Orientifold Planar Equivalence: the chiral condensate

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(with A. Armoni, A. Patella, C. Pica [hep-th/0804.4501])

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Orientifold planar equivalence

- The antisymmetric and the antifundamental representations coincide for $SU(3)$ (but not in general for $SU(N)$) $\Rightarrow$ different SU(N) generalizations of QCD.
- In the planar limit, the (anti)symmetric representation is equivalent to another gauge theory with the same number of Majorana fermions in the adjoint representation (in a common sector). In particular, QCD with one massless fermion in the antisymmetric representation is equivalent to $\mathcal{N} = 1$ SYM in the planar limit $\Rightarrow$ copy analytical predictions from SUSY to QCD.
- The orientifold planar equivalence holds if and only if the C-symmetry is not spontaneously broken in both theories $\Rightarrow$ a calculation from first principles is mandatory.
- Assuming that planar equivalence works, how large are the $1/N$ corrections?

A. Armoni, M. Shifman and G. Veneziano. *SUSY relics in one-flavor QCD from a new 1/N expansion.* Phys. Rev. Lett. 91, 191601, 2003.

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M. Unsal and L. G. Yaffe. *(In)validity of large N orientifold equivalence.* Phys. Rev. D74:105019, 2006.

A. Armoni, M. Shifman and G. Veneziano. *A note on C-parity conservation and the validity of orientifold planar equivalence.* arXiv:hep-th/0701229, 2007.

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Dynamical fermions difficult to simulate $\Rightarrow$ start with the quenched theory.

Outline

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Condensates on the lattice

Aim

To measure the bare quark condensate with staggered fermions in the two-index representations of the gauge group, in the quenched lattice theory.

- Wilson action.
- Staggered Dirac operator.
- The two-index representations.
- The bare condensate.

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$$S_{YM} = -\frac{2N}{\lambda} \sum_p \Re\mathrm{e}\, \mathrm{tr}\, U(p)$$

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$$D_{xy} = m\delta_{xy} - K_{xy} =$$
$$= m\delta_{xy} + \frac{1}{2}\sum_{\mu} \eta_{\mu}(x) \left\{ R[U_{\mu}(x)]\delta_{x+\hat{\mu},y} - R[U_{\mu}(x-\hat{\mu})]^{\dagger}\delta_{x-\hat{\mu},y} \right\}$$

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$$\mathrm{tr}\,\mathrm{Adj}[U] = |\,\mathrm{tr}\,U|^2 - 1$$

$$\mathrm{tr}\,\mathrm{S/AS}[U] = \frac{(\mathrm{tr}\,U)^2 \pm \mathrm{tr}(U^2)}{2}$$

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For S/AS representations:

$$\langle \bar{\psi}\psi \rangle_q = \frac{1}{V} \langle \mathrm{Tr}(m - K)^{-1} \rangle_{YM}$$

For the adjoint representation:

$$\langle \lambda\lambda \rangle_q = \frac{1}{2V} \langle \mathrm{Tr}(m - K)^{-1} \rangle_{YM}$$

Outline

Proof of the "quenched" equivalence

Equivalence

$$\lim_{N\to\infty} \frac{1}{VN^2} \langle \mathrm{Tr}(m - K_{\mathrm{S/AS}})^{-1} \rangle = \lim_{N\to\infty} \frac{1}{2VN^2} \langle \mathrm{Tr}(m - K_{\mathrm{Adj}})^{-1} \rangle$$

- Expand in m^{-1}.
- Replace the two-index representations.
- Take the large-N limit.
- Mathematical details. The condensate is an analytical function of each real mass. The large-N limit can be exchanged with the series.

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$$\frac{1}{VN^2} \langle \mathrm{Tr}(m-K)^{-1} \rangle = \frac{1}{VN^2} \sum_{n=0}^{\infty} \frac{1}{m^{n+1}} \langle \mathrm{Tr}\, K^n \rangle =$$

$$= \frac{1}{VN^2} \sum_{\omega \in C} \frac{c(\omega)}{m^{L(\omega)+1}} \langle \mathrm{tr}\, \mathrm{R}[U(\omega)] \rangle$$

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A convenient parameterization

$$\frac{1}{N^2}\langle\bar{\psi}\psi\rangle_{\mathrm{S/AS}} = \frac{1}{2V}\sum_{\omega\in\mathcal{C}}\frac{c(\omega)}{m^{L(\omega)+1}}\,\frac{\langle[\mathrm{tr}\,U(\omega)]^2\rangle \pm \langle\mathrm{tr}[U(\omega)^2]\rangle}{N^2}$$

$$\frac{1}{N^2}\langle\lambda\lambda\rangle_{\mathrm{Adj}} = \frac{1}{2V}\sum_{\omega\in\mathcal{C}}\frac{c(\omega)}{m^{L(\omega)+1}}\,\frac{\langle|\,\mathrm{tr}\,U(\omega)|^2\rangle - 1}{N^2}$$

