(Electric) Polarizabilities from Lattice QCD

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14 July, 2008

Lattice 2008

Outline

- **Motivation:**

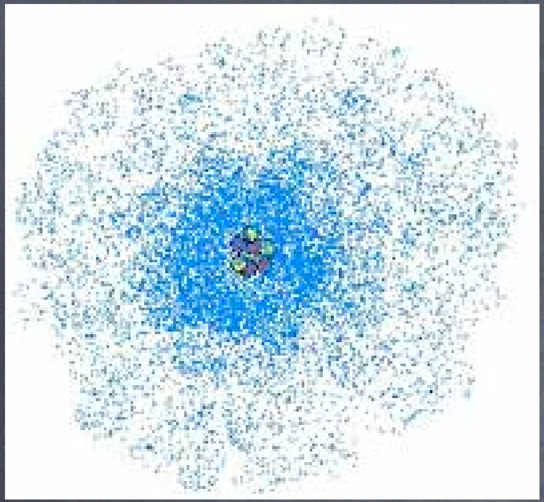
  Polarizabilities, Chiral Symmetry, Experimental Effort

- **Computational Details:**

  Hybrid Lattice Action, Background Fields

- **Results:**

  Neutral & Charged Particle Electric Polarizabilities



Electromagnetic Polarizabilities

$$\vec{p} = -\alpha_E \vec{E}$$

$$\Delta H = -\frac{1}{2}\alpha_E \vec{E}^2$$

- In external EM fields, hadrons polarize against strong chromodynamic forces
- Chiral dynamics: charged pion cloud deforms

Hadronic polarizabilities are tightly constrained by chiral dynamics

Pion $\alpha_E^\pi = N \frac{e^2}{m_\pi \Lambda^2}$ Proton $\alpha_E^p = N' \frac{e^2}{m_\pi \Lambda^2}$

$$\Delta \overline{H}_N = -\frac{1}{2}\alpha_E \boldsymbol{E}^2 - \frac{1}{2}\beta_M \boldsymbol{B}^2$$

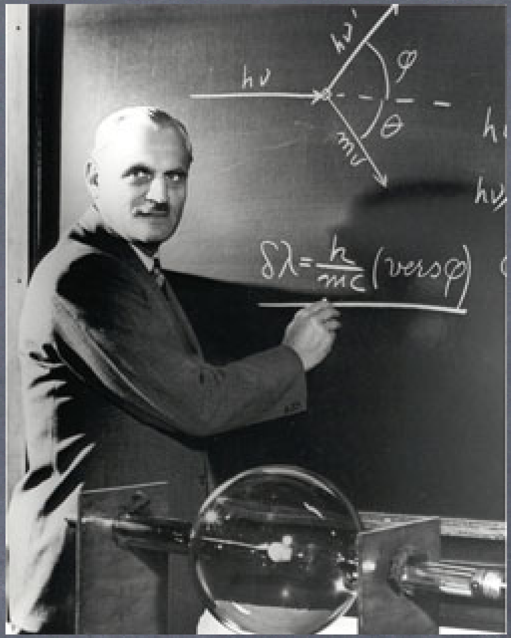
Spin Polarizabilities

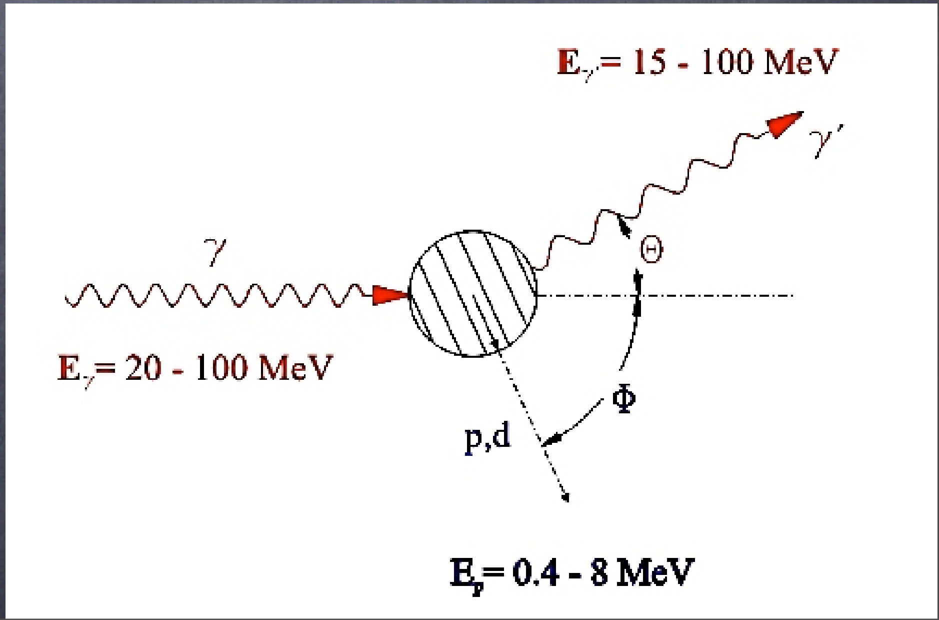
$$\begin{aligned}\Delta H_N(\sigma) \;=\; & -\mu_N \boldsymbol{\sigma}\cdot\boldsymbol{B} - \frac{1}{2}\gamma_{E1E1}\boldsymbol{\sigma}\cdot\left(\boldsymbol{E}\times\frac{\partial}{\partial t}\boldsymbol{E}\right) - \frac{1}{2}\gamma_{M1M1}\boldsymbol{\sigma}\cdot\left(\boldsymbol{B}\times\frac{\partial}{\partial t}\boldsymbol{B}\right) \\ & + \frac{1}{2}\gamma_{M1E2}\left(\nabla_i E_j + \nabla_j E_i\right)\sigma_i B_j - \frac{1}{2}\gamma_{E1M2}\left(\nabla_i B_j + \nabla_j B_i\right)\sigma_i E_j\end{aligned}$$

Physics Motivation

Compton

Scattering





RBC/UKQCD 2+1 DWF Lattices

1 $141: \ 16^3 \times 32 \quad m_s = 0.04 \quad m_u = 0.01$

2 $228: \ 24^3 \times 64 \quad m_s = 0.04 \quad m_u = 0.01$

3 $174: \ 24^3 \times 64 \quad m_s = 0.04 \quad m_u = 0.005$

$\beta = 2.13$

Valence: large number of prop. inversions
--> Tadpole-improved Clover

κ-tuning:

1 $\kappa_s = 0.13810 \quad \kappa_u = 0.13939 \quad \kappa_c = 0.14000$

2 $\kappa_s = 0.13806 \quad \kappa_u = 0.13934 \quad \kappa_c = 0.13993$

3 $\kappa_s = 0.13811 \quad \kappa_u = 0.13957 \quad \kappa_c = 0.13993$

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1 $m_\pi = 400\,\mathrm{MeV}$ ~ 10

