Decay constants from twisted mass QCD

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Twisted mass fermions

- Extrapolation of the mass of rho meson.
- leptonic decay constants of the ρ meson
- decay constants in charmonium

Use the correlators from the ETM collaboration's n_f = 2 unquenched lattice QCD calculations. Wilson twisted mass quarks and tree level improved Symanzik gauge action (details in Urbach's talk). Silvano Simula and Petros Dimopoulos have also worked on this analysis (some of which was in Vladikas's talk yesterday).

Data sets used

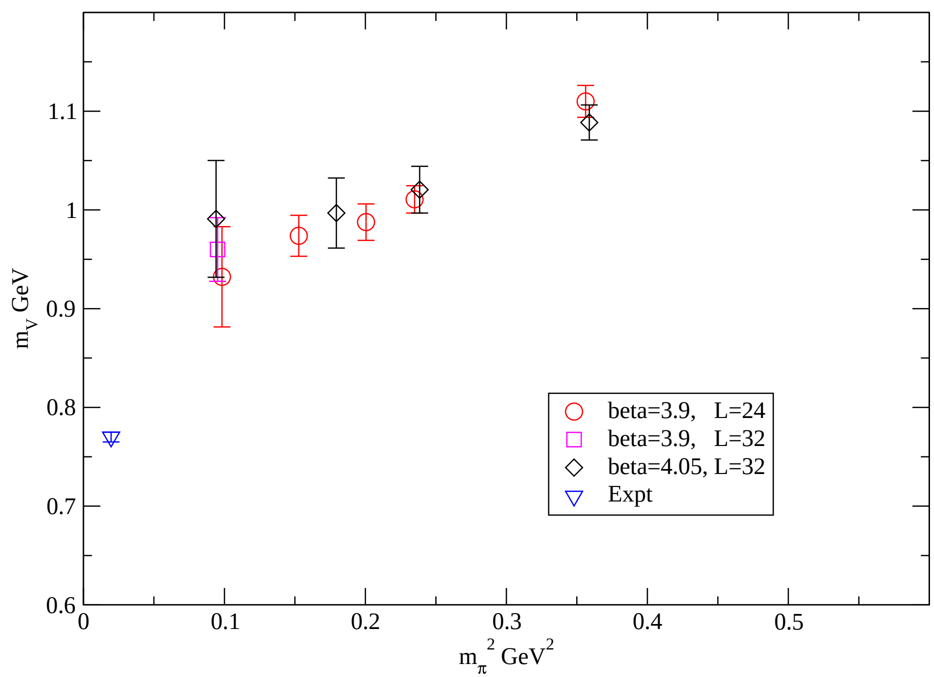
The following data sets from the ETM collaboration were considered

Ensemble	β	μ	$L^3 \times T$	Measurements
B_1	3.9	0.004	$24^3 \times 48$	111×8
B_2	3.9	0.0064	$24^3 \times 48$	78×32
B_3	3.9	0.0085	$24^3 \times 48$	66×32
B_4	3.9	0.01	$24^3 \times 48$	38×32
B_5	3.9	0.015	$24^3 \times 48$	44×32
B_6	3.9	0.004	$32^3 \times 64$	$81 \times$ ‘6
C_1	4.05	0.003	$32^3 \times 64$	64×8
C_2	4.05	0.006	$32^3 \times 64$	66×8
C_3	4.05	0.008	$32^3 \times 64$	61×8
C_4	4.05	0.012	$32^3 \times 64$	40×8

Lattice spacing for B_i (C_i) 0.0855 fm (0.0667 fm). Using results from fits to 4 by 4 correlation matrices done by Chris Michael.

Mass of light vector mesons

Plot ETMC masses for charged ρ, use f_π to set the scale.



A fit linear in the square of the pion mass gives m_ρ = 867(29) MeV ($\beta = 3.9$ data).

(Experiment 770 MeV). Naively ρ data prefers different lattice spacing to that from f_π.

