Minkowskian Dynamics of a Polyakov Loop Model under a Heating Quench

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Glauber Dynamics

- Glauber dynamics is the non-relativistic diffusive dynamics found in the MCMC process for local updating algorithms. Metropolis and heatbath belong to the Glauber (also called model A) dynamical universality class.
- In equilibrium LGT simulations the notion of time is lost. The Euclidean and Minkowskian equilibrium systems agree, because the fourth direction of the lattice just serves to define the temperature.
- An artificial time scale is introduced by using Glauber dynamics to study the *response* of the system to rapidly changing an external parameter (quench) and tracing its evolution to a new equilibrium state.[1]

[1] A. Bazavov, B.A. Berg and A. Velytsky, PRD 74 (2006) 014501

Structure Factors (SFs)

Two-point correlation function:

$$\langle u_0(0) u_0^\dagger(\vec{j}) \rangle_L = \frac{1}{N_s^3} \sum_{\vec{i}} u_0(\vec{i}) u_0^\dagger(\vec{i} + \vec{j}) \,.$$

The structure function:

$$F(\vec{p}) = \sum_{\vec{j}} a^3 \, \langle u_0(0) u_0^\dagger(\vec{j}) \rangle_L \, e^{i\, \vec{k}\, \vec{j}}, \quad a\, \vec{p}\, \vec{i} \, = \, \vec{k}\, \vec{i} \, = \frac{2\pi}{N_s}\, \vec{n}\, \vec{i}\,,$$

$$F(\vec{p}) = \frac{a^3}{N_s^3} \left| \sum_{\vec{i}} e^{-i\, \vec{k}\, \vec{i}}\, u_0(\, \vec{i}\,) \right|^2$$

or (divided by volume)

$$S(\vec{p}) = \frac{F(\vec{p})}{V} = \left| \frac{1}{N_s^3} \sum_{\vec{i}} e^{-i\, \vec{k}\, \vec{i}}\, u_0(\, \vec{i}\,) \right|^2$$

SFs Evolution

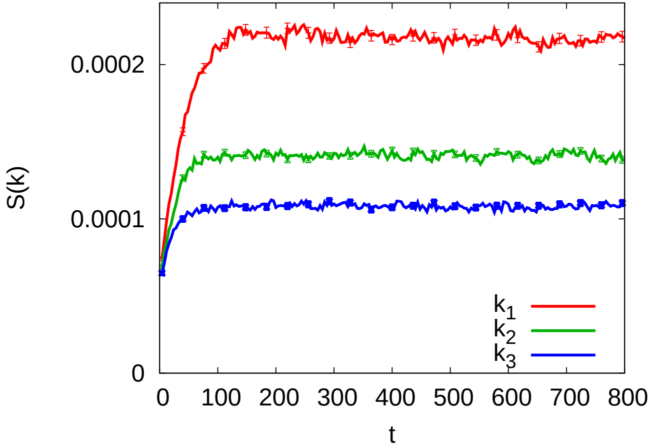


Figure: Subcritical $T_f < T_c$ quench in the 3D 3-state Potts model (T in LGT language, 40^3 lattice, $\beta = 0.2 \rightarrow 0.27$, $\beta_c = 0.2752\ldots$).

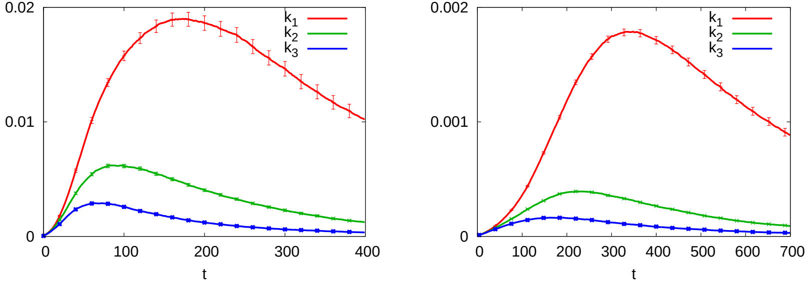


Figure: $T_f > T_c$ quench: Time evolution of the first three structure factors $S(\vec{k})$ in the 3D 3-state Potts model $\beta = 0.2 \rightarrow 0.28$ on 40^3 lattice (left) and SU(3) gauge theory with the Wilson action $6/g^2 = 5.5 \rightarrow 5.92$ on 4×32^3 lattice (right).

