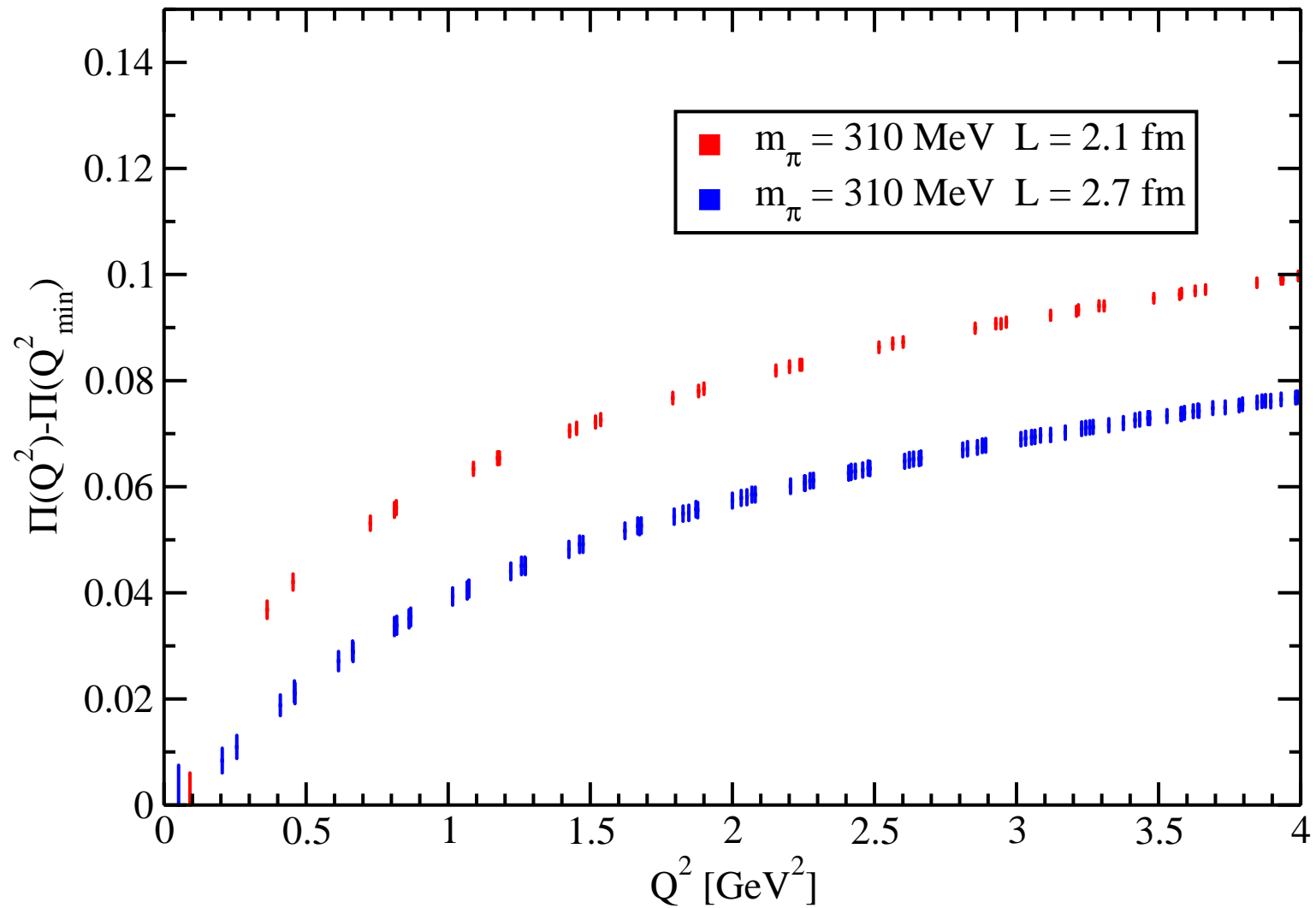
## $L$ Dependence



- no noticeable finite size effects except for lowest  $q^2$