2 $m_\pi = 420\,\mathrm{MeV}$ Exceptionals! ~ 2

3 $m_\pi = 330\,\mathrm{MeV}$ ~ 15

Now: Toss, Focus on 2

Future: HYP smear

Background Fields

Couple classical $\vec{E}$ field to valence quarks

$$U_\mu(x) \longrightarrow U_\mu(x) U_\mu^{\text{cl}}(x)$$

as phases $U_\mu^{\text{cl}}(x) = \delta_{\mu 3} \exp(-iqEx_4)$

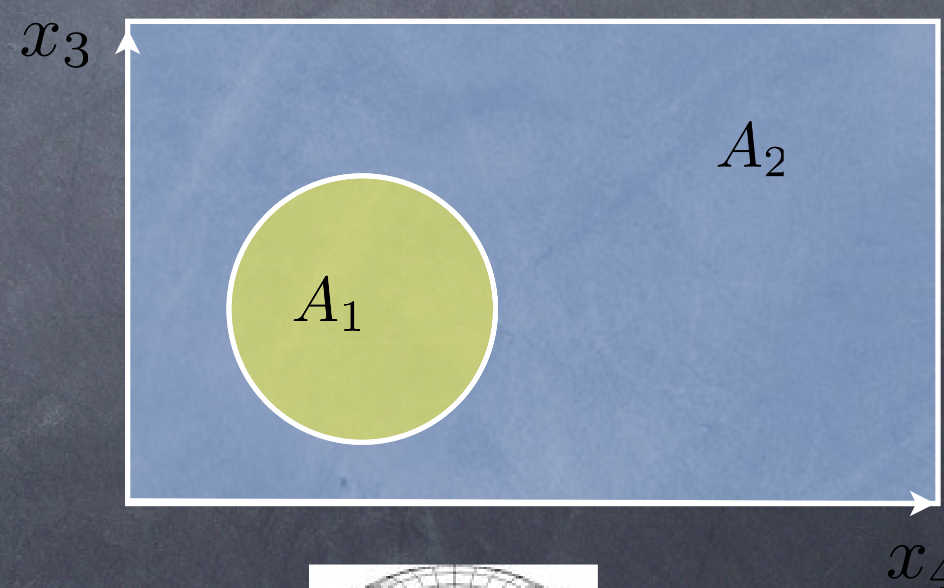
Analytic Continuation:
perturbative exp. = obvious
nonperturbative = absent

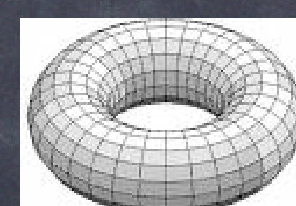
Periodic Action --> Torus

$$e^{iqEA_1} = e^{-iqEA_2}$$

Quantization: $q_d E = \dfrac{2\pi n}{\beta L}$

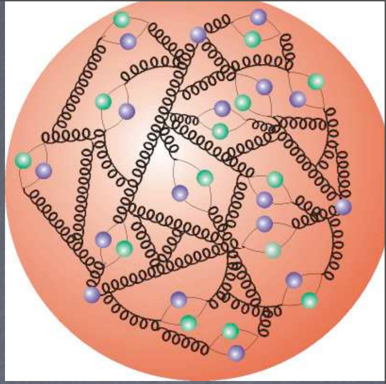
Fields: n = 1, -2, 3, 4, 5, -6, -7

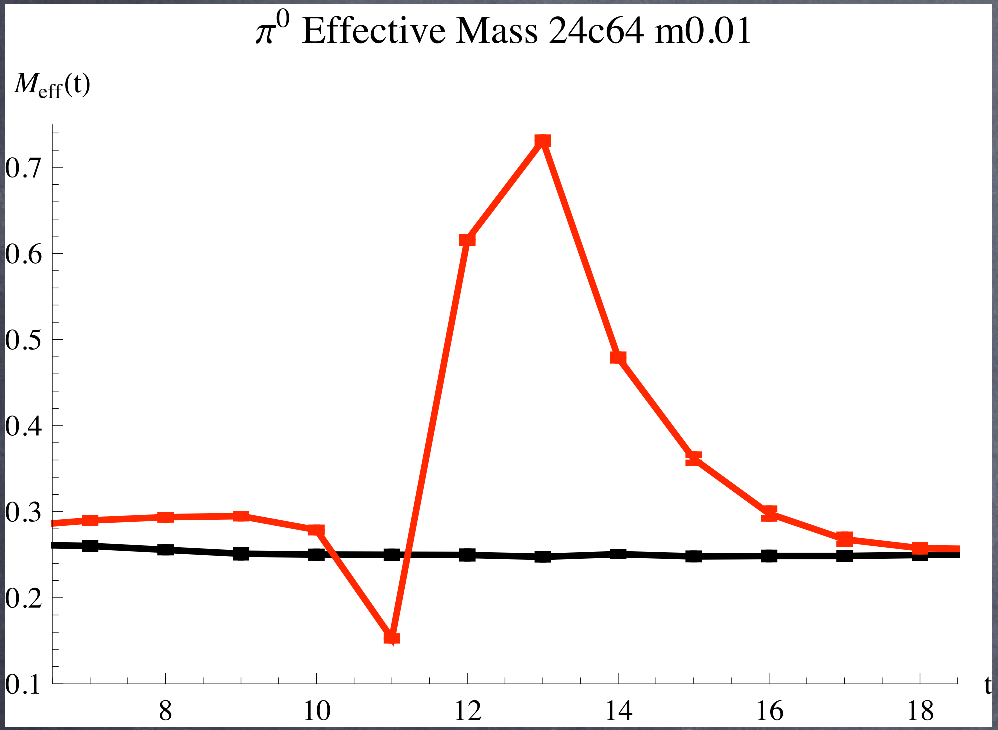




Background Fields

- Quantized fields (Q) not everywhere constant
- Gauge fix transverse links (GF)





$$n = 3, \ t_{\text{src}} = 0$$

$$n = 3, \ t_{\text{src}} = 52$$

(Q)

(GF)

$$n = e, \ t_{\text{src}} = 52$$

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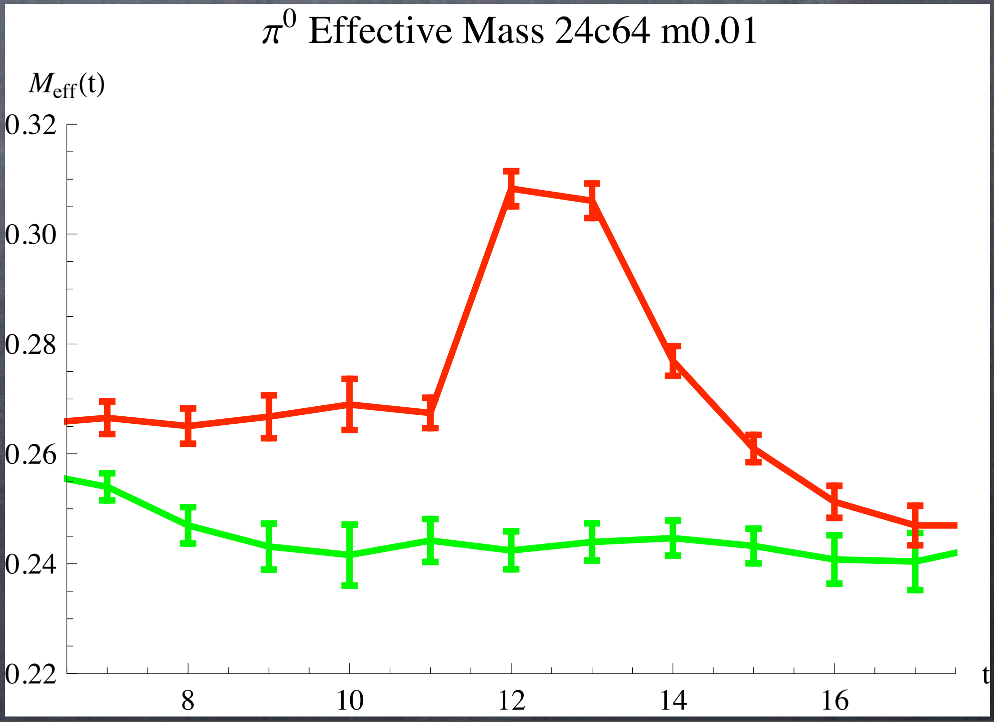
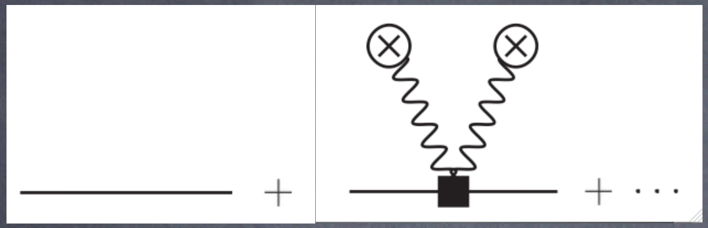