A convenient parameterization

$$\frac{1}{N^2}\langle\bar{\psi}\psi\rangle_{\mathrm{S/AS}} = f\left(m, \frac{1}{N^2}\right) \pm \frac{1}{N} g\left(m, \frac{1}{N^2}\right)$$

$$\frac{1}{N^2}\langle\lambda\lambda\rangle_{\mathrm{Adj}} = \tilde{f}\left(m, \frac{1}{N^2}\right) - \frac{1}{2N^2}\langle\bar{\psi}\psi\rangle_{free}$$

Planar equivalence: $f(m,0) = \tilde{f}(m,0)$.

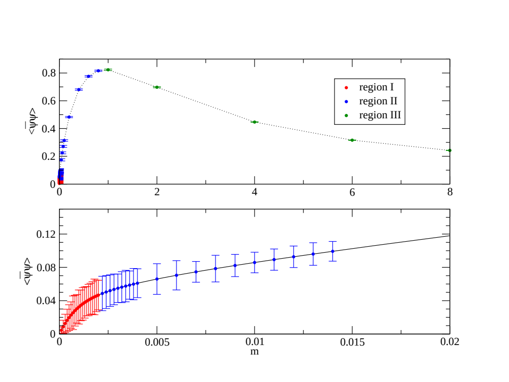
Strategy

1. Simulate the condensates at various values of the mass.
2. Extract the functions f, g, $\tilde{f}$.
3. Fit at fixed mass:

$$\tilde{f} = a_0 + \frac{b_0}{N^2} \qquad g = a_1 + \frac{b_1}{N^2} \qquad f - \tilde{f} = \frac{a_2}{N^2} + \frac{b_2}{N^4}$$

Simulation details

- $N = 2, 3, 4, 6, 8$
- $\beta(N)$ chosen in such a way that $(aT_c)^{-1} = 5$ ($a \simeq 0.145$ fm)
- 14^4 lattice, which corresponds to $L \simeq 2.0$ fm
- 22 values of the bare mass in the range $0.012 \cdots 8.0$



Chiral condensate
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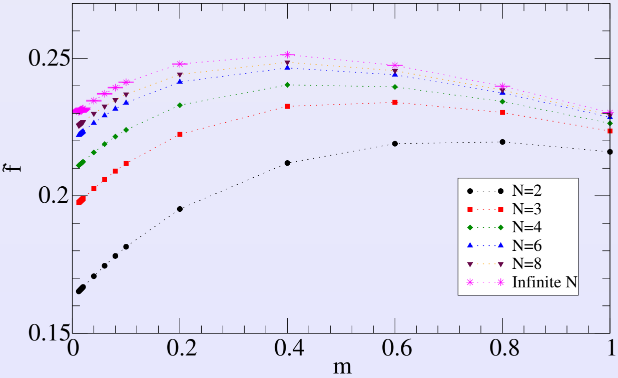
Motivations
Condensates on the lattice
Proof of the "quenched" equivalence
Lattice setup
Results
Conclusions and perspectives

Outline

Function $\tilde{f}$



For $m \leq 0.2$ we get $\chi^2/\mathrm{dof} \leq 0.53$ (we use $N = 4, 6, 8$).

Function f

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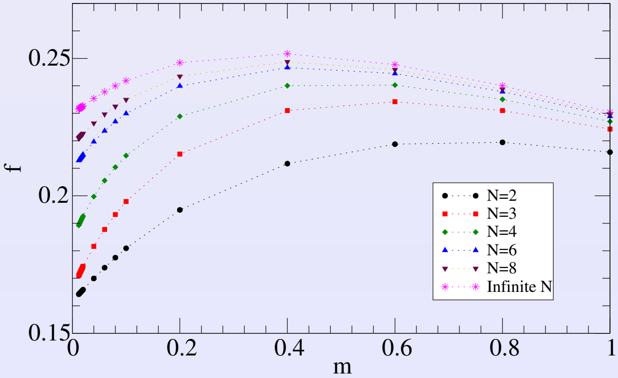
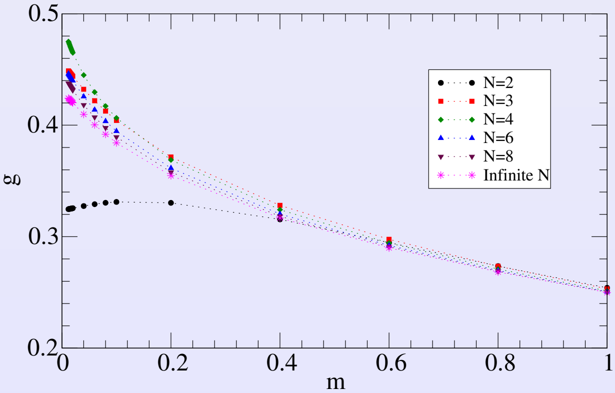