Reading between the lines

Today we are looking at the ρ meson, but we are really thinking (since we are so close to JLAB) of future lattice qcd calculations of exotic 1^{-+} mesons [a].

One approach [b] for dealing with resonances is to

- Determine decay widths from lattice calculations (for example hep-lat/0603007) and then feed that information into chiral extrapolation models. Decay widths of exotics will not be known from experiment for predictions.
- Lattice QCD calculations have not really started to study the mass dependence of the coupling constants for strong decays of existing states (CP-PACS, hep-lat/06100201).

Last year Michael and Urbach (arXiv:0709.4564) used a lattice calculation of $g_{\rho\pi\pi}$ coupling to estimate the mixing of ρ with two pions. They estimated a 5% shift for the mass of the lightest rho meson.

The vector data is noisy.

- $\beta = 3.9$ signal to noise $\sim e^{-(m_V - m_\pi)t}$. No new physics making correlators noisy. Just need to tune stochastic measurements

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[a] Thanks to J. Dudek for discussions and NUCLATT presentation

[b] this was either covered, or not covered, by Colin this morning.

Chiral extrapolation of the ρ

The effective field theory of vector mesons “predicts” a chiral extrapolation model of

$$m_\rho = m_\rho^0 + d_1 m_\pi^2 + d_2 m_\pi^3$$

(from Jenkins, Manohar and Wise. Phys.Rev.Lett 75, 2272 (1995)).
"New" chiral extrapolation formulae for the ρ meson by Bruns and Meißner (hep-ph/0411223). This uses a modified $\overline{MS}$ regulator. (Becher and Leutwyler’s method applied to the ρ hep-ph/9901384).

$$M_\rho = M_\rho^0 + c_1 M_\pi^2 + c_2 M_\pi^3 + c_3 M_\pi^4 \ln(\frac{M_\pi^2}{M_\rho^2})$$

Good to check that size of c_i is consistent with other estimates, they claim phemenology prefers $\mid c_i \mid < 3$. Leinweber et al. (hep-lat/0104013) claim to know sign of c_2 (negative) and magnitude (eg. from $\omega\rho\pi$ coupling).
Too many parameters to determine with this data, so use an augmented χ^2 with the above constraints built in (hep-lat/0110175).

$$\chi_{aug}^2 = \chi^2 + \sum_{j=2}^{3} \frac{(c_i - 0)^2}{3^2}$$

Chiral extrapolations

Also tried the fit model suggested by the Adelaide group (Leinweber et al. hep-lat/0104013)

$$m_\rho = c_0 + c_2 m_\pi^2 + \frac{\Sigma_{\pi\omega}(\Lambda_{\pi\omega}, m_\pi) + \Sigma_{\pi\pi}(\Lambda_{\pi\pi}, m_\pi)}{2(c_0 + c_2 m_\pi^2)}$$

where $\Sigma_{\pi\omega}$ and $\Sigma_{\pi\pi}(\Lambda_{\pi\pi}, m_\pi)$ are the self energies from the $\pi\pi$ and $\pi\omega$ states.

$$\Sigma_{\pi\pi} = -\frac{f_{\rho\pi\pi}^2}{6\pi^2} \int_0^\infty \frac{dk k^4 u_{\pi\pi}^2(k)}{(\sqrt{k^2 + m_\pi^2})(k^2 + m_\pi^2 - m_{\rho\ phys}^2/4)}$$

$$\Sigma_{\pi\omega} = -\frac{g_{\rho\omega\pi}^2 m_{\rho\ phys}}{12\pi^2} \int_0^\infty \frac{dk k^4 u_{\pi\omega}^2(k)}{k^2 + m_\pi^2}$$

The $u_{\pi\pi}$ and $u_{\pi\omega}$ functions are dipole regulators.