Electric Polarizabilities: Neutral Particles

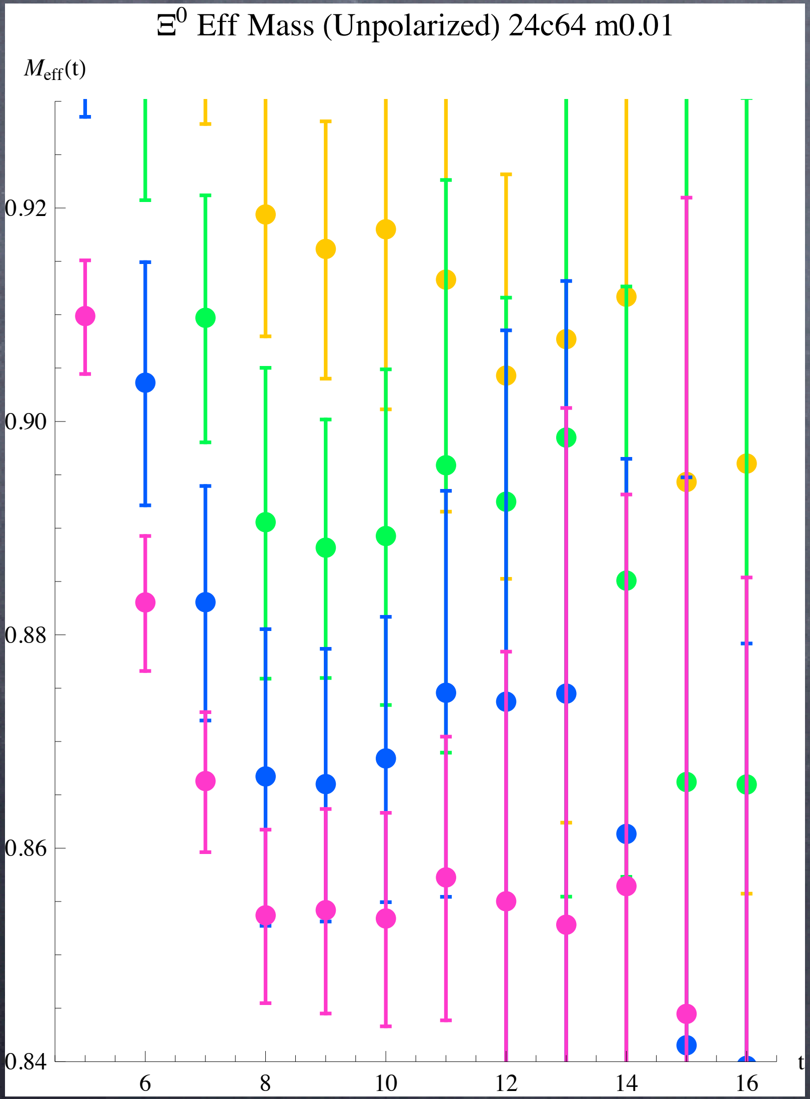


$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x},\tau)\chi^\dagger(\vec{0},0)\rangle = Z\exp(-E\tau)$$

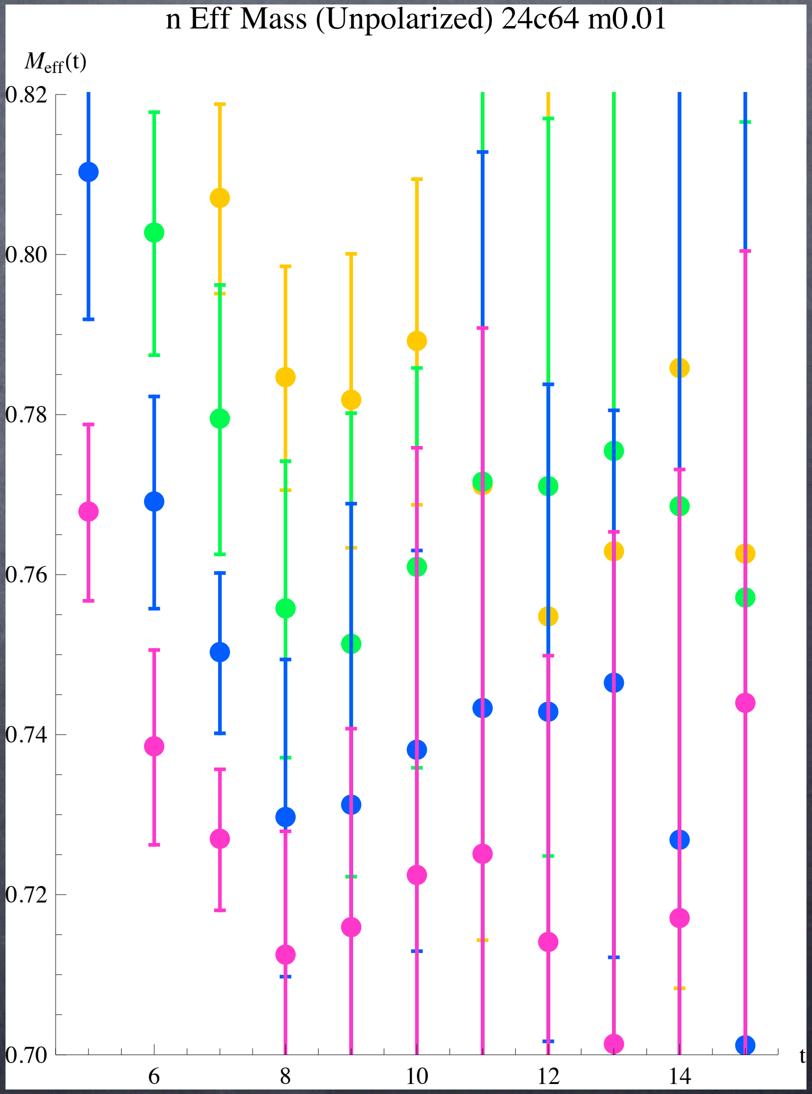
$$E = M + \frac{1}{2}\alpha_E \vec{E}^2 + \frac{1}{4!}\overline{\alpha}_E \vec{E}^4 + \ldots$$

E.g.

