# 2+1 flavor QCD calculation of $< \mathbf{x} >$, $< \mathbf{x}^2 >$ and form factors.

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# OUTLINE

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**2.** Formalism

**3.** Results

**4.** Current projects in the pipeline

**5.** Future (plans)

# Introduction

One of the main area of HEP is to study hadron structure.

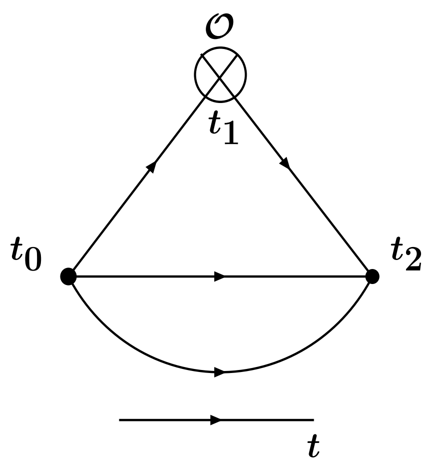
- In lattice qcd lot of calculations have been done in this area for the "quenched approximation". It's quite interesting to calculate various physical quantities for full qcd (including vaccum polarization effects) and compare those with the quenched one.

My work involves calculation of first and second moments of unpolarised structure function, calculation of form factors for various currents (local and point split) and quark angular momentum for 2+1 flavor Clover fermion action. Calculation is restricted only to "connected insertions".

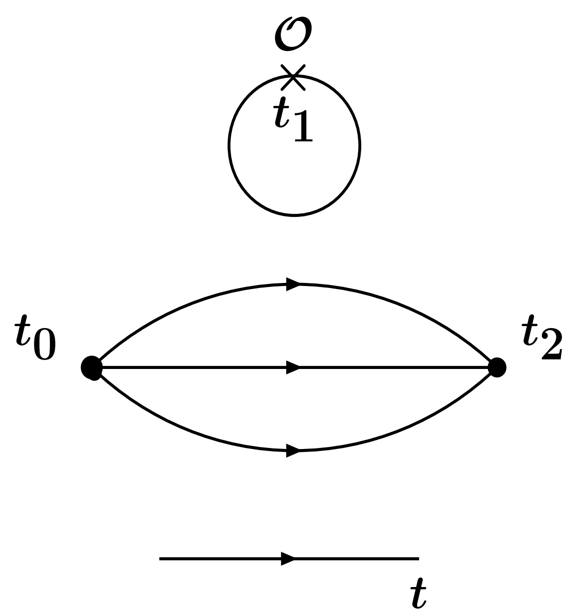
# Formalism

- According to quantum field theory, Wick's contraction gives us both connected and disconnected diagrams.

**Connected diagram:**



**Disconnected diagram:**



# Formalism

Two index operator:

$$O_{\mu\nu} = \frac{1}{2}[\bar{\psi}^{(f)}(x)\gamma_{\{\mu}\overrightarrow{D}_{\nu\}}\psi^{(f)}(x) - \bar{\psi}^{(f)}(x)\gamma_{\{\mu}\overleftarrow{D}_{\nu\}}\psi^{(f)}(x)] \quad (1)$$

$$\begin{aligned}
O_{\mu\nu} &= \frac{1}{8\,a}[\bar{\psi}^{(f)}(x)\gamma_\mu U_\nu(x)\psi^{(f)}(x+a_\nu) - \bar{\psi}^{(f)}(x)\gamma_\mu \\
& U_\nu^\dagger(x-a_\nu)\psi^{(f)}(x-a_\nu) \\
&+ \bar{\psi}^{(f)}(x-a_\nu)\gamma_\mu U_\nu(x-a_\nu)\psi^{(f)}(x) - \bar{\psi}^{(f)}(x+a_\nu)\gamma_\mu \\
& U_\nu^\dagger(x)\psi^{(f)}(x) \\
&+ (\mu \leftrightarrow \nu)]
\end{aligned} \quad (2)$$