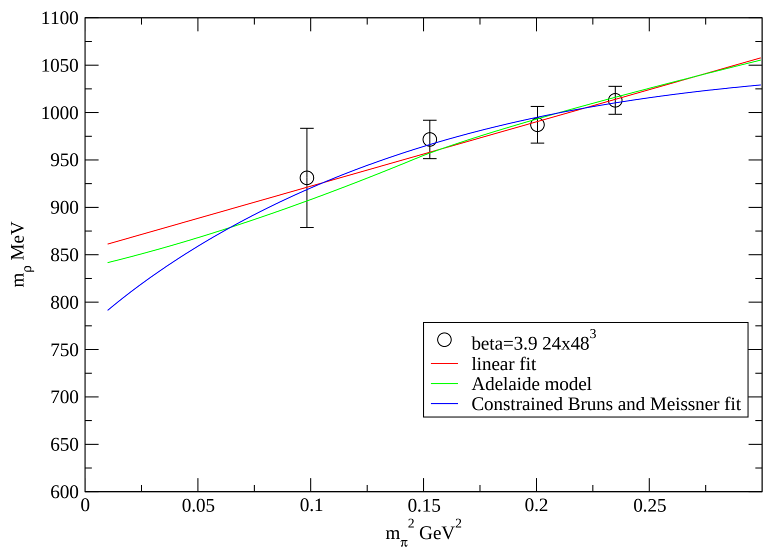
$$u_{\pi\pi}(k) = \left(\frac{\Lambda_{\pi\pi}^2 + m_{\rho\ phys}^2}{\Lambda_{\pi\pi}^2 + 4(m_\pi^2 - k^2)} \right)^2$$

$$u_{\pi\omega}(k) = \left(\frac{\Lambda_{\pi\omega}^2 - m_{\pi\ phys}^2}{\Lambda_{\pi\omega}^2 + k^2} \right)^2$$

Fit parameters $\Lambda_{\pi\omega}$, c_0, and c_2. The couplings $g_{\rho\omega\pi}^2$ and $f_{\rho\pi\pi}^2$ are input.

Some results for the chiral extrapolation.

Just fit the β = 3.9 24^3 48 data.



Fit the Bruns-Meißner model to the data with constraints on the c_2 and c_3 coefficients the result is m_ρ = 807(8) MeV. For the Adelaide method we get m_ρ = 847(72) MeV.

Leptonic decay constants

In the continuum the decay constant of the ρ meson is defined via

$$\langle 0 \mid \overline{\psi}(x)\gamma_\mu\psi(x) \mid \rho\rangle = m_\rho f_\rho \epsilon_\mu$$

The transverse decay constant ($f_V^T(\mu)$) of the ρ meson is defined by

$$\langle 0 \mid \overline{\psi}\sigma_{\mu\nu}\psi \mid \rho\rangle = i f_V^T(\mu)(p_\mu\epsilon_\nu - p_\nu\epsilon_\mu)$$

where $\sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$. The transverse decay constant is used in light cone sum rule calculations.
For the charmonium part of the analysis will use:

$$f_{PS} = (2\mu)\frac{\mid \langle 0 \mid P^1(0) \mid P\rangle \mid}{M_{PS}^2}$$

Light cone sum rules

The non-perturbative input to light cone sum rules are the light cone wave functions of mesons (rather than condensates). Complementary to pure lattice calculation.
Motivation from light cone sum rule for $B \rightarrow \rho \ell \nu$, required for $\mid V_{ub} \mid$ from Becirevic et al. (hep-lat/0301020)

$$A_1(q^2) = \frac{m_b f_\rho \mathrm{e}^{(m_B^2 - m_b^2)/M^2}}{m_B^2 f_B (m_B + m_\rho)} \int_{u_0}^{1} \frac{du}{u} \mathrm{e}^{(1-1/u)(q^2 - m_b^2 - u m_\rho^2)/M^2} \times$$

$$\left\{ \frac{m_b^2 + u^2 m_\rho^2 - q^2}{2u} \frac{f_\rho^T}{f_\rho} \phi_T(u) + \frac{m_b m_\rho}{2} \left[\int_0^u dv \frac{\phi_\parallel(v)}{1-v} + \int_u^1 dv \frac{\phi_\parallel(v)}{v} \right] \right\},$$

