Finite volume study of the Δ magnetic moments using dynamical Clover fermions

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Outline

- Delta Magnetic Moments (PDG)
- Background Field:
  - Periodicity issues
  - Finite Volume effects
- Quenched Results
- Preliminary Dynamical Results
- Conclude

From the PDG

$$\mu_{\Delta^{++}} = (4.52 \pm 0.50 \pm 0.45)\mu_N$$

$$\mu_{\Delta^{+}} = (2.7\,^{+1.0}_{-1.3} \pm 1.5 \pm 3)\mu_N$$

Theoretical input

Best way to get this theoretically is on the lattice!

Using Form Factors

Difficult to reach low q^2 values,
due to the finite volume of the lattice

$$p_{\min} = \frac{2\pi}{aL}$$

Solutions:
Bigger Volumes
Twisted Boundary Conditions

Alternative: Background Field method

In a background magnetic field, B
the ground-state energy is shifted to

$$m_B = m_\Delta \pm \mu B$$

2pt function: $C_{B,s}(t) \sim A_{s,B} e^{-m_B t}, \quad t \gg 1$

One can also calculate other EM properties
(see all the other talks in this session)

Studies with this technique go all the way back to C. Bernard, et al., PRL 49:1076 (1982)

Up until now, studies are quenched and do not use magnetic fields which satisfy the necessary periodicity constraint...

The U(1) gauge fields take the form

$$U_\mu(x,y,z,t) = \exp\left[ieaA_\mu(x,y,z,t)\right]$$

For constant B-field in the z-direction:

$$A_\mu(x,y,z,t) = \begin{cases} aBx & \mu = y \\ 0 & \mu = x,z,t \end{cases}$$

(naive) Periodicity

We wish to make sure the particles don't see a discontinuity on the boundaries of the lattice.

Thus, the link at x=L-1 must equal the link at x=0 for all y, so

$$\Rightarrow B = \frac{2\pi n}{L}$$

(naive) Periodicity

For current lattices, the minimally allowed field is often too large

$$L = 20, a^{-1} = 2 \text{ GeV}$$

$$\Rightarrow B = 314 \text{ MeV}$$

This field is large enough to distort our hadrons

Again, we can solve this by using larger lattices, but this is expensive

Try smaller B fields (non-periodic), place the baryon far from the boundaries. (and hope the discontinuity is not noticeable)

Or, modify implementation of B field to change the constraint [Damgaard, Heller, NPB309 (1988)]

First, recognize that it is not the vector potential that must be periodic.

We want the magnetic flux, or plaquette to be continuous over the boundary

In other words, on the boundary, the x-links become:

$$U_x(L, y, z, t) = e^{-iaBy(L+1)}$$

or

$$B = \frac{2\pi n}{L^2}$$

But is this sufficient?

$$B_{\text{min}}^{\text{patched}} = \frac{1}{L} B_{\text{min}}^{\text{unpatched}}$$

But we want more than one field to simulate (higher n means much stronger field)

In fact, we need at least three (if we want two fields and to do the Δ^+)

Is it safe not to satisfy the periodicity?

