On Scale Determination in Lattice QCD with Dynamical Quarks

arXiv:0803.1281 [hep-lat]

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Outline

The Problem

Simulations of Lattice QCD with dynamical quarks show:

Sommer parameter r_c/a depends on sea quark mass am_q

The Questions are

- Is this a cut-off effect or a physical effect?
- How is the lattice scale to be determined?
  - Should the scale a be taken as dependent on the quark mass m_q?
  - Since the quark mass is a scale-dependent quantity, how to do chiral extrapolations of hadronic quantities like masses?

NO theoretical understanding yet !

Our Simulation

- Wilson (**unimproved**) gauge and fermion actions ($\mathcal{O}(a)$ cutoff effects)
- $N_F = 2$ degenerate sea quarks
- $\beta = 6/g^2 = 5.6$, $16^3 32$ lattice
- 8 values of sea quark masses, $am_q \approx 0.07 - 0.014$

- Standard HMC updating so far (DDHMC runs in progress)
- 5000 trajectories at each sea quark mass
- Gaussian smearing at both mesonic source and sink, highly optimized, arXiv:0712.4354 [hep-lat]
- APE smearing used to extract static potential from $< W >$

- All errors shown are single-omission JK errors from 200 independent configurations at each quark mass

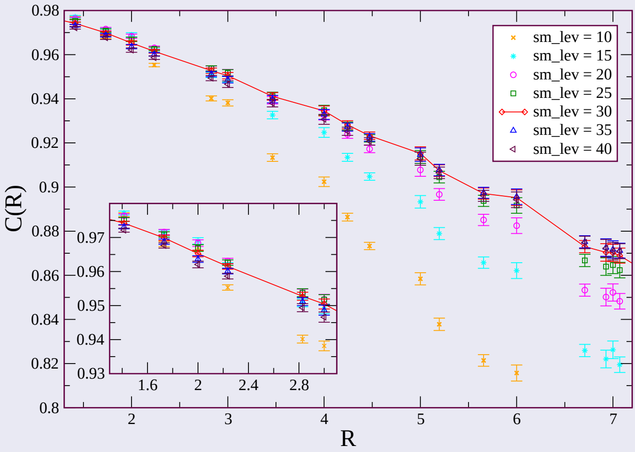
Analysis of $< W(R,T) >$

- Smearing level up to 40 with $\varepsilon = 2.5$ where $c/4 = 1/(\varepsilon + 4)$ is the coefficient of the staples
- $< W(R,T) >$ measured up to $T = 16$ and $R = 8\sqrt{3}$
- Reasonable plateau obtained in effective potential plots between $T = 3$ and $T = 5$
- Static potential V(R) extracted from single exponential fits between $[T_{min}, T_{max}] = [3,4]$, $[3,5]$, $[4,5]$

$$< W(R,T) >= C(R) \exp[-aV(R)T]$$

- Optimum smearing level determined at a given quark mass by observing the ground state overlap $C(R)$ as a function of R

$\beta = 5.6$, $\kappa = 0.15775$ $(am_q \approx 0.02)$



Analysis of $aV(R)$

At each β and quark mass, the static potential obtained is analyzed with the following parameterization:

$$aV(R) = aV_0 + a^2\sigma R - \frac{\alpha}{R} - \delta_{\mathrm{ROT}}\left(\left[\frac{1}{R}\right] - \frac{1}{R}\right)$$

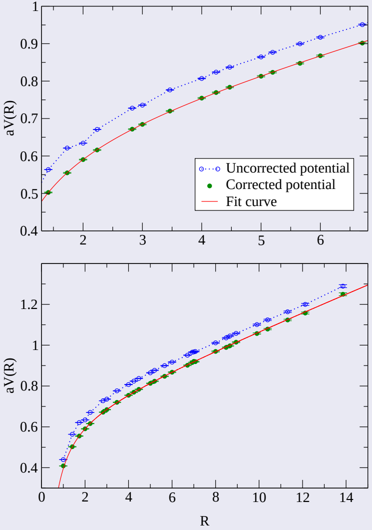
where δ_{ROT} is the coeff of the lattice correction term with

$$\left[\frac{1}{R}\right] = \frac{4\pi}{L^3}\sum_{q_i\neq 0}\frac{\cos(aq_i\cdot R)}{4\sin^2(aq_i/2)}$$

being the lattice fourier transform of the gluon propagator.

