# Transverse Momentum Distributions of Partons in the Nucleon

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presenting work in collaboration with LHPC and

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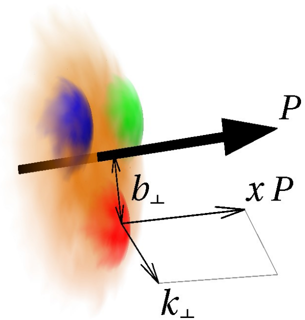
Emmy Noether-Programm
Deutsche Forschungsgemeinschaft DFG

Technische Universität München

# motivation: parton picture




Fast nucleon: Quarks look like “partons”. Distribution depends on

- momentum fraction $\boldsymbol{x} \equiv k^+/P^+$
  of the nucleon momentum $\boldsymbol{P}$,
  - intrinsic transverse momentum $\boldsymbol{k}_\perp$,
  - transverse position $\boldsymbol{b}_\perp$ (impact parameter).

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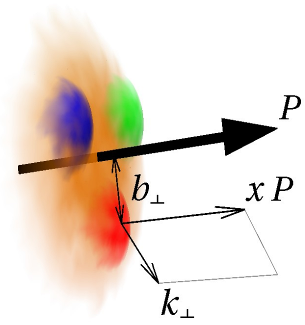


**How are the quarks distributed with respect to $\boldsymbol{x}$ and $\boldsymbol{k}_\perp$?**

**TMDPDFs**

transverse momentum dependent parton distribution functions

e.g. $\boldsymbol{f_1}(\boldsymbol{x}, \boldsymbol{k}_\perp)$

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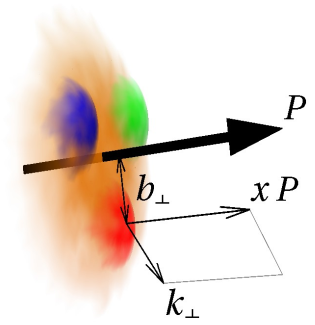


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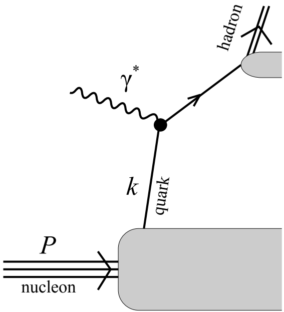
Fast nucleon: Quarks look like “partons”. Distribution depends on

- momentum fraction $\boldsymbol{x} \equiv k^+/P^+$ of the nucleon momentum $\boldsymbol{P}$, $\Rightarrow$ PDFs
- intrinsic transverse momentum $\boldsymbol{k}_\perp$, $\Rightarrow$ TMDPDFs
- transverse position $\boldsymbol{b}_\perp$ (impact parameter). $\Rightarrow$ GPDs

# $k_\perp$-dependence and factorization

example: semi inclusive deep inelastic scattering experiment (**SIDIS**)



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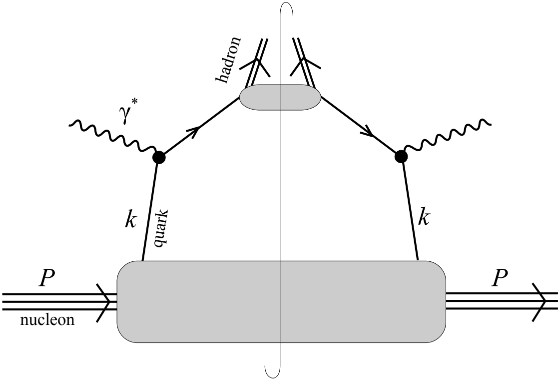
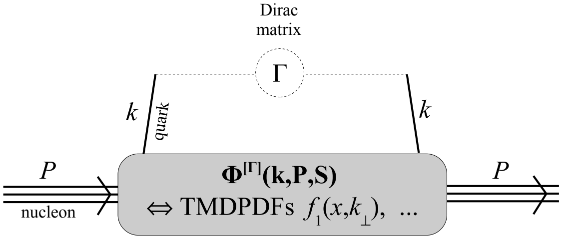



factorization $\Longrightarrow$ hard process + soft blobs (non-perturbative)

[Collins, Soper, Sterman PLB 83, NPB 85]
[Ji, Ma, Yuan PRD (2005)], [Mulders, Tangerman NPB (1996)]

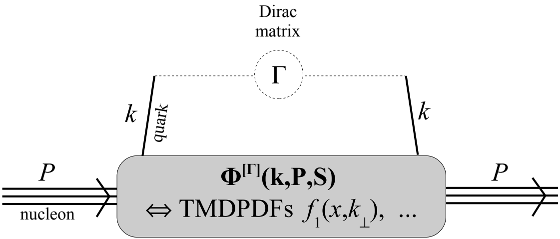
# $k_\perp$-dependence and factorization