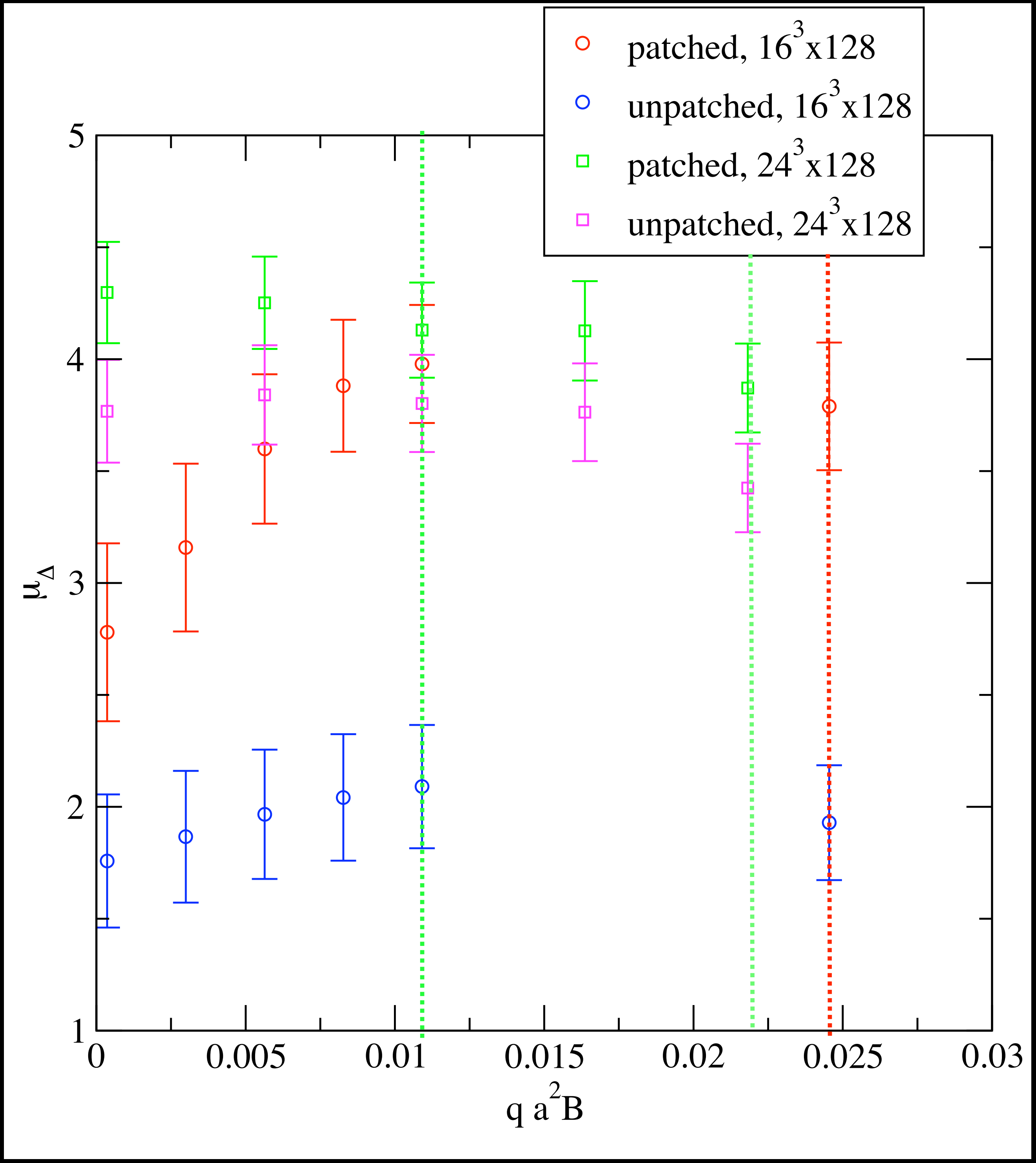
Answer: Yes, sort of

The worry is that the particle may propagate to the boundary and see the discontinuity

Safe, for large enough volume!

First a quenched test
(Clover quarks)

Volume	a_s, a_t	Pion Mass (MeV)	$\frac{2\pi}{L}$	$\frac{2\pi}{L^2}$
16^3x128	0.1 fm, 0.03fm	750	0.39	0.025
24^3x128	0.1 fm, 0.03fm	750	0.26	0.011

