A non-perturbative test of the chirally rotated Schrödinger functional

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Motivation

SF = Schrödinger functional

standard SF, $m=0$ ⟺ rotated SF, $m=0$

rotated SF:

- ✔ automatic O(a) improvement in the bulk for $\alpha = \pi/2$
- ✔ automatic O(a) improvement for parity even observables (just like tmQCD)
- ✗ only n_f even so far

thus:

- ➔ no need for c_{SW} , c_A , c_V for n_f =4
- ➔ expect reduced cutoff effects for (parity odd) 4-fermion operators
- ➔ SF more attractive then ever

Lattice action of the rotated SF

- gauge part of the action same as standard SF
- Dirac operator in the fermion action:

$$aD_W\psi(x) = -U(x,0)P_-\psi(x+a\hat{0}) + K\psi(x) - U(x-a\hat{0},0)^\dagger P_+\psi(x-a\hat{0})$$

where $\psi(x) = 0 \quad \text{for} \quad x_0 \leq 0 \quad \text{and} \quad x_0 \geq T-a$

$$K\psi(x) = (1 + am_0 + a\ \text{spatial Wilson}\ + c_{SW}a\ \text{SW term})\,\psi(x)$$

$$+\,\delta_{x_0,a}\ i\gamma_5\tau^3 P_-\psi(x) + \delta_{x_0,T-a}\ i\gamma_5\tau^3 P_+\psi(x)$$

O(1) and O(a) boundary counter terms: $D_W \;\rightarrow\; D_W + \delta D_W$

$$\delta D_W\psi(x) = (\delta_{x_0,a} + \delta_{x_0,T-a})\big[(z_f-1) + (d_s-1)a\ \text{spatial Wilson}\big]\psi(x)$$

Correlation functions

standard SF

$$\zeta(\mathrm{x}) = U(x,0)|_{x_0=a} P_- \psi(x)|_{x_0=a}$$

$$.\bar{\zeta}(\mathrm{x}) = \bar{\psi}(x)|_{x_0=a} P_+ U(x,0)^{-1}|_{x_0=a}$$

$$\mathcal{O}^a = a^6 \sum_{\mathrm{y},\mathrm{z}} \bar{\zeta}(\mathrm{y}) \gamma_5 \tfrac{1}{2}\tau^a \zeta(\mathrm{z})\, e^{i\mathrm{p}(\mathrm{y}-\mathrm{z})}$$

$$f_X^{ab}(x_0) = -\langle X^a(x) \mathcal{O}^b \rangle$$

rotated SF

$$\zeta(\mathrm{x}) = U(x,0)|_{x_0=a} \psi(x)|_{x_0=a}$$

$$.\bar{\zeta}(\mathrm{x}) = \bar{\psi}(x)|_{x_0=a} U(x,0)^{-1}|_{x_0=a}$$

$$\mathcal{Q}_\pm^a = a^6 \sum_{\mathrm{y},\mathrm{z}} \bar{\zeta}(\mathrm{y}) \gamma_5 \tfrac{1}{2}\tau^a Q_\pm \zeta(\mathrm{z})\, e^{i\mathrm{p}(\mathrm{y}-\mathrm{z})}$$

$$g_X^{ab}(x_0)_\pm = -\langle X^a(x) \mathcal{Q}_\pm^b \rangle$$

where $X^a = A_0^a, V_0^a, S^a, P^a$

Strategy for a quenched scaling test

For a line of constant physics $(L = 1.436\, r_0)$: [M.Guagnelli et al., hep-lat/0505002]

1. Fix the parameters of the action

Tune κ : $$m_{\mathrm{PCAC},-} \equiv \frac{\widetilde{\partial}_0 g_A(T/2)_-}{2 g_P(T/2)_-} = 0$$

Tune z_f : $$g_A(T/2)_- = 0$$

Set d_s : $$d_s = d_s^{(0)} = 3/2$$

2. Check

Universality: $$\frac{g_P(x_0)_-}{g_P(T/4)_-} = \frac{f_P(x_0)}{f_P(T/4)} + \mathrm{O}(a^2)$$

Boundary conditions: $$g_P(x_0 > a)_+ = 0 + \text{cutoff effects}$$

Strategy for a quenched scaling test

For a line of constant physics $(L = 1.436\, r_0)$: [M.Guagnelli et al., hep-lat/0505002]

1. Fix the parameters of the action

Set κ : values from standard SF with clover Wilson [M.Guagnelli et al., hep-lat/0505002]