Changing f_ρ^T / f_ρ from 0.75 to 1 changes $A_1(q^2)$ by up to 20%.
Almost [a] all lattice groups have stopped work on this decay, preferring to look at $B \rightarrow \pi \nu l$ to get $\mid V_{ub} \mid$. (Still need these decay constants for $B \rightarrow \rho \gamma$ $B \rightarrow K^\star \gamma$ Ball and Zwicky (hep-ph/0603232).) Plan is to try to study the ρ decay on these decay constants.
Test/validate lattice techniques on f_ρ and then “predict” $f_V^T(\mu)$.

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[a] new work by Van de Water et al. at this conference

Renormalisation and chiral stuff

Use the results from the Rome-Southampton method, reported by ETMC in arXiv:0710.0975, Dimopoulos et al for light decay constants. Used updated analysis by Frezzotti at Trento meeting 2008 for charm data (1.4% (3%) increase Z_A (Z_T)).

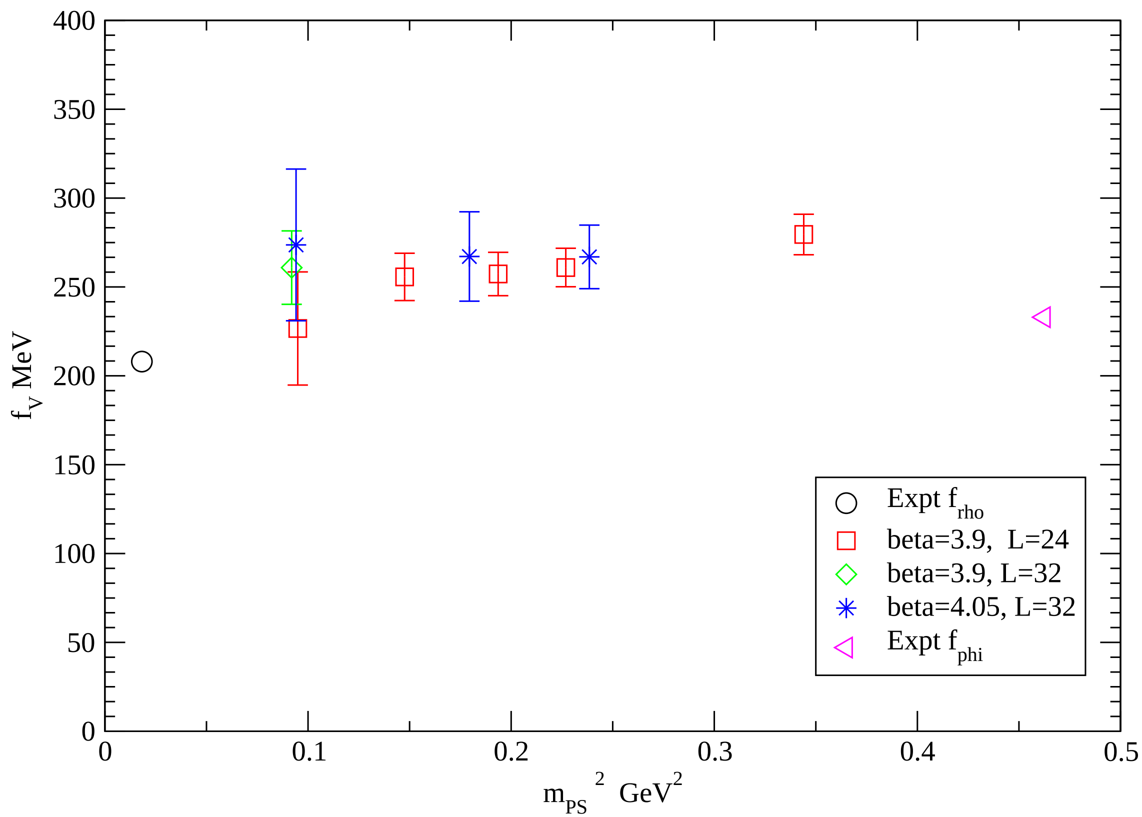
β	Z_A	Z_V	$Z_T(\mu = 1/a)$
3.90	0.76	0.65	0.75
4.10	0.77	0.67	0.79