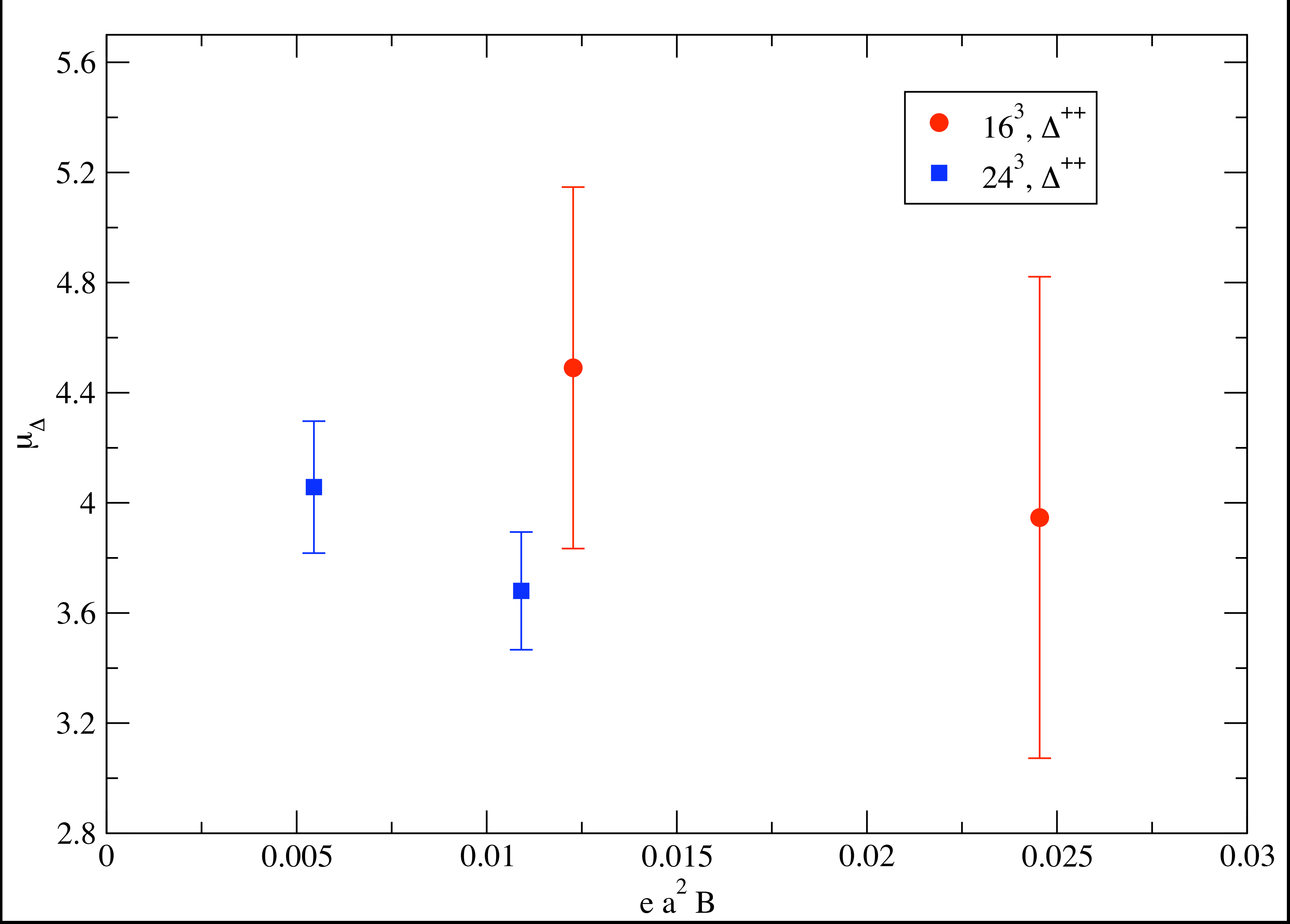

patched, 16^3x128
unpatched, 16^3x128
patched, 24^3x128
unpatched, 24^3x128
μ_Δ
q a^2B
1
2
3
4
5
0
0.005
0.01
0.015
0.02
0.025
0.03

