On the phase diagram of the Higgs $SU(2)$ model

C. Bonati, G. Cossu, A. D'Alessandro,

M. D'Elia, A. Di Giacomo

Summary

- the model (notations)

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- features of the model

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- previous results in literature

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- previous results in literature
- new results
- conclusions

The model

$SU(2)$ gauge theory coupled with a Higgs doublet in fundamental representation

$$S = S_W[U] - \frac{\kappa}{2}\sum_{x,\mu}\{\Phi^\dagger(x)U_\mu(x)\Phi(x+\hat{\mu}) +$$

$$+\Phi^\dagger(x+\hat{\mu})U_\mu^\dagger(x)\Phi(x)\} + \lambda\sum_x[\Phi^\dagger(x)\Phi(x) - 1]^2$$

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in this work $\lambda = \infty$

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$$\tilde{\Phi}(x) = i\sigma_2\Phi(x)^* \quad \phi(x) = \begin{pmatrix} \tilde{\Phi}_1(x) & \Phi_1(x) \\ \tilde{\Phi}_2(x) & \Phi_2(x) \end{pmatrix}$$

$$\Phi^\Lambda = \Lambda\Phi \longrightarrow \phi^\Lambda = \Lambda\phi$$

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$$S = \beta \sum_{x,\mu<\nu} \left\{1 - \frac{1}{2} \mathrm{Re}\mathrm{Tr} P_{\mu\nu}(x)\right\} - -\frac{\kappa}{2} \sum_{x,\mu} \mathrm{Tr}[\phi^\dagger(x) U_\mu(x+\hat{\mu}) \phi(x+\hat{\mu})]$$

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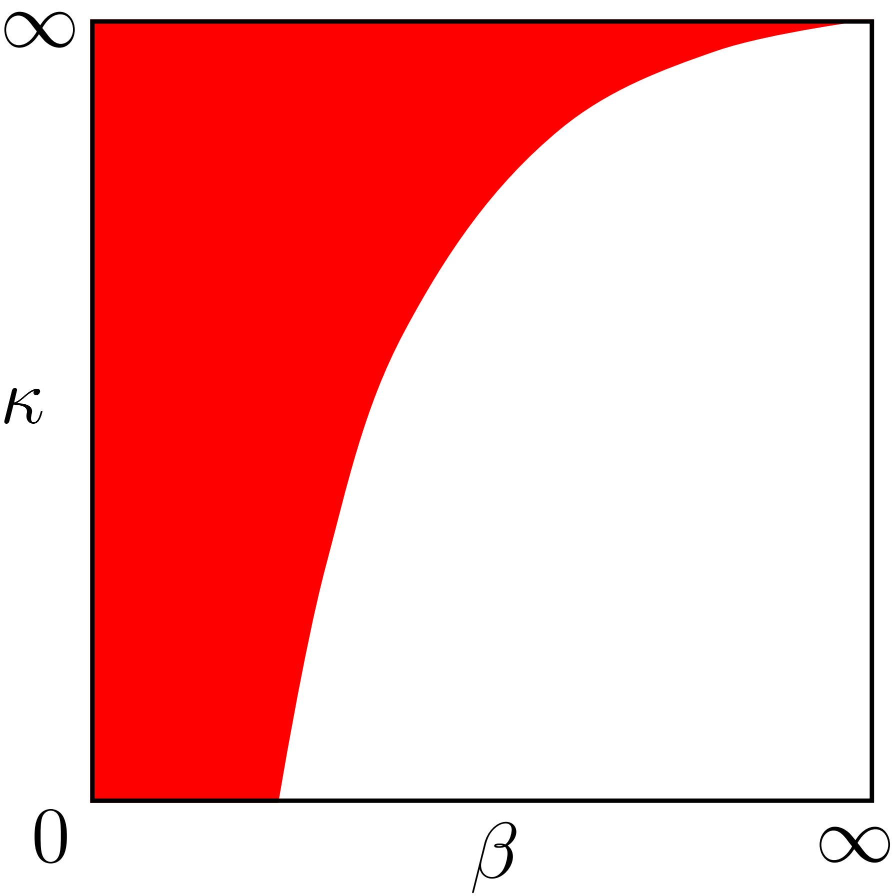
- $\beta = 0$ & unitary gauge ⟶ indipendent $U_\mu(x)'s$
- $\kappa = \infty$ & unitary gauge ⟶ no dynamics

Features of the model

Fradkin-Shenker theorem:
Phys. Rev. D **19**, 3682 (1979)

in the red region
local observables
are analytic



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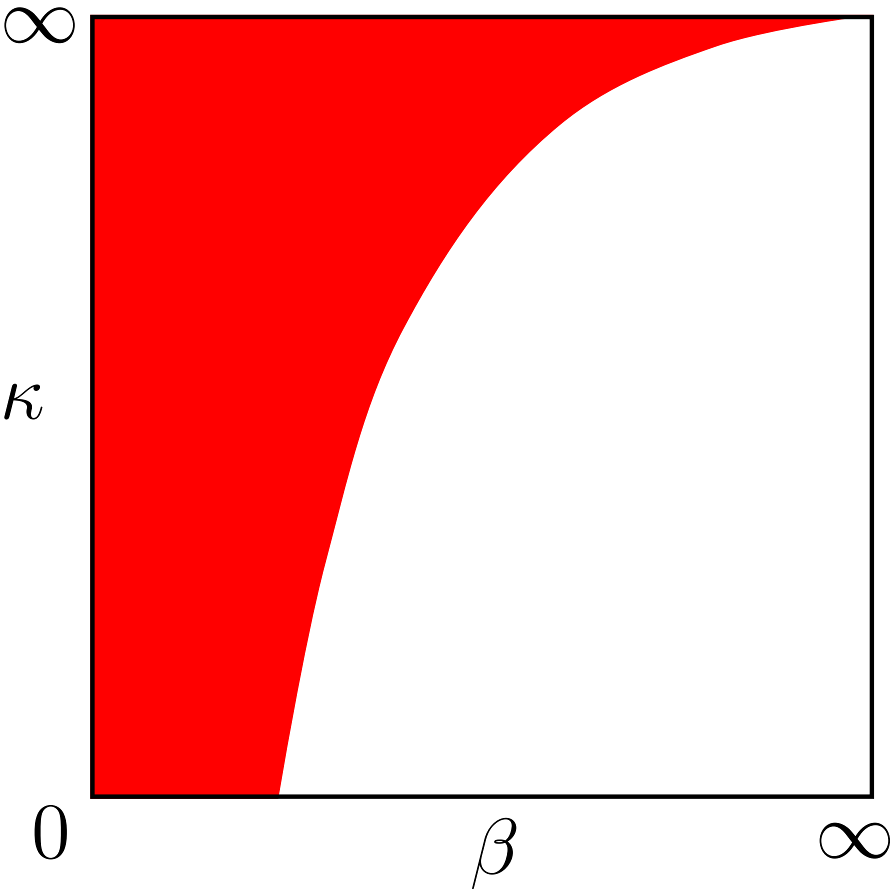