Ξ^0



n



1,3,5,-7

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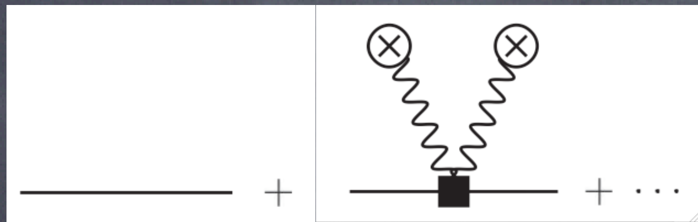

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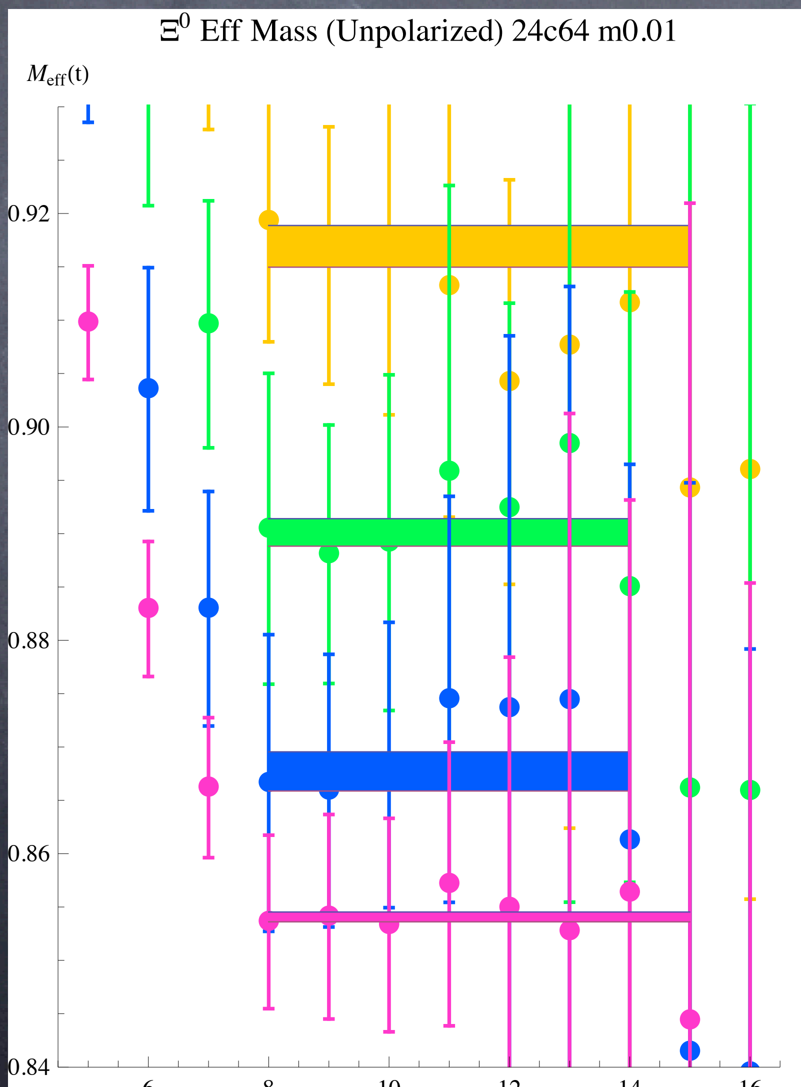

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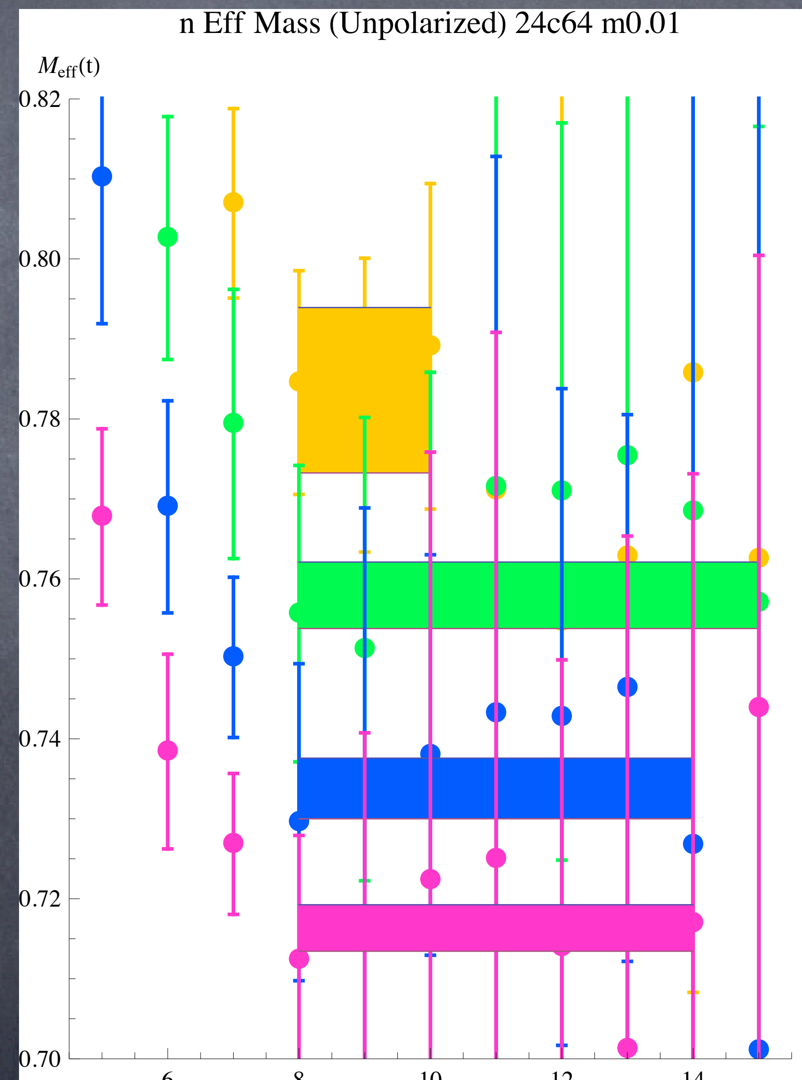

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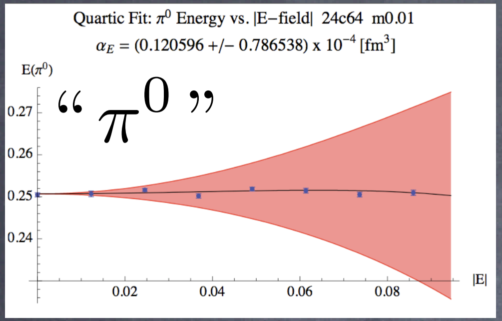
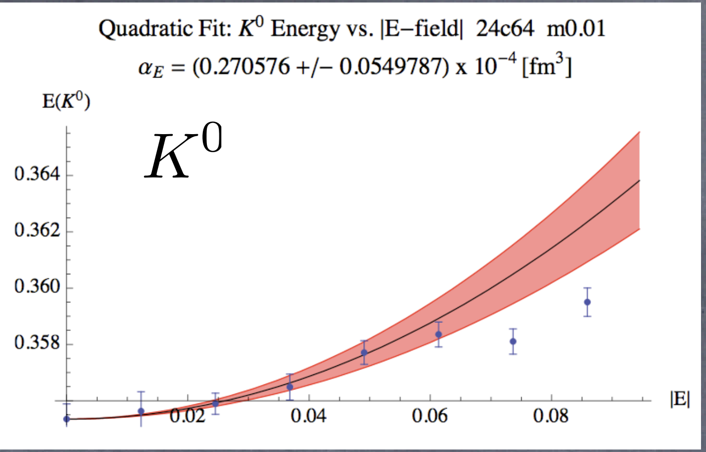
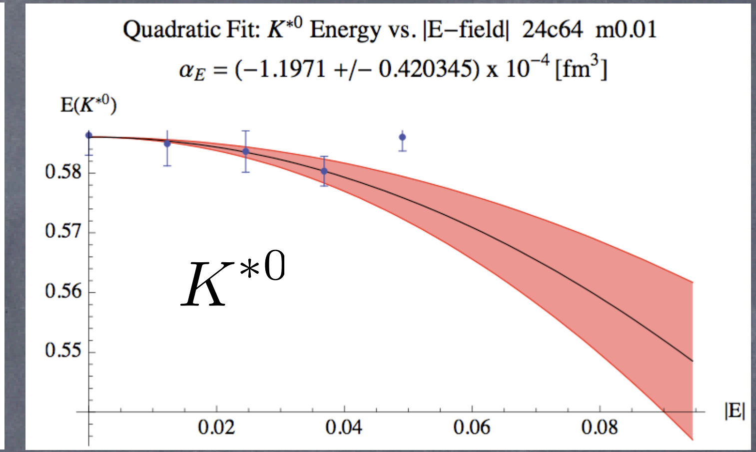
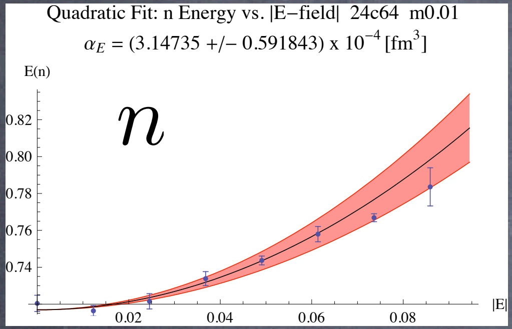
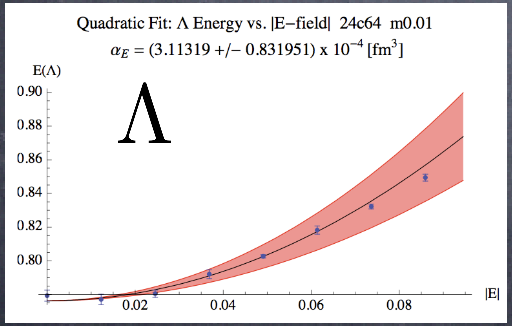
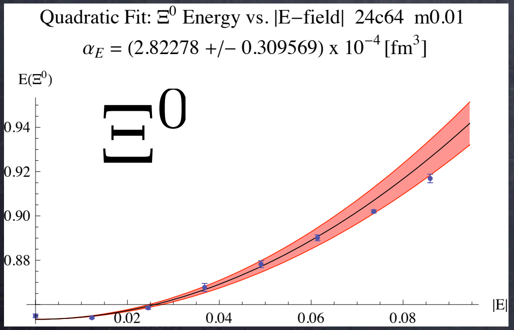


