Epsilon regime calculations with reweighted clover fermions

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Dynamical simulations with light quarks are (still) difficult:

- Expensive
- Large autocorrelation
- Stability problems with Wilson-type fermions
- Configurations with small eigenmodes give the largest contribution to correlators, yet they are infrequently sampled

Reweighting in the quark mass can help

Reweighting in the quark mass

Generate configurations with mass m_1 and reweight it to $m_2 < m_1$ by assigning a weight factor

$$w_i \propto \det \frac{\mathrm{D}_2^\dagger[\mathrm{U_i}]\mathrm{D}_2[\mathrm{U_i}]}{\mathrm{D}_1^\dagger[\mathrm{U_i}]\mathrm{D}_1[\mathrm{U_i}]} = e^{-\Delta S_f}$$

to each congfiguration.
Calculate expectation values as

$$\langle O \rangle_2 = \frac{\sum_i w_i O[U_i]}{\sum_i w_i}$$

Reweighting helps as the heavy mass controls

- Computational expense
- Autocorrelation
- Algorithmic stability
- Largest contributions to the correlators are over-sampled and reweighted

General belief:

Reweighting cannot work

- $w \propto e^{-\Delta S}$, $\Delta S \propto$ Volume
- not enough overlap between generated and desired configurations
- weight is difficult to calculate

Reweighting works

in a wide mass range and volumes

- Only the fluctuations of ΔS matter
  $\delta(\Delta S) \sim V^{\alpha}$ and α is small

- Most of the UV fluctuations can be absorbed
  by a pure gauge action term

- $\Delta S \propto (m_1 - m_2)$: reweight at small masses

- There is an inverse correlation between correlators and weights

- Calculating the weight stochastically is fast
  and does not introduce systematic errors

Stochastic estimator

$$w \propto \det A = \langle e^{-\xi^\dagger (A-1)\xi} \rangle_\xi$$

Take $\langle ... \rangle_\xi$ together with configuration average
$\longrightarrow$ no systematic error from the weight factor

To reduce statistical fluctuations of w
★ separate low eigenmodes
★ determinant breakup
★ UV subtraction (absorb as pure gauge term in action)

Cost: $\approx$30-40 D^{-1} per 5MeV reduction in quark mass

Examples of reweighted ensembles:

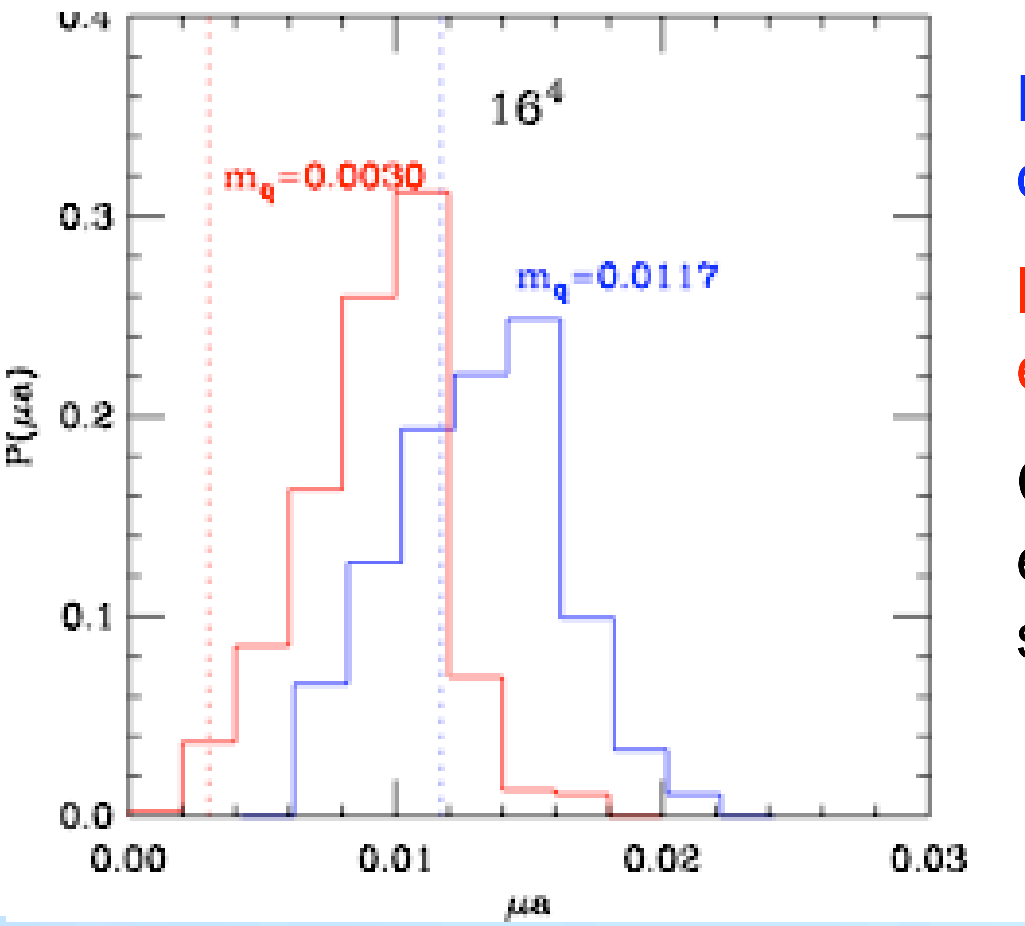
Original simulations: $n_f = 2$ flavor nHYP smeared tree level improved Wilson fermions with $a \sim 0.12$fm