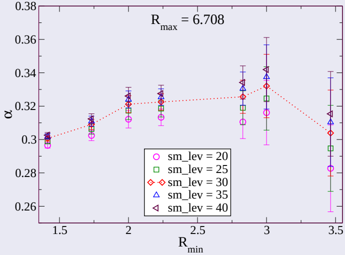
The first 3 terms of $aV(R)$ above is differentiated to obtain the Sommer parameter: $a/r_c = 1/R_c = a\sigma^{1/2}/\sqrt{(\mathcal{N}_c - \alpha)}$

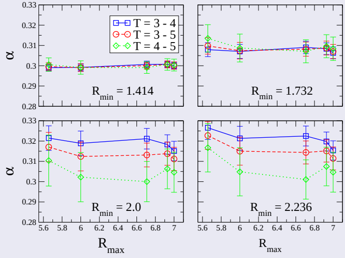
$\beta = 5.6$, $\kappa = 0.1575$ ($am_q \approx 0.03$)

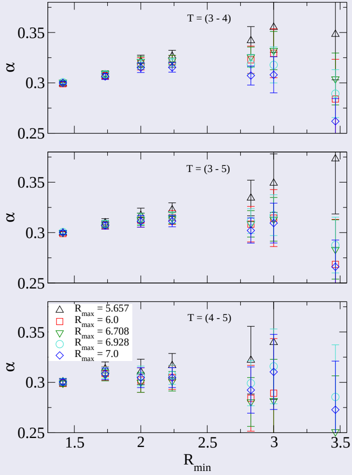


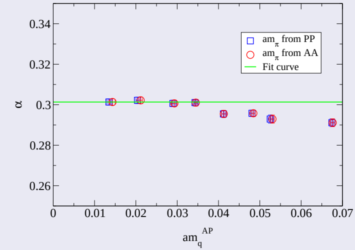
- the difference ($[1/R] - 1/R$) is never negligible on a finite lattice
- α is expected to run with R at these intermediate length scales
- can only estimate an average α over the values of R where the static potential is fit
- perturbative running is generally applicable at scales $\gtrsim 2$ GeV which translates into $R \lesssim 1$ in our case

$\beta = 5.6,\ \kappa = 0.1575$
$(am_q \approx 0.03)$

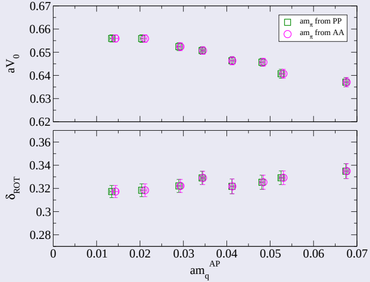






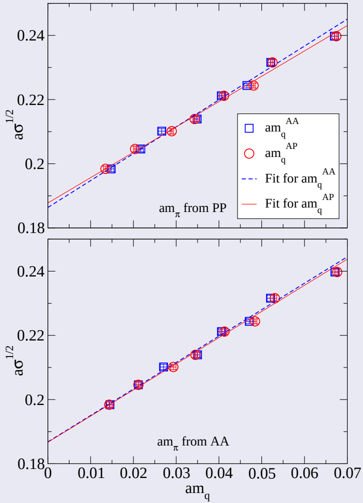


α
0.34
0.32
0.3
0.28
0.26
0 0.01 0.02 0.03 0.04 0.05 0.06 0.07
am_q^{AP}
am_π from PP
am_π from AA
Fit curve



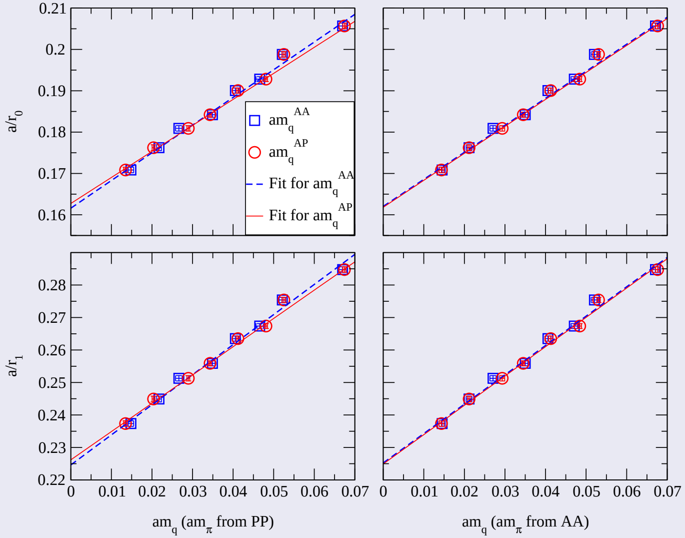
aV_0
0.67 0.66 0.65 0.64 0.63 0.62
am_π from PP
am_π from AA
δ_{ROT}
0.36 0.34 0.32 0.3 0.28
0 0.01 0.02 0.03 0.04 0.05 0.06 0.07
am_q^{AP}