**non-perturbative correlator, defined as**

$$\Phi^{[\Gamma]}(k,P,S) \;\equiv\; \text{“}\,\langle P|\,\bar{q}(k)\,\Gamma\,q(k)\,|P\rangle\,\text{”}$$

# $k_\perp$-dependence and factorization




**non-perturbative correlator, defined as**

$$\Phi^{[\Gamma]}(k,P,S) \equiv \frac{1}{2}\int \frac{d^4\ell}{(2\pi)^4}\, e^{-ik\cdot\ell}\ \langle P,S|\ \bar{q}(\ell)\,\Gamma\ \mathcal{U}\ q(0)\ |P,S\rangle$$

# gauge link operator $\mathcal{U}$

$\langle P|\ \overline{q}(\ell)\,\Gamma\,\mathcal{U}\,q(0)\ \ |P\rangle$ is gauge invariant.

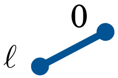
**continuum**

$$\mathcal{U} \equiv \mathcal{P}\exp\left(-ig\int_0^\ell d\xi^\mu A_\mu(\xi)\right)$$

along path from 0 to $\ell$

- factorization in SIDIS :
  path runs to infinity and back


- ***here*** (up to now):
  straight path




$\longrightarrow$ retains probability interpretation! e.g., [Bacchetta et al., PRL85,712 (2000)]

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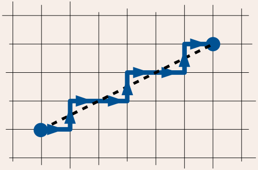
**lattice**



product of link variables

- factorization in SIDIS :
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- ***here*** (up to now):
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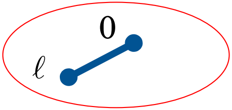


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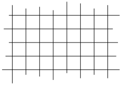
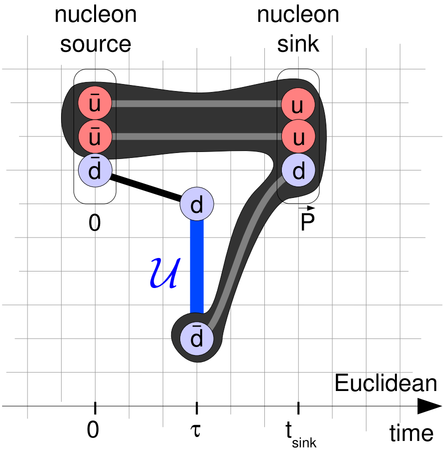
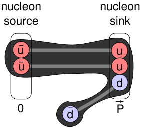
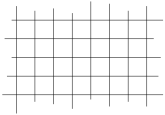
# extracting nucleon structure from the lattice

| Ingredients | | Output : 3-point correlator $C_{3\text{pt}}$ |
|---|---|---|
|  | gauge configs. |  |
|  | quark propagators | |
|  | nucleon sequential propagators | |

[We neglect "disconnected contributions" (absent for up minus down).]

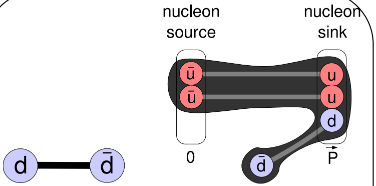
We use the Chroma library [Edwards, Joo (2005)] to process



**MILC gauge configurations**

staggered Asqtad action,
2+1 flavors, $a \approx 0.124$ fm,
$m_\pi \approx 500,\ 610,$ and $760$ MeV

[Orginos, Toussaint PRD (1999)]




**LHPC propagators**

domain wall valence fermions,
$m_\pi$ adjusted to staggered sea,

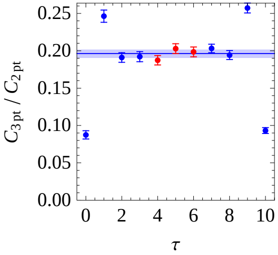
nucleon momenta:
$\vec{P} = 0$ and $|\vec{P}| = 500$ MeV

e.g., [Hägler et al. PRD (2008)]