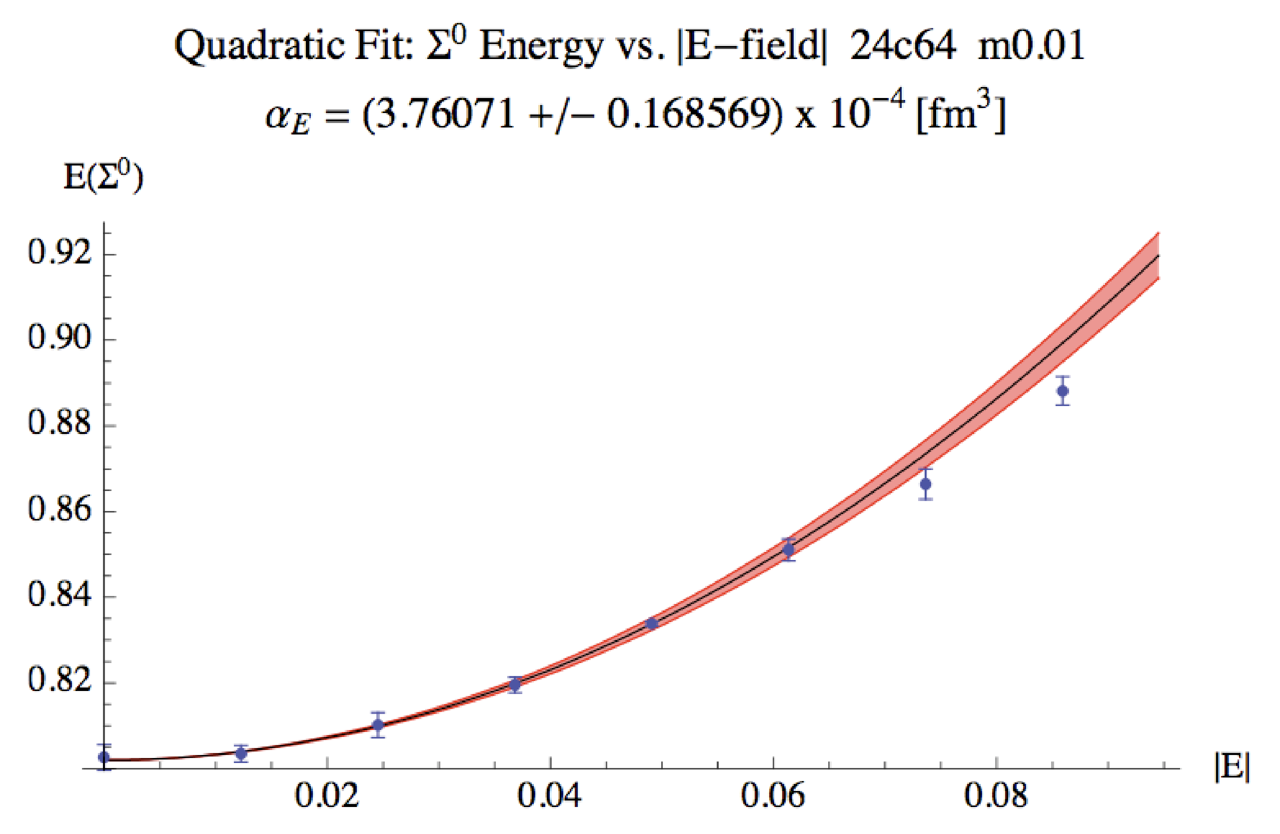




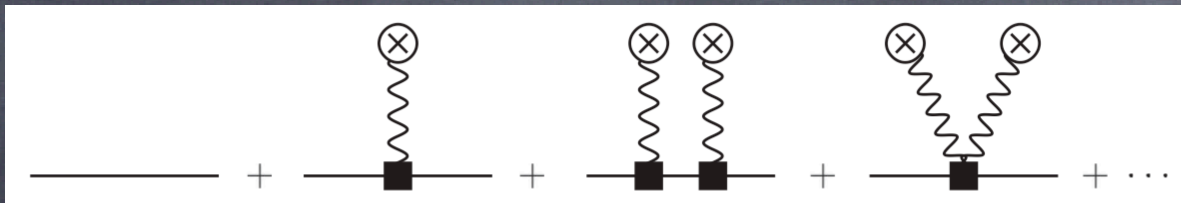








Electric Polarizabilities: Charged Particles



$$G(\tau) = Z\, g(E, \vec{E}, \tau)$$

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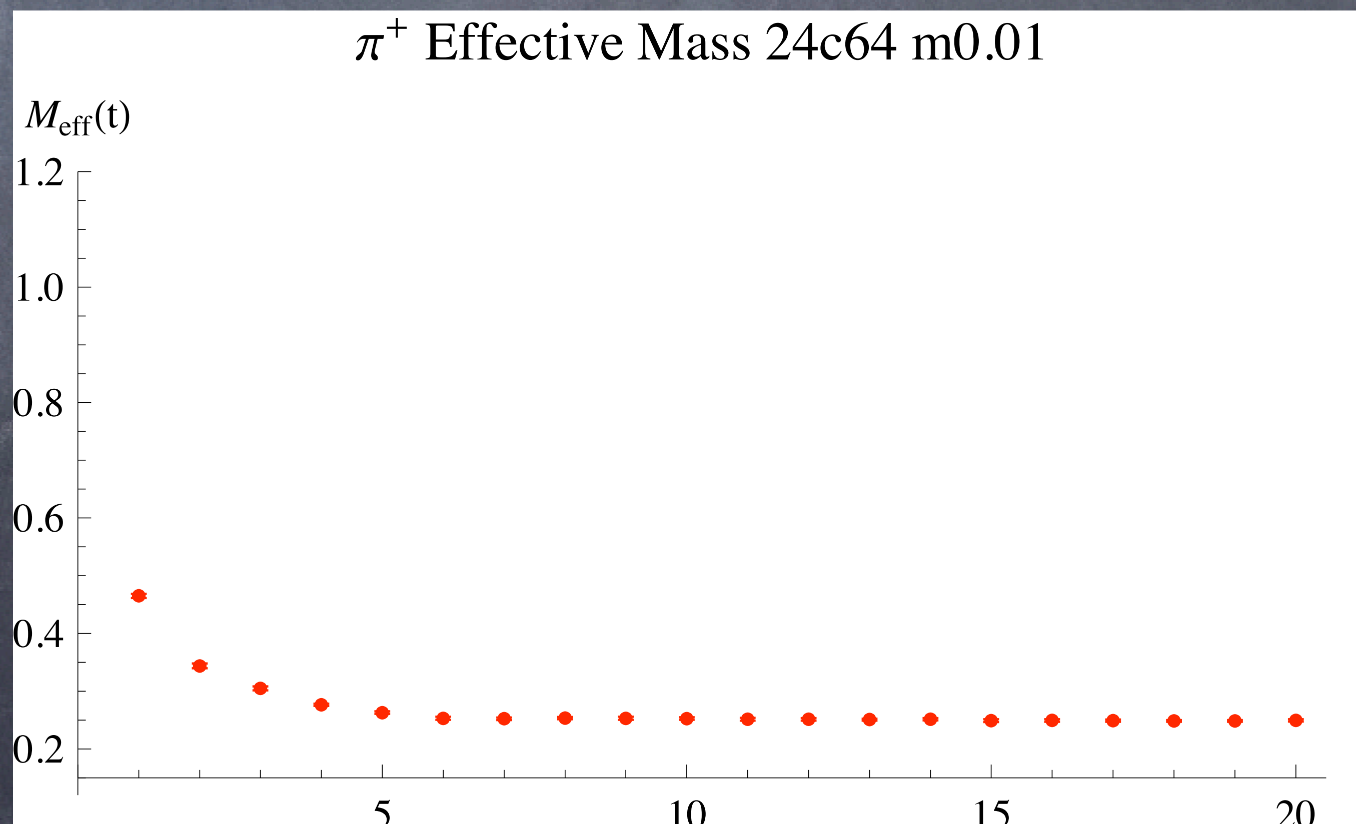
$$g(E, \vec{E}, \tau) \stackrel{\text{NR}}{\rightarrow} \exp\left(- E\tau - \frac{Q^2 \vec{E}^2 \tau^3}{6M} \right)$$

$$-\log \frac{G(\tau+1)}{G(\tau)}$$

$$= C + t(t+1)\frac{Q^2 \vec{E}^2}{2M}$$



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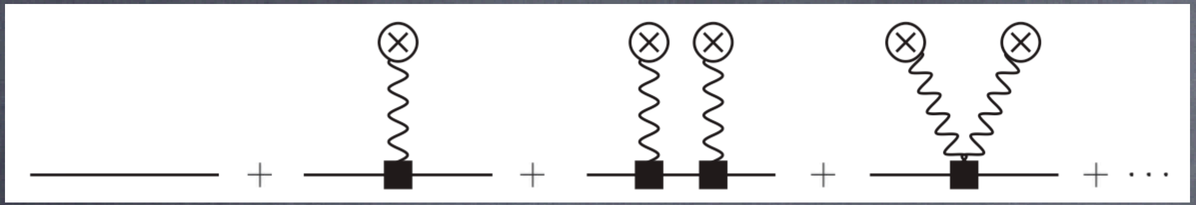

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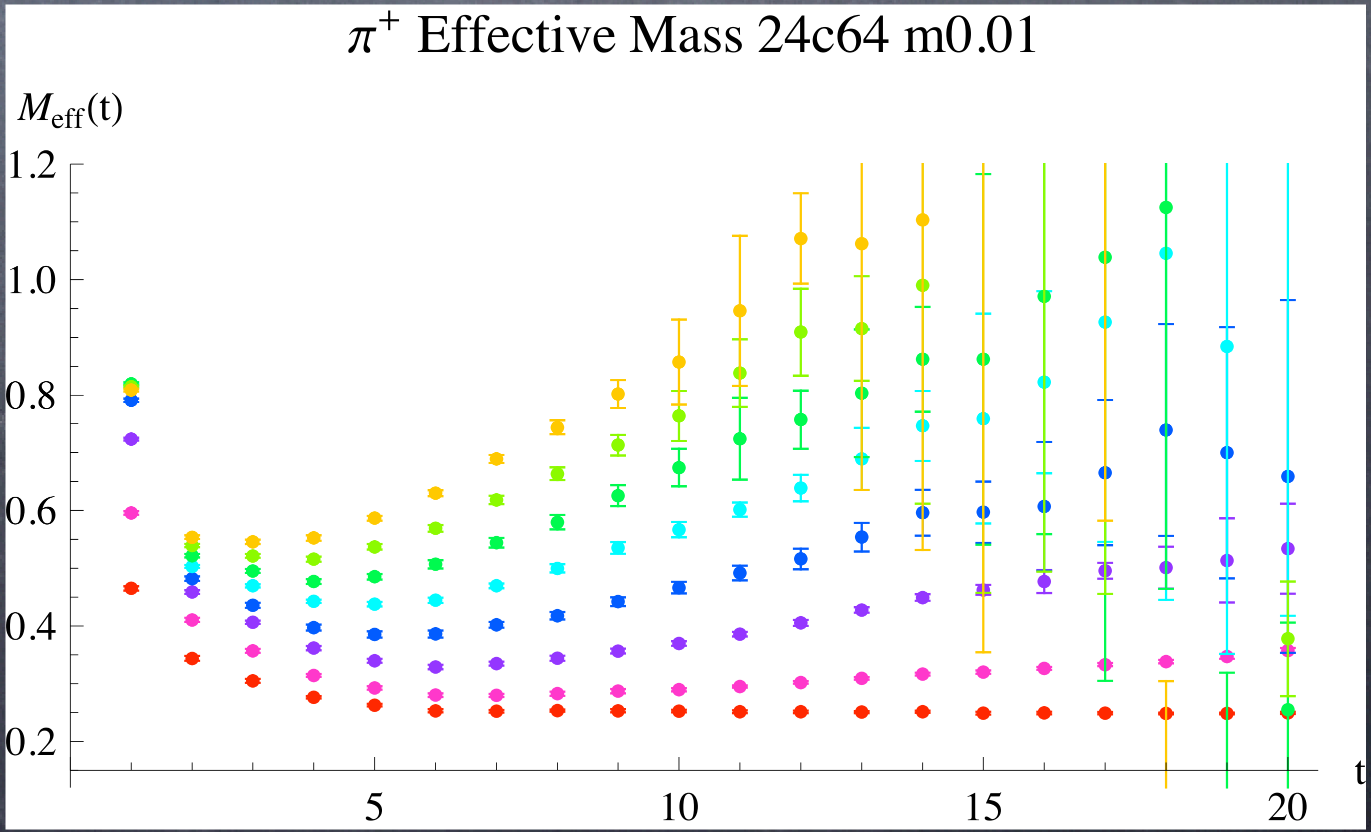