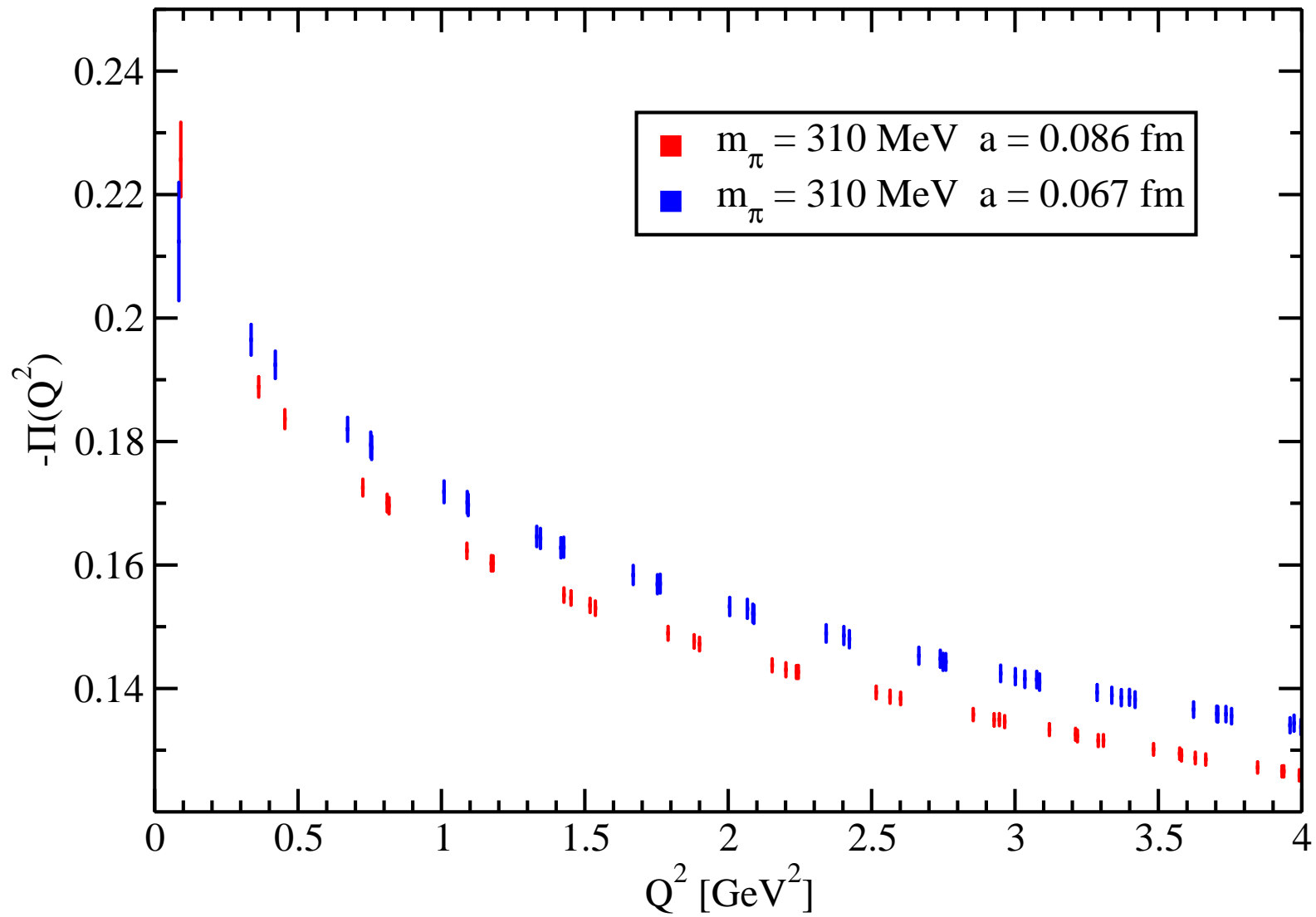


## $L$ Dependence



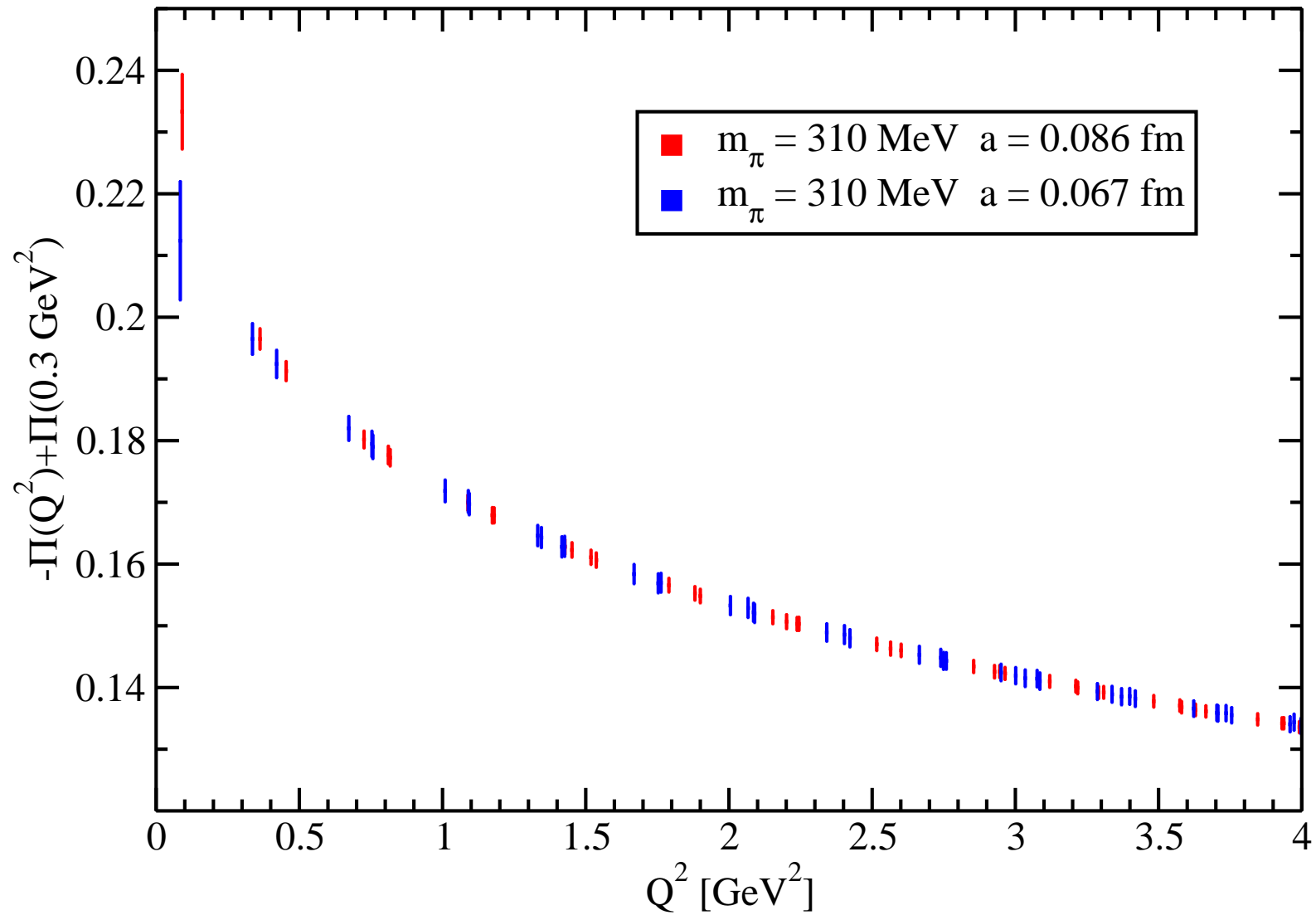
- finite size effect is much larger in renormalized result