## Formalism

Three index operator:

$$
\begin{aligned}
O_{4ii} &= -\frac{1}{24\,a^2}[\bar{\psi}^{(f)}(x)\gamma_i\{U_4(x)U_i(x+a_4)+U_i(x) \\
&\quad U_4(x+a_i)\}\psi^{(f)}(x+a_4+a_i) \\
&- \bar{\psi}^{(f)}(x)\gamma_i\{U_4(x)U_i^\dagger(x+a_4-a_i)+U_i^\dagger(x-a_i) \\
&\quad U_4(x-a_i)\}\psi^{(f)}(x+a_4-a_i) \\
&- \bar{\psi}^{(f)}(x)\gamma_i\{U_4^\dagger(x-a_4)U_i(x-a_4)+U_i(x) \\
&\quad U_4^\dagger(x+a_i-a_4)\}\psi^{(f)}(x-a_4+a_i) \\
&+ \bar{\psi}^{(f)}(x)\gamma_i\{U_4^\dagger(x-a_4)U_i^\dagger(x-a_4-a_i)+U_i^\dagger(x-a_i) \\
&\quad U_4^\dagger(x-a_4-a_i)\}\psi^{(f)}(x-a_4-a_i) \\
&+ \bar{\psi}^{(f)}(x-a_4)\gamma_i\{U_4(x-a_4)U_i(x)+U_i(x-a_4) \\
&\quad U_4(x+a_i-a_4)\}\psi^{(f)}(x+a_i)
\end{aligned}
$$

$$
\begin{aligned}
- \quad & \bar{\psi}^{(f)}(x-a_4)\gamma_i\{U_4(x-a_4)U_i^\dagger(x-a_i)+U_i^\dagger(x-a_4-a_i) \\
& U_4(x-a_4-a_i)\}\psi^{(f)}(x-a_i) \\
- \quad & \bar{\psi}^{(f)}(x+a_4)\gamma_i\{U_4^\dagger(x)U_i(x)+U_i(x+a_4)U_4^\dagger(x+a_i)\}\psi^{(f)}(x+a_i) \\
+ \quad & \bar{\psi}^{(f)}(x+a_4)\gamma_i\{U_4^\dagger(x)U_i^\dagger(x-a_i)+U_i^\dagger(x-a_i+a_4) \\
& U_4^\dagger(x-a_i)\}\psi^{(f)}(x-a_i) \\
+ \quad & 2\bar{\psi}^{(f)}(x)\gamma_4 U_i(x)U_i(x+a_i)\psi^{(f)}(x+2a_i)-4\bar{\psi}^{(f)}(x)\gamma_4\psi^{(f)}(x) \\
+ \quad & 2\bar{\psi}^{(f)}(x)\gamma_4 U_i^\dagger(x-a_i)U_i^\dagger(x-2a_i)\psi^{(f)}(x-2a_i)]
\end{aligned} \tag{3}
$$

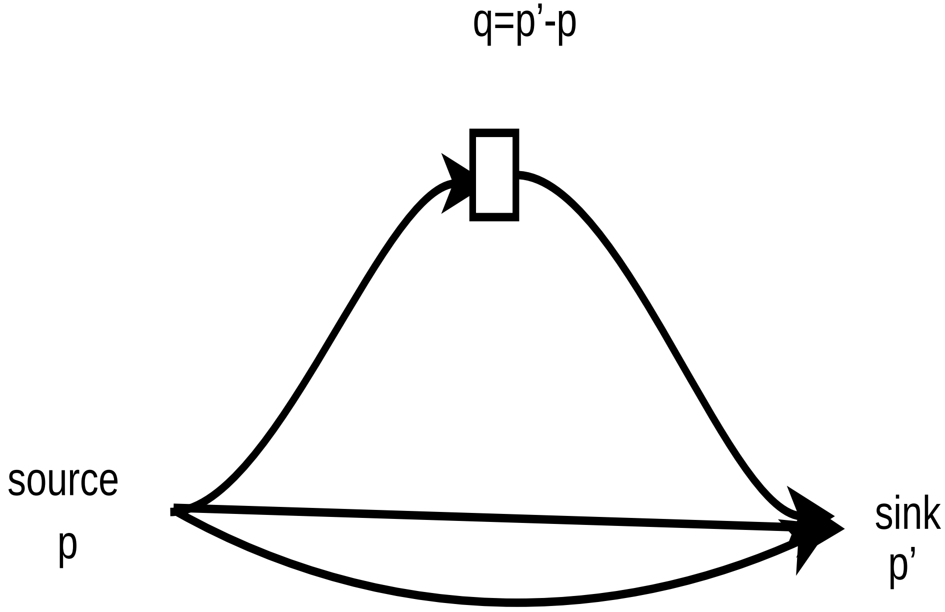
## SIMULATION PARAMETERS:

