Parton Distribution Amplitudes

An Update on Results and Non-perturbative Renormalisation

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Lattice 2008

Williamsburg

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Meson distribution amplitudes

- contain non-perturbative QCD effects appearing in hard exclusive processes
- are universal hadronic properties
- important for form factors at large q^2, B-decays

$\longrightarrow$ Lattice can provide the lowest moments of the distribution amplitudes

This talk will cover

- an update on our previous results for pseudo-scalar distribution amplitudes: Kaon, π PoS(LAT07)369
- our first results on vector distribution amplitudes: K^*, ρ, ϕ (for longitudinal polarisation)

Introduction

Quantities like quark masses and weak matrix elements require renormalisation

$\longrightarrow$ to obtain meaningful physical results, convert bare lattice results to a continuum scheme like $\overline{\mathrm{MS}}$.

Up to now we only renormalise perturbatively (known caveat)

We now use the Rome-Southampton RI-MOM technique – underlying idea: find a simple renormalisation condition useful for any regularisation, facilitating scheme changes. NPB445, 81

Use continuum perturbation theory to remove scale dependence and to perform matching to $\overline{\mathrm{MS}}$. NPB662, 247, NPB667, 242

To improve statistics: use momentum sources. NPB544, 699

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The presented work is part of the UKQCD/RBC programme, so we use

- $N_f = 2 + 1$ Domain Wall Fermions, $24^3 \times 64 \times 16$ and $16^3 \times 32 \times 16$

  $$am_s = 0.04,\ am_q = 0.03, 0.02, 0.01, (0.005)$$

  corresponding to pion masses of

  $$672\,\mathrm{MeV},\ 557\,\mathrm{MeV},\ 419\,\mathrm{MeV},\ \text{and } 331\,\mathrm{MeV}$$

  Physical strange quark is $\underline{am_s = 0.0343(16)}$

- Iwasaki gauge action, lattice spacing of $a^{-1} = 1.729(28)\,\mathrm{GeV}$, resulting in spatial volumes $(2.74\mathrm{fm})^3$ and $(1.83\mathrm{fm})^3$

Details on the ensembles: PRD76, 014504 and arXiv:0804.0473

For our other NPR projects, see(n) talks of

→ Y. Aoki (non-exceptional momenta) and

→ C. Kelly (B_K)

Distribution Amplitudes

DAs are matrix elements of suitable non-local light cone operators, e.g. for pseudo-scalars

$$\langle 0|\, \overline{q}(z)\gamma_\mu\gamma_5\,\mathcal{U}\, u(-z)\, |\Pi^+(p)\rangle\Big|_{z^2=0} = \mathrm{i} f_\Pi p_\mu \int_{-1}^{1} \mathrm{d}\xi\, e^{\mathrm{i}\xi pz}\, \phi_\Pi(\xi,\mu)\,, \quad \xi = 2x-1\,,$$

$$\int_{-1}^{1} \mathrm{d}\xi\, \phi_\Pi(\xi,\mu) = 1\,, \quad \phi_\Pi(\xi,\mu) = \tfrac{3}{4}(1-\xi^2)\left(1 + \sum a_n^\Pi C_n^{3/2}(\xi)\right)\,.$$

The lattice can only access moments thereof

$$\boxed{\langle \xi^n \rangle_\Pi(\mu) = \int \mathrm{d}\xi\, \xi^n\, \phi_\Pi(\xi,\mu)\,,}$$

related to local matrix elements

$$\langle 0|\, \mathcal{O}^5_{\{\mu\mu_1\cdot\cdot\mu_n\}}(0)\, |\Pi^+\rangle = -\mathrm{i}^{n+1} f_\Pi\, p_{\{\mu}\cdot\cdot p_{\mu_n\}}\, \langle \xi^n \rangle_\Pi\,,$$

$$\mathcal{O}^5_{\{\mu\mu_1\cdot\cdot\mu_n\}}(0) = \overline{q}(0)\gamma_{\{\mu}\gamma_5 \overleftrightarrow{D}_{\mu_1}\cdot\cdot\overleftrightarrow{D}_{\mu_n\}}\, u(0)\,.$$