- 16^4 , $La \sim 1.85$fm, $m_q = 20$MeV $\rightarrow$ 5MeV
- 24^4 , $La \sim 2.7$fm, $m_q = 8$MeV $\rightarrow$ 4MeV
- $16^3 \times 32$, $La \sim 2.0$fm, $m_q = 30$MeV $\rightarrow$ 5MeV

Any in-between quark mass is automatically available
The lowest possible quark mass is limited by the spread of the Dirac operator eigenmodes

Reweighting:

Distribution of the Hermitian gap



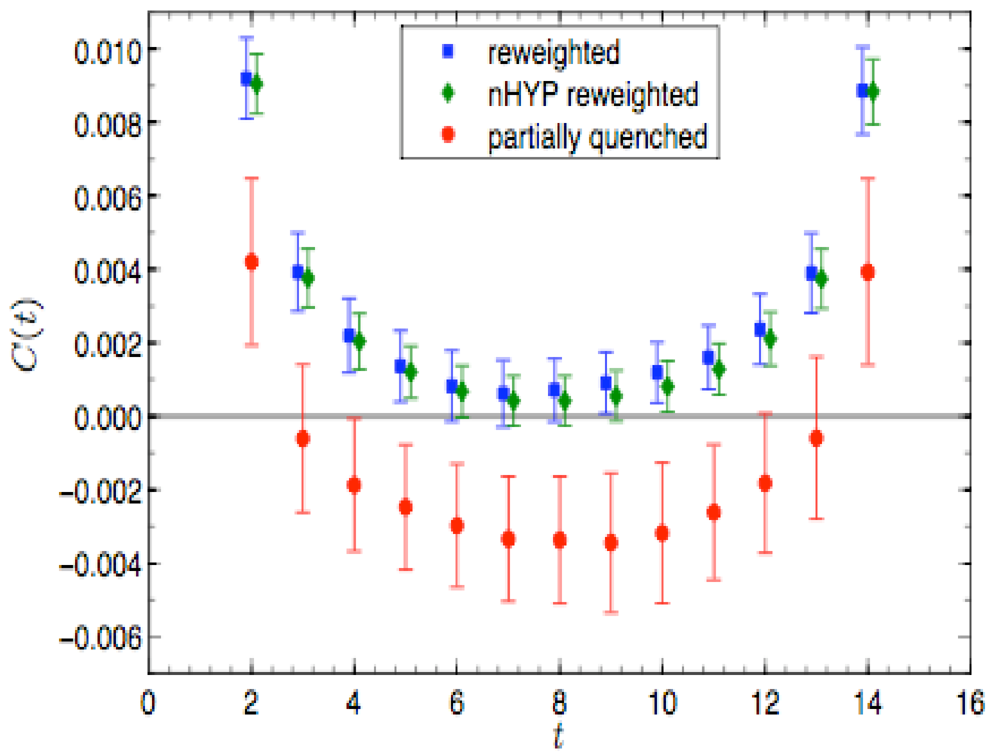
Lowest eigenmode on original (20 MeV) and