Tune z_f : $g_A(T/2)_- \;=\; 0$

Set d_s : $d_s = d_s^{(0)} = 3/2$

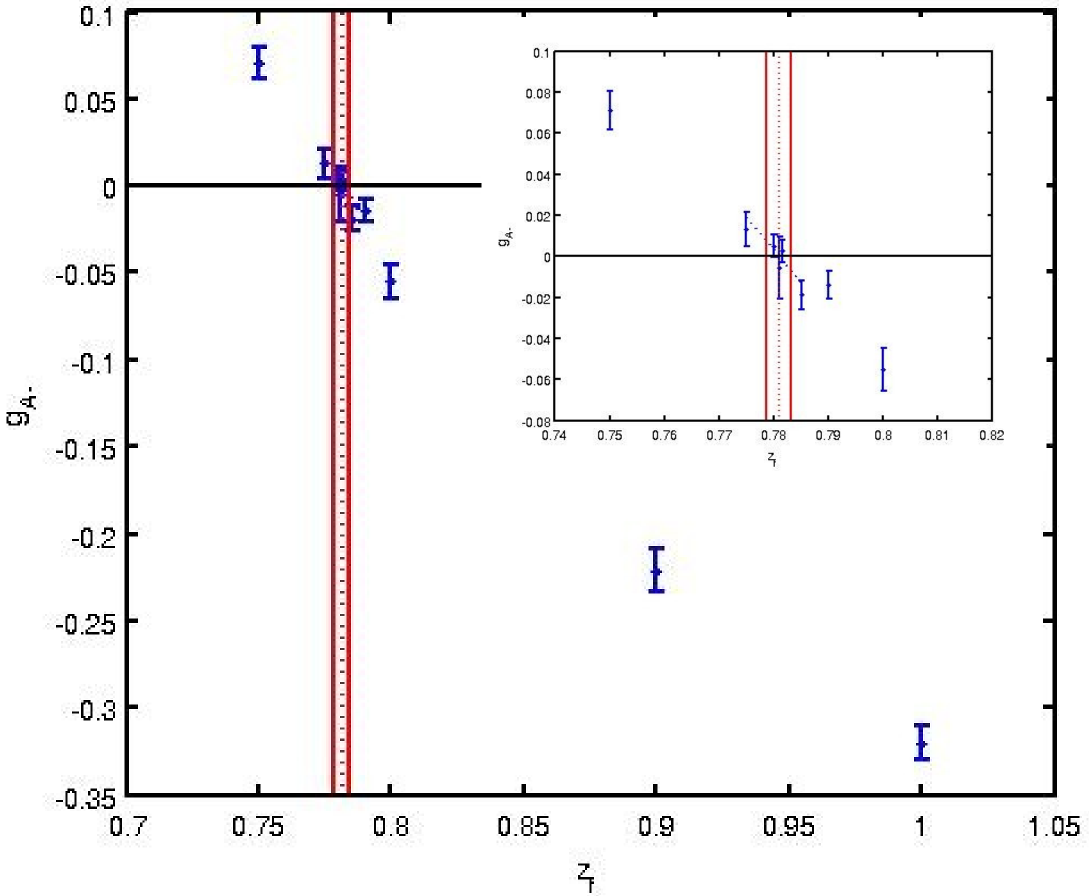
2. Check

Universality: $$\frac{g_P(x_0)_-}{g_P(T/4)_-} = \frac{f_P(x_0)}{f_P(T/4)} + \mathrm{O}(a^2)$$

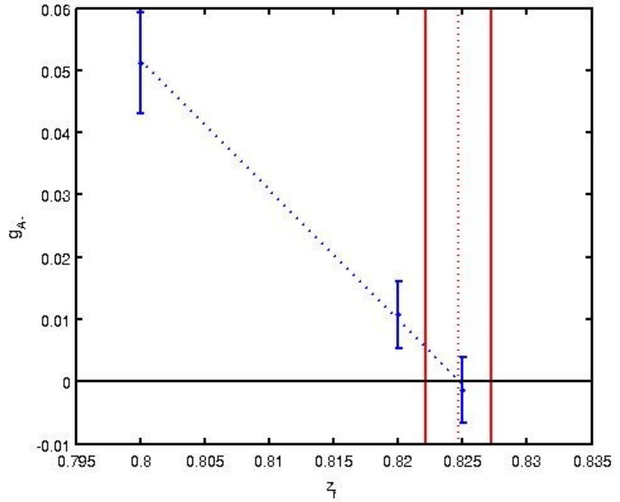
Boundary conditions: $g_P(x_0 > a)_+ = \; 0 + \text{cutoff effects}$