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The theorem does not hold for *non-local* observables!
Grady, Phys. Lett. B **626**, 161 (2005)

Features of the model

Supposed phase diagram at zero temperature

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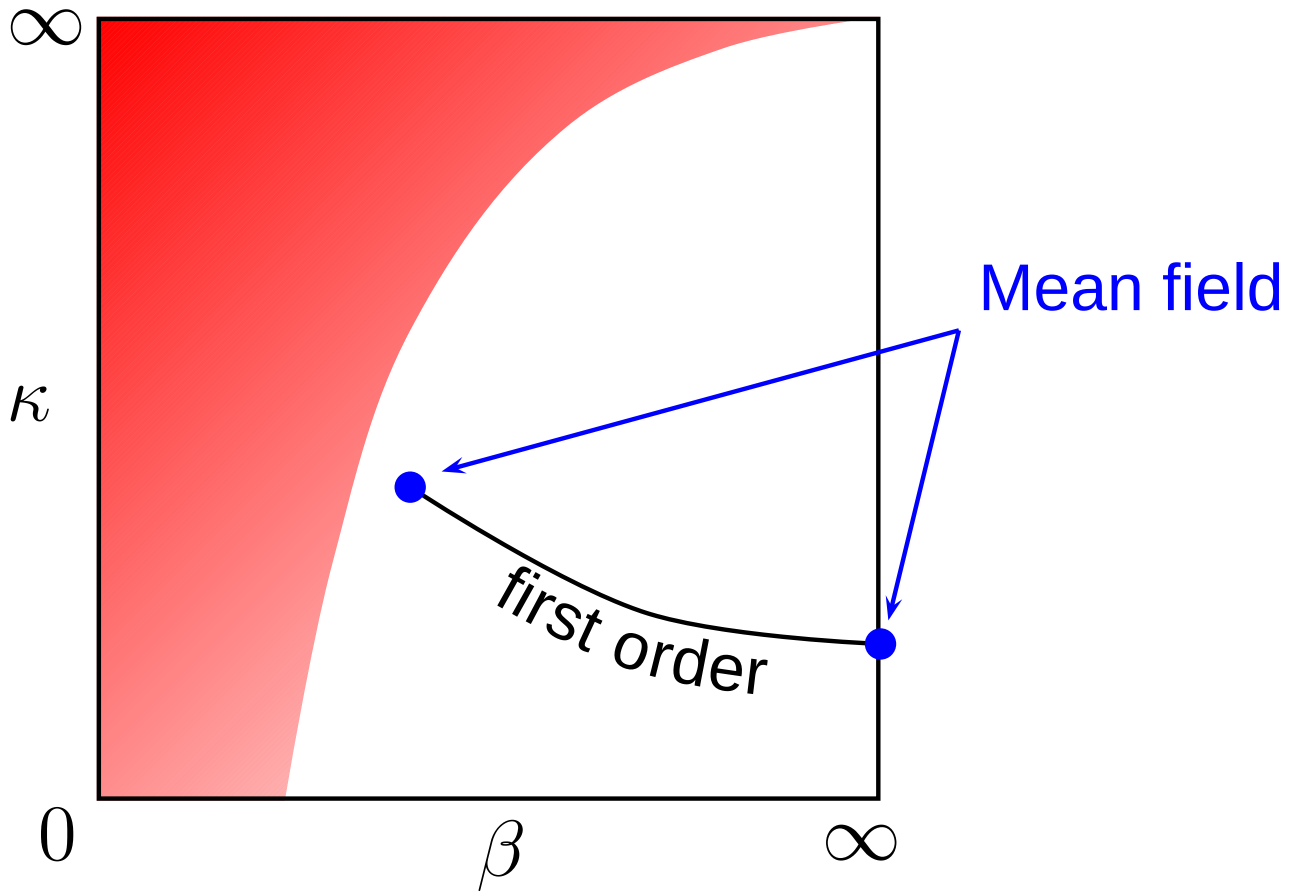