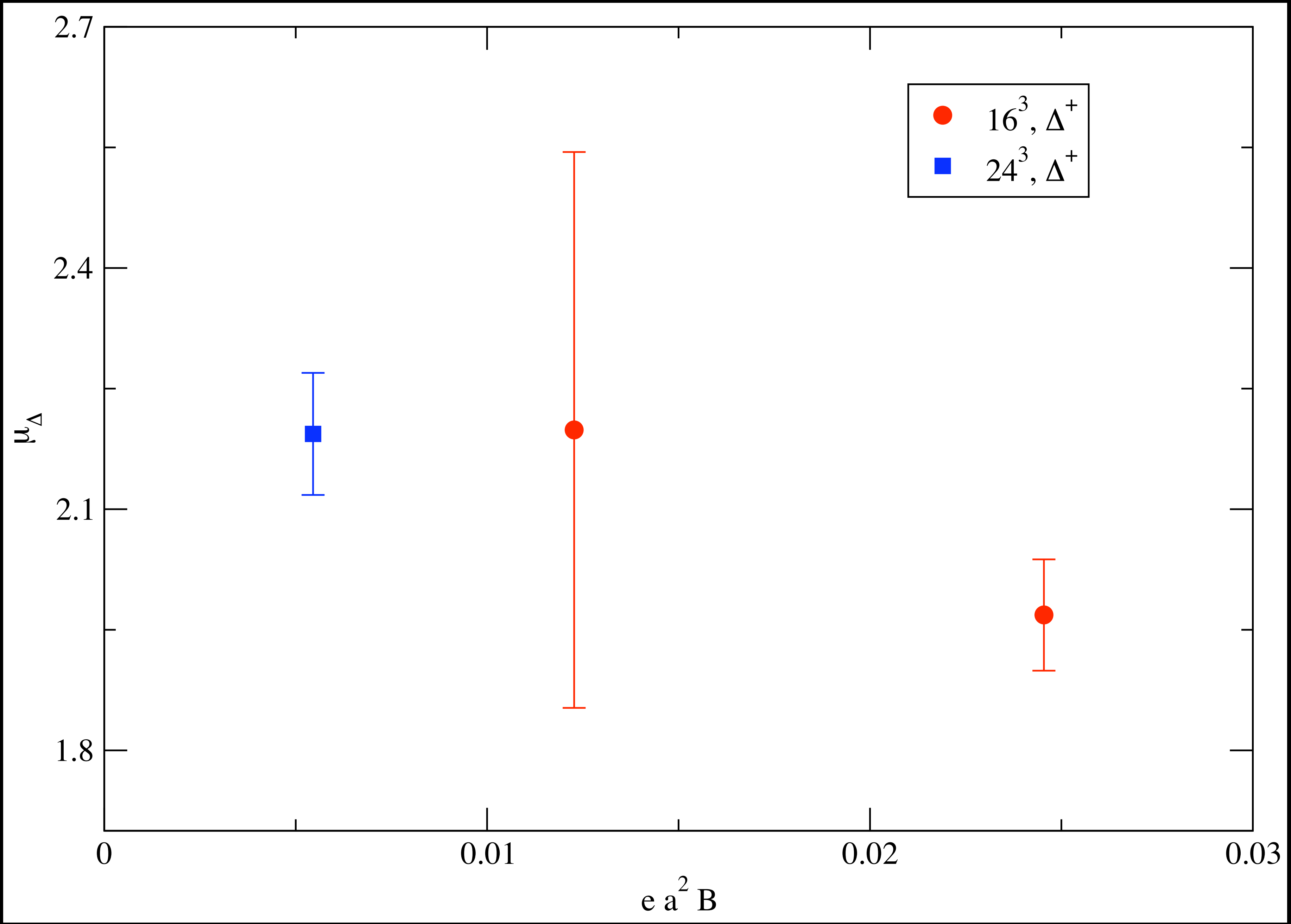
Preliminary Dynamical Results

Volume	a_s, a_t	Pion Mass (MeV)	#confs
16^3x128	0.1 fm, 0.036 fm	366	39
24^3x128	0.1 fm, 0.036 fm	366	120 (s) 147 (u)