Tuning of z_f

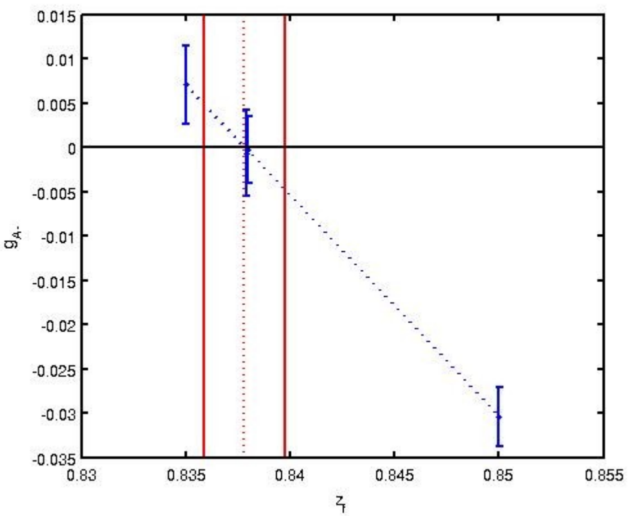
8 x 8 x 8 x 8, $\beta = 6.0219$



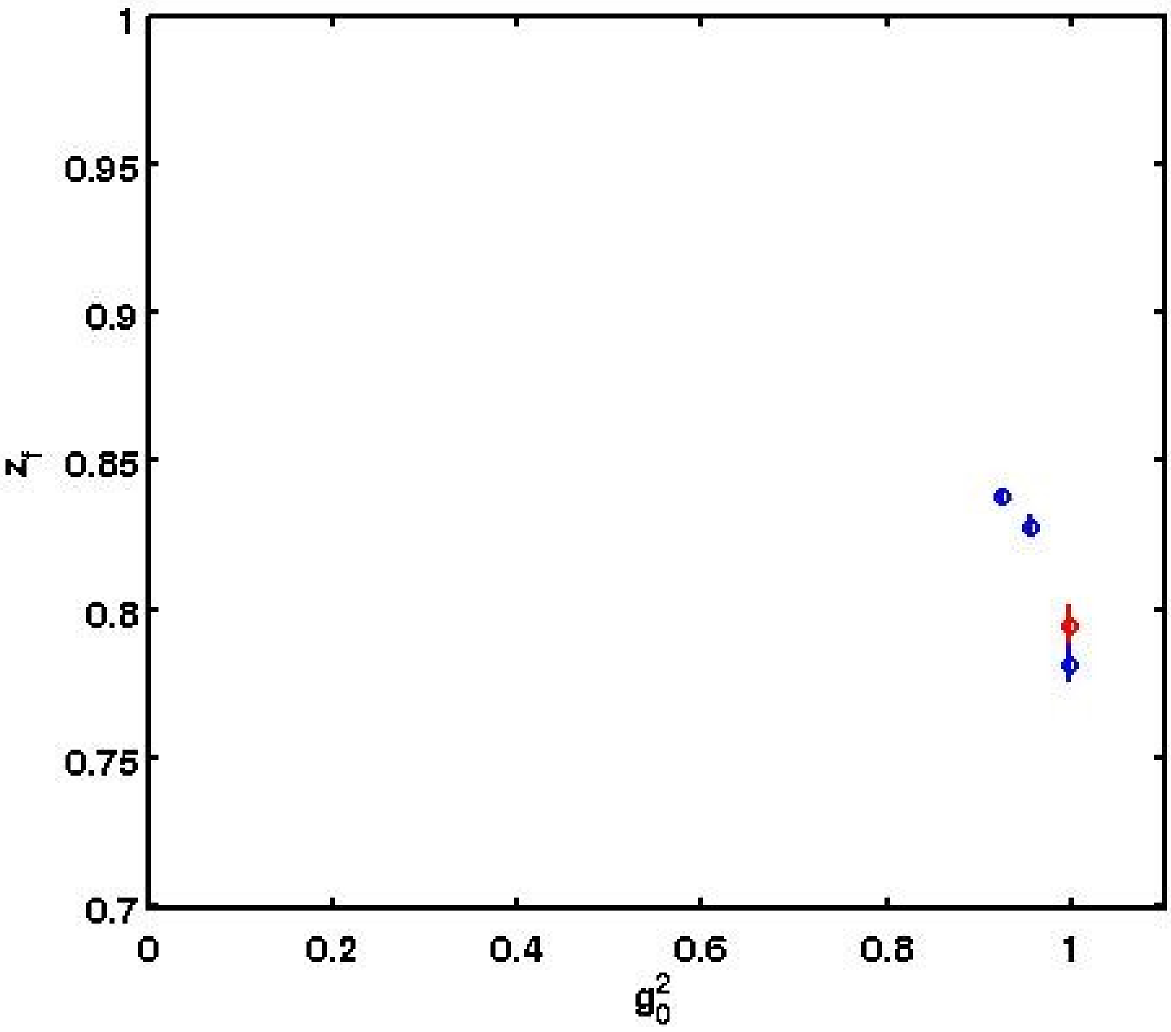
12 x 12 x 12 x 12, $\beta = 6.2885$



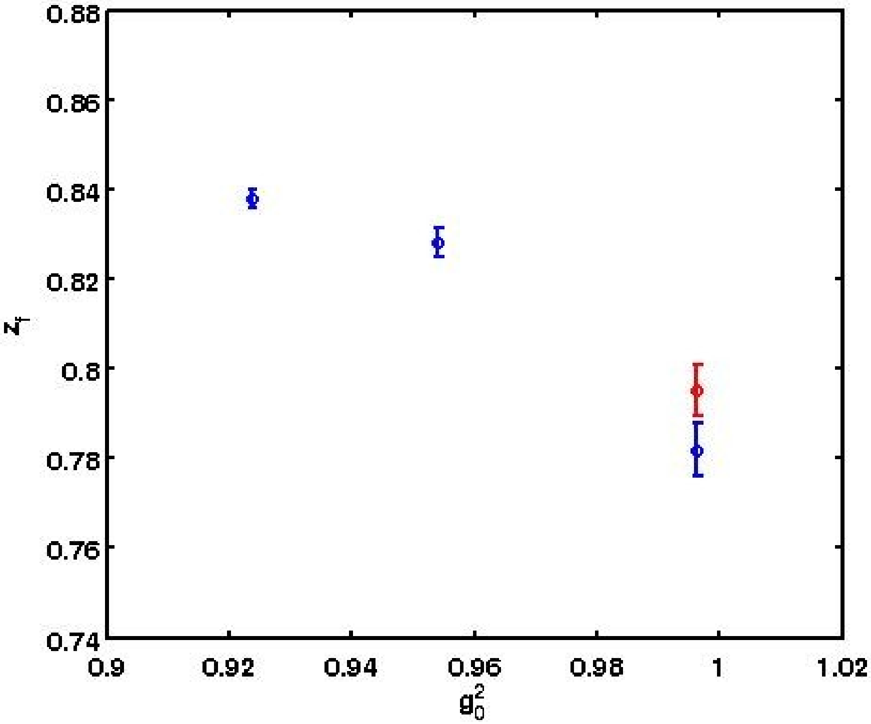