$\beta = 5.6$



$a\sigma^{1/2}$
0.24 0.22 0.2 0.18
am_q^{AA}
am_q^{AP}
Fit for am_q^{AA}
Fit for am_q^{AP}
am_π from PP
am_π from AA
0 0.01 0.02 0.03 0.04 0.05 0.06 0.07
am_q

$\beta = 5.6$



a/r0
a/r1
0.21
0.2
0.19
0.18
0.17
0.16
0.28
0.27
0.26
0.25
0.24
0.23
0.22
0 0.01 0.02 0.03 0.04 0.05 0.06 0.07
am_q^AA
am_q^AP
Fit for am_q^AA
Fit for am_q^AP
am_q (am_π from PP)
am_q (am_π from AA)

From the numerical results on α, $a\sigma^{1/2}$ and a/r_c in dependence of am_q, at fixed β, we conclude:

- The dimensionless parameter α does NOT significantly depend on am_q for small enough am_q ($\lesssim 0.035$)
- Scaling violations (= cutoff effects) are negligible for small enough am_q
- $a\sigma^{1/2}$ is linear in am_q for small enough am_q:

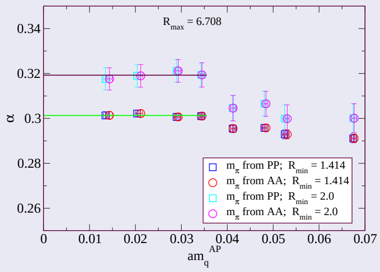
$$a\sigma^{1/2} = C_1 + C_2 am_q$$

- a/r_c is linear in am_q for small enough am_q:

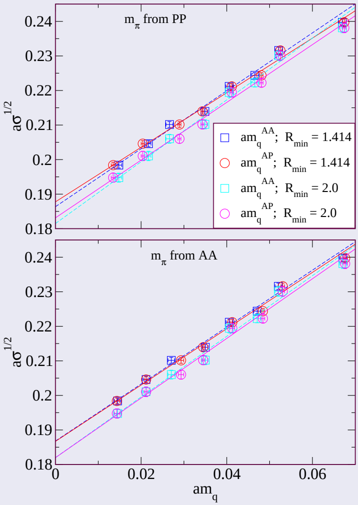
$$a/r_c = A_c + B_c am_q$$

- The qualitative content of the above conclusions does NOT change with any sensible change of the parameters of the analysis like T_{min}, T_{max}, R_{min}, R_{max} and the smearing level

Check with a different
$R_{min} = 2.0$

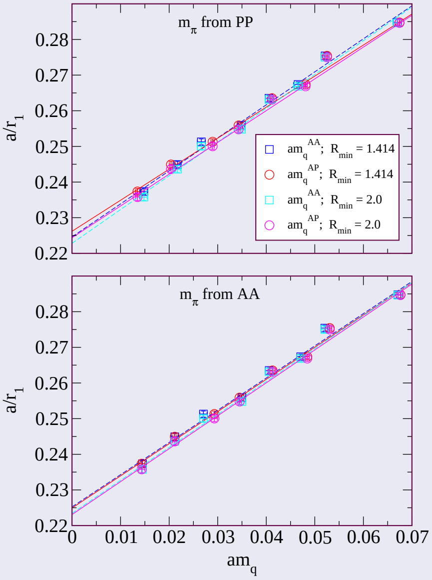


$\beta = 5.6$



$\beta = 5.6$

- a/r_c is relatively independent of the choice of R_{min}
- For our final analysis, we settled for APE smearing level = 30 (for the lightest 3 quark masses) and 25 for the rest of the quark masses
  $[T_{min}, T_{max}] = [3, 4]$
  $[R_{min}, R_{max}] = [\sqrt{2}, 3\sqrt{5}]$



We interpret our results **at fixed β**

for small $am_q \lesssim 0.035$:

- α independent of am_q
- $a\sigma^{1/2} = C_1 + C_2 am_q$
- $a/r_c = A_c + B_c am_q$

to be a **physical dependence** of $\sigma^{1/2}$ and $1/r_c$ on m_q:

- $\sigma^{1/2} = \mathscr{C}_1 + C_2 m_q$ with $C_1 = a\mathscr{C}_1$
- $1/r_c = \mathscr{A}_c + B_c m_q$ with $A_c = a\mathscr{A}_c$

In other words, for data points with small enough am_q, **at fixed β**, the scale is taken to be the same for all quark masses $\Rightarrow$ a mass-independent scheme and a valid linear chiral extrapolation of a/r_c and $a\sigma^{1/2}$ in the small am_q region

For chiral extrapolation of a/r_c to the physical point, use dependence on $(am_\pi)^2$, instead of $(r_c m_\pi)^2$ or $(m_\pi/m_\rho)^2$:
$a/r_c = P_c + Q_c(am_\pi)^2$