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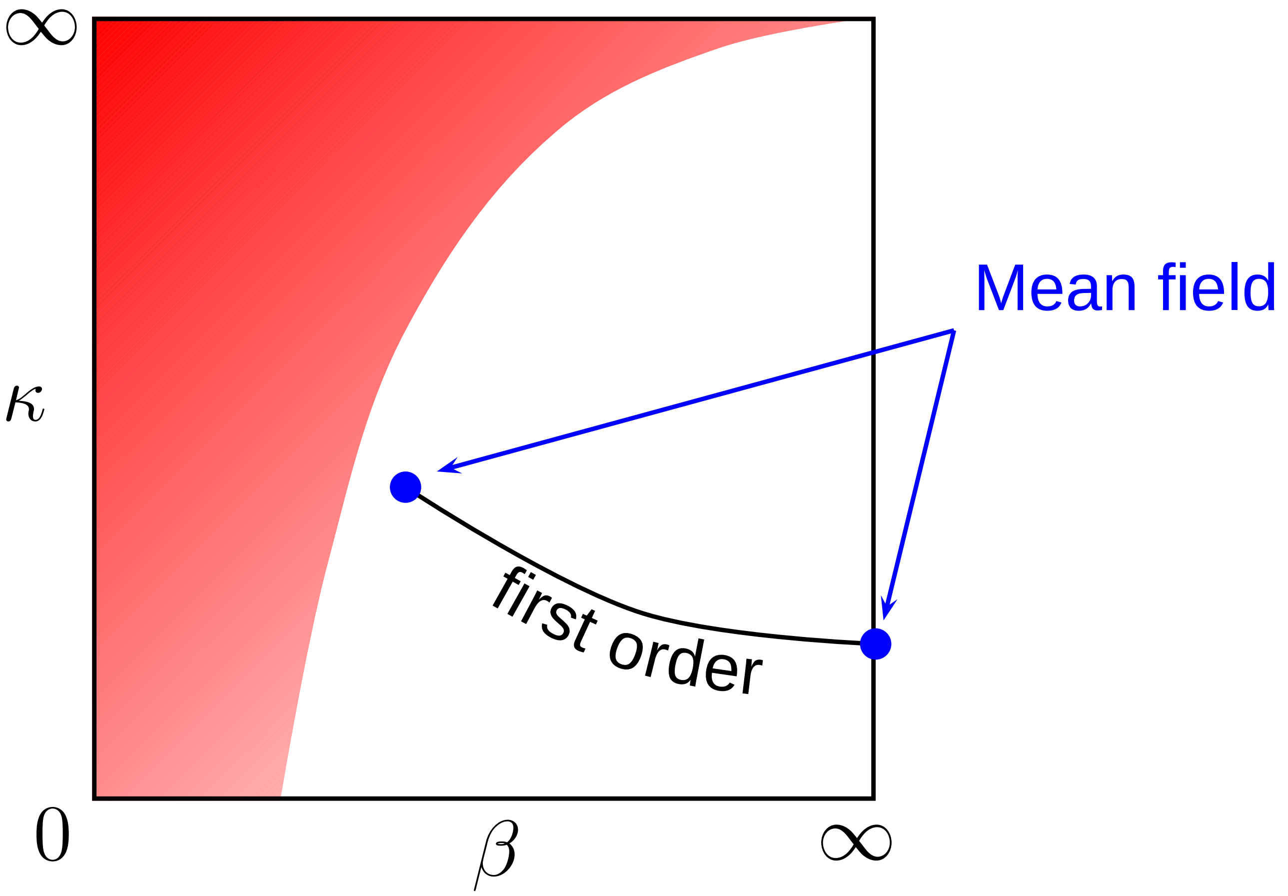

Confermed in simulations at $\lambda = 0.5$

Bock et alt. Phys. Rev. D **41**, 2573 (1990)

Results in literature for the $\lambda = \infty$ case

- first numerical study on 4^4 lattice seem to confirm theoretical prediction

  Lang, Rebbi & Virasoro Phys. Lett. B **104**, 294 (1981)

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- "the system exhibits a transient behavior up to $L = 24$ along which the order of the transition cannot be discerned" (also in this case $\beta = 2.3$)

  Campos Nucl. Phys. B **514**, 336 (1998)

New results

Local observables analized

- plaquette
- Higgs-gauge interaction:
  $\frac{1}{2}\mathrm{Tr}[\phi^\dagger(x)U_\mu(x+\hat{\mu})\phi(x+\hat{\mu})]$
- Z_2 monopoles

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Studied using

- susceptibilities $L^4[\langle O^2\rangle - \langle O\rangle^2]$
- Binder fourth-order cumulant $1 - \langle O^4\rangle/[3\langle O^2\rangle^2]$

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resulted the most sensitive one

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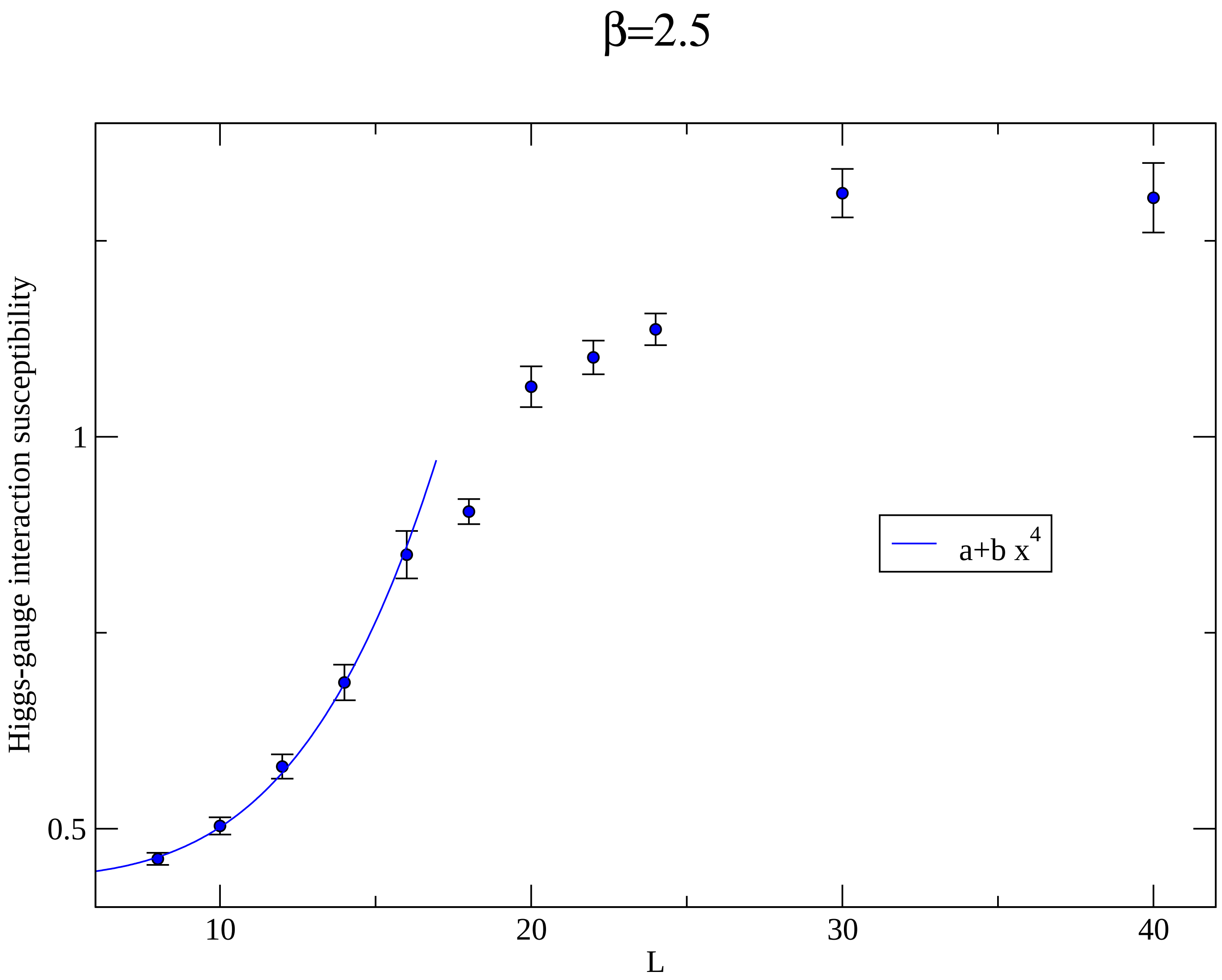