16 x 16 x 16 x 16, $\beta = 6.4956$



Tuning of z_f

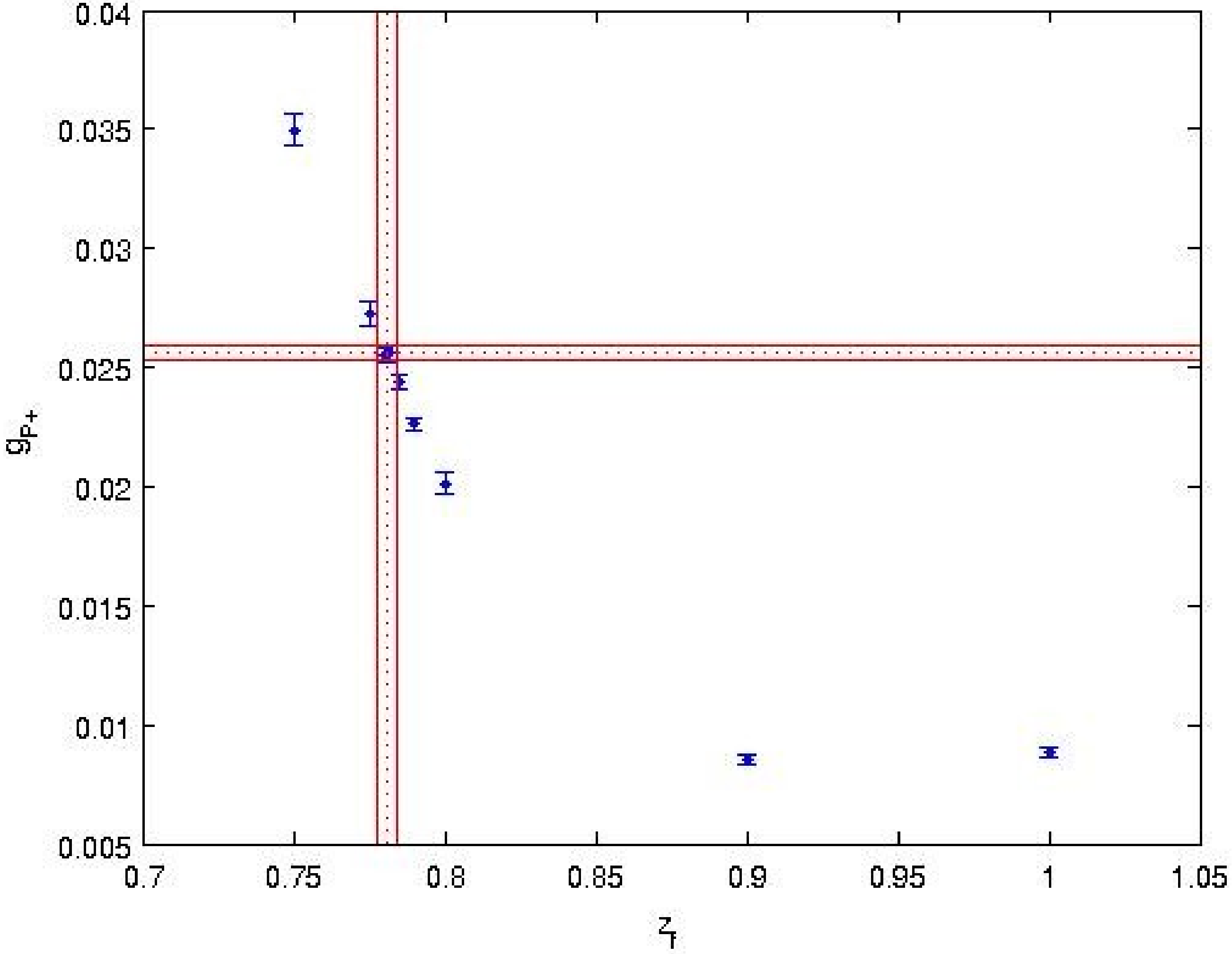


1
0.95
0.9
0.85
0.8
0.75
0.7
0
0.2
0.4
0.6
0.8
1
z_f
g_0^2