## $a$ Dependence



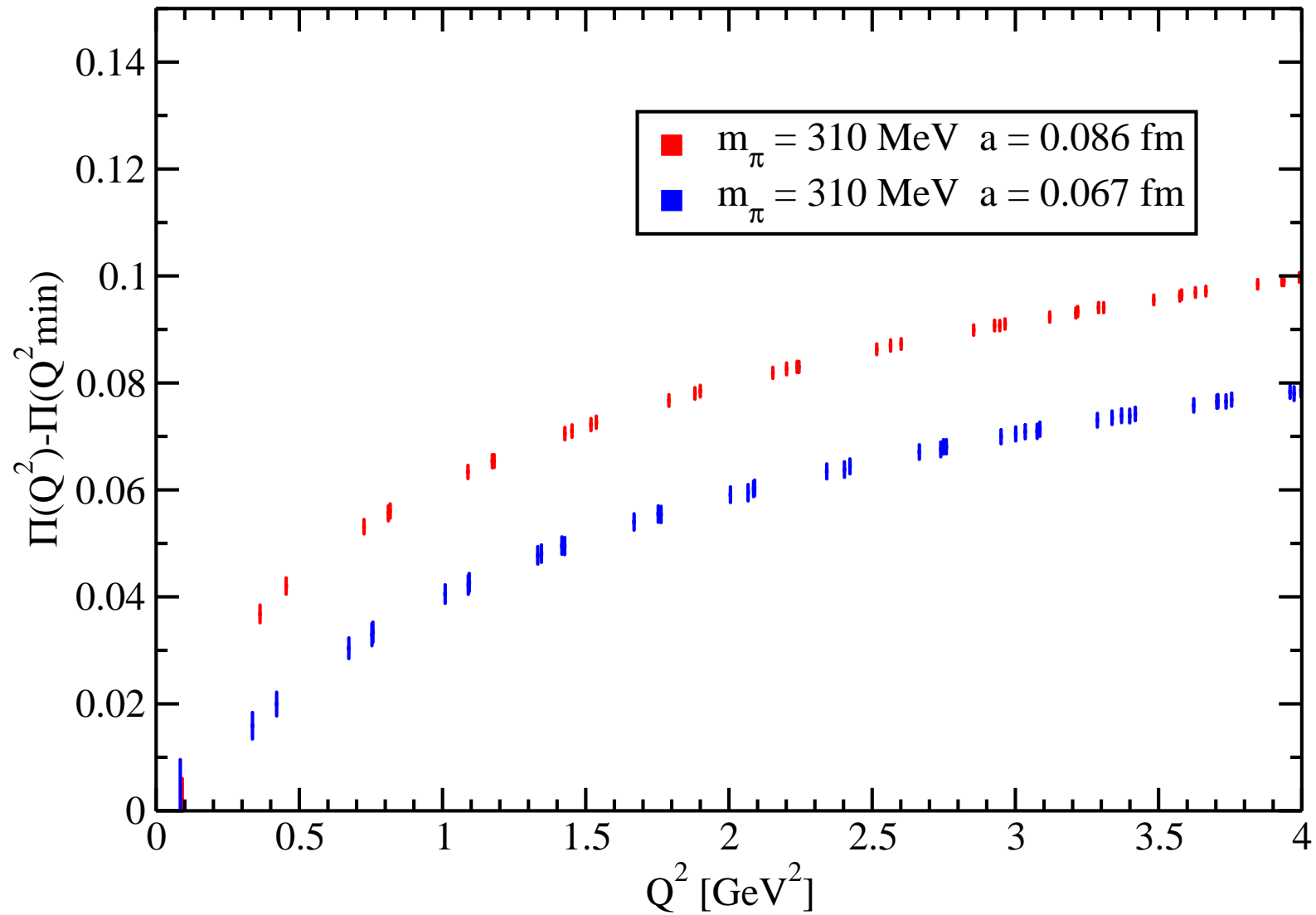
- bare results differ for different  $a$