- Action: 2+1 Clover fermion (CP-PACS/JLQCD)
  - Lattice size: $16^3 \times 32$
  - $\beta$=1.83, $\kappa = 0.13760(800\text{config}), 0.13800(810\text{config}), 0.13825(813\text{config})$
  - $a = 0.1219(19)$ fm, $m_{ps}/m_v \approx$ 0.6-0.78, $M_\pi^{lowest} = 601$ Mev
  - $C_{SW} = 1.7610$, $K_S = 0.13760$, Source position: (0,0,0,0)
- Point source with no smearing for the 1st inversion and the 2nd inversion. Sequential source (fixed sink) method was implemented for the 2nd inversion.
- Sink mometum: (0,0,0), (1,0,0), (-1,0,0)
- Polarisation: z-direction (for calculation of $L_q$ and magnetic moments)
- Operators:
  $O_{44}, O_{33}, O_{22}, O_{11}, O_{41}, O_{42}, O_{43}, O_{411}, O_{422}, O_{433}$, angular momentum ,
  $\bar{\psi}\Gamma\psi, \bar{\psi}\Gamma\gamma_5\psi$ where $\Gamma = \gamma_1, \gamma_2, \gamma_3, \gamma_4$

# Formalism

**Sequential Source Method:**

- For quenched data sink momentum was (-1,0,0) and for dynamical data we took average over (1,0,0) and (-1,0,0) to reduce noise.



# Formalism

Three point function:

$$C^{2pt}(\tau, p) = \sum_{j,k} (\Gamma_{unpol})_{jk} < N_k(\tau, p) \bar{N}_j(\tau_{src,p}) > \quad (4)$$

$$C^{3pt}(\tau, p', p) = \sum_{j,k} (\Gamma_{pol})_{jk} < N_k(\tau, p_{snk}) \Theta(\tau, q) \bar{N}_j(\tau_{src}, p) > \quad (5)$$

$$\text{where,} \quad \Gamma_{unpol} = \frac{(1+\gamma_4)}{2} \quad (6)$$

$$\text{and} \quad \Gamma_{pol} = \frac{(1+\gamma_4)\gamma_3\gamma_5}{2} \quad (7)$$

To get $< x >$ and $< x^2 >$ we need to take ratio of 3pt to 2pt function.

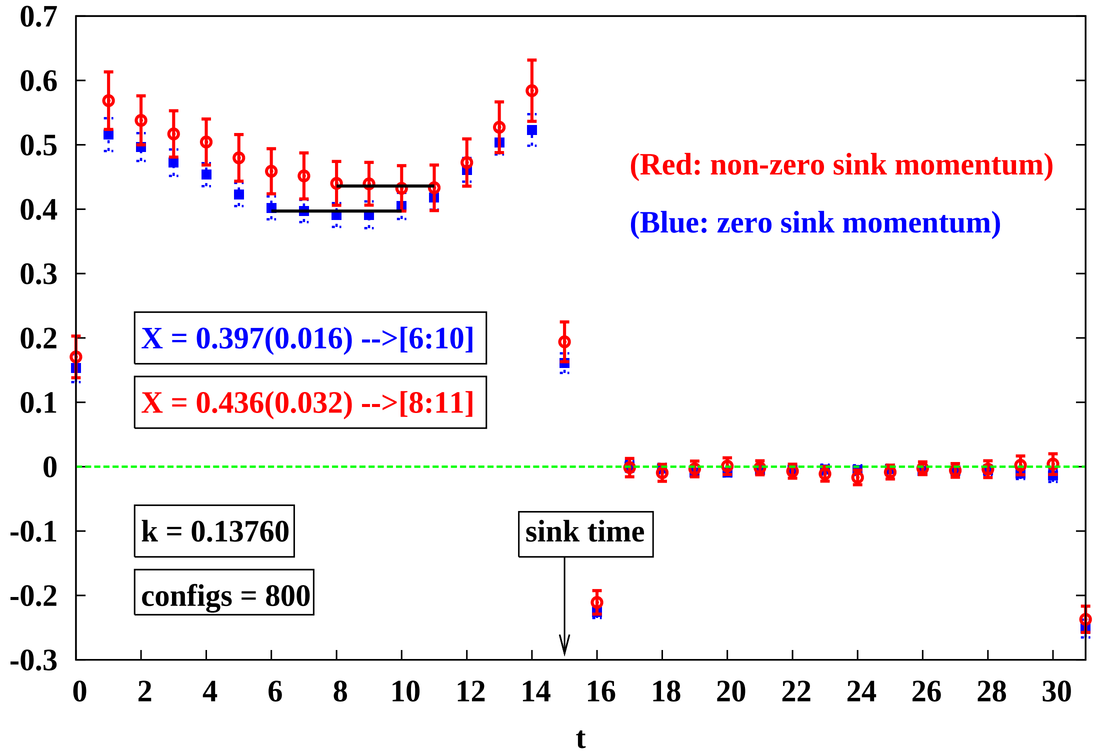
$$\frac{E_p^o}{\sqrt[2]{(E_p^o)^2 - \frac{1}{3}(p')^2}} \frac{C^{3pt}(\tau, p', p)}{C^{2pt}(\tau_{sink}, p)} = < x > \text{ for } \Theta_{\mu\mu} \text{ operator} \quad (8)$$