# extracting TMDPDFs from the lattice

**ratio of correlators far away from nucleon source and sink**

$$\frac{C_{3\mathrm{pt}}(\tau, t_{\mathrm{sink}}, P, \ldots)}{C_{2\mathrm{pt}}(t_{\mathrm{sink}}, P, \ldots)} \xrightarrow{0 \ll \tau \ll t_{\mathrm{sink}}} \langle P, S| \; \overline{q}(\ell)\, \Gamma\, \mathcal{U}\, q(0) \; |P, S\rangle$$




matrix element
extracted from plateau value

# extracting TMDPDFs from the lattice

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## isolation of Lorentz-invariant amplitudes

compare [Mulders, Tangerman NPB (1996)]

$$\langle P, S| \;\overline{q}(\ell)\, \gamma_\mu\, \mathcal{U}\, q(0)\; |P, S\rangle = 4\, \tilde{A}_2\, P_\mu + 4i\, m_N{}^2\, \tilde{A}_3\, \ell_\mu$$

$$\langle P, S| \;\overline{q}(\ell)\, \gamma_\mu \gamma^5\, \mathcal{U}\, q(0)\; |P, S\rangle = -4\, m_N\, \tilde{A}_6\, S_\mu - 4i\, m_N\, \tilde{A}_7\, P_\mu (\ell \cdot S) + 4\, m_N{}^3\, \tilde{A}_8\, \ell_\mu (\ell \cdot S)$$

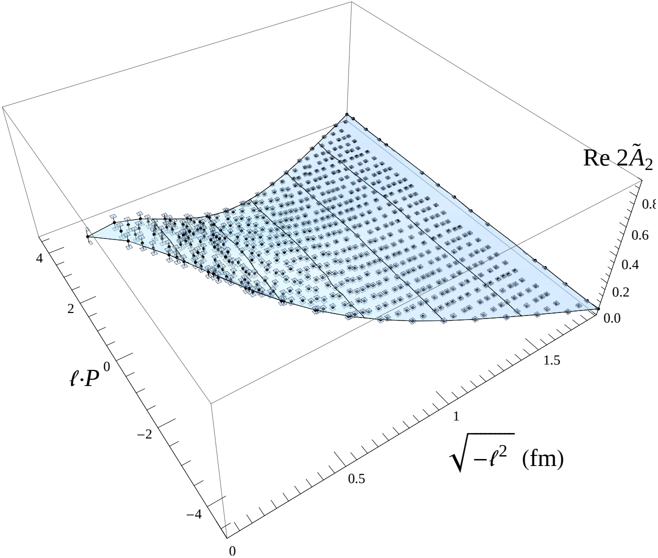
The amplitudes fulfill $\tilde{A}_i(\ell^2, \ell \cdot P) = \left[\tilde{A}_i(\ell^2, -\ell \cdot P)\right]^*$.

Lattice restriction: $\ell_0 = \ell_4 = 0 \quad \Rightarrow \quad \ell^2 \leq 0,\; |\ell \cdot P| \leq |\vec{P}|\sqrt{-\ell^2}$

# First Results

(Renormalization is preliminary.)

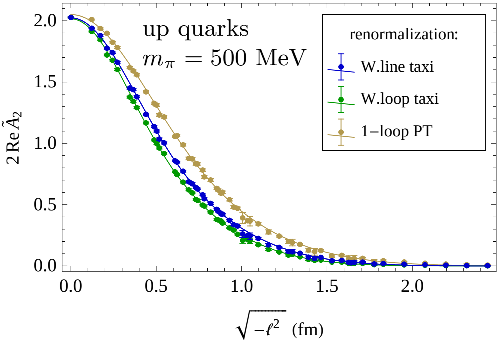
# Re $\tilde{A}_2(\ell^2, \ell\cdot P)$ from the lattice




$$\langle P|\ \bar{q}(\ell)\,\gamma_4\,\mathcal{U}\,q(0)\ |P\rangle \longrightarrow \tilde{A}_2(\ell^2, \ell\cdot P) \xrightarrow[\text{trans.}]{\text{Fourier}} \underbrace{f_1^{\text{lat}}(x, k_\perp)}_{\text{TMDPDF}}$$

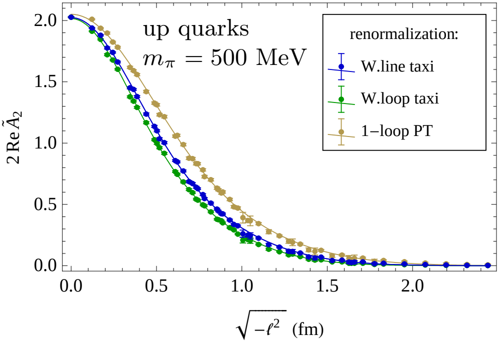
# Re $\tilde{A}_2(\ell^2, \ell\cdot P=0)$ and the TMDPDF $f_1(x, k_\perp)$

$$f_1(x, k_\perp) \equiv \int dk^- \, \Phi^{[\gamma^+]}(k, P, S)$$



# Re $\tilde{A}_2(\ell^2, \ell\cdot P=0)$ and the TMDPDF $f_1(x, k_\perp)$

1st Mellin moment $f_1^{(1)}(\quad k_\perp) \equiv \int dx \int dk^- \; \Phi^{[\gamma^+]}(k, P, S)$