## $a$ Dependence



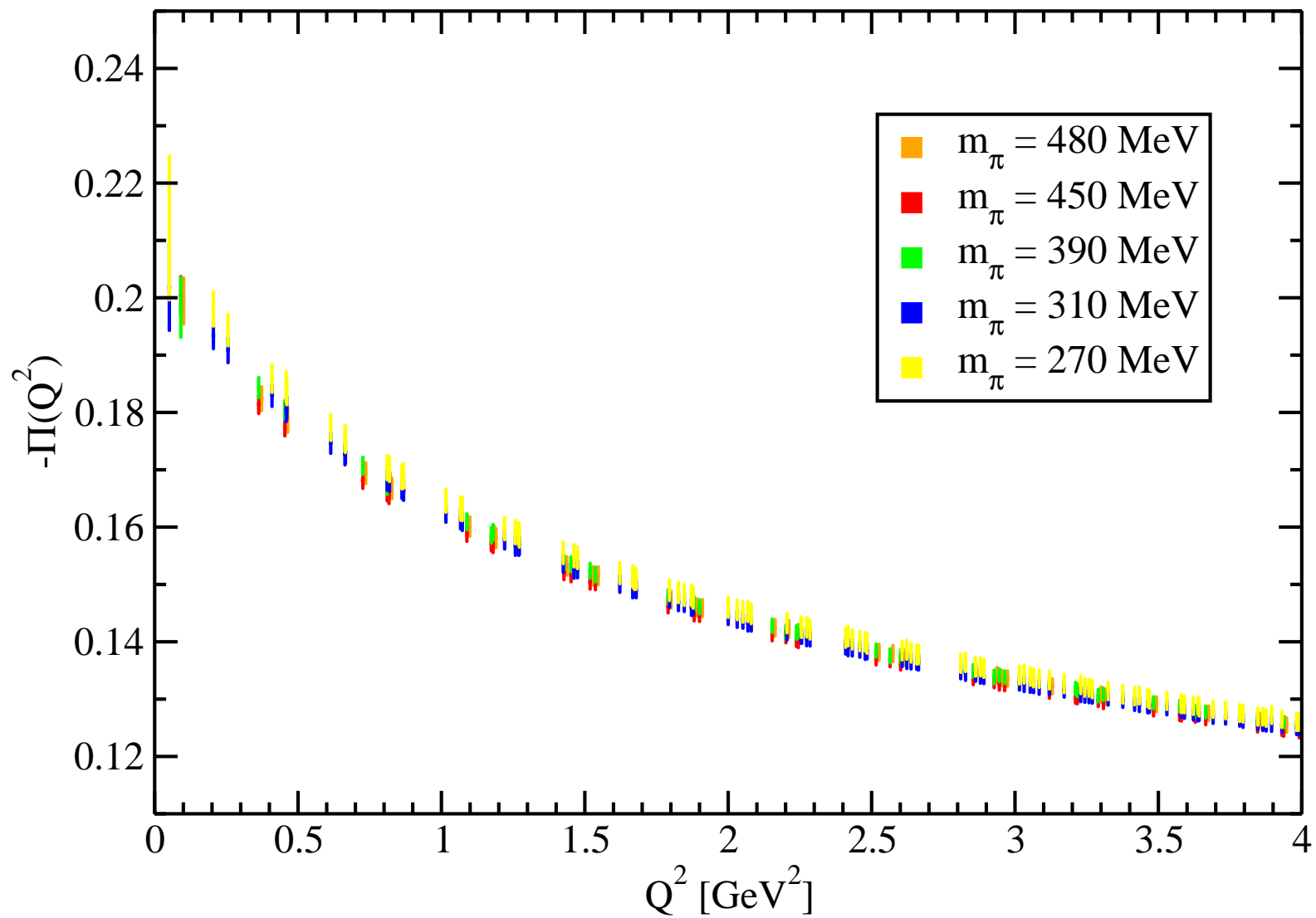
- shape agrees for all but the lowest  $q^2$

## $a$ Dependence



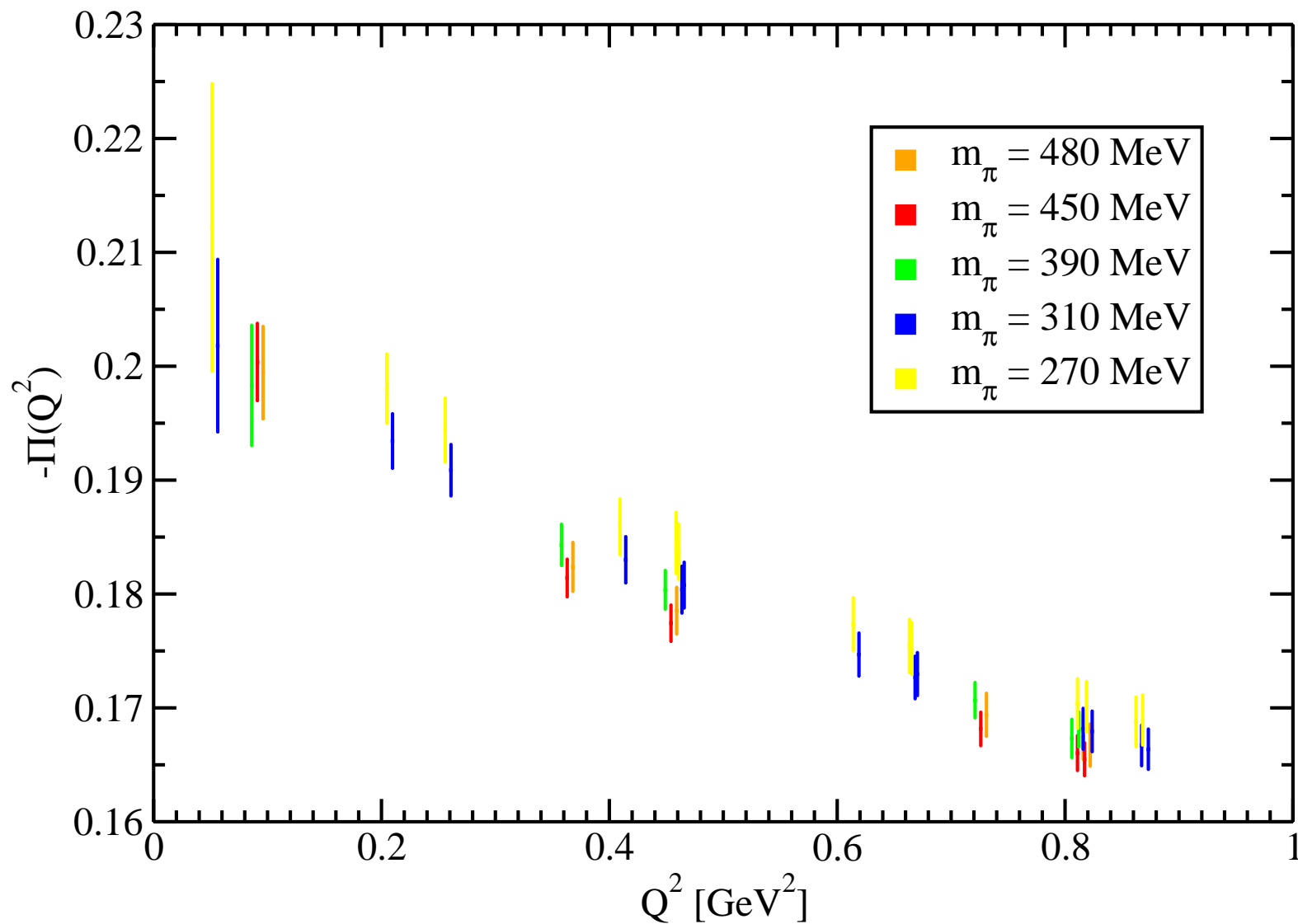
- lattice artifact is much larger in renormalized result