The Z_T renormalisation factor depends on the scale. Run Z_T to 2 GeV from the scale of the inverse lattice spacing. Use method described by Becirevic et al. (hep-lat/0301020). There are updated expressions due to Gracey's three loop computation in the RI' scheme (hep-ph/0304113) that will be incorporated later.

- Just use linear fits in quark mass
- Wait for improved statistics to use Chiral perturbation theory for decay constants developed by Bijnens et al. (hep-ph/9801418). The corrections due loops start at $m_q \log m_q$ and $m_q^{3/2}$.
- Formalism for tensor currents developed (arXiv:0705.2948, Cata and Mateu), but no loop results exist.

Results for f_ρ

The leptonic decay constant of the vector meson as a function of the square of the pion mass. The experimental points for the ρ and ϕ are also included.



Leptonic decay constant of the ρ

How close have previous lattice calculations got to the experimental number

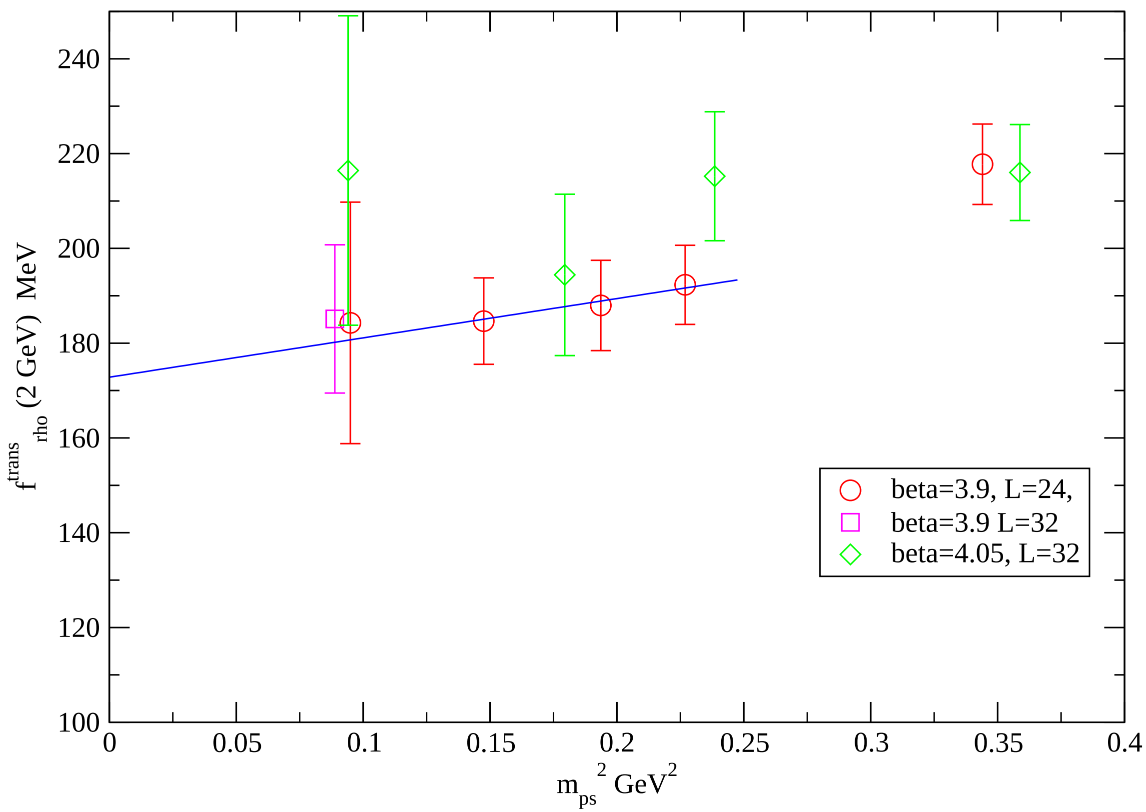
$$f_{\rho^+}^{expt} \sim 208 MeV$$

from

- Lewis and Woloshyn (hep-lat/9610027) obtained a result close to experimental result for the f_ρ in a quenched QCD calculation using the D234 improved action,
- CP-PACS (hep-lat/0105015) even with non-perturbative renormalisation (conserved current) didn't quote a value for f_ρ in the continuum limit in their $n_f = 2$ calculations with the tadpole improved clover action.
- QCDSF obtained (hep-lat/0509196) f_ρ = 256(9) MeV from n_f =2 lattice calculations with non-perturbatively improved clover action.
- Recently Hashimoto and Izubuchi (arXiv/0803.0186) obtained f_ρ^{phys} = 210(15) MeV from a n_f = 2 unquenched calculations that used domain wall fermions. However this calculation also found that r_0^{phys} = 0.549(9) fm.
- ETMC (prelim) f_ρ^{phys} = 230(35) MeV (linear fits and only used the β - 3.9 data.)

Results for f_ρ^{trans} (2 GeV)

The transverse decay constant of the vector meson as a function of the square of the pion mass.



Summary of transverse decay constant

Summary of results for transverse decay constants of the ρ and ϕ meson. I only include the result from the finest lattice of Braun et al.

Group	Method	f_ρ^T(2 GeV) MeV	f_ϕ^T(2 GeV) MeV	$\frac{f_\rho^T}{f_\rho}$
Ball et al.	sum rule	155(10)	208(15)	0.74(3)
Becirevic et al.	quenched	150(5)	177(2)	$0.72(2)^{+2}_{0}$
Braun et al.	quenched	154(5)	182(2)	0.74(1)
QCDSF 1999	quenched	149(9)	-	
QCDSF 2005	unquenched	168(3)	-	
RBC-UKQCD	unquenched	143(6)	175(2)	0.69(3)
ETMC (prelim)	unquenched lattice	173(26)	-	0.69(7)

- Only QCDSF compute f_ρ^T on its own, all the others compute $\frac{f_\rho^T}{f_\rho}$ and then multiply by experiment value for f_ρ.
- If I use (reasonable) constant fits to the ETMC data I can get $\frac{f_\rho^T}{f_\rho}$ = 0.74(2).

Decay constants in charmonium

One way to understand charmonium production is to use the NRQCD formalism, where non-perturbative information is coded in a few matrix elements. NRQCD mostly produces a good description of experiment [a]
However consider recent result from from Belle (hep-ex/0205104)

$$\sigma[e^+e^- \rightarrow J/\psi + \eta_c]\mathcal{B} = 25.6 \pm 2.8 \pm 3.4 \ fb$$

