The light hadron spectrum II

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The Budapest-Marseille-Wuppertal collaboration

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Outline

WHY THE LIGHT HADRON SPECTRUM?

- Goal:
  - Firmly establish (or invalidate?) QCD as the theory of strong interaction in the low energy region
- Method:
  - Post-diction of light hadron spectrum
    - Octet baryons
    - Decuplet baryons
    - Vector mesons
- Challenge:
  - Minimize and control all systematics
    - 2+1 dynamical fermion flavors (→S. Krieg)
    - Physical quark masses
    - Continuum
    - Infinite volume (treatment of resonant states)

SETUP

Goal:

- Stable formulation/algorithm for $N_f = 2 + 1$ QCD

Method:

- Tree level Symanzik improved gauge action
- Tree level $O(a)$ improved smeared Wilson fermions
  - 6-step stout-smearing, $\rho = 0.11$
- Stable HMC-RHMC algorithm
  - Multiple timescale omelyan integrator, mass preconditioning, mixed precision solver (Sexton, Weingarten, 1992; Hasenbusch, 2001; Omelyan, 2003; Urbach et. al. 2006; BMW 2008)

stable, excellent scaling (➛S. Krieg, T. Kurth)

QUARK MASS DEPENDENCE

Goal:

- Extra-/Interpolate M_X (baryon/vector meson mass) to physical point (M_π, M_K)

Method:

- Use M_Ξ or M_Ω to set the scale
- Variables to parametrize M_π^2 and M_K^2 dependence of M_X:
  - Use bare masses aM_y, $y \in \{X, \pi, K\}$ and a (bootstrapped)
  - Use dimensionless ratios $r_y := \frac{M_y}{M_{\Xi/\Omega}}$ (cancellations)

We use both procedures ➛systematic error

QUARK MASS DEPENDENCE (ctd.)

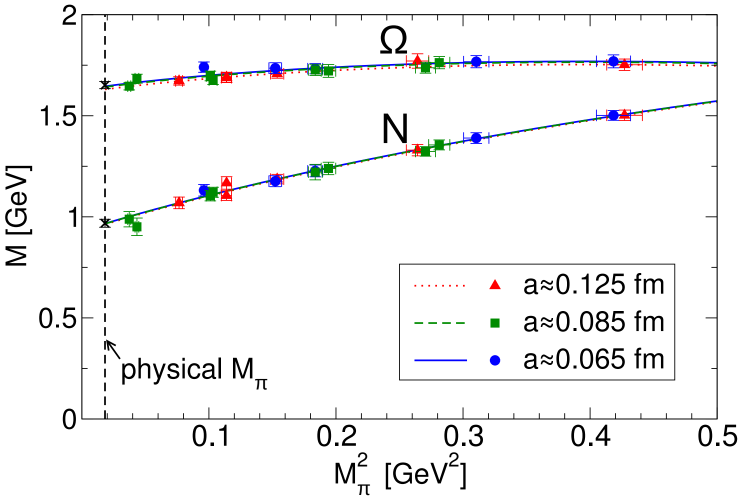
Method (ctd.):

- Parametrization: $M_X = M_X^{(0)} + \alpha M_\pi^2 + \beta M_K^2 +$ higher orders
  - Leading order sufficinet for M_K^2 dependence
  - We include higher order term in M_π^2
    - Next order χPT (around $M_\pi^2 = 0$): $\propto M_\pi^3$
    - Taylor expansion (around $M_\pi^2 \neq 0$): $\propto M_\pi^4$