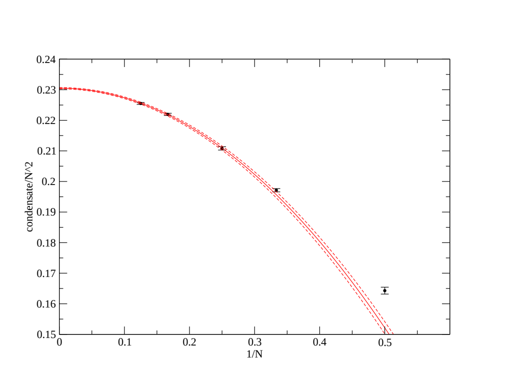
For $m \leq 0.2$ we get $\chi^2/\mathrm{dof} \leq 0.37$ (we are fitting here $f - \tilde{f}$; we use $N = 4, 6, 8$).

Function g



For $m \leq 0.2$ we get $\chi^2/\mathrm{dof} \leq 0.17$ (we use $N = 4, 6, 8$).

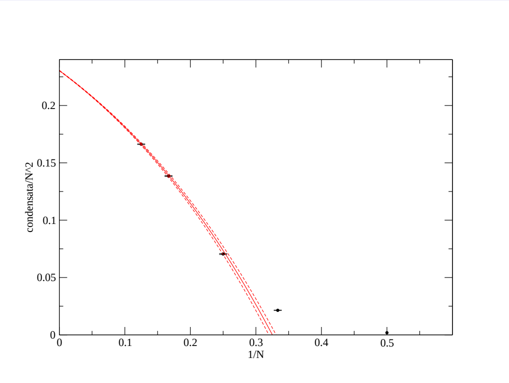
Condensate in the adjoint representation



$$\frac{\langle \lambda\lambda \rangle_{\mathrm{Adj}}(m = 0.012)}{N^2} = 0.23050(22) - \frac{0.3134(72)}{N^2}$$

At $N = 3$, relative error $\simeq 0.8\%$.

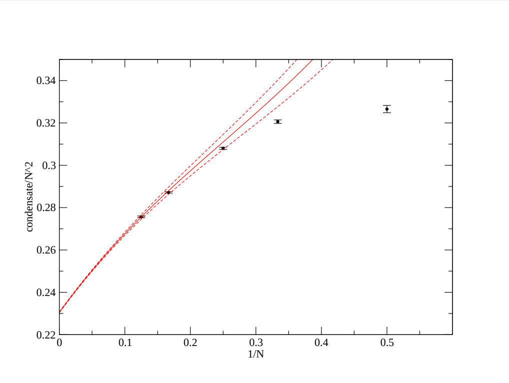
Condensate in the antisymmetric representation



$$\frac{\langle\bar{\psi}\psi\rangle_{\mathrm{AS}}(m=0.012)}{N^2} = 0.23050(22) - \frac{0.4242(11)}{N} - \frac{0.612(43)}{N^2} - \frac{0.811(25)}{N^3}$$

At $N = 3$, condensate < 0!

Condensate in the symmetric representation



$$\frac{\langle\bar{\psi}\psi\rangle_S(m=0.012)}{N^2} = 0.23050(22) + \frac{0.4242(11)}{N} - \frac{0.612(43)}{N^2} + \frac{0.811(25)}{N^3}$$

At $N = 3$, relative error $\simeq 4\%$.

Conclusions and perspectives

- First lattice calculation involving fermions in the two-index representations at $N \geq 4$.
- Check of the orientifold planar equivalence in a simple case.
- Computation of the quark condensate
  - For fermions in the adjoint and symmetric representations, the leading $1/N^2$ correction describes the data at $N \geq 3$ with an accuracy of a few percents;
  - For fermions in the antisymmetric representation higher order corrections play a major role.
- Current and future developments
  - Dynamical fermions;
  - Renormalization of the condensate and continuum limit.