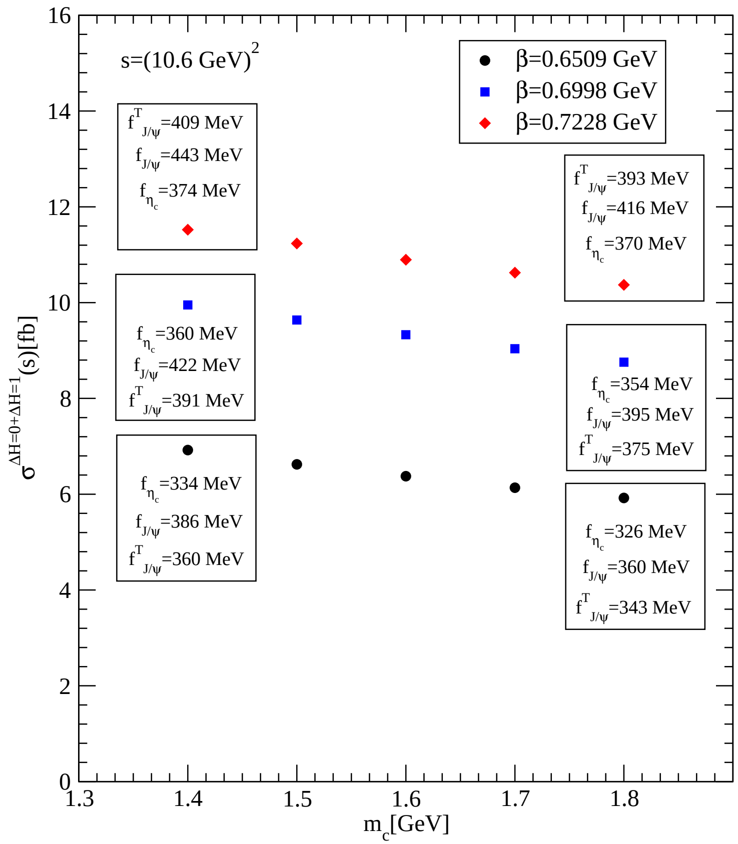
where $\mathcal{B} < 1$. (BaBar has a similar result). Compare to the prediction from leading order NRQCD by Bodwin et al. (hep-ph/0212352).

$$\sigma[e^+e^- \rightarrow J/\psi + \eta_c]_{(NRQCD;LO)} = 3.78 \pm 1.26 \ fb$$

- It is claimed that calculations that use light cone wave functions of charm mesons agree with the Belle result (eg. hep-ph/0412335, Bondar and Chernyak)