$$\frac{1}{p'} \frac{C^{3pt}(\tau, p', p)}{C^{2pt}(\tau_{sink}, p)} = < x > \text{ for } \Theta_{\mu\mu} \text{ and } \Theta_{\mu\nu} \text{ operator} \quad (9)$$

$$\frac{1}{p'^2} \frac{C^{3pt}(\tau, p', p)}{C^{2pt}(\tau_{sink}, p)} = < x^2 > \text{ for } \Theta_{4ii} \text{ operator} \quad (10)$$

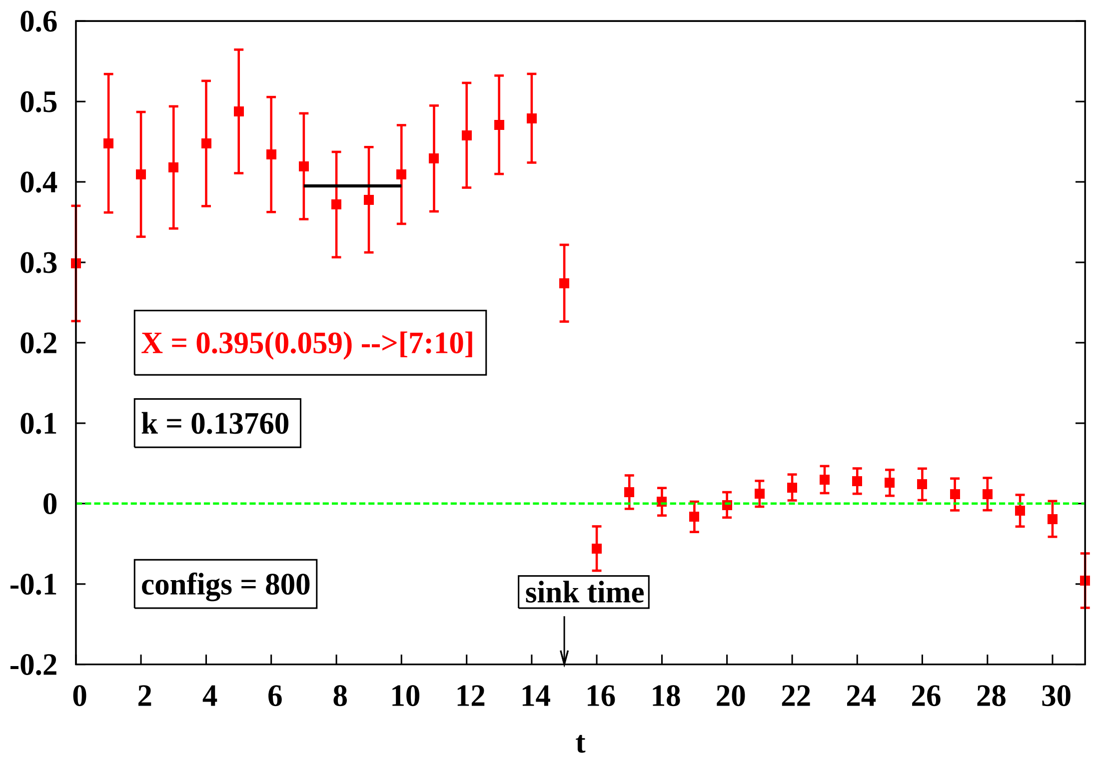
# RESULTS

**First moment $(O_{44} - \frac{1}{3}(O_{11} + O_{22} + O_{33}))$: (UP)**



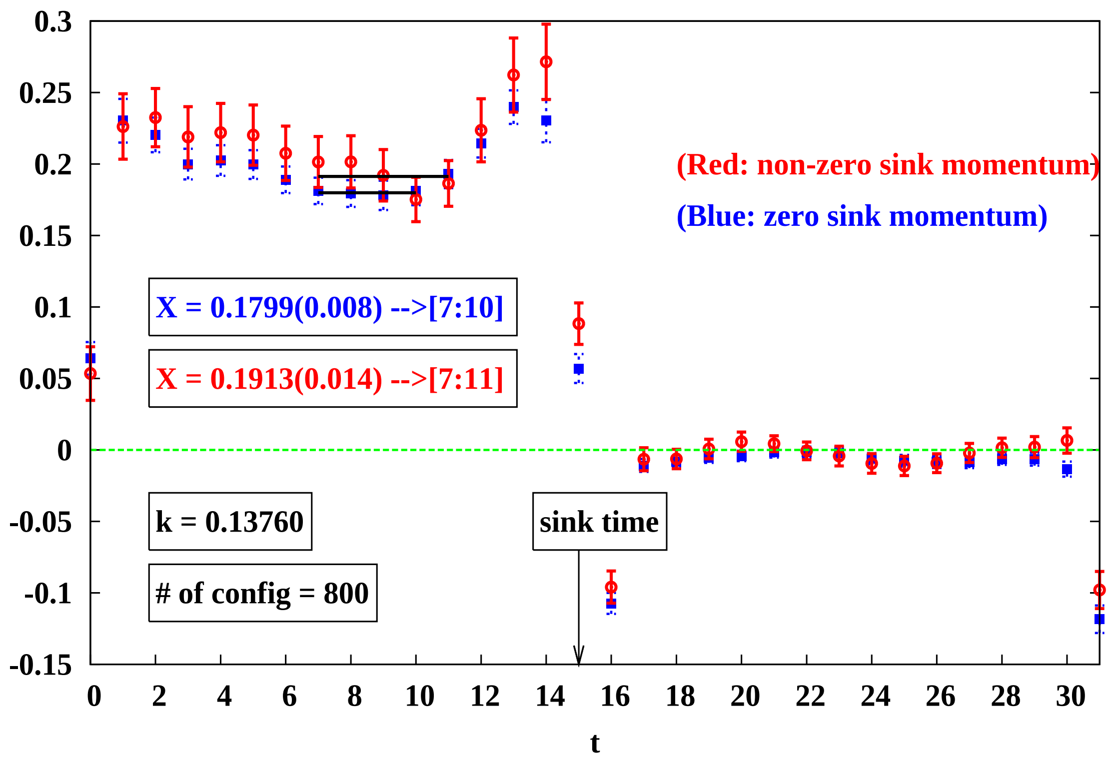
# RESULTS

**First moment ($O_{41}$): (UP)**



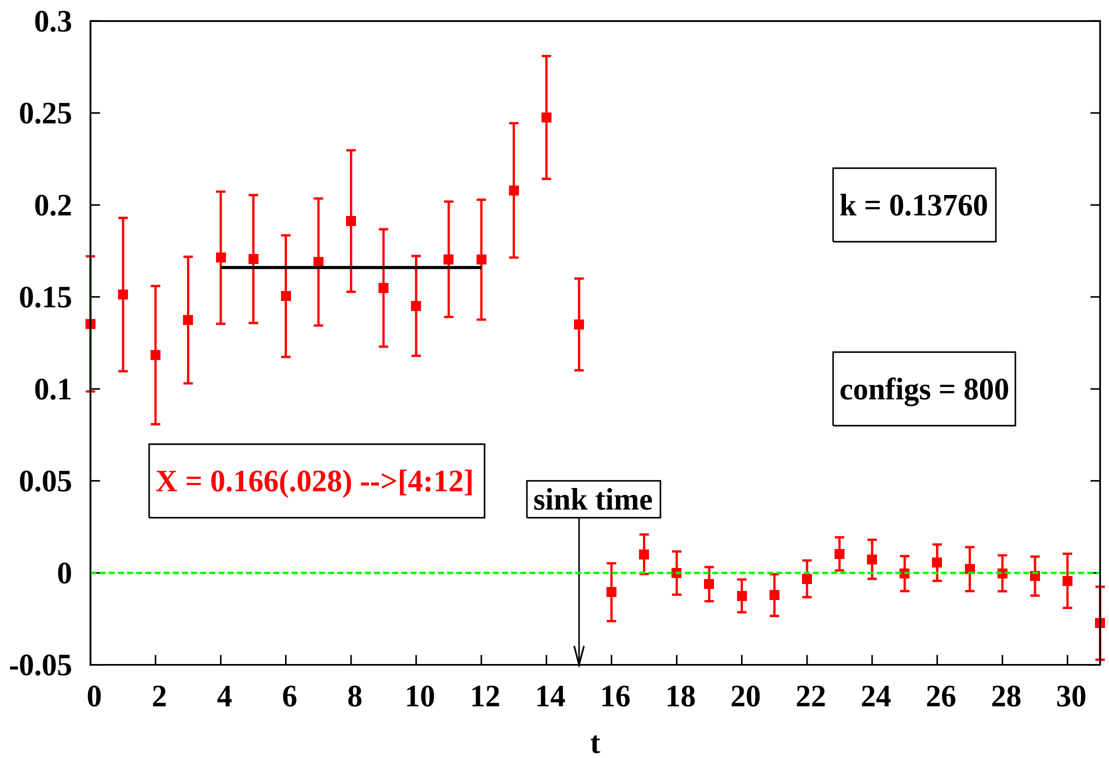
# RESULTS

**First moment $(O_{44} - \frac{1}{3}(O_{11} + O_{22} + O_{33}))$: (DOWN)**



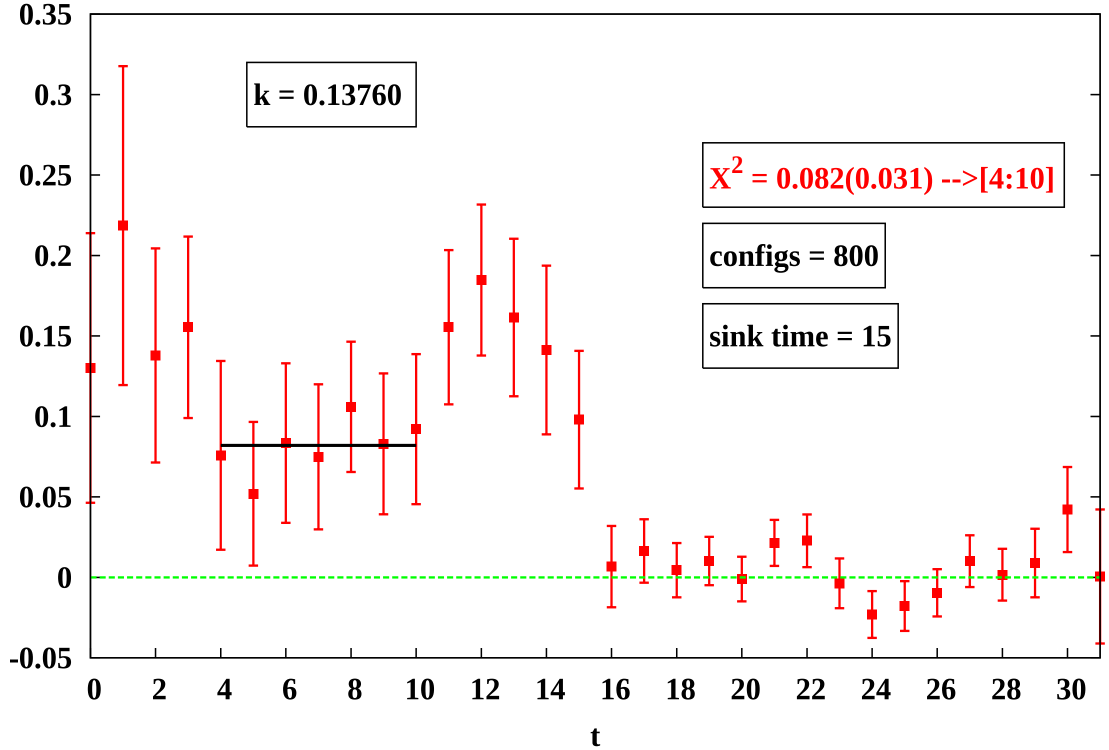