Distribution Amplitudes

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Extracting Bare Values from Lattice Data – Pseudo-Scalars

$$\frac{\sum_x e^{\mathrm{i}\vec{p}\vec{x}} \langle 0| \mathcal{O}^5_{\{\mu\nu\}}(t,\vec{x}) P^\dagger(0) |0\rangle}{\sum_x e^{\mathrm{i}\vec{p}\vec{x}} \langle 0| \mathcal{O}^5_{\rho}(t,\vec{x}) P^\dagger(0) |0\rangle} \longrightarrow \mathrm{i}\frac{p_\mu p_\nu}{p_\rho} \langle \xi \rangle^{\mathrm{bare}}$$

with directions $\mu = 1, 2, 3$, $\nu, \rho = 4$ and one unit of momentum

$$\frac{\sum_x e^{\mathrm{i}\vec{p}\vec{x}} \langle 0| \mathcal{O}^5_{\{\rho\mu\nu\}}(t,\vec{x}) P^\dagger(0) |0\rangle}{\sum_x e^{\mathrm{i}\vec{p}\vec{x}} \langle 0| \mathcal{O}^5_{\sigma}(t,\vec{x}) P^\dagger(0) |0\rangle} \longrightarrow -\frac{p_\rho p_\mu p_\nu}{p_\sigma} \left\langle \xi^2 \right\rangle^{\mathrm{bare}}$$

directions $\rho, \mu = 1, 2, 3\,(\rho \neq \mu)$, $\nu = \sigma = 4$ and two units of momenta

$$\longrightarrow \boxed{\langle \xi \rangle_K, \left\langle \xi^2 \right\rangle_K, \left\langle \xi^2 \right\rangle_\pi}$$

Distribution Amplitudes

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Extracting Bare Values from Lattice Data – Vectors

$$\frac{\sum_x e^{i\vec{p}\vec{x}} \langle 0| \mathcal{O}_{\{\mu\nu\}}(t,\vec{x}) V_\rho^\dagger(0) |0\rangle}{\frac{1}{3}\sum_i \sum_x e^{i\vec{p}\vec{x}} \langle 0| \mathcal{O}_i(t,\vec{x}) V_i^\dagger(0) |0\rangle} \longrightarrow -i \langle \xi \rangle^{\parallel,\text{bare}}$$

$$\times \tanh((t-T/2)E_V)\,\frac{1}{2}\left(-g_{\mu\rho}p_\nu - g_{\nu\rho}p_\mu + \frac{2p_\mu p_\nu p_\rho}{m_V^2}\right)$$

directions $\mu = \rho = 1,2,3$, $\nu = 4$ and $\vec{p} = 0$

$$\frac{\sum_x e^{i\vec{p}\vec{x}} \langle 0| \mathcal{O}_{\{\rho\mu\nu\}}(t,\vec{x}) V_\sigma^\dagger(0) |0\rangle}{\frac{1}{3}\sum_i \sum_x e^{i\vec{p}\vec{x}} \langle 0| \mathcal{O}_i(t,\vec{x}) V_i^\dagger(0) |0\rangle} \longrightarrow -i \langle \xi^2 \rangle^{\parallel,\text{bare}} \tanh((t-T/2)E_V)$$

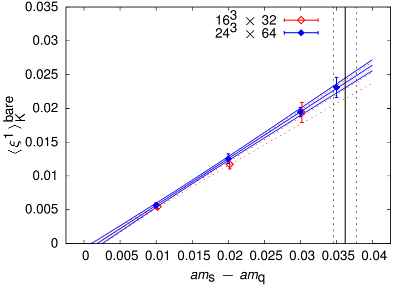
$$\times \frac{1}{3}\left(-g_{\rho\sigma}p_\mu p_\nu - g_{\mu\sigma}p_\rho p_\nu - g_{\nu\sigma}p_\rho p_\mu + \frac{3p_\rho p_\mu p_\nu p_\sigma}{m_V^2}\right)$$

directions e.g. $\mu = \sigma = 2$, $\nu = 4$, $\rho = 1$ and one unit of momentum ($p_i \neq 0$, one unit)

$$\longrightarrow \boxed{\langle \xi \rangle_{K^*}, \langle \xi^2 \rangle_{K^*}, \langle \xi^2 \rangle_\rho, \langle \xi^2 \rangle_\phi}$$