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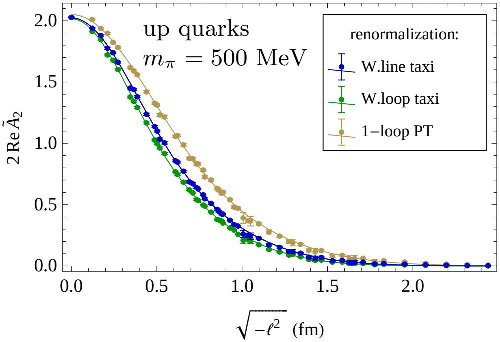
$$= \int \frac{d^2\ell_\perp}{(2\pi)^2}\; e^{ik_\perp\cdot\ell_\perp}\; 2\,\tilde{A}_2(-\ell_\perp^{\,2}, 0)$$



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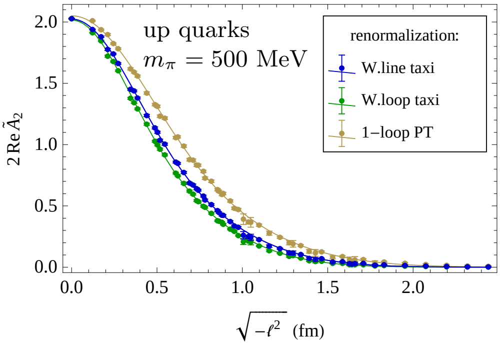


**fit function**

$$C_1 \exp(-|\ell|^2/\sigma_1^2) + C_2 \exp(-|\ell|^2/\sigma_2^2)$$

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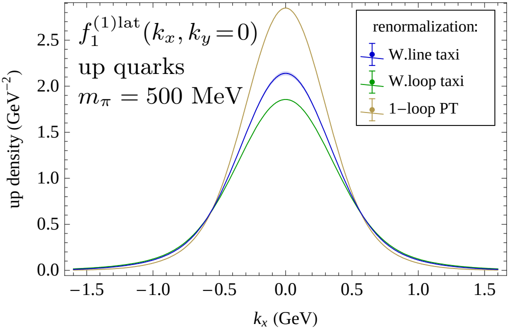

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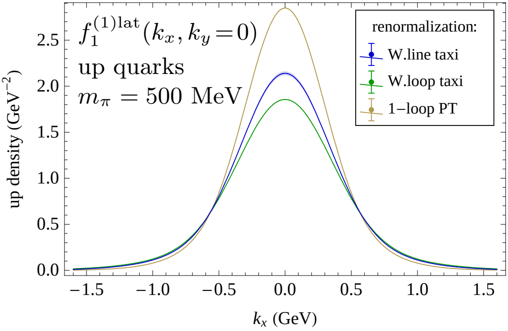


$f_1^{(1)\text{lat}}(k_\perp)$ gives the **density** of quarks with an intrinsic transverse momentum $k_\perp = (k_x, k_y)$

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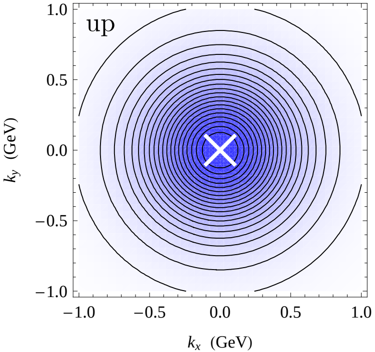


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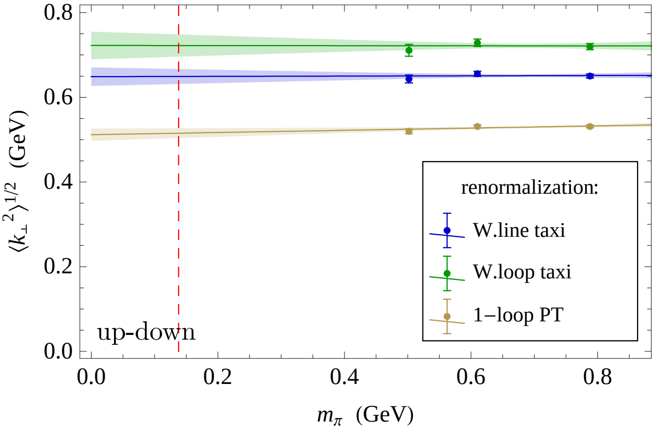
# linear extrapolation $\langle k_\perp^2 \rangle^{1/2}$ to physical pion mass