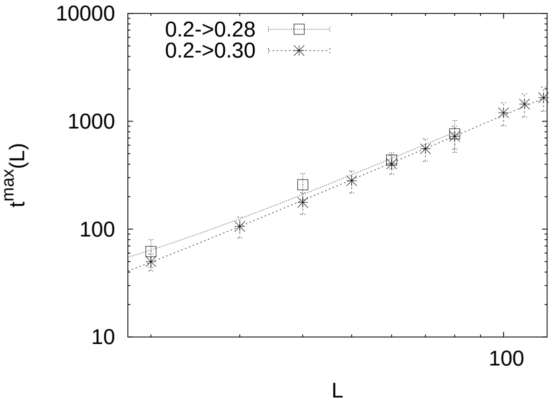


Figure: Time positions of SF F_1 maxima versus lattice size.

Due to order-order domain walls: $t^{\max}(L) \sim L^2$.
No convergence in finite time.

Minkowskian Dynamics

- In Pisarski's effective model[2] the deconfined phase of a pure gauge theory is described as a condensate of Polyakov loops

$$\mathcal{L} \;=\; T^2 \left(|\partial_t \ell|^2 + |\partial_i \ell|^2 \right) - \mathcal{V}(\ell)$$

- The Z(3)-symmetric effective potential takes the form

$$\mathcal{V}(\ell) = \left(-\frac{b_2}{2} |\ell|^2 - \frac{b_3}{6} (\ell^3 + (\ell^*)^3) + \frac{1}{4} (|\ell|^2)^2 \right) b_4 \; T^4 \; .$$

- The energy scale is set by T^4, the mass coefficient $b_2 = b_2(T)$ is temperature dependent, while b_3 and b_4 are constants. These couplings are chosen to reproduce lattice data for the SU(3) pressure and energy density above T_c.[3]

[2] Pisarski, Phys. Rev. D 62 (2000) 111501(R); Dumitru, Pisarski, Phys. Lett. B 504 (2001) 282), Phys. Rev. D 66 (2002) 096003

[3] Scavenius, Dumitru and Lenaghan, Phys. Rev. C 66 (2002) 034903

Effective Potential

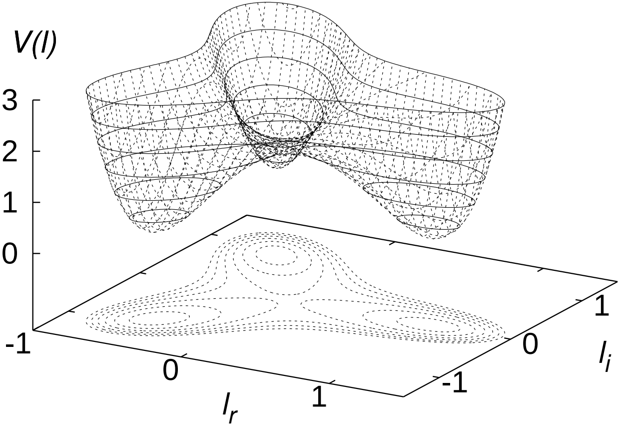


Figure: Effective potential at temperatures $T_f > T_c$. In the plane below equal height contours are shown.

Molecular Dynamics Simulations of the Model

- Polyakov loop fields are defined on the sites of a spatial cubic lattice of size N_s^3 with periodic boundary conditions and lattice spacing a.[4]
- Static, non-expanding metric.
- Time evolution with a leapfrog algorithm (e.g., Frenkel and Smit) according to the Euler-Lagrange equations derived from the Lagrangian.
- When integrating the hyperbolic differential equations we use time steps $\triangle t/a = 0.01$ (due to Lorentz invariance and with the choice $c = 1$, units of time and length agree.).

[4]Scavenius, Dumitru, and Jackson, PRL 87 (2001) 182302

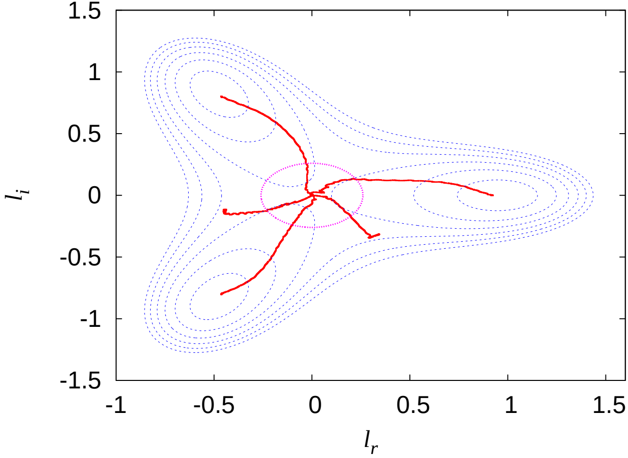
Heating Quench

- At $t = 0$ Polyakov loops are initialized in the confined phase (Gaussian fluctuations and counterterms).
- A physical length scale is introduced through coarse graining of the initial field configuration:

$$\ell(\vec{x}) \rightarrow \ell'(\vec{x}) = N_{\mathrm{cg}}^{-3} \sum_{\vec{x}\,' \in \mathrm{cg}} \ell(\vec{x}) \ .$$

- Then the temperature entering the Lagrangian is set to a value $T_f > T_c$ above the transition temperature.
- Molecular Dynamics (MD) time evolution.