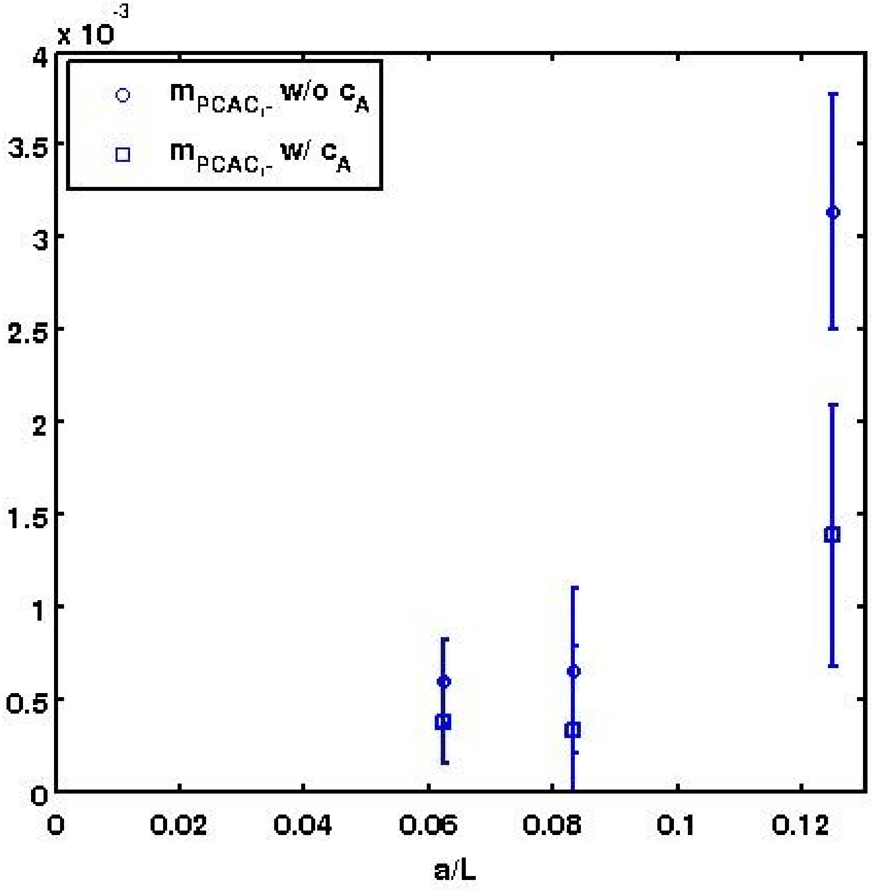


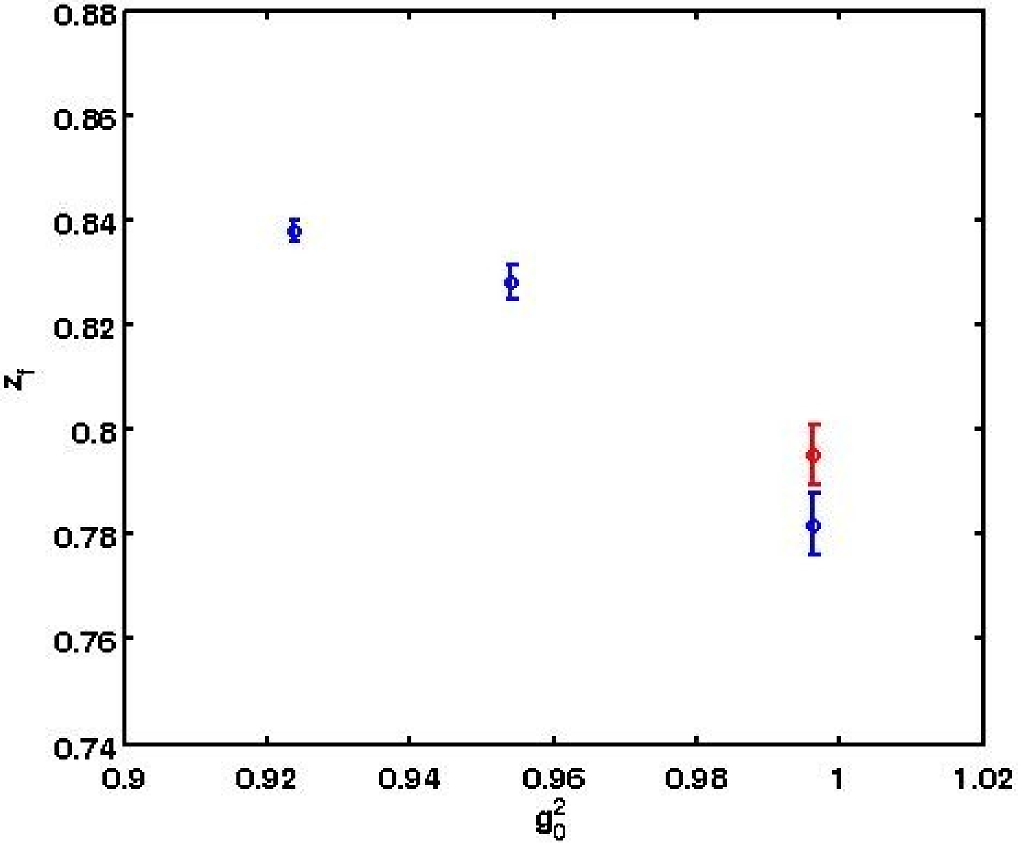
0.88
0.86
0.84
0.82
0.8
0.78
0.76
0.74
0.9
0.92
0.94
0.96
0.98
1
1.02
z_f
g_0^2

Tune z_f by demanding $g_{P+} =$ minimal ?