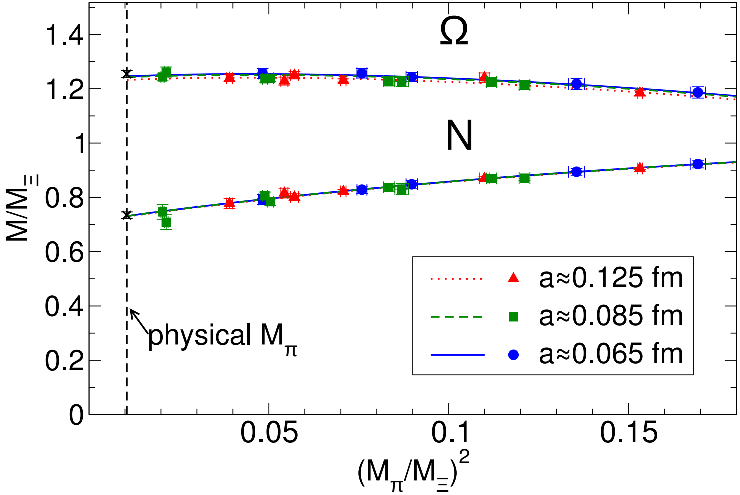
  Both procedures fine ➛systematic error
  No sensitivity to any order beyond these
- Vector mesons: higher orders not significant
- Baryons: higher orders significant
  - Restrict fit range to further estimate systematics:
    - full range, $M_\pi < 550/450$MeV

  We use all 3 ranges ➛systematic error

CHIRAL FIT


Ω
N
M [GeV]
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
M$_\pi^2$ [GeV^2]
physical M$_\pi$
a≈0.125 fm
a≈0.085 fm
a≈0.065 fm

CHIRAL FIT USING RATIOS



CONTINUUM EXTRAPOLATION

Goal:

- Eliminate discretization effects

Method:

- Formally in our action: $O(\alpha_s a)$ and $O(a^2)$
- Discretization effects are tiny
  - Not possible to distinguish between $O(a)$ and $O(a^2)$

➛include both in systematic error

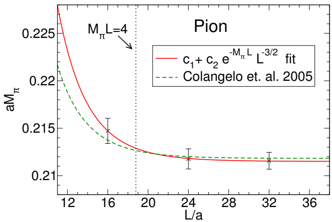
FINITE VOLUME EFFECTS FROM VIRTUAL PIONS

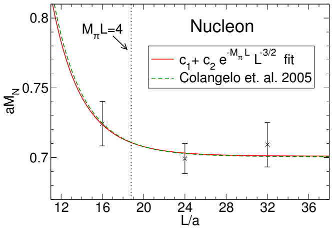
Goal:

- Eliminate virtual pion finite V effects

Method:

- Best practice: use large V
  - We use $M_\pi L \gtrsim 4$ (and one point to study finite V)
  - Effects are tiny and well described by
    $\frac{M_X(L)-M_X}{M_X} = cM_\pi^{1/2} L^{-3/2} e^{M_\pi L}$ (Colangelo et. al., 2005)





FINITE VOLUME EFFECTS IN RESONANCES

Goal:

- Eliminate spectrum distortions from resonances mixing with scattering states

Method:

- Stay in region where resonance is ground state
  - Otherwise no sensitivity to resonance mass in ground state
- Systematic treatment (Lüscher, 1985-1991)
  - Conceptually satisfactory basis to study resonances
  - Coupling as parameter (related to width)
- Fit for coupling (assumed constant, related to width)
  - No sensitivity on width (compatible within large error)
  - Small but dominant FV correction for light resonances

SYSTEMATIC UNCERTAINTIES

Goal:

- Accurately estimate total systematic error

Method:

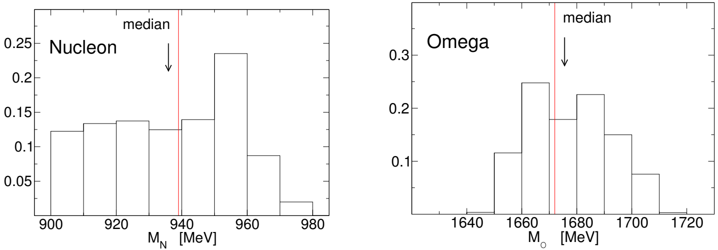
- We account for all the above mentioned effects
- When there are a number of sensible ways to proceed, we take them: Complete analysis for each of
  - 18 fit range combinations
  - ratio/nonratio fits (r_X resp. M_X)
  - $O(a)$ and $O(a^2)$ discretization terms
  - NLO χPT M_π^3 and Taylor M_π^4 chiral fit
  - 3 χ fit ranges for baryons: $M_\pi < 650/550/450$ MeV

  resulting in 432 (144) predictions for each baryon (vector meson) mass with each 2000 bootstrap samples for each Ξ and Ω scale setting