## $m_\pi$ Dependence



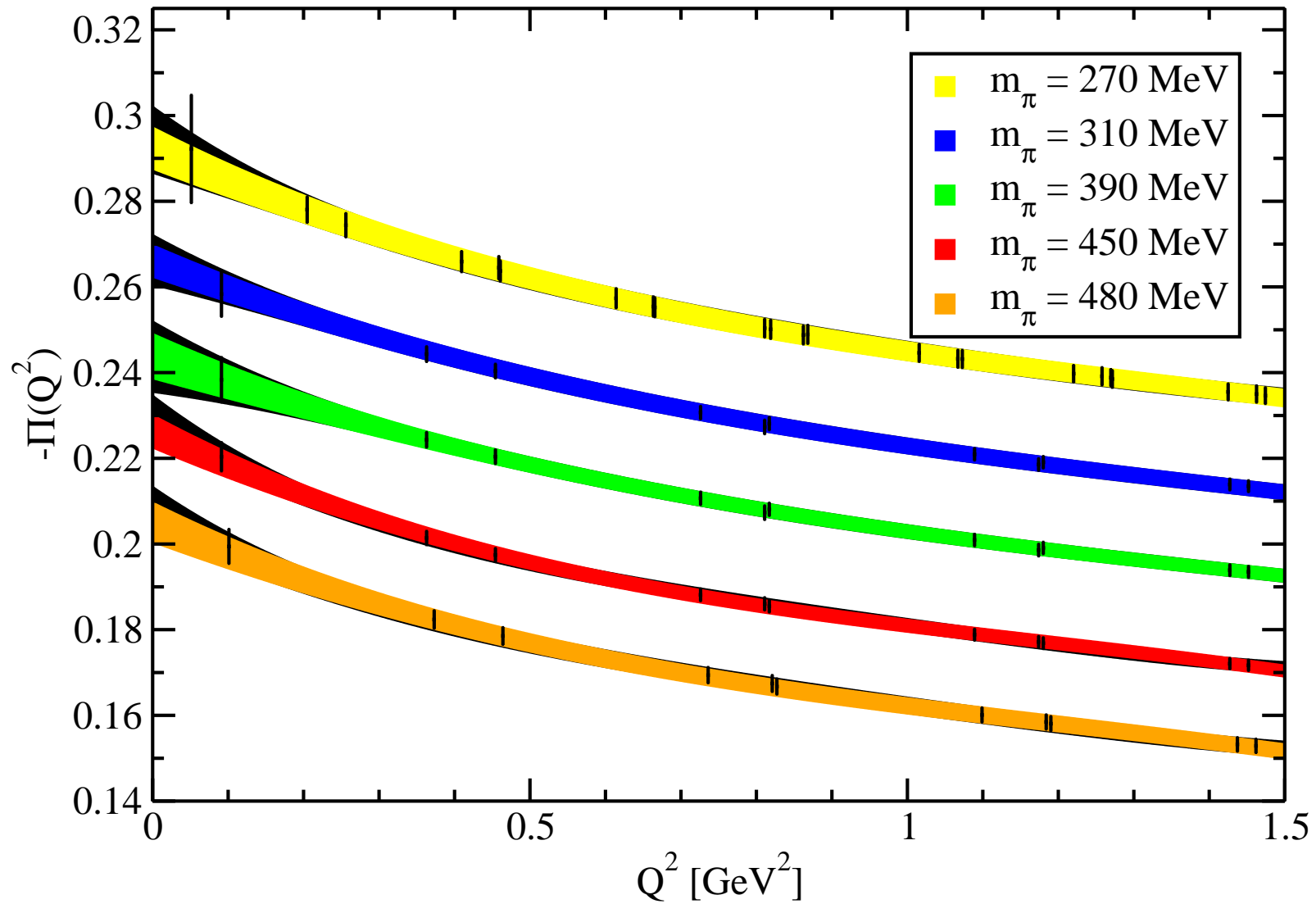
- no  $m_\pi$  dependence for large  $q^2$