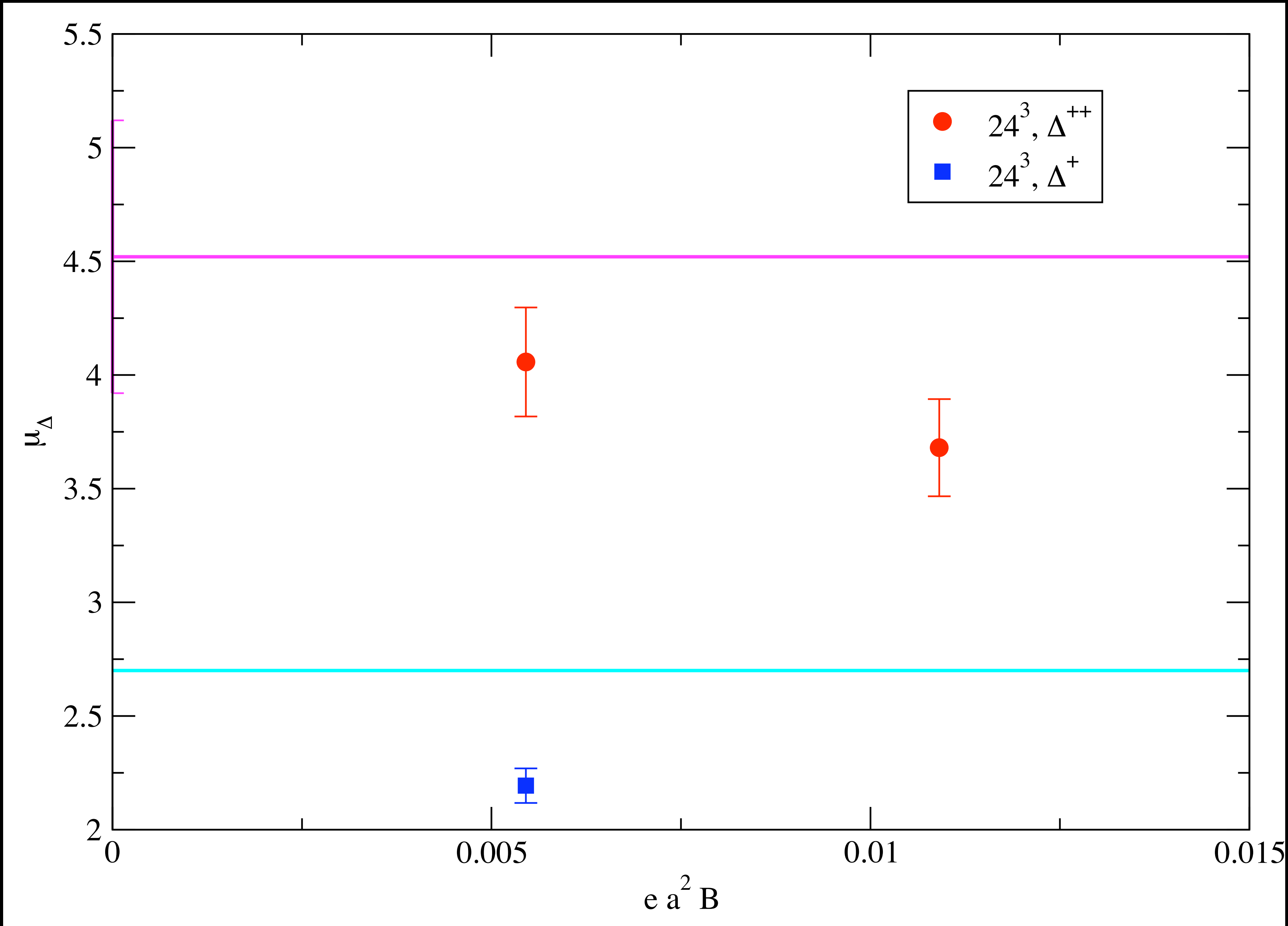
See talks by R. Edwards & M. Peardon

$m_\Delta = 1.408$ GeV

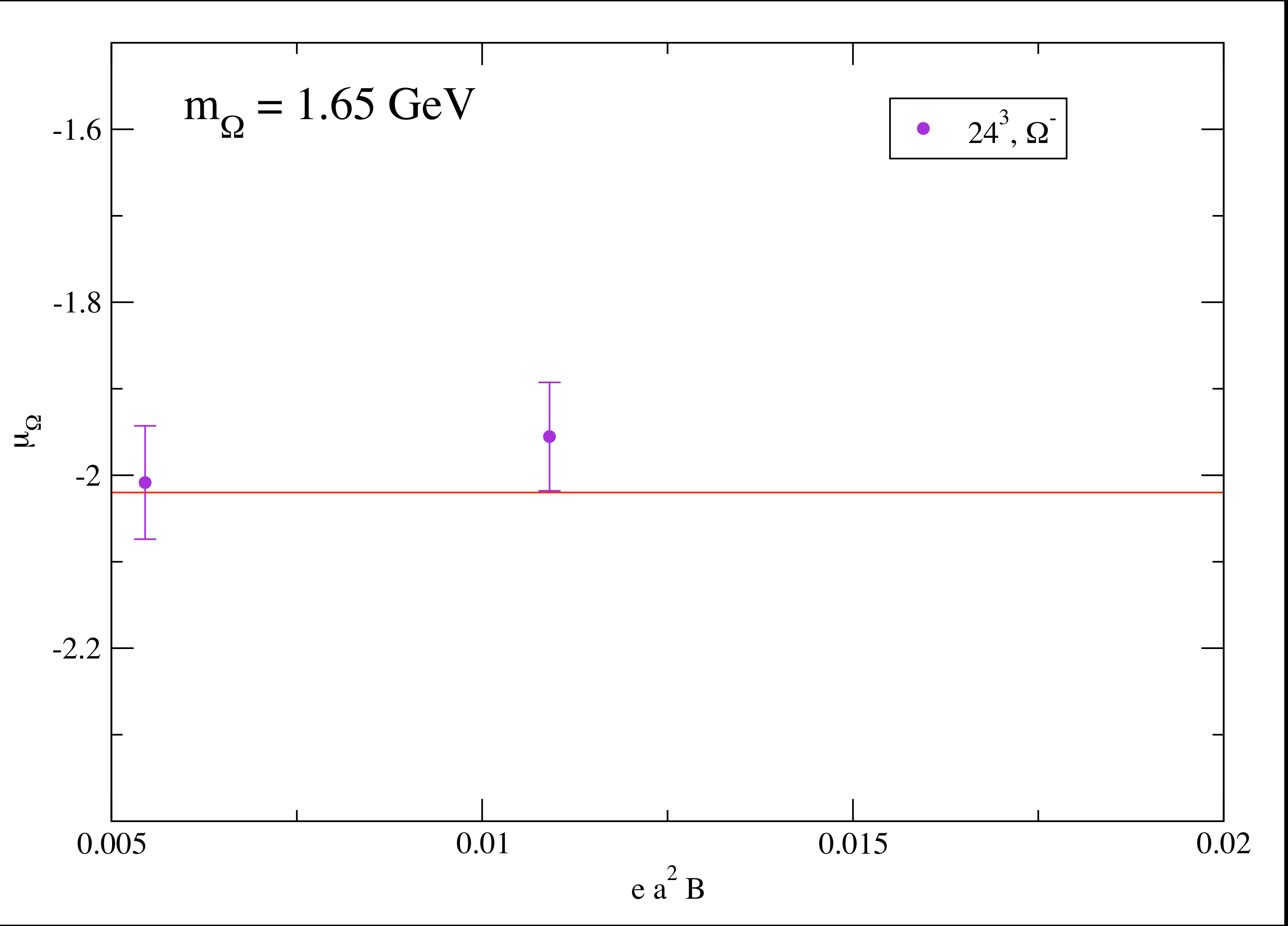




2.7
2.4
2.1
1.8
μ_Δ
0
0.01
0.02
0.03
e a^2 B
16^3, Δ^+
24^3, Δ^+



5.5
5
4.5
4
3.5
3
2.5
2
0
0.005
0.01
0.015
μ_Δ
$e\,a^2\,B$
24^3, Δ^{++}
24^3, Δ^{+}



m_Ω = 1.65 GeV
24^3, Ω^-
-1.6
-1.8
-2
-2.2
μ_Ω
0.005
0.01
0.015
0.02
e a^2 B

To conclude...

Using the "patched" B-field, FV errors from BF are not very large

Must patch!

Need more 16^3 statistics

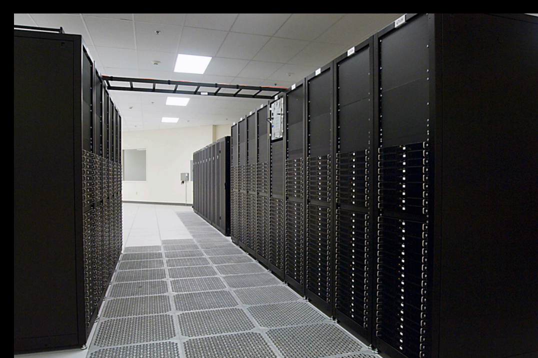
Also: diff. masses for chiral extrap.

Other moments with BF? No..
Cost is comparable with FF approach, and theoretically challenging

Computing resources

NERSC

cyclades cluster
@ WM



JLab