Distribution Amplitudes

Updated Results – Pseudo-scalars



$$\langle \xi \rangle_K^{\text{bare}} = 0.0238(7)(11) \; 0.0228(14)(11)$$

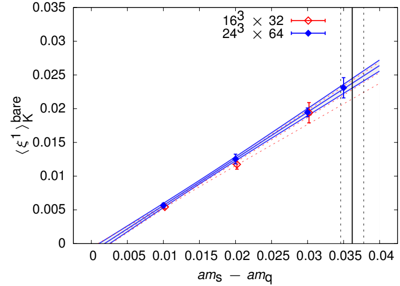
Results compatible with prediction from lowest order χPT

$\longrightarrow$ $\langle \xi \rangle_K$ proportional to $m_s - m_q$

this makes m_s correction easy

Distribution Amplitudes

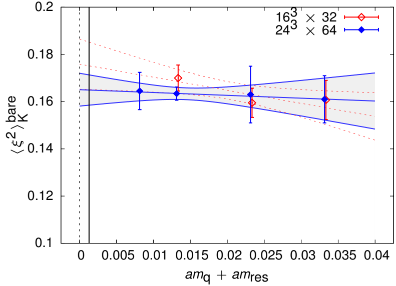
Updated Results – Pseudo-scalars

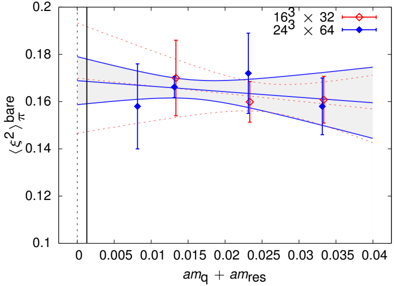


$$\langle \xi \rangle_K^{\text{bare}} = 0.0238(7)(11) \; 0.0228(14)(11)$$

$$\langle \xi^2 \rangle_K^{\text{bare}} = 0.1651(70) \; 0.176(11)$$

$$\langle \xi^2 \rangle_\pi^{\text{bare}} = 0.169(10) \; 0.170(23)$$





Distribution Amplitudes

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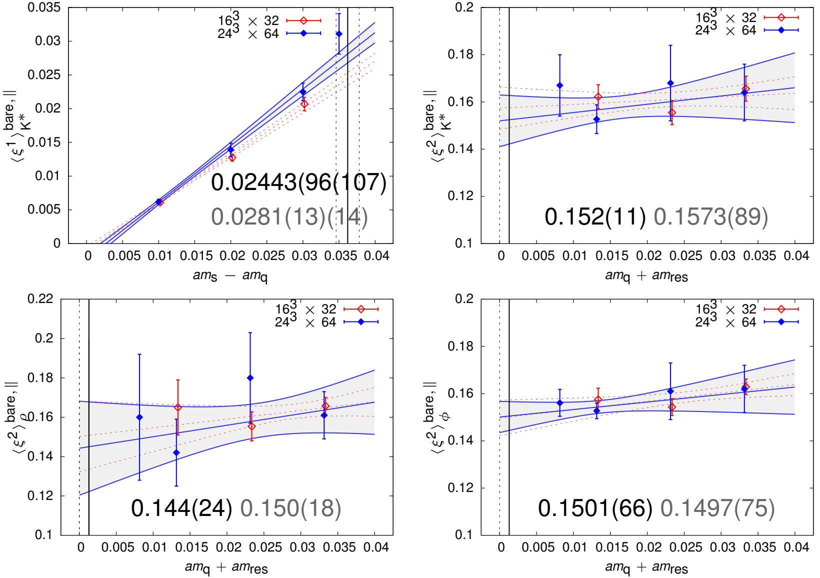
PRELIMINARY Results – Vectors



Non-perturbative Renormalisation

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$\psi^{\text{ren}}(x) = Z_q^{\frac{1}{2}} \psi^{\text{bare}}(x)$ $\quad m^{\text{ren}} = Z_m m^{\text{bare}}$ $\quad \mathcal{O}^{\text{ren}} = Z_{\mathcal{O}} \mathcal{O}^{\text{bare}}$

$$\mathcal{O}_\Gamma(q) = \sum_{x,x'} e^{iqx}\, \overline{\psi}(x) J_\Gamma(x,x') \psi(x')\,,$$