RMS transverse momentum $\langle k_\perp^2 \rangle^{1/2} = (649 \pm 18_{\text{stat}})$ MeV

based on double Gaussian Ansatz

compare phenomenology [Anselmino et al., PRD71, 074006 (2005)]:

$\langle k_\perp^2 \rangle^{1/2} \approx 500$ MeV

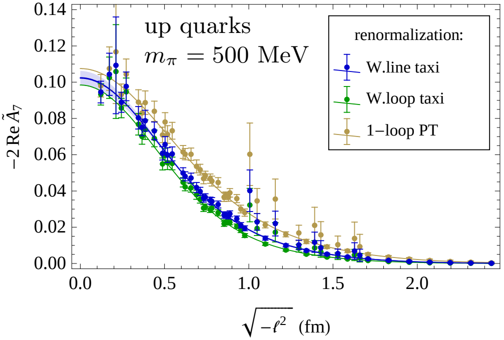
based on single Gaussian Ansatz

# polarized quark densities: the TMDPDF $g_{1T}(x, k_\perp)$

In a transversely spin polarized nucleon ($\vec{S} \perp \vec{P}$):

$$\frac{1}{2} \int dx \int dk^- \; \Phi^{[\gamma^+ \frac{1}{2}(\mathbb{1}+\gamma^5)]}(k, P, S) =$$

$$\frac{1}{2} \left( \; f_1^{(1)\mathrm{lat}}(k_\perp) + \frac{k_\perp \cdot S_\perp}{m_N} \, g_{1T}^{(1)\mathrm{lat}}(k_\perp) \; \right)$$




$g_{1T}^{(1)\mathrm{lat}}$ is obtained from amplitude $\tilde{A}_7$

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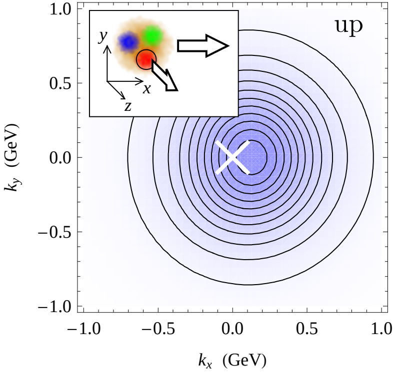


**density** of quarks with positive helicity in a proton with spin pointing in x direction

net transverse momentum $k_x$

$\langle k_x \rangle = (135 \pm 10_{\mathrm{stat}} \pm 6_{\mathrm{renorm.}})$ MeV

@ $m_\pi = 500$ MeV

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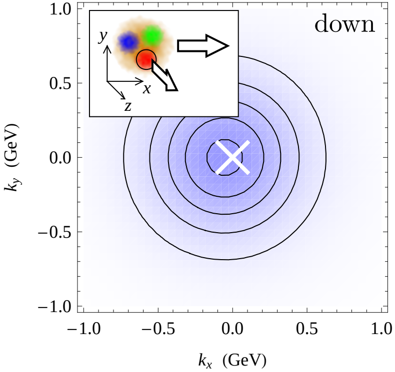


**density** of quarks with positive helicity in a proton with spin pointing in x direction

net transverse momentum $k_x$

$\langle k_x \rangle = (-24 \pm 5_{\mathrm{stat}} \pm 3_{\mathrm{renorm.}})$ MeV

@ $m_\pi = 500$ MeV

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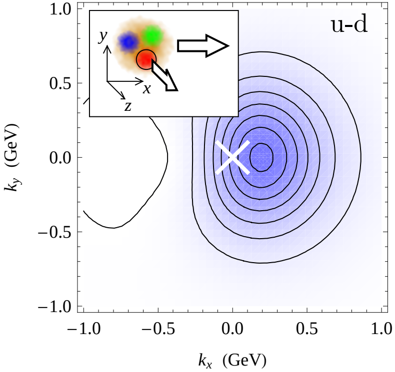



**density** of quarks with positive helicity in a proton with spin pointing in x direction

$k_\perp$-densities analogous to impact parameter densities

[Diehl, Hägler EPJC44 (2005)],

[QCDSF PRL98, 222001 (2007)]

see also [G. Miller PRC76, 065209 (2007)]

# Link Renormalization

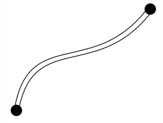
Thanks to
Gunnar Bali and Vladimir Braun
(Univ. Regensburg)
for helpful discussions

# continuum renormalization and Taxi Driver Method

## continuum renormalization of Wilson lines

[Craigie, Dorn NPB185,204 (1981)]

smooth path



$$\langle \mathcal{U}_{\text{ren}} \rangle = Z_z^{-1} \exp( -\delta m \, L ) \, \langle \mathcal{U} \rangle$$