- Bodwin (hep-ph/0509203) not clear that model light cone wave functions are to close to true quarkonuim wave functions.

[a] see review Heavy quarkonium physics, hep-ph/0412158

From Choi and Ji (arXiv:0707.1173)



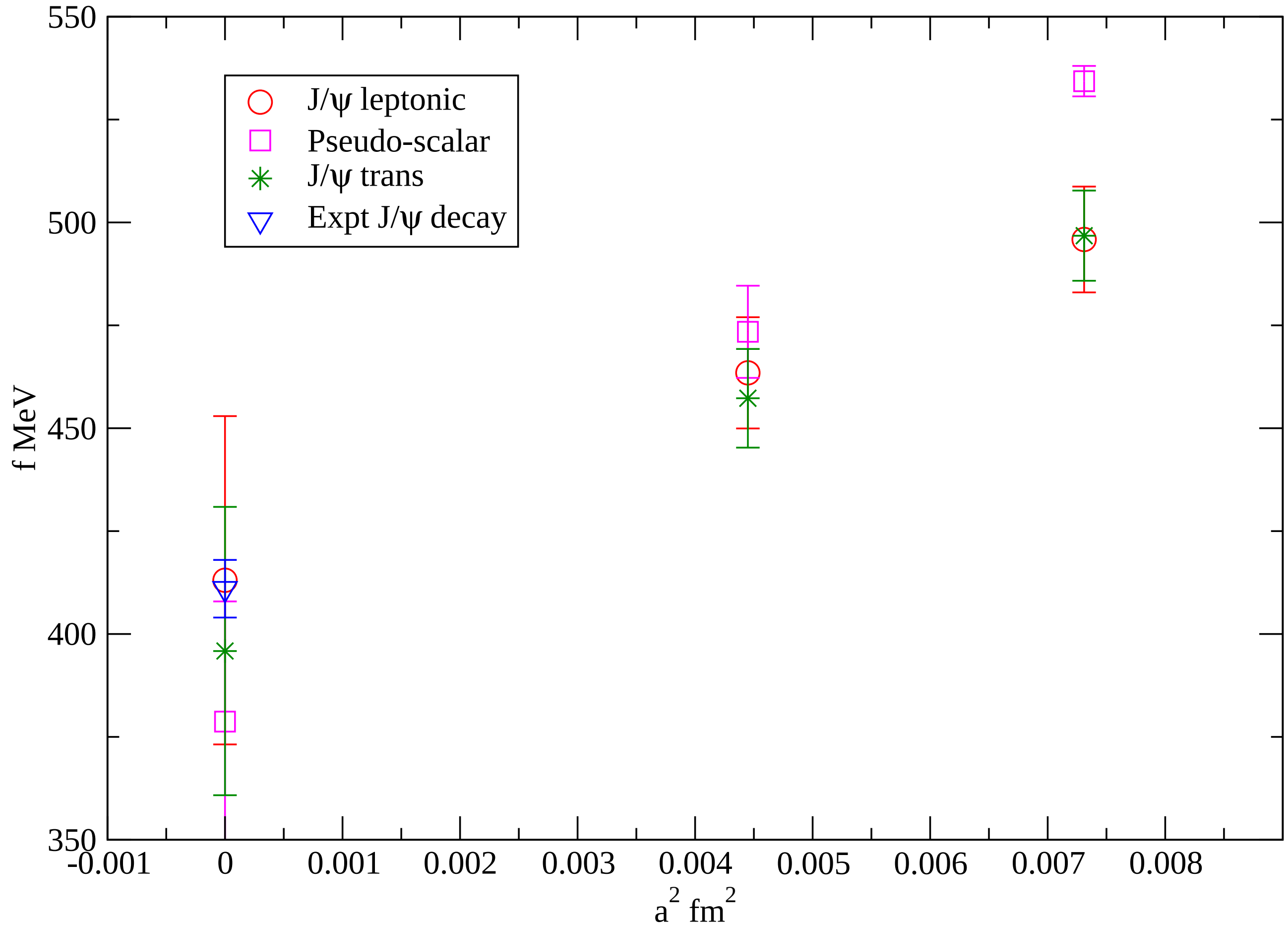
The above is from a quark model calculation. Probably also need higher order QCD corrections to get agreement with Belle.