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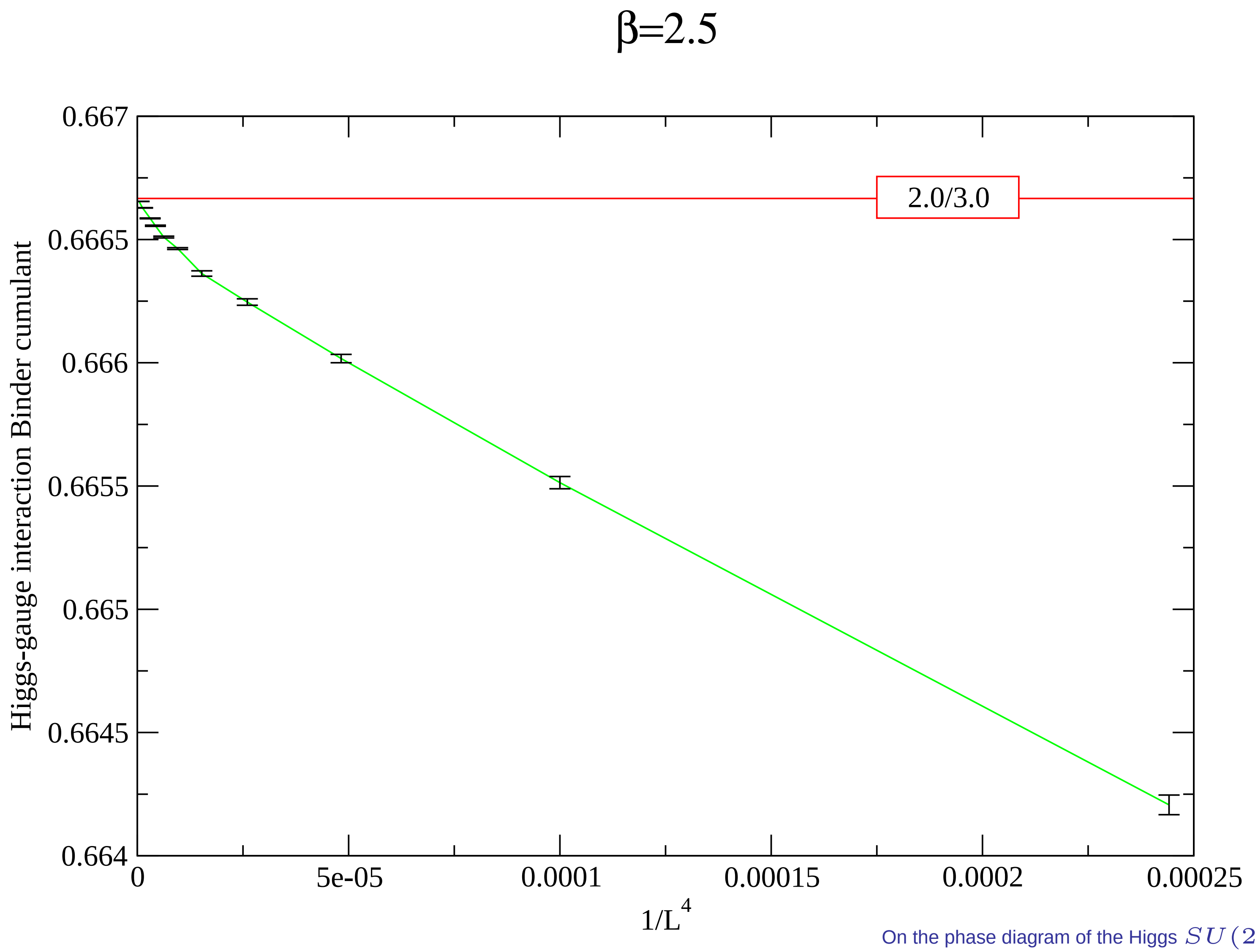

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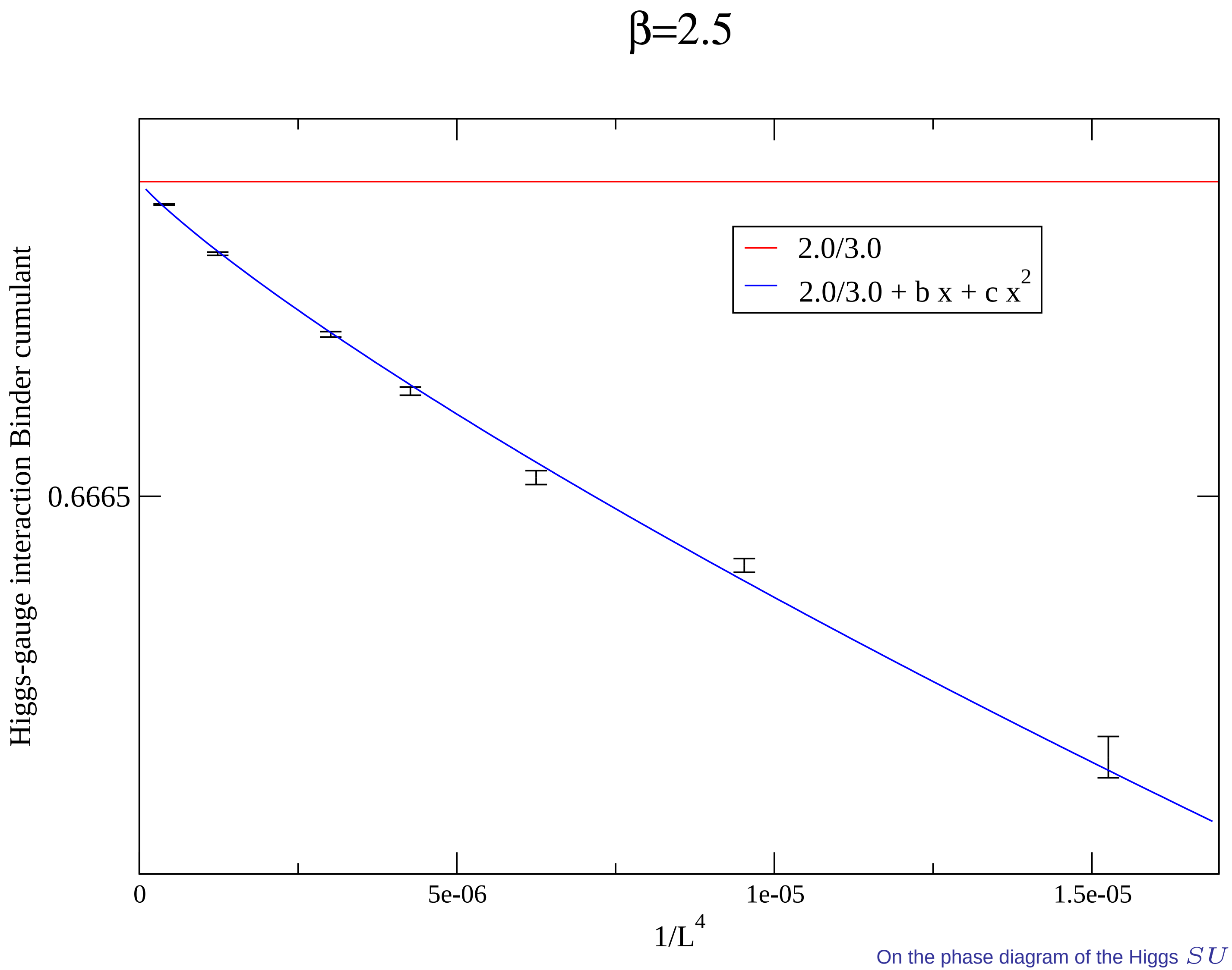