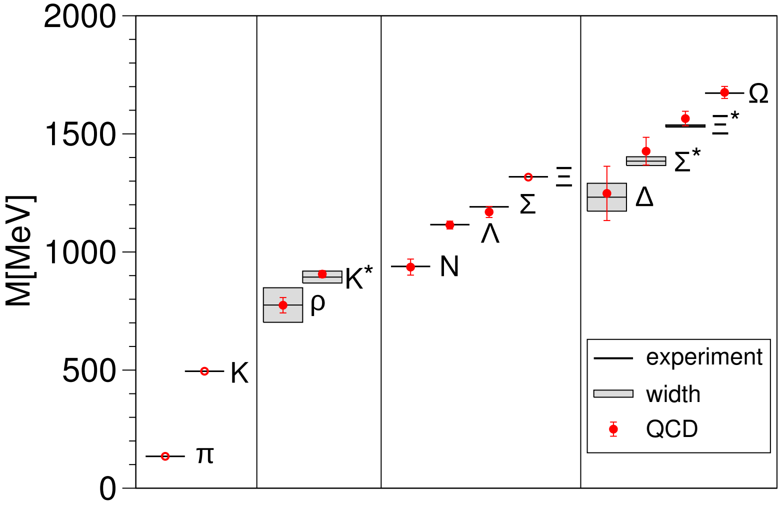
SYSTEMATIC UNCERTAINTIES II

Method (ctd.):

- Weigh each of the 432 (144) central values by fit quality Q
  - Median of this distribution ➛final result
  - Central 68% ➛systematic error
- Statistical error from bootstrap of the medians



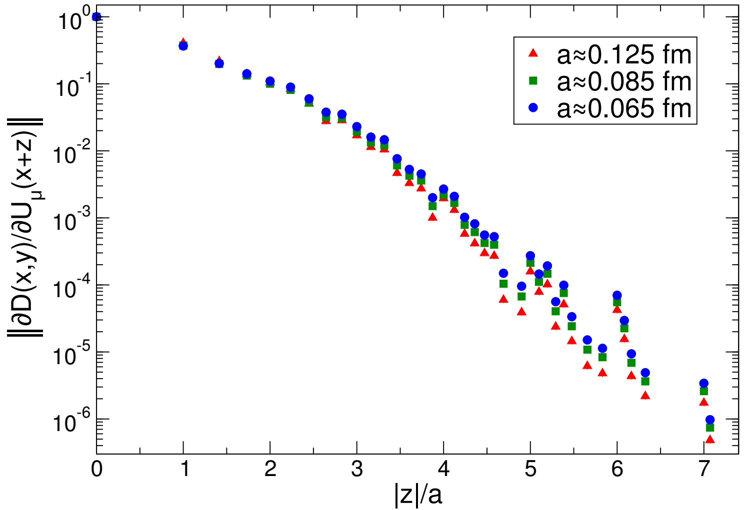
THE LIGHT HADRON SPECTRUM



Mass predictions in GeV

	Exp.	Ξ scale	Ω scale
ρ	0.775	0.775(29)(13)	0.778(30)(33)
K^*	0.894	0.906(14)(4)	0.907(15)(8)
N	0.939	0.936(25)(22)	0.953(29)(19)
Λ	1.116	1.114(15)(5)	1.103(23)(10)
Σ	1.191	1.169(18)(15)	1.157(25)(15)
Ξ	1.318		1.317(16)(13)
Δ	1.232	1.248(97)(61)	1.234(82)(81)
Σ^*	1.385	1.427(46)(35)	1.404(38)(27)
Ξ^*	1.533	1.565(26)(15)	1.561(15)(15)
Ω	1.672	1.676(20)(15)	

Locality



a≈0.125 fm
a≈0.085 fm
a≈0.065 fm
‖∂D(x,y)/∂U_μ(x+z)‖
|z|/a
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
0
1
2
3
4
5
6
7