# RESULTS

**First moment ($O_{41}$): (DOWN)**



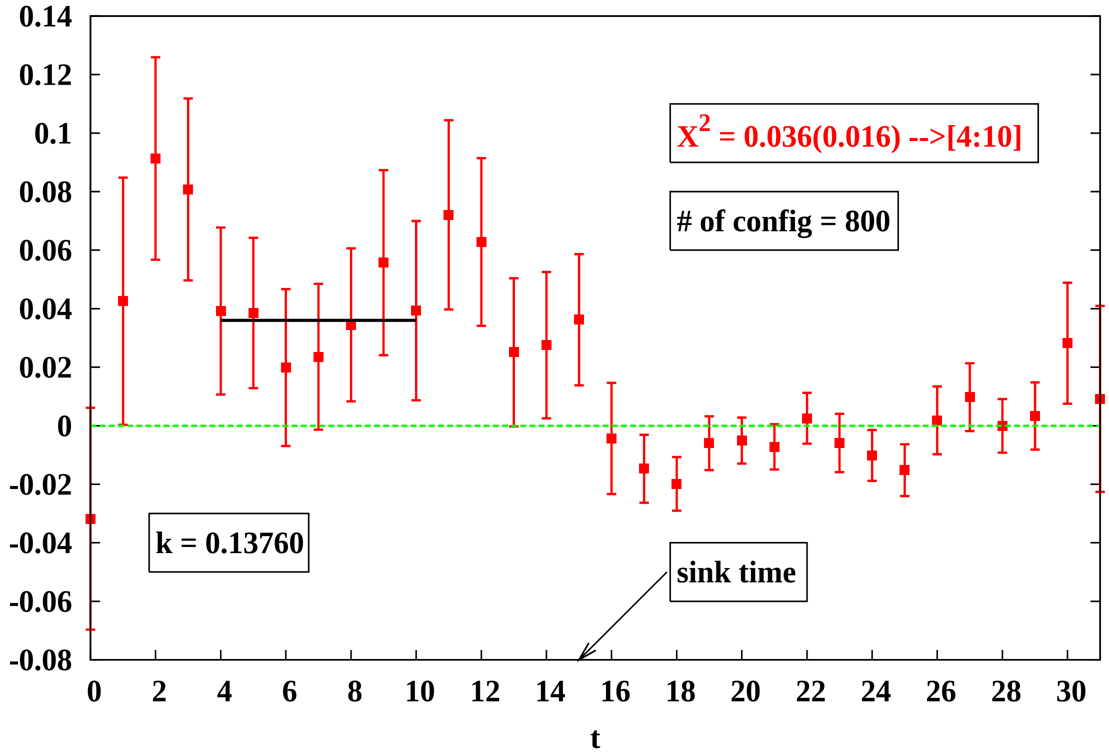
# RESULTS

**Second moment** $(O_{411} - \frac{1}{2}(O_{422} + O_{433}))$**: (UP)**



# RESULTS

**Second moment** $(O_{411} - \frac{1}{2}(O_{422} + O_{433}))$**: (DOWN)**



## Systematic errors

- Operator mixing (though there is less mixing for the lowest moments and perticular choice of operartors, gluonic mixing can not be avoided for the unquenched case).
- Excited states contribution:
  - Unsmeared source and sink.
  - Same projection operator was used at sink for zero and non-zero sink momentum .
- Finite size effects.
- Oprator improvement was not considered.

**Improvements:**

- Planning to smear source and sink to minimize contamination from the excited states.
- Planning to improve the statistics by considering more (around 4/6) source positions.
- Carefully picking up the sink time for all masses.
- Planning to use proper projection operator at sink.

# Current projects in the pipeline

**Production:**

- Currently production for the remaing two masses is going on. I'm generating and calculating primary and secondary propagators with zero and non-zero sink momentum for unpolarised and polarised sink and corresponding three-point functions.

- Since spinors are kept open (did not contract spinors when calculated three-point function), various physical quantities for different currents namely, $O_{\mu\nu}, O_{4ii}, \bar{\psi}\Gamma\psi$ where, $\Gamma = \gamma_\mu, \gamma_\mu\gamma_5, I, \gamma_5, \sigma_{\mu\nu}$(local and point-split) and angular momentum can be calculated.

## Future (Plans)

- We are planning to repeat the same analysis for the larger lattice (for which pion mass is significantly small $\approx$ 200Mev).
- Planning to include deflation so that inversion time can be reduced and hence many sources can be considered to increase statistics.