$L$ is the total length of the Wilson line

$Z_z^{-1}$, $\delta m$, $\nu(\theta)$ are renormalization constants

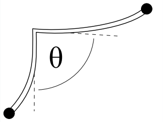
$\delta m \propto \mu \mathrel{\hat{=}} \frac{1}{a}$ removes linear divergence

*This linear divergence is a long-standing problem in heavy-light calculations.*

# continuum renormalization and Taxi Driver Method

## continuum renormalization of Wilson lines

[Craigie, Dorn NPB185,204 (1981)]




$$\langle \mathcal{U}_{\text{ren}} \rangle = Z_z^{-1} \exp(\, -\delta m\, L \; - \nu(\theta) \,)\, \langle \mathcal{U} \rangle$$

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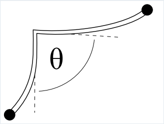


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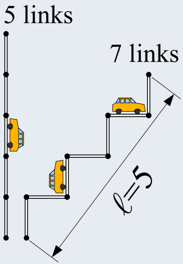
## link renormalization on the lattice: Taxi Driver Method



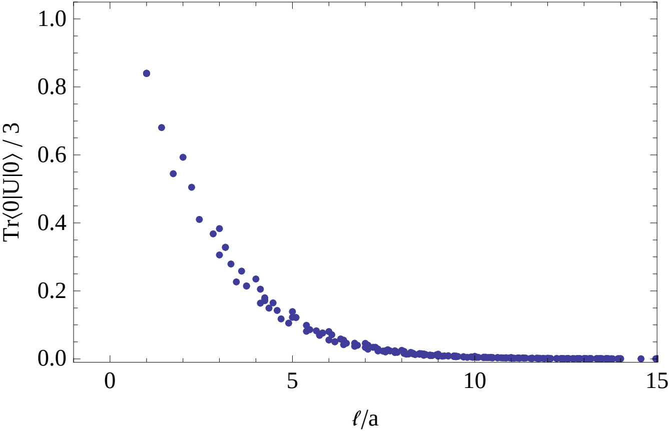

working hypothesis: like continuum theory

$$\langle \mathcal{U}^{\text{lat}}_{\text{ren}} \rangle = Z_z^{-1} \exp(-a\delta m\, \#\text{links} - \nu\, \#\text{corners}\ ) \langle \mathcal{U}^{\text{lat}} \rangle$$

*Idea:* Evaluate straight and step like link paths $\text{Tr}\,\langle 0 | \mathcal{U}^{\text{lat}}\, | 0 \rangle$ on Landau gauge fixed ensemble. Adjust $a\delta m$, $\nu$ such that $\text{Tr}\,\langle 0 | \mathcal{U}^{\text{lat}}_{\text{ren}}\, | 0 \rangle$ depends smoothly on $\ell$ only.

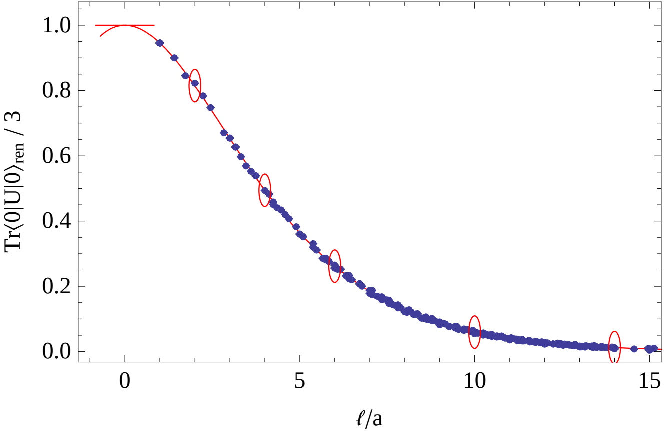
# Taxi Driver Renormalization

$\frac{1}{3}\text{Tr } \langle 0| \mathcal{U} |0\rangle$ on Landau gauge fixed ensemble (no link smearing)



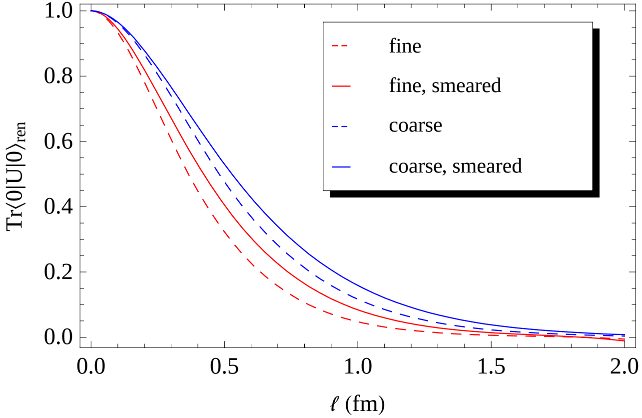
# Taxi Driver Renormalization

$\frac{1}{3}\mathrm{Tr}\ \langle 0|\,\mathcal{U}_{\mathrm{ren}}\,|0\rangle$ renormalized requiring smoothness