bare Green's function between off-shell quarks for $p = p'$, e.g. vector current

$$\begin{aligned} G_{\gamma_\mu}(p) &= \left\langle \psi(p) \mathcal{O}(0) \overline{\psi}(p) \right\rangle_G \\ &= \sum_x \left\langle \gamma_5 \left[\sum\nolimits_y S(x|y) e^{ipy} \right]^\dagger \gamma_5\, \gamma_\mu \, \left[\sum\nolimits_z S(x|z) e^{ipz} \right] \right\rangle_G . \end{aligned}$$

realise: $S(p)_x = \sum_y S(x|y) e^{ipy} \longrightarrow \sum_{x'} D(x,x') S(p)_{x'} = e^{ipx}$

$$G_{\gamma_\mu}(p) = \sum_x \left\langle \gamma_5 S(p)_x^\dagger \gamma_5 \gamma_\mu S(p)_x \right\rangle_G$$

Non-perturbative Renormalisation

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Find the amputated Green's function

$$\Pi_{\mathcal{O}}(p) = \langle S(p)\rangle_G^{-1} \langle G_{\mathcal{O}}(p)\rangle_G \langle S(p)\rangle_G^{-1} ,$$

and the bare vertex amplitude

$$\Lambda_{\mathcal{O}}^{\text{bare}}(p) = \frac{1}{12}\text{Tr}\left(\Pi_{\mathcal{O}}(p)\widehat{P}_{\mathcal{O}}\right) .$$

The renormalisation condition, leading to $Z_{\mathcal{O}}$, then is

$$\boxed{\Lambda_{\mathcal{O}}^{\text{ren}}(p) = \frac{Z_{\mathcal{O}}(\mu)}{Z_q}\Lambda_{\mathcal{O}}^{\text{bare}}(p)\Big|_{p^2=\mu^2} = 1 ,} \qquad \Lambda_{\text{QCD}} \ll \mu \ll 1/a$$

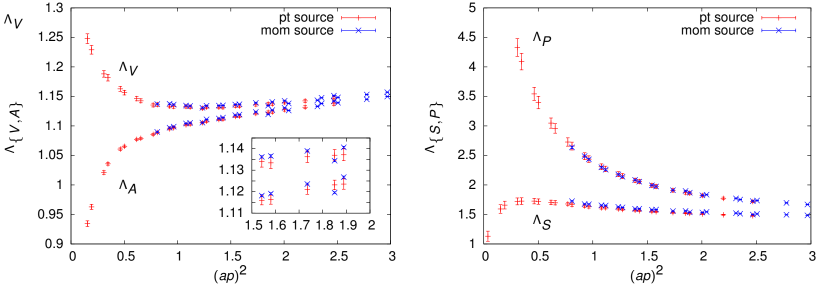
Projectors for bilinears: $\mathbb{1}$, γ_5, $\sum \gamma_\mu$, $\sum \gamma_5\gamma_\mu$

Projection for $\overline{\psi}(x)\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\psi(x)$

$$\Lambda_{V2}(p) = \frac{1}{6}\sum_{\substack{\mu,\nu \\ \mu\leq\nu}}\left[\frac{\text{Tr}\left[\Pi_{\gamma_\mu D_\nu}(p)(\gamma_\mu+\gamma_\nu)\right]}{12(\hat{p}^\mu+\hat{p}^\nu)} - \frac{\sum_{\rho\neq\mu,\nu}\text{Tr}\left[\Pi_{\gamma_\mu D_\nu}(p)\gamma_\rho\right]}{12\sum_{\rho\neq\mu,\nu}\hat{p}^\rho}\right] .$$

Non-perturbative Renormalisation

Point vs Momentum Sources (I) – Amputated Vertex Functions



Statistical errors are drastically reduced.