Obtain the scale a by solving the quadratic equation in a:

$$\frac{a}{r_c^{\rm Ph}} = P_c + Q_c(am_\pi^{\rm Ph})^2$$

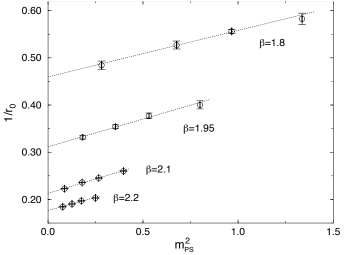
where $r_c^{\rm Ph}$ and $am_\pi^{\rm Ph}$ are the physical values in physical units

Check chiral limits with am_q and $(am_\pi)^2$ extrapolations:

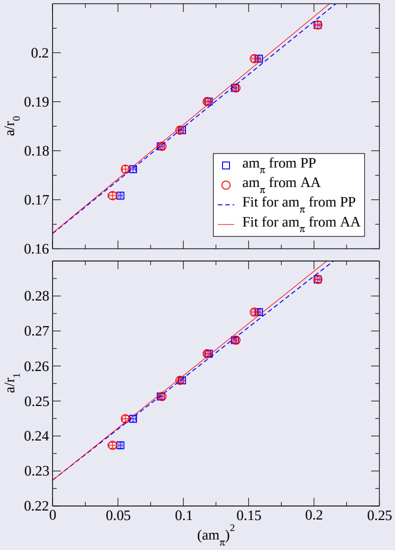
Extrapolation	**Chiral limit of a/r_0**			
	am_π **from PP**		am_π **from AA**	
	am_q^{AA}	am_q^{AP}	am_q^{AA}	am_q^{AP}
am_q	**0.1616(13)**	**0.1627(10)**	**0.1620(13)**	**0.1618(12)**
$(am_\pi)^2$	**0.1631(16)**		**0.1632(16)**	

Our fit (right) drops the lowest $(am_\pi)^2$ points for FS effects and also drops some of the largest mass points for possible scaling violation

Below is shown a similar plot from CPPACS (2002)



$\beta = 5.6$



The scale a and a^{-1}

	am_π from PP		am_π from AA	
	a (fm)	a^{-1} (GeV)	a (fm)	a^{-1} (GeV)
a/r_0 fit	**0.08027(77)**	**2.458(23)**	**0.08032(76)**	**2.457(23)**
a/r_1 fit	**0.08053(70)**	**2.450(21)**	**0.08053(71)**	**2.450(22)**

Contrast that with our own fully hadronic scale determination from linear dependence of am_ρ on $(am_\pi)^2$: $am_\rho = F_1 + F_2(am_\pi)^2$

am_ρ fit	0.07932(135)	2.488(41)	0.07995(195)	2.468(60)

Concluding Remarks

The Problem

- All lattice actions (including all improved gauge and fermion actions) have shown the Sommer parameter in lattice units (r_c/a) depends on the sea quark mass in lattice units (am_q)
- All of this dependence then cannot be a scaling violation (positive power of the scale a). It must partly be a physical effect (see McNeile and Bernard et al, Lattice 2007)
- How to get rid of the scale-violating part?

Our simulation

- Our approach was to take Wilson action with $\mathcal{O}(a)$ effects and investigate the quark mass dependence at as many small enough quark masses as possible (8 values ~ 0.014 to 0.07) at a small enough lattice spacing (0.08 fm)

Importance of α

- Essential to determine α, the coeff of the $1/R$ term as carefully as possible. Its behavior with respect to changes of smearing level and R_{min} should come out as expected.

Our Interpretation

- The dimensionless α being independent of am_q for small am_q is interpreted as a signal for getting rid of the scale-violating region.
- For the same range of am_q, $a\sigma^{1/2}$ and a/r_c are both linear in am_q. This is interpreted as physical linear m_q dependence of $\sigma^{1/2}$ and $1/r_C$, all in physical units.
- For our β (=5.6), this region of quark mass is approximately $m_q < 85$ MeV

Chiral Extrapolation

- With the basic premises set, accurate chiral extrapolation needed to determine the lattice scale
  - Have used $(am_\pi)^2$ for extrapolation to the physical point
  - Only linear extrapolation in $(am_\pi)^2$ is done only for small masses (generally consistent with $am_q < 0.035$). Larger masses show deviation from linear behavior and in our experience these are scaling violations and should NOT be included in the fit.
  - Have checked the chiral limit with am_q extrapolation
  - Extrapolations with $(r_c m_\pi)^2$ and $(m_\pi/m_\rho)^2$ are better avoided. Introduce uncertainty and inaccuracy.
  - The whole procedure is testable with larger volumes and smaller quark masses (simulations underway with DDHMC)