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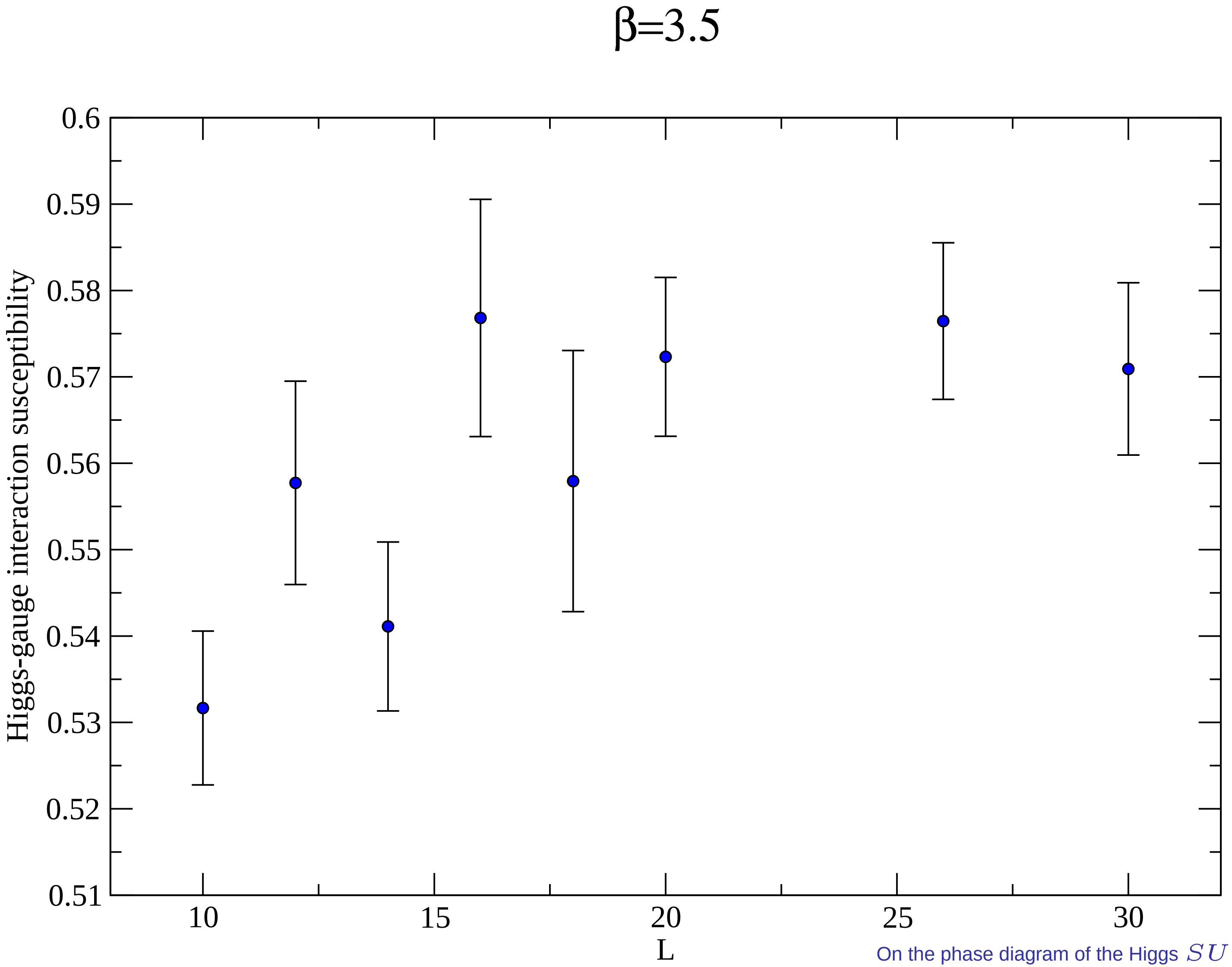