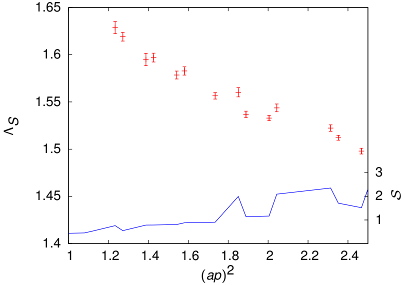
Before: 4 sources, 75 configurations, momenta with **many** directions

Now: 19 - 25 configurations, 1 direction

Λ_A

Non-perturbative Renormalisation

Discretisation Errors (I) – Amputated Vertex Functions

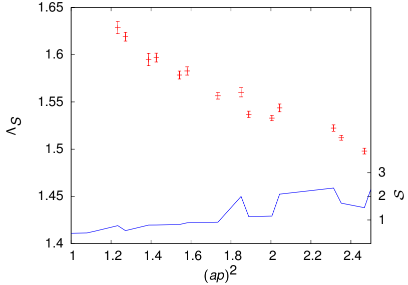


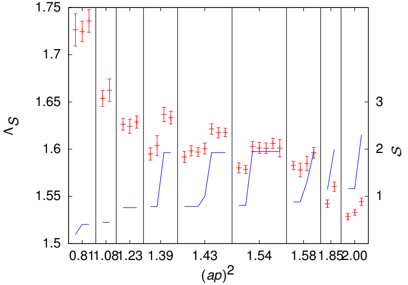
assume terms like ap and $(ap)^2$
$ap \to a\hat{p} = \sin(ap)$ and expanding $a\hat{p}$

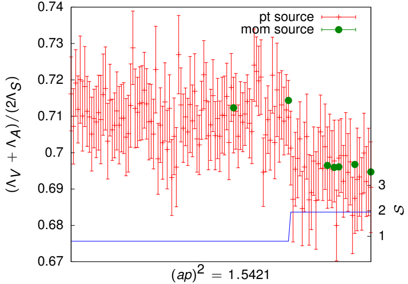
$$\Longrightarrow \mathcal{S} = \sum_\mu \frac{2\pi}{L_\mu} p_\mu^4$$

Non-perturbative Renormalisation

Discretisation Errors (I) – Amputated Vertex Functions







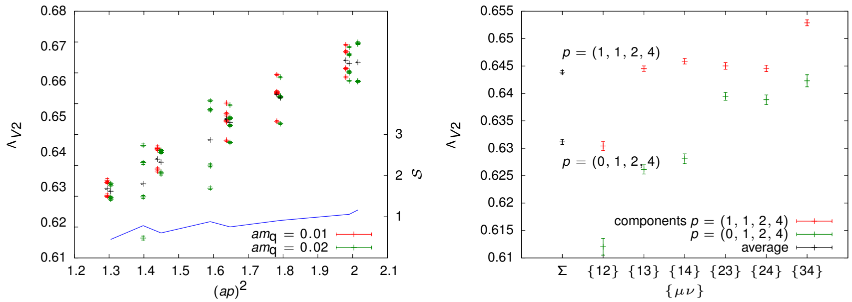
assume terms like ap and $(ap)^2$

$ap \rightarrow a\hat{p} = \sin(ap)$ and expanding $a\hat{p}$

$$\Longrightarrow \mathcal{S} = \sum_\mu \frac{2\pi}{L_\mu} p_\mu^4$$

Non-perturbative Renormalisation

Discretisation Errors (II) – Amputated Vertex Functions

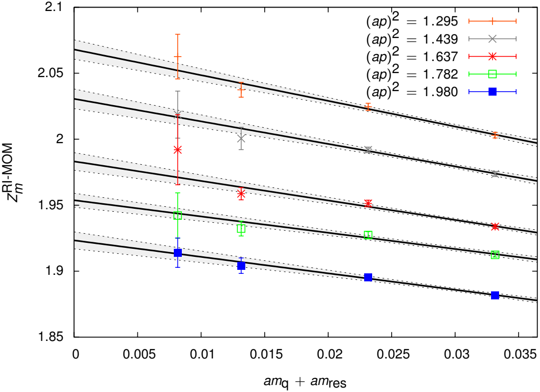


Different directions of the derivative w.r.t. momentum not compatible.

More severe discretisation errors? Dependence on m_q much smaller.