lightest reweighted (5MeV) ensembles

Configurations with small eigenmodes are suppressed

Reweighting:

The scalar correlator



sea quarks: 20MeV

valence quarks: 10MeV

- Reweighted correlator stays positive
- Statistical errors are reduced wrt partial quenched

Epsilon regime with Wilson fermions

finite volume region requires

$$\left.\begin{array}{l} \bullet \text{ light quarks : } m_\pi L \ll 1 \\ \bullet \text{ large volume : } FL \gg 1 \end{array}\right\} \; m\Sigma V = \mathcal{O}(1)$$

At NLO-χPT the 2-point functions are parabolic in time depend only on Σ and F. The other low energy constants enter only at the next order.

- Pseudo scalar correlator

$$G_{PP} = \Sigma^2 \Big(a_p + \frac{b_P}{(FL)^2} h_1(t/L) + \mathcal{O}(\frac{1}{(FL)^4}) \Big)$$

- Axial vector correlator

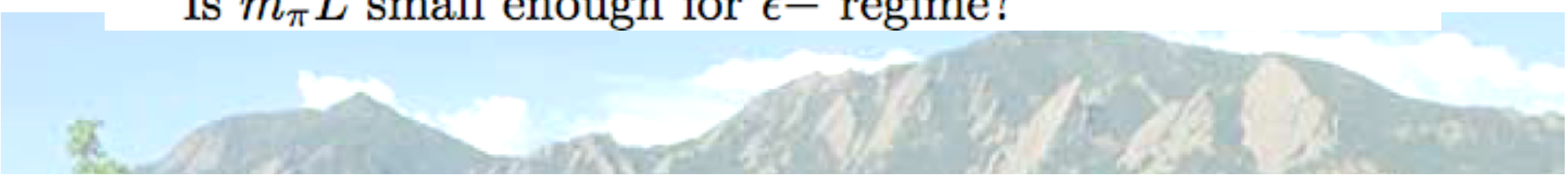
$$G_{AA} = \frac{F^2}{V} \Big(a_A + \frac{b_A}{(FL)^2} h_1(t/L) + \mathcal{O}(\frac{1}{(FL)^4}) \Big)$$

- The expansion is in terms of $1/(FL)^2$
  $F = 86\text{MeV}, \;\; L = 1.85 \text{ fm} \longrightarrow 1/(FL)^2 \sim 1.6$
  $F = 86\text{MeV}, \;\; L = 2.70 \text{ fm} \longrightarrow 1/(FL)^2 \sim 0.7$