Individual Trajectories



Numerical Results

- We quench[5] to $T_f/T_c = 1.5,\ 1.75,\ 2.0,\ 2.25,\ 2.5$ and average over 200 replica. We set our length scale by

  $$a\,N_{cg} = 1/T_c = 0.736\,\mathrm{fm}\,,\ \text{using}\ N_{cg} = 4,\ a = 0.184\,\mathrm{fm}\,.$$

- We take lattices that accommodate at least ten times the coarse grained length N_{cg}: $N_s = 40,\ 48,\ 64,\ 80,\ 96$. MD time step is $\triangle t = 0.00184\,\mathrm{fm/c}$, and we ran trajectories of 15000 to 25000 steps (27.6 fm/c to 46 fm/c).
- SF behavior is qualitatively the same as before.

[5] Bazavov, Berg, and Dumitru, ArXiv:0805.0784

SF evolution

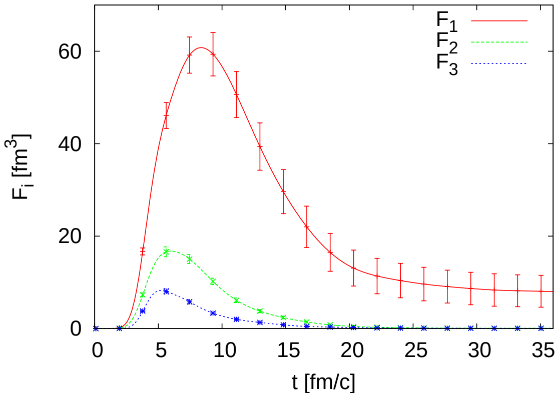
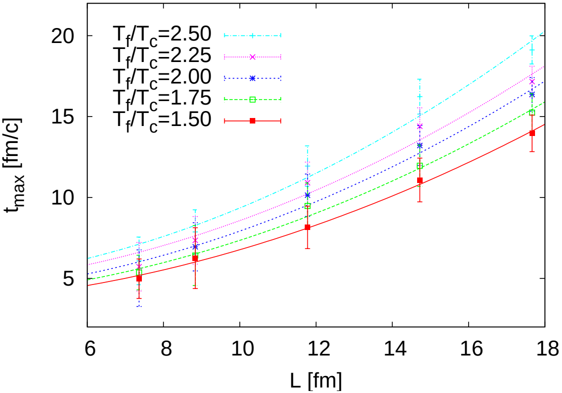


Figure: Structure factors for Minkowski dynamics (64^3 lattice, $T_f / T_c = 1.5$, lattice size $L_s = 11.8$ fm). F_n corresponds to the n^{th} lattice mode with $|\vec{k}| = 2\pi\sqrt{n}/L_s = \sqrt{n}\times 105.3$ MeV.

Time to reach SF maxima



As with Glauber dynamics

$$t^{\max}(L) \sim L^2 \,.$$

Other fits are also possible within the accuracy of the data.

Time to reach SF maxima

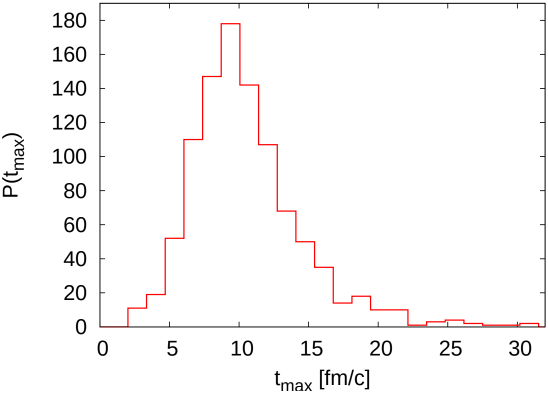


Figure: Histogram of t^{max} times ($T_f = 2\,T_c$ quench on 64^3 lattice).

Broad distribution. So, individual times may largely differ.

Summary and Conclusions

- Non-perturbative, large variations of the Z(3) phase within domain walls arise during the conversion of the confined to the deconfined vacuum ensemble.
- For the parameters studied, SF maxima indicate spinodal transitions.
- Domains of different Z(3) triality slow down equilibration and a scenario emerges for which the system can get stuck around SF maxima.
- Minkowskian dynamics of a simple model for Polyakov loops reproduces qualitatively the features found with Glauber dynamics and sets a physical time scale. E.g., for a $(10\,\mathrm{fm})^3$ box, $t^{max} \approx 8\,\mathrm{fm/c}$ is found, and equilibration takes another $\approx 20\,\mathrm{fm/c}$.