Non-perturbative Renormalisation

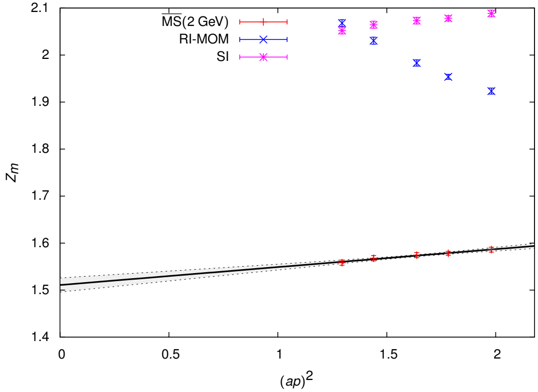
Extrapolation in m_q



linear extrapolation of Z_m^{RI-MOM} to the chiral limit

Non-perturbative Renormalisation

Reducing Discretisation Effects



$Z_m^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 1.656(48)$

arXiv:0712.1061

PRELIMINARY

$Z_m^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 1.511(15) \quad 24^3$

$Z_m^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 1.544(34) \quad 16^3$

only stat. errors...

We extrapolate $(ap)^2 \to 0$ to remove lattice artefacts.

Non-perturbative Renormalisation

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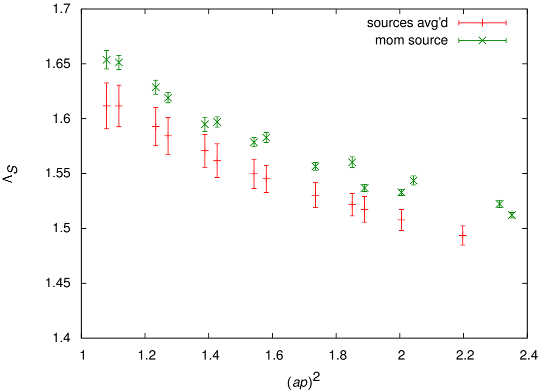
Point vs Momentum Sources (II) – Amputated Vertex Functions



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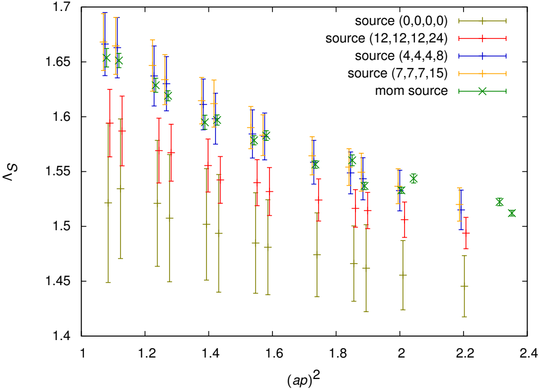
Point vs Momentum Sources (II) – Amputated Vertex Functions



The number and position of sources seems to explain the the difference between the results of the point/momentum source technique.

$\longrightarrow$ better off when volume averaging

Summary and Outlook

PRELIMINARY Numbers

Summary of results, $\overline{\text{MS}}$ ($\mu = 1.729\,\text{GeV}$). Statistical and systematic errors. Still perturbatively renormalised.

$\langle \xi^2 \rangle_\pi$	0.278(15)(13)	0.280(35)(13)	0.269(39)
$\langle \xi \rangle_K$	0.03039(91)(174)	0.0291(18)(17)	0.0272(5)
$\langle \xi^2 \rangle_K$	0.272(11)(13)	0.289(17)(14)	0.260(6) PRD74:074501
$\langle \xi^2 \rangle_\rho$	0.240(36)(12)	0.249(27)(12)	
$\langle \xi \rangle_{K^*}^{\parallel}$	0.0359(17)(22)	0.0312(12)(17)	0.033(2)(4) PoS(LAT07)144
$\langle \xi^2 \rangle_{K^*}^{\parallel}$	0.252(17)(12)	0.260(13)(13)	
$\langle \xi^2 \rangle_\phi^{\parallel}$	0.250(10)(12)	0.249(11)(12)	
	24^3	16^3	

Summary and Outlook

- showed updates of our results for pseudo-scalar and vector DAs
  $\longrightarrow$ anticipate to use non-perturbative renormalisation soon

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- Rome-Southampton method with momentum sources has very small statistical errors even with moderate computational effort
  $\longrightarrow$ possible to clearly see and investigate discretisation errors
- artefacts (esp. for derivatives) not yet fully understood
- we have other interesting projects, see(n) talks of

  $\rightarrow$ Y. Aoki (non-exceptional momenta) and
  $\rightarrow$ C. Kelly (B_K)

Authors

UKQCD/RBC Collaborations

especially:

P.A. Boyle, D.B., M. Donnellan, J. Flynn,

A. Jüttner, C. Kelly, C. Sachrajda