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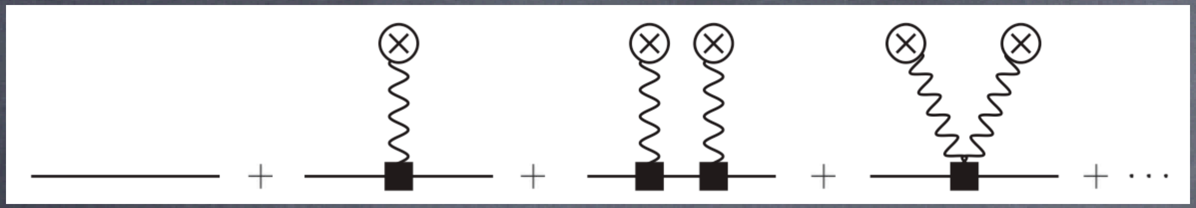

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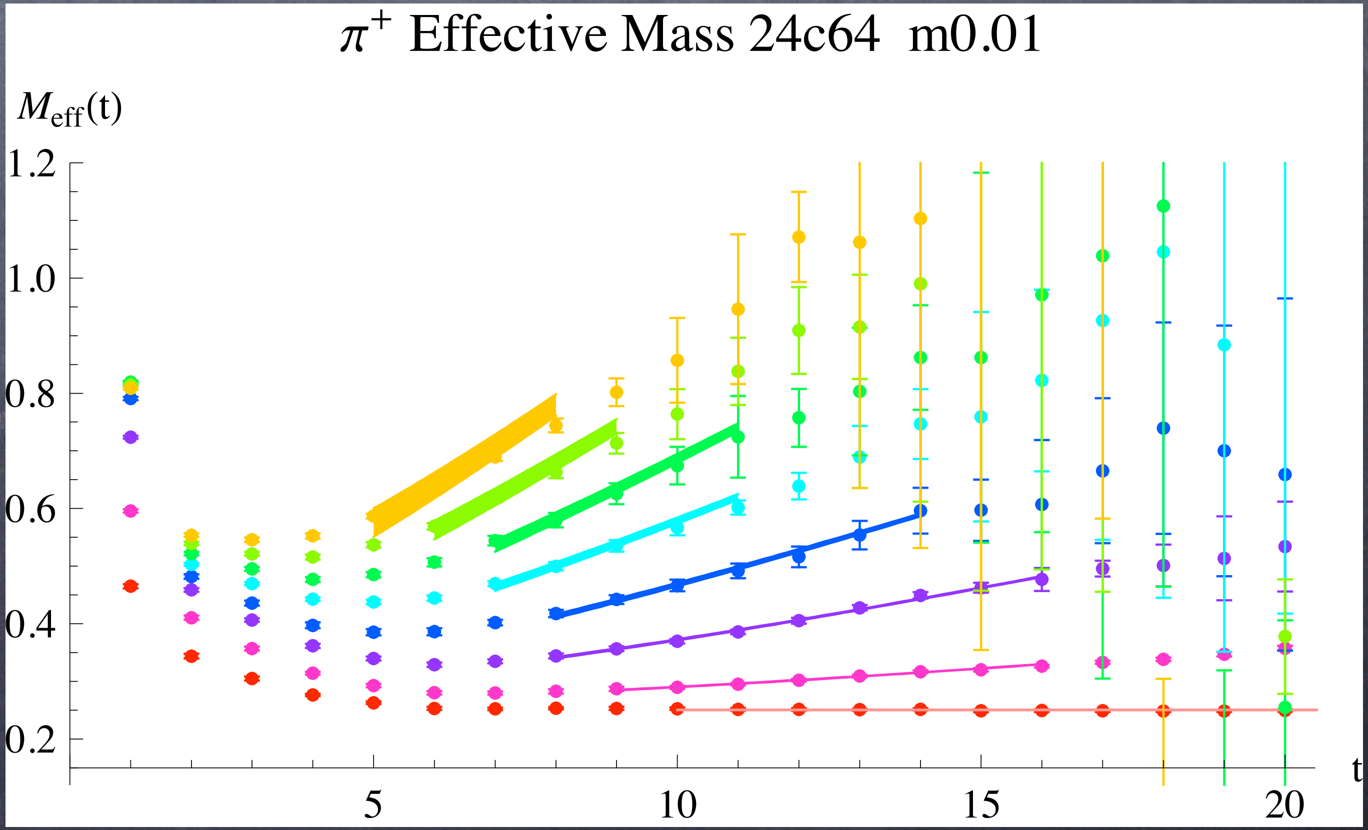


Electric Polarizabilities: Charged Particles

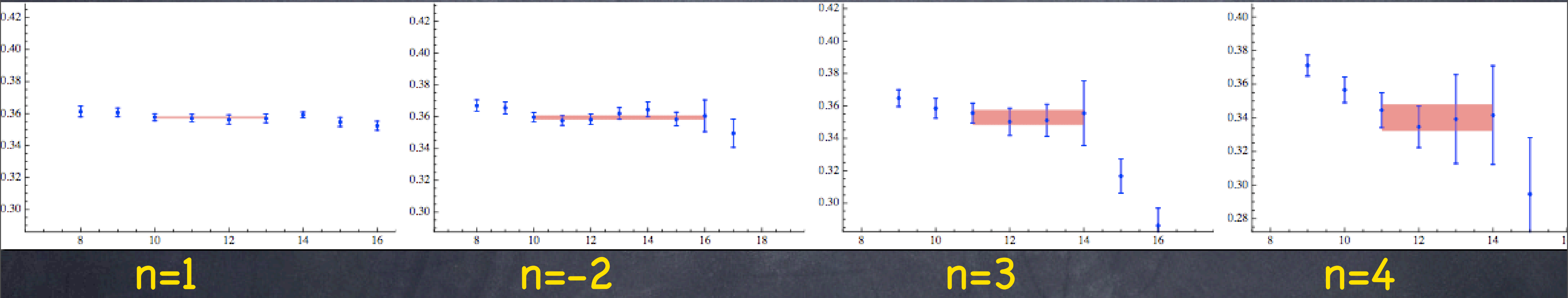
Non-constant Energy Fits $M_{\text{eff}}(t)$ guides the eye...

"Effective Energy" solve for E_{eff} successively, look for plateau

$$\frac{g(E_{\text{eff}}, \vec{E}, \tau + 1)}{g(E_{\text{eff}}, \vec{E}, \tau)} = \frac{C(\tau + 1)}{C(\tau)}$$

$g(E, \vec{E}, \tau)$ expressible as a one-dimensional integral

$E_{\text{eff}}(K^+)$



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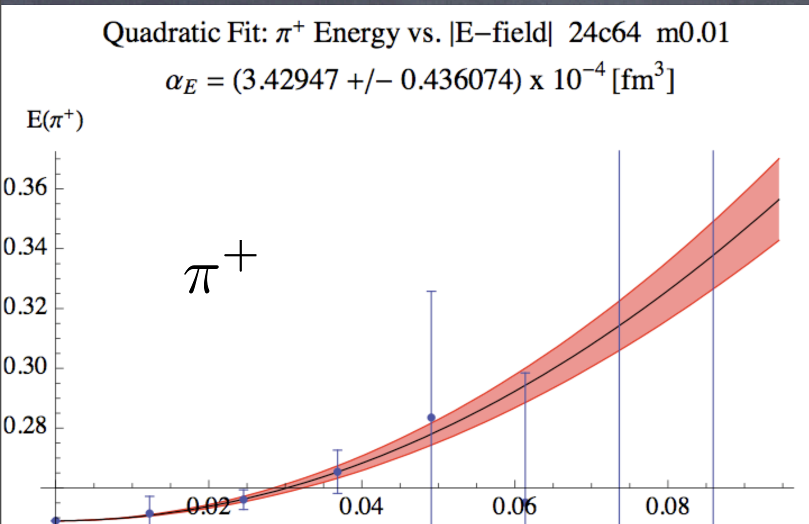