## $m_\pi$ Dependence

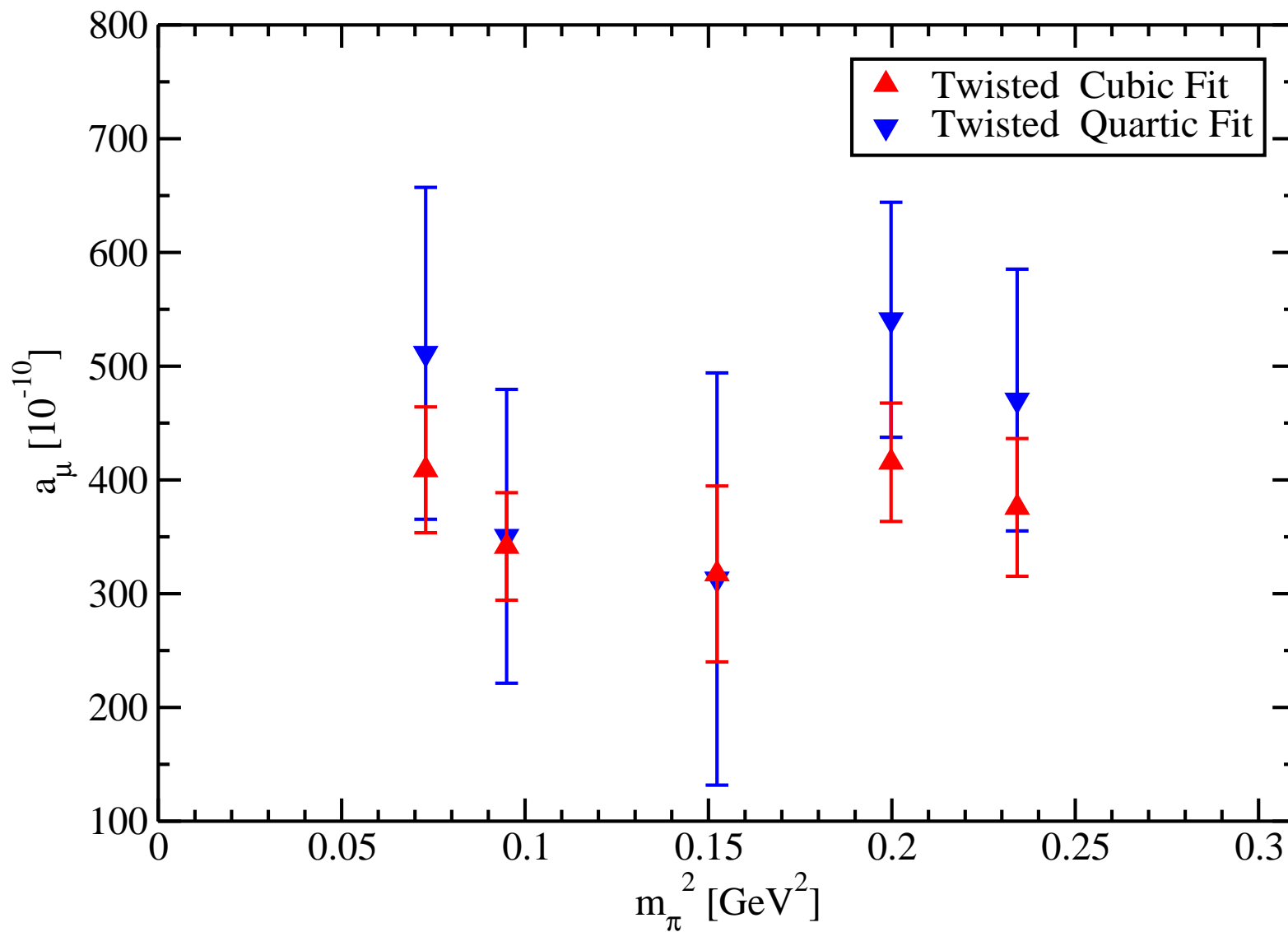


- systematic (but within errors) shift upward for  $m_\pi \leq 450$  MeV

# Cubic and Quartic Fits



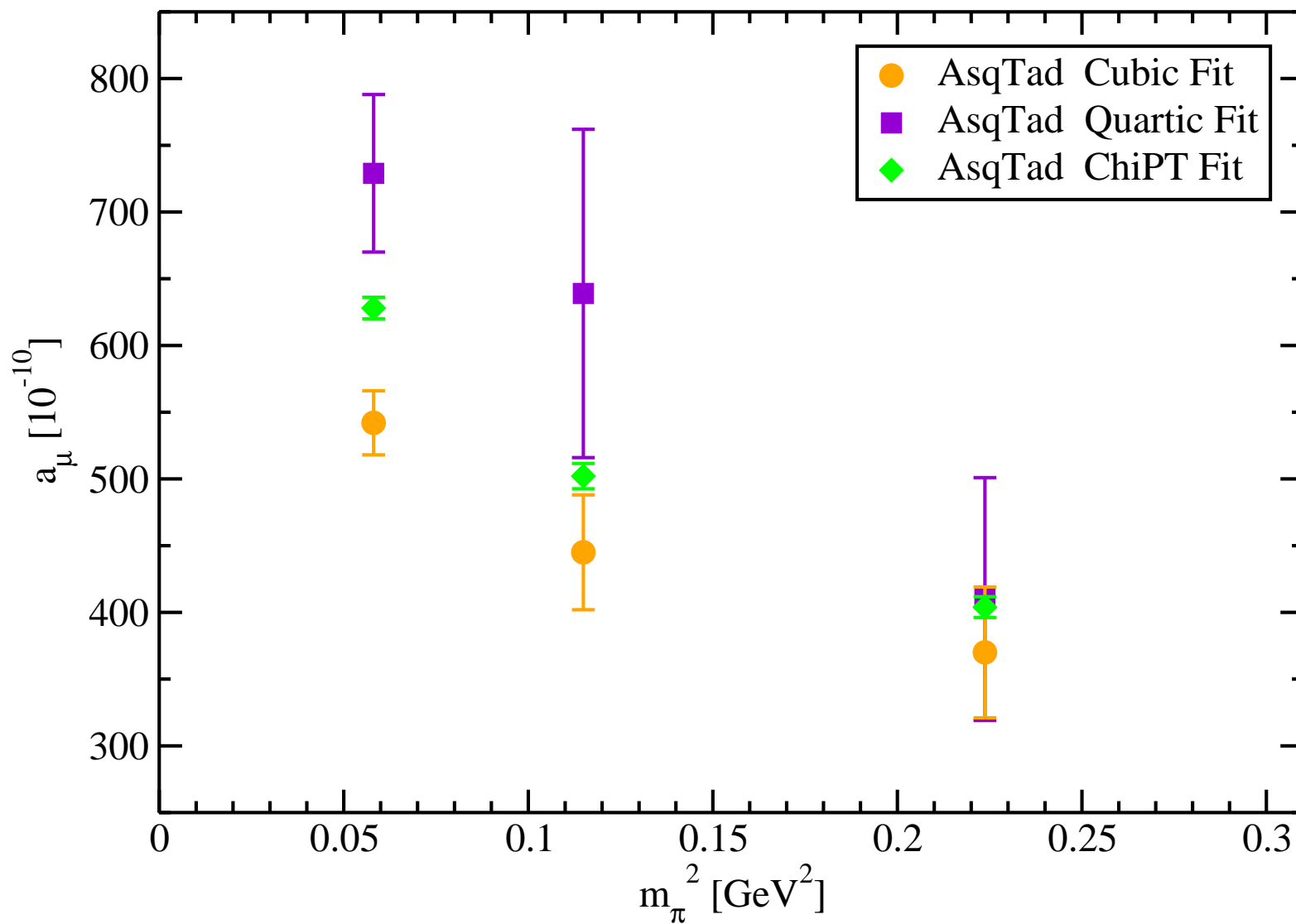
- cubic fits are colored, quartic fits are black