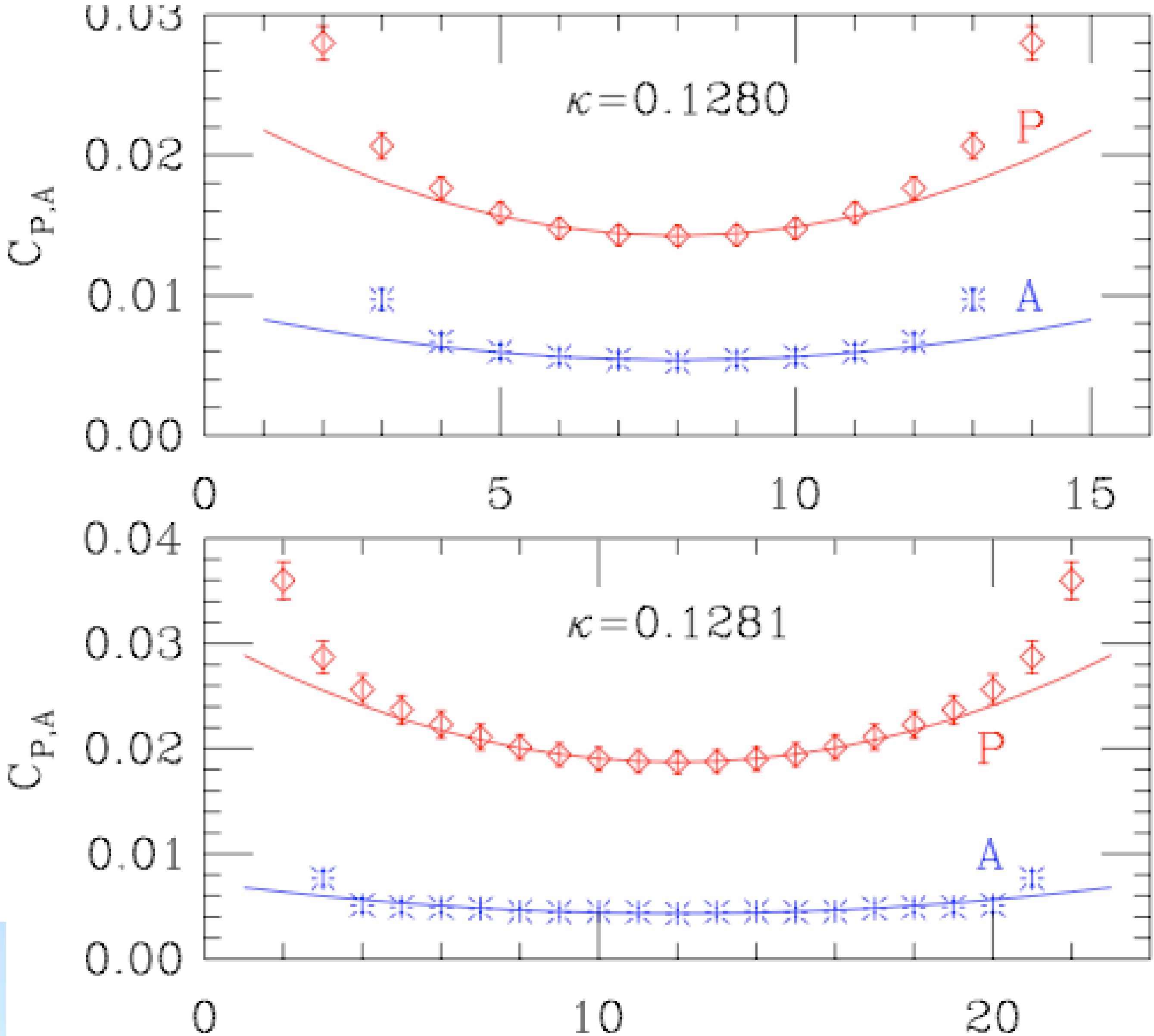
Parameters of the simulations

κ	κ_{rew}	L	N_{conf}	$a\, m_{\mathrm{PCAC}}$	m[MeV]
0.1278	0.1278	16	180	0.0117(3)	22
	0.1279	16	180	0.0088(5)	16.5
	0.1280	16	180	0.0058(7)	11
	0.12805	16	180	0.0047(8)	9
	0.1281	16	180	0.0028(11)	5
0.12805	0.12805	24	154	0.0044(3)	8.5
	0.12810	24	154	0.0030(3)	5.8
	0.128125	24	154	0.0024(3)	4.2
	0.12815	24	154	0.0019(4)	3.8

Is $m_\pi L$ small enough for $\epsilon-$ regime?



Combined fit to PP and AA correlators



16^4

10MeV quarks

t : 5-11