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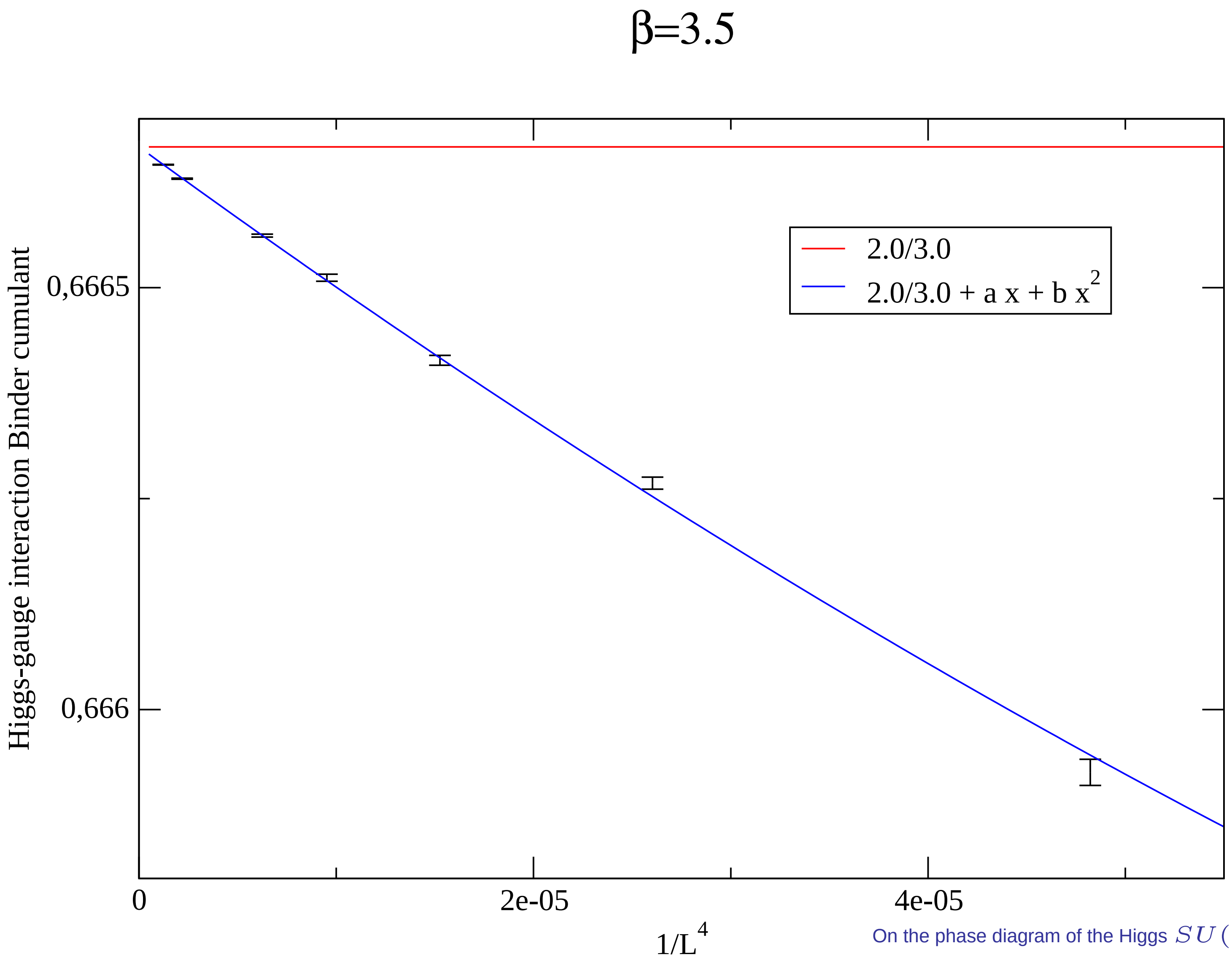
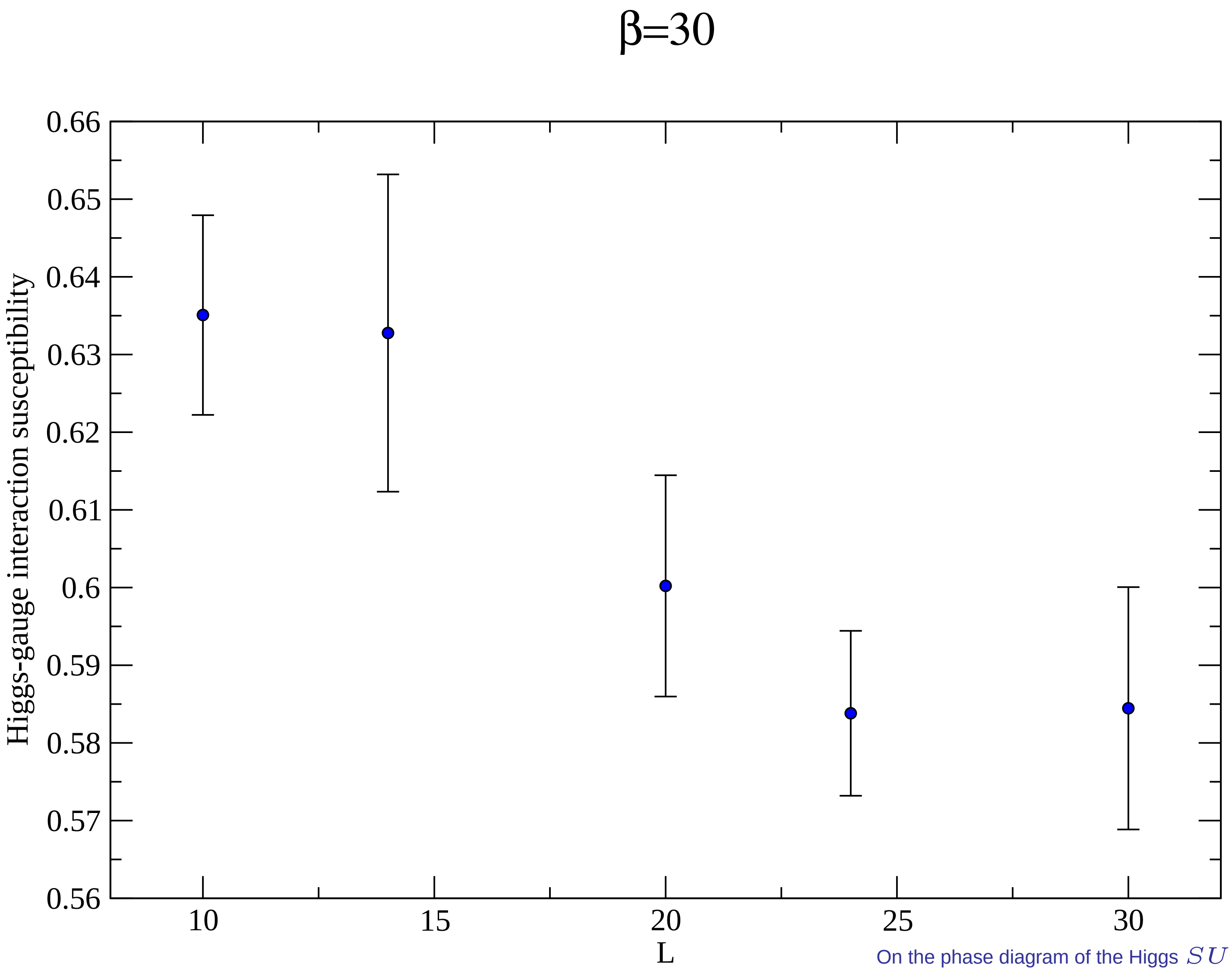