# Taxi Driver Renormalization on different lattices

- fine: MILC $a = 0.084$ fm, $m_\pi \approx 760$ MeV
- coarse: MILC $a = 0.121$ fm, $m_\pi \approx 790$ MeV
- with and without HYP smearing (reduces $a\delta m$ drastically)




Still $a$-dependence. Renormalization incomplete or $O(a^2)$-effects?

# Summary

Results:

- First lattice calculation of quark distributions $f_1^{\text{lat}}$ and $g_{1T}^{\text{lat}}$ as a function of transverse momentum.
- Densities of longitudinally polarized quarks in a transversely polarized proton are deformed.

Outlook:

- Analysis of further amplitudes and TMDPDFs.
- Need for improved renormalization of the non-local operators.
- Study of non-straight gauge links similar as in SIDIS.

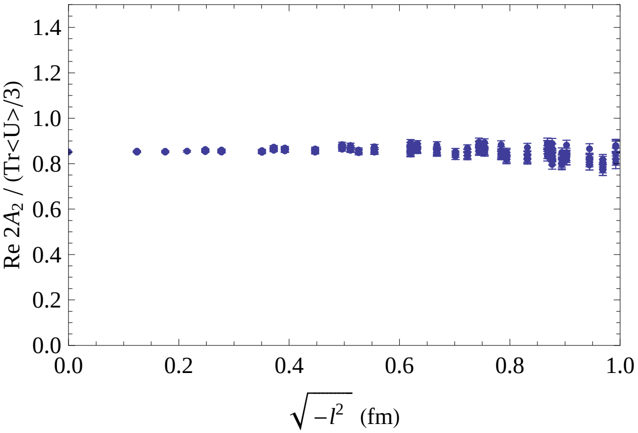
# Backup Slides

# Renormalization constants on different lattices

| $a$ (fm) | | method | $a\delta m$ | $\nu$ | $Z_z$ |
|---|---|---|---|---|---|
| 0.12 | | taxi | $-0.2058(17)$ | 0.04648(80) | 1.107(19) |
| 0.12 | | perturb. | $-0.1987$ | | |
| 0.08 | | taxi | $-0.1804(17)$ | 0.04173(69) | 1.098(23) |
| 0.08 | | perturb. | $-0.1908$ | | |
| 0.12 | smeared | taxi | $-0.01228(42)$ | 0.00104(16) | 1.021(17) |
| 0.12 | smeared | perturb. | $-0.0659$ | | |
| 0.08 | smeared | taxi | $-0.00825(32)$ | 0.00081(11) | 1.017(16) |
| 0.08 | smeared | perturb. | $-0.0631$ | | |

Coarse lattice: $a = 0.121$ fm, $m_\pi \approx 790$ MeV, $m_u = m_d = m_s$
Fine lattice: $a = 0.084$ fm, $m_\pi \approx 760$ MeV, $m_u = m_d = m_s$

# Amplitude $\tilde{A}_2$ compared to VEV of gauge link

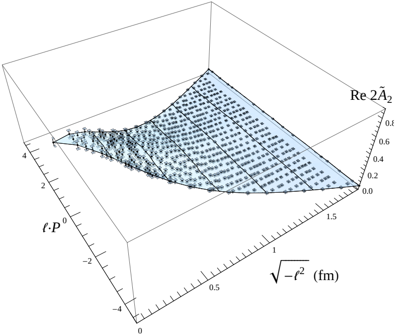



$$\frac{\left\langle \vec{P}=0 \right| \bar{q}(\ell)\,\gamma^+\,\mathcal{U}\,q(0) \left| \vec{P}=0 \right\rangle}{\frac{1}{3}\,\mathrm{Tr}\,\langle 0|\,\mathcal{U}\,|0\rangle}$$ turns out constant

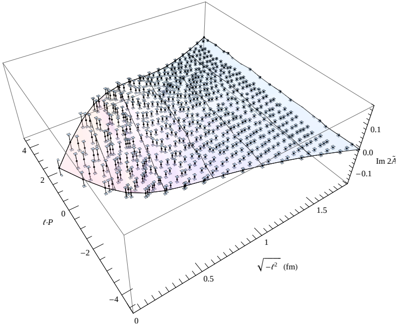
# $(\boldsymbol{x}, \boldsymbol{k}_\perp)$ – factorization hypothesis