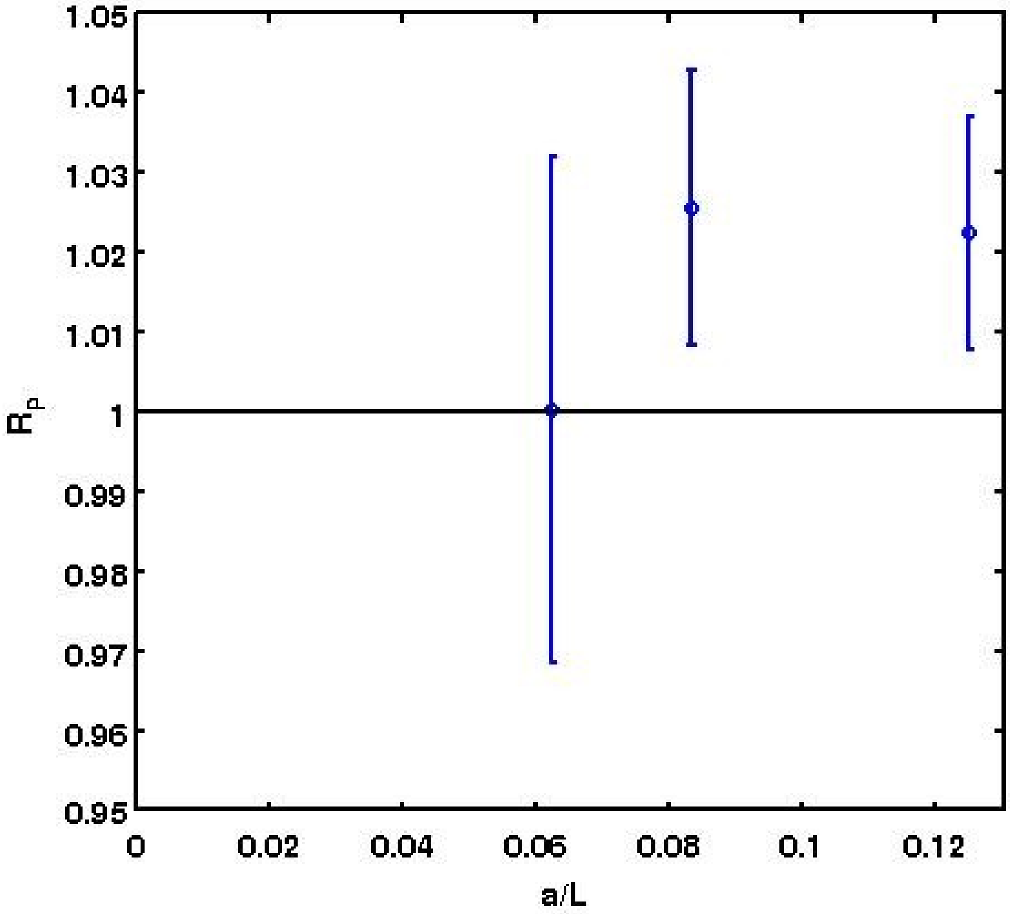


How critical is κ_c



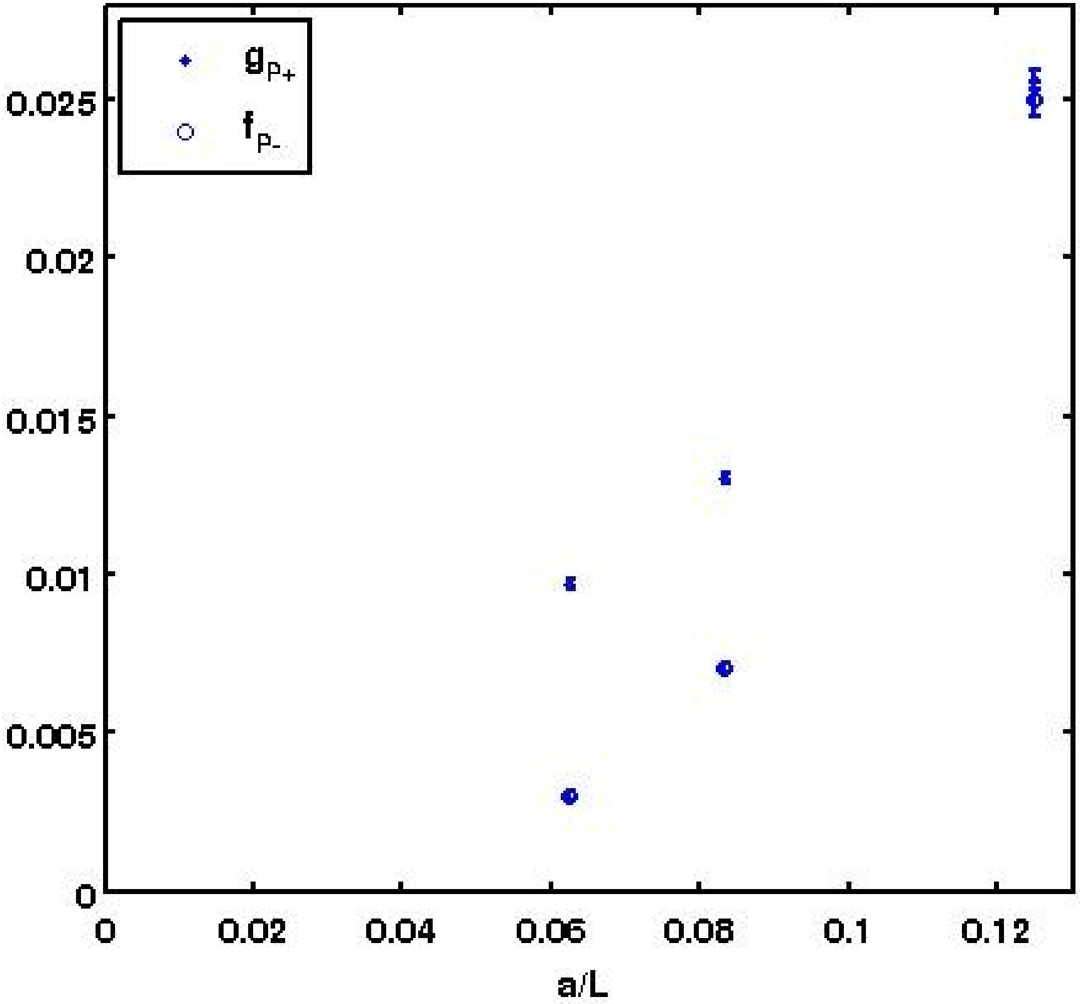


Check: Universality



$$R_P = \frac{g_P(x_0)_-}{g_P(T/4)_-} \Bigg/ \left(\frac{f_P(x_0)}{f_P(T/4)} \right) = 1 + \mathrm{O}(a^2)$$