Lattice calculation

- Use correlators generated using stochastic point sources (“one end trick”).
- Use data at two lattice spacings (beta=3.9 and beta=4.05).
- Interpolate to charm with three heavy masses
- Simple chiral extrapolation in sea quark mass
- Used charged interpolating operators.
- No disconnected diagrams included

As stressed by FNAL and HPQCD collaborations (PoS LATTICE2007:353,2007) the computation of the decay constant (known from experiment) of the J/ψ meson is an important validation test.

Scaling of decay constants (preliminary)



Results for decay constants

Preliminary results. Needs more work on continuum extrapolation.

Decay	f MeV (preliminary)
$f_{J/\psi}$	413(40)
$f_{J/\psi;trans}$	396(35)
f_{η_c}	379(29)

- Compare to experiment $f_{J/\psi}$= 411(7) MeV (PDG, Dudek et al. hep-ph/0601137).
- “Sort of experiment result” f_{η_c} = 335(75) MeV from $B \rightarrow \eta_c K$ with factorization assumption from CLEO (hep-ex/0007012).
- Jlab group obtained (hep-ph/0601137) $f_{J/\psi}$ = 399(4) MeV f_{η_c} = 429(4)(28) MeV from quenched QCD

Conclusions

- Difficult to believe that we can get the masses of the scalars, exotics, Roper resonance, correct, if we can't get the mass of the humble ρ meson correct.
- Choice of scale important for getting non-analytic corrections to rho mass and decay constants correct. Checking the lattice from 2S-1S mass splitting in Upsilon using NRQCD in progress.
- Need to reduce statistical error on ρ meson correlators (color dilution, more sources, probably need more CPU time).
- I showed why computing the decay constants of charmonuim mesons is useful.
- Perhaps need a twisted FNAL heavy quark formalism to deal with $O((aM_Q)^n)$ $n > 1$ corrections.
- Results for the ground state decay constants in charmonium.
- May also need to compute second moment of η_c. Also possible with twisted mass fermions (Baron et al. arXiv:0710.1580 for the pion).