New results: $\beta = 30$



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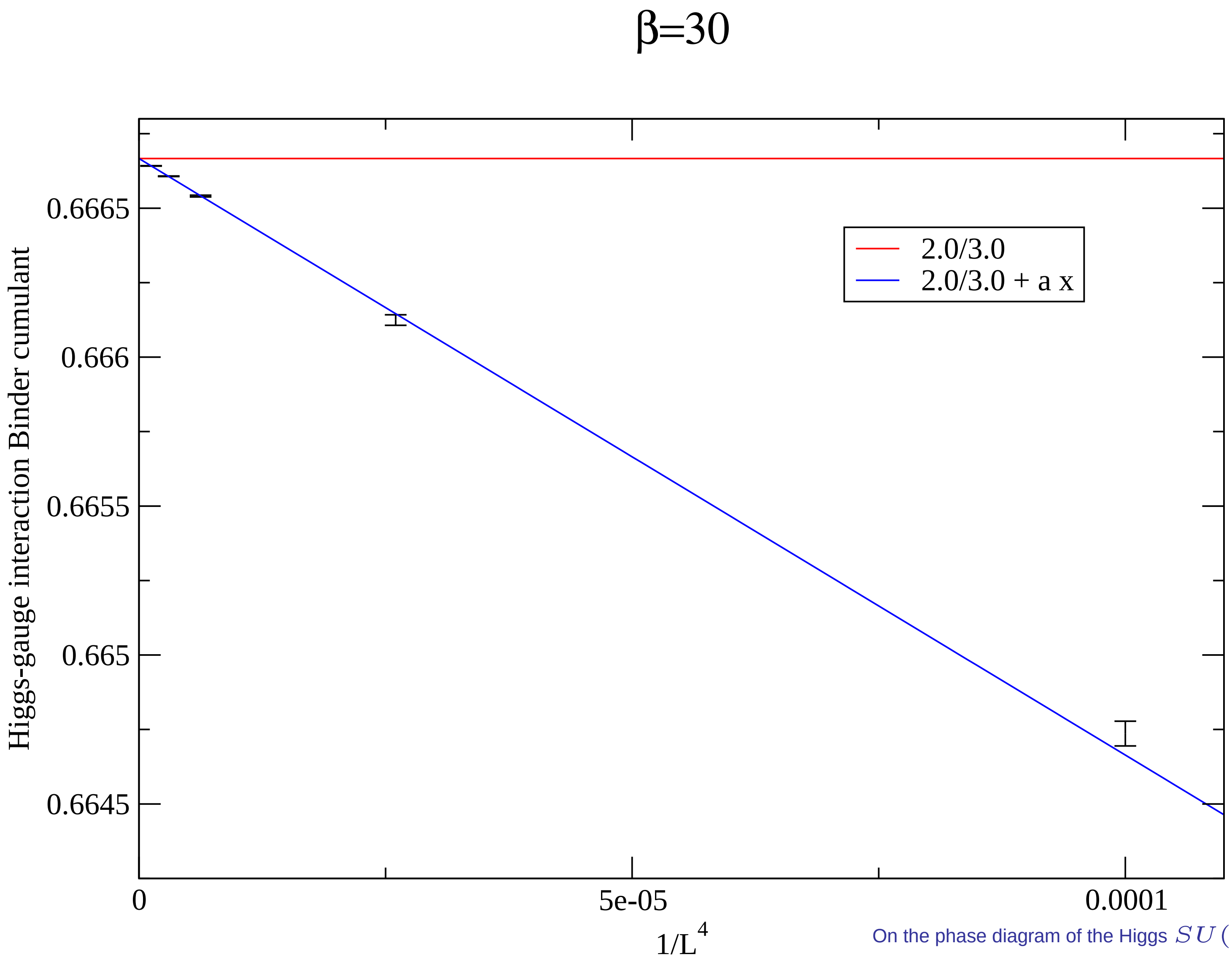