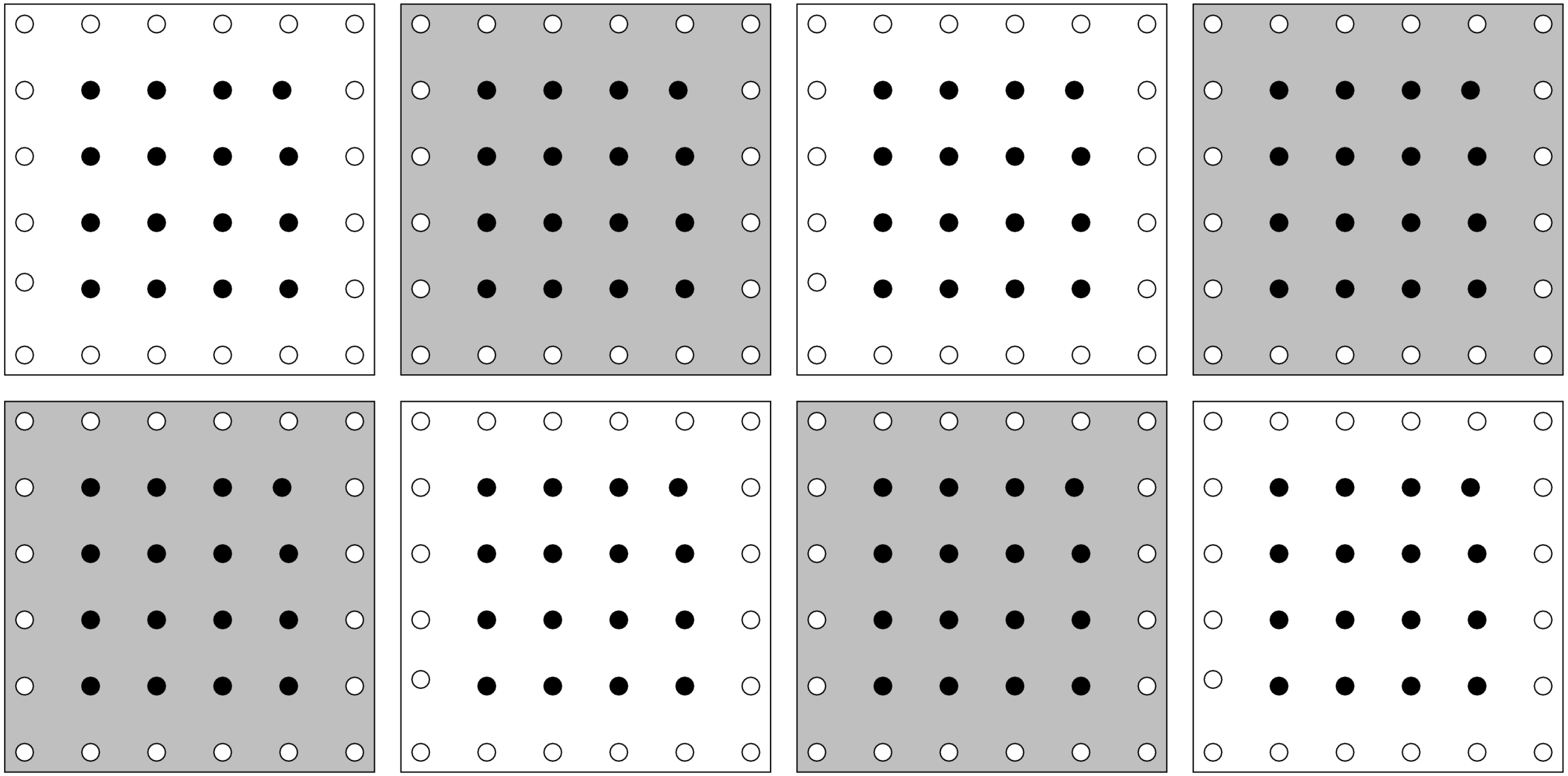
Check: Boundary conditions



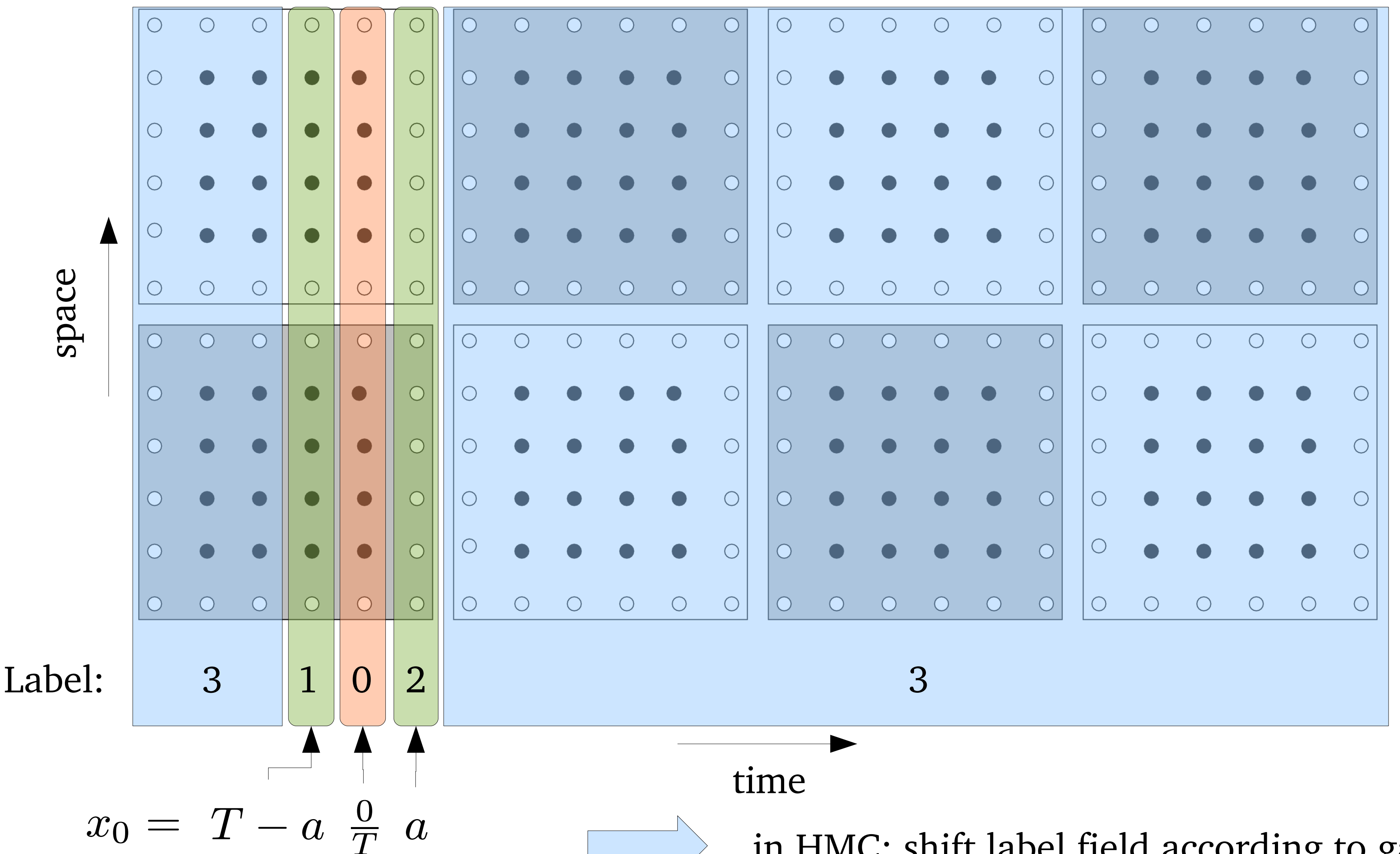
standard SF: faster than a^2

rotated SF: linear in a ?

DD-HMC-SF



DD-HMC-SF



in HMC: shift label field according to gauge field

Status of the DD-HMC-SF

Done:

- Dirac operator (block/full) for standard and rotated SF
- even-odd preconditioning for standard SF
- Cabibbo-Marinari gauge update algorithm
- SAP solver and deflated SAP solver for standard SF

ToDo:

- even-odd for rotated SF
- forces for the HMC

Code publication:

- standard SF code will be published under the GNU license in fall
- rotated SF code will be spring 2009

Summary

- demonstrated tuning of the O(1) boundary counter term in quenched QCD
- checked that rotated SF is in same universality class as standard SF
- used setup (source and boundary terms at x0=a) gives rise to finite normalisation of source fields
- bulk O(a) improvement seems to work
- DD-HMC package has been extended to SF boundary conditions
- DD-HMC-SF will be published soon