- cubic and quartic fits agree to within errors

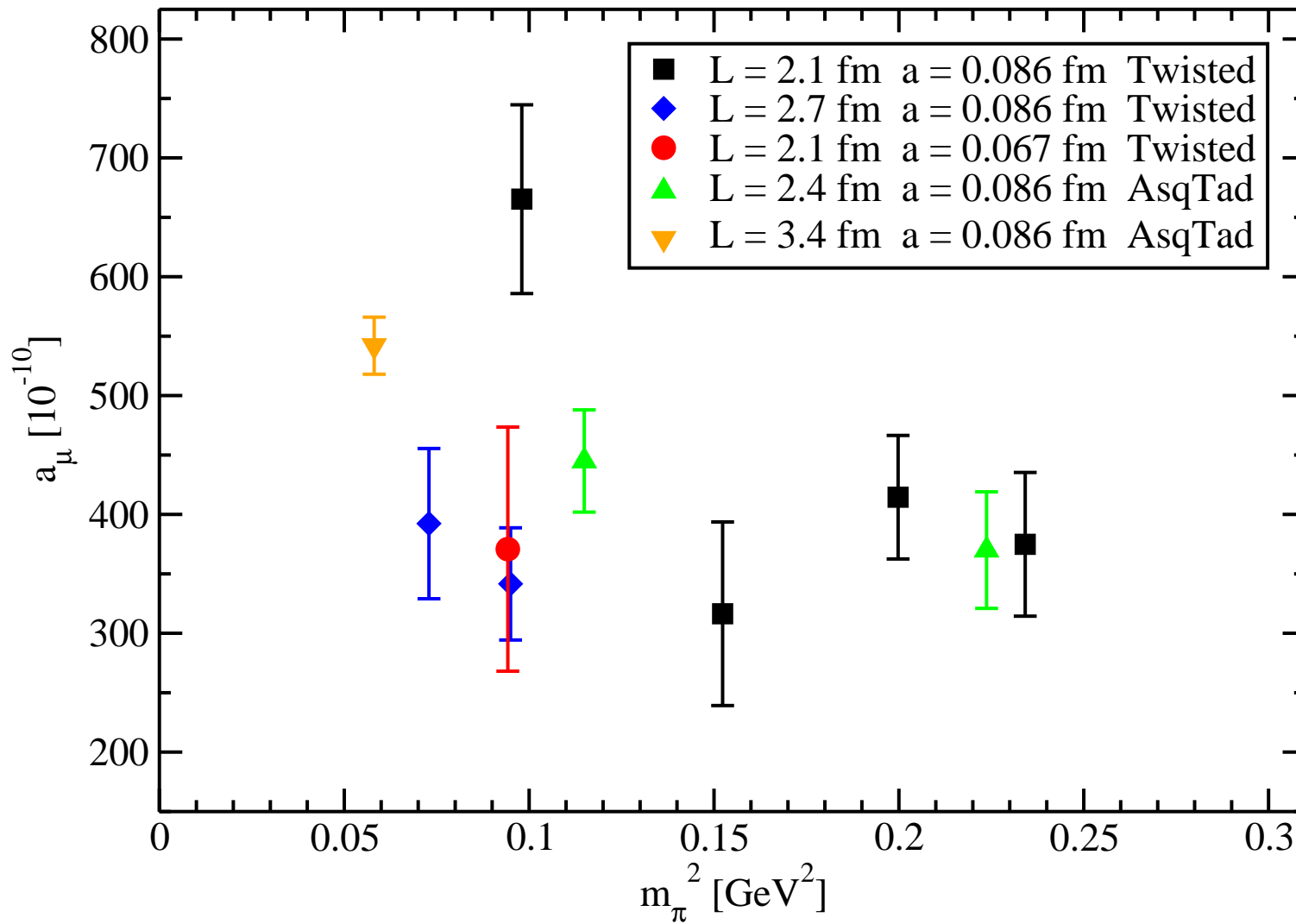


$$a_\mu^{\text{had}}$$



- systematic shift between fits at lightest  $m_\pi$

$$a_\mu^{\text{had}}$$



- comparison of cubic fits only

## Conclusions

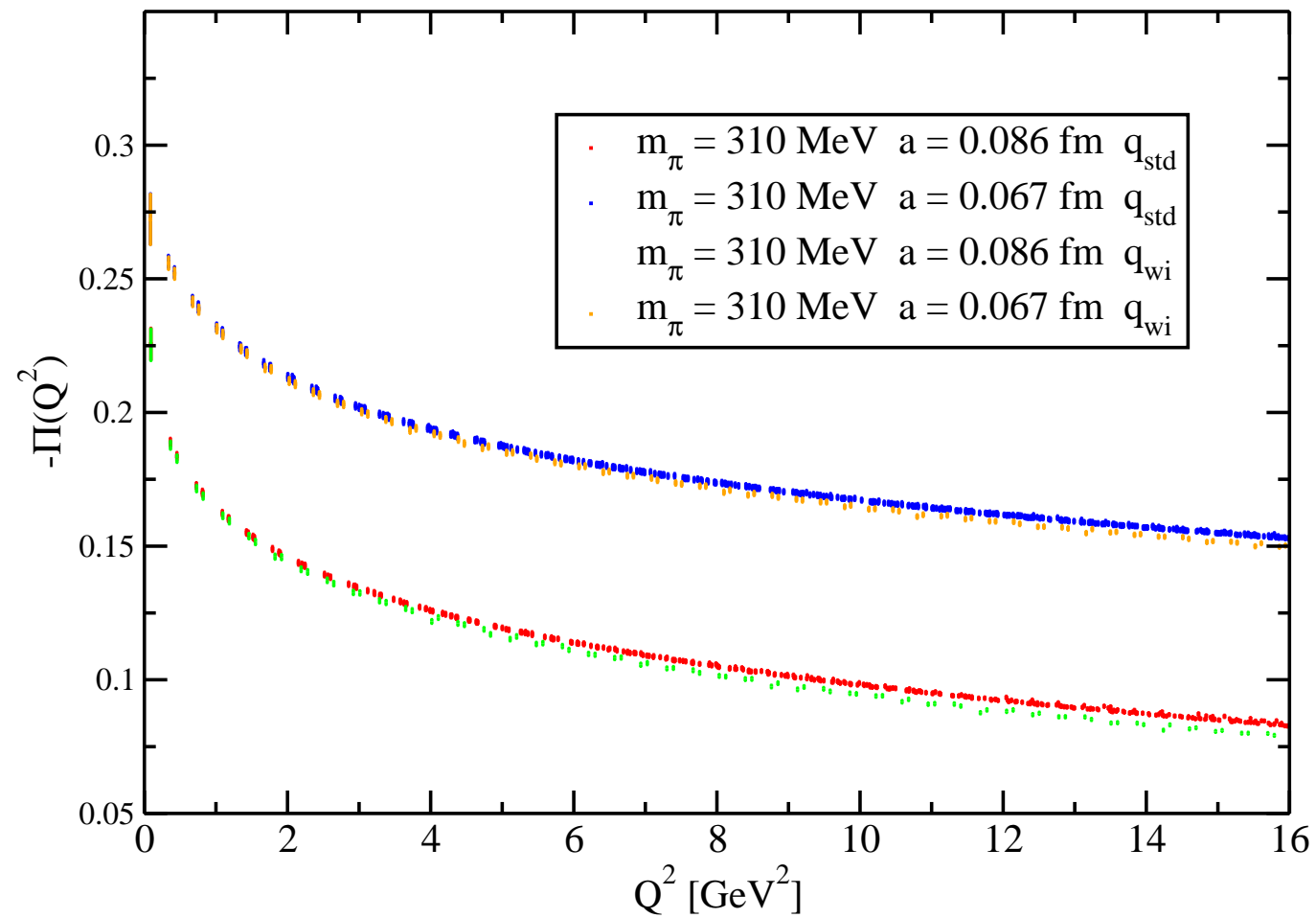
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- we calculate the leading order hadronic contribution to the muon anomalous magnetic moment with  $N_F = 2$  maximally twisted fermions
- the impact of explicit flavor (parity and time-reversal) breaking on the calculation are understood
- we emphasize the need to carefully examine systematic errors due to not only the low  $q^2$  extrapolation but also the finite size effects and lattice artifacts
- we have a finer lattice spacing, additional volumes at the current spacing, and more statistics available

## Extra Slides

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## $q^2$ and $\hat{q}^2$ Dependence



$$a_\mu^{\text{had}}$$