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Characteristic behavior

- smooth crossover ⟶ no singularity
- at transition ⟶ $\min\rho \to -\infty$

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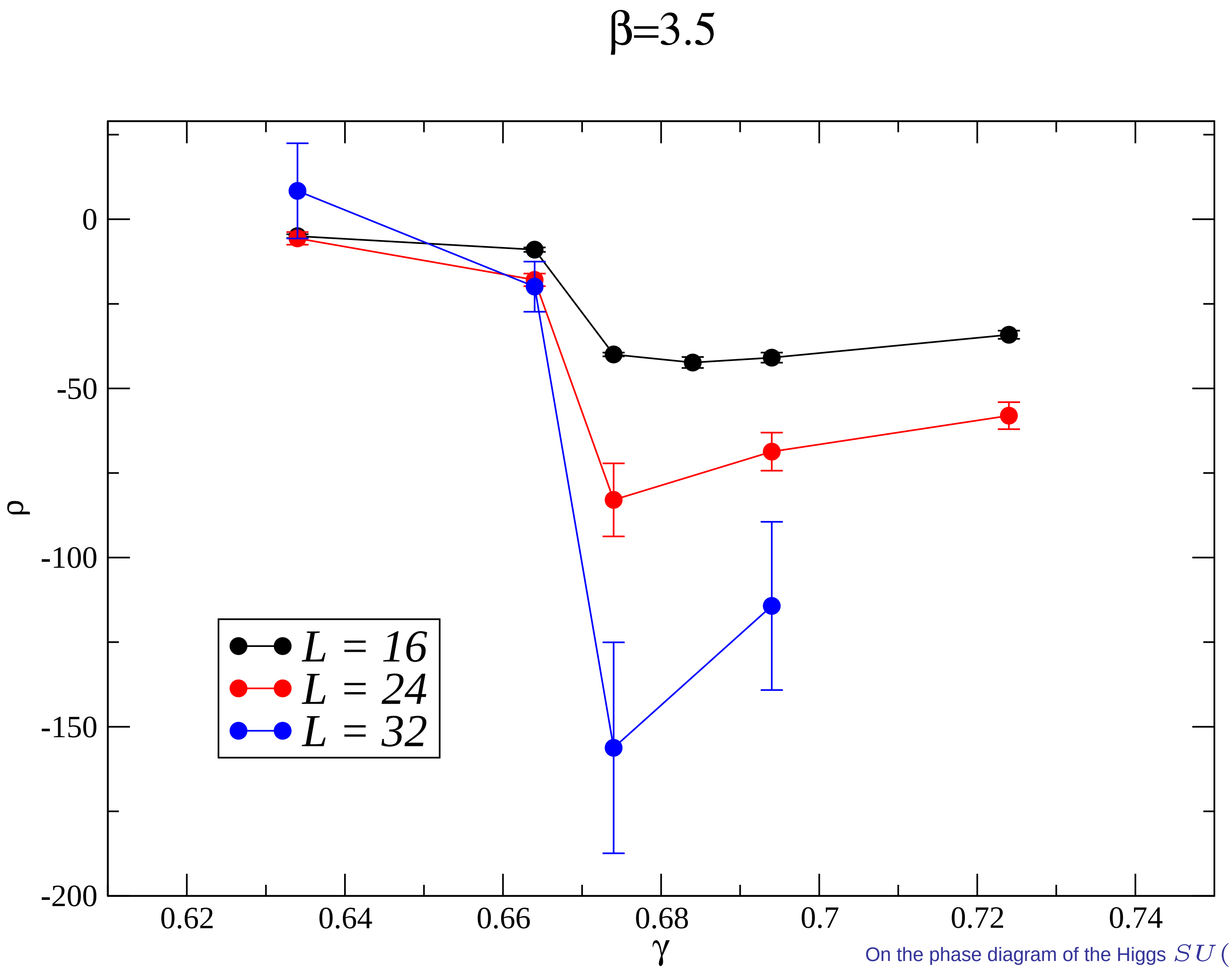

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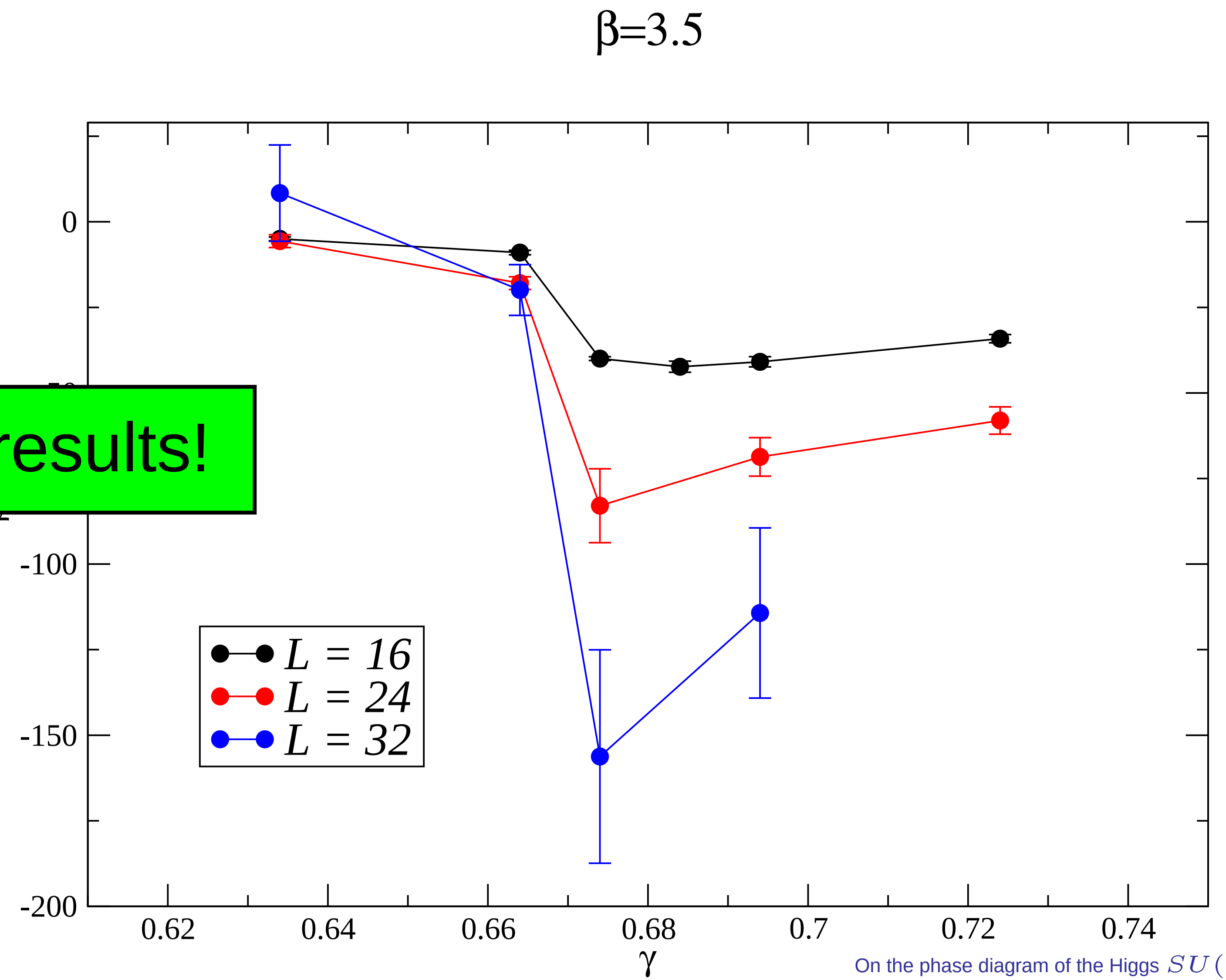

Not yet firm results!

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- for all the β's simulated only a smooth crossover is seen
- it seem natural to suppose that the first-order line of transitions is not present in the $\lambda = \infty$ case
- conservative point of view: we have shown that, if it exists, the line of first order transitions ends for β much bigger than the value $\beta_c \approx 2$ previously thought as critical