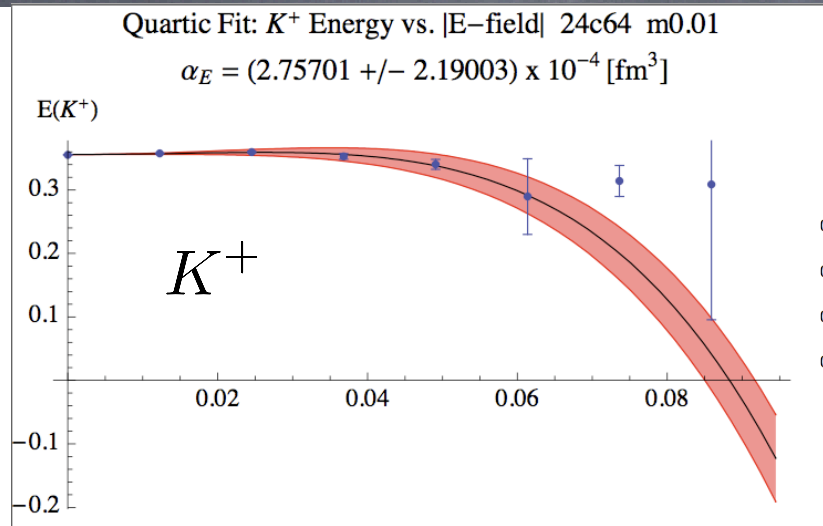

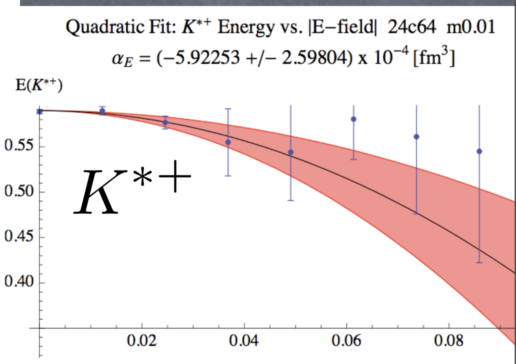

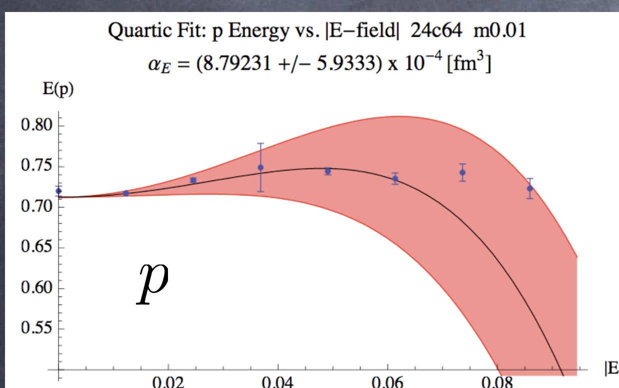

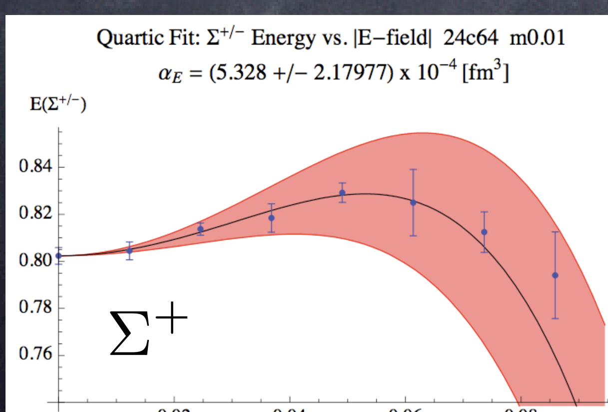

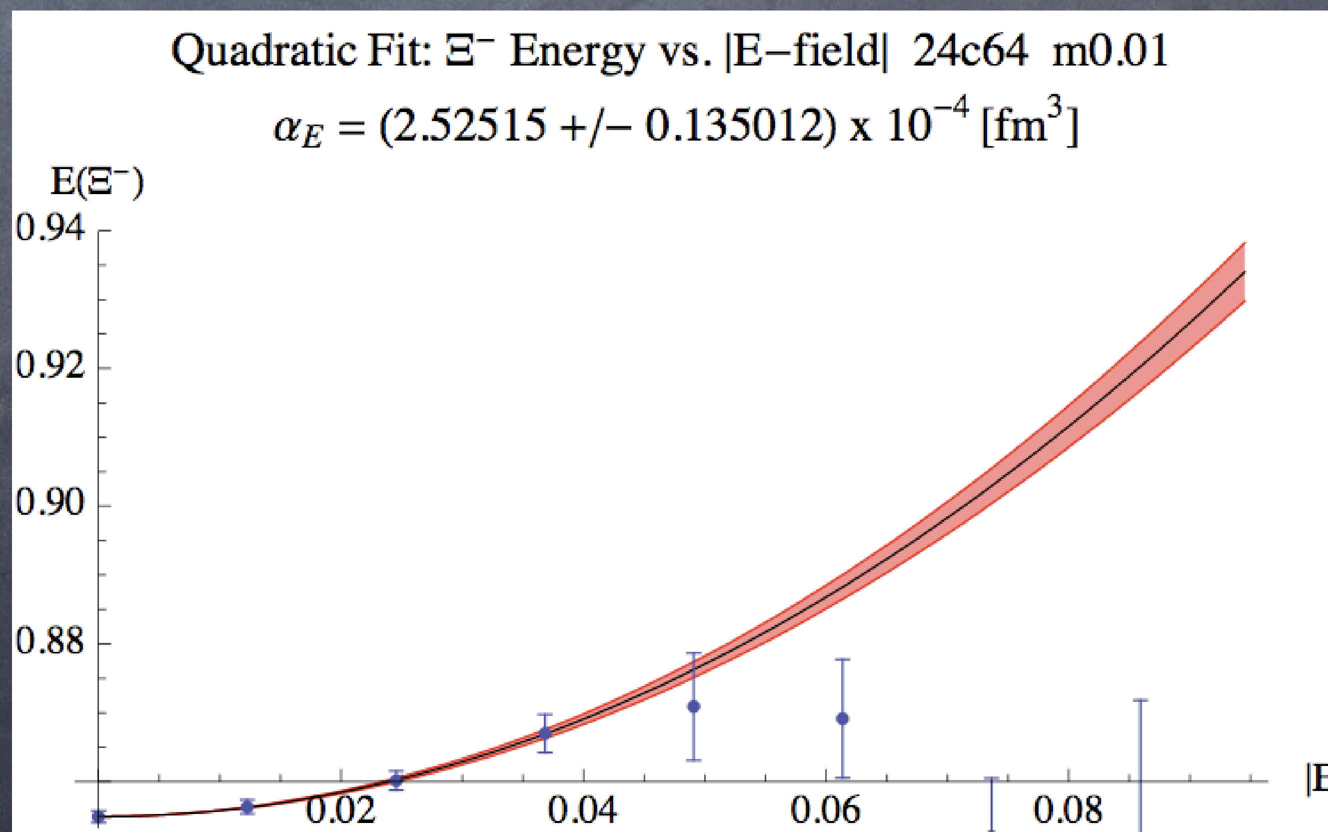

Summary

- Hadrons deform in applied fields

  QCD dynamics, chiral symmetry, experiments, lattice simulations

- Background field electric polarizability calculations

  Neutral and charged particles accessible, behavior of correlators

- Plenty of refinements to be made

  Exceptionals, statistics, lattice spacings, volumes, sea-quark charges, . . .