# $\ell\cdot P$ - dependence of $\tilde{A}_2(\ell^2, \ell\cdot P)$

2 Re $\tilde{A}_2(\ell^2, \ell\cdot P)$

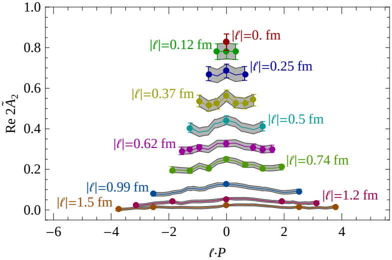



2 Im $\tilde{A}_2(\ell^2, \ell\cdot P)$

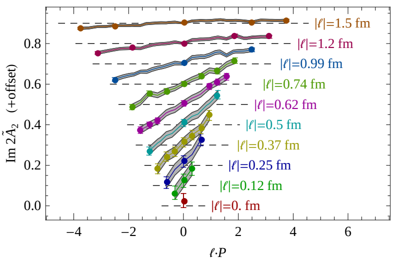


# $\ell \cdot P$ - dependence of $\tilde{A}_2(\ell^2, \ell \cdot P)$

$2 \text{ Re } \tilde{A}_2(\ell^2, \ell \cdot P)$




$2 \text{ Im } \tilde{A}_2(\ell^2, \ell \cdot P)$



# $(x, k_\perp)$-factorization hypothesis

## factorization hypothesis

$$f_1^{\text{lat}}(x, \vec{k}_\perp) \;=\; \hat{\mathbf{f}}_1^{\text{lat}}(x) \;\; f_1^{(1)\text{lat}}(\vec{k}_\perp)$$

as in phenomenological applications,
e.g., [ANSELMINO PRD (2005)]

Then $\tilde{A}_2$ factorizes, too:

$$\tilde{A}_2(\ell^2, \ell\cdot P) = \hat{\mathbf{A}}_2(\ell\cdot P)\; \tilde{A}_2(\ell^2, 0).$$

To test this, we define a **scaled** amplitude

$$\boxed{\hat{\mathbf{A}}_2(\ell^2, \ell\cdot P) \equiv \frac{\tilde{A}_2(\ell^2, \ell\cdot P)}{\text{Re}\;\tilde{A}_2(\ell^2, 0)}}$$

If factorization holds, $\hat{\mathbf{A}}_2$ should be $\ell^2$-independent.

# $(x, k_\perp)$-factorization hypothesis

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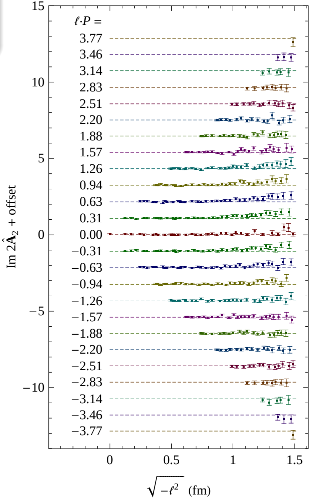
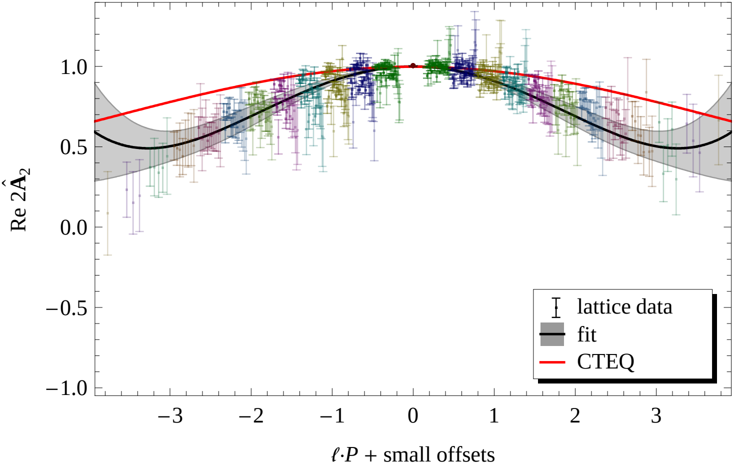
✓ within statistics



# comparison to CTEQ parton distributions

All our data for $\hat{\mathbf{A}}_2(\ell^2, \ell\cdot P)$ at $m_\pi \approx 610$ MeV

compared to a Fourier transform of $f_1(x)$
from CTEQ5 [Lai et al., EPJ C12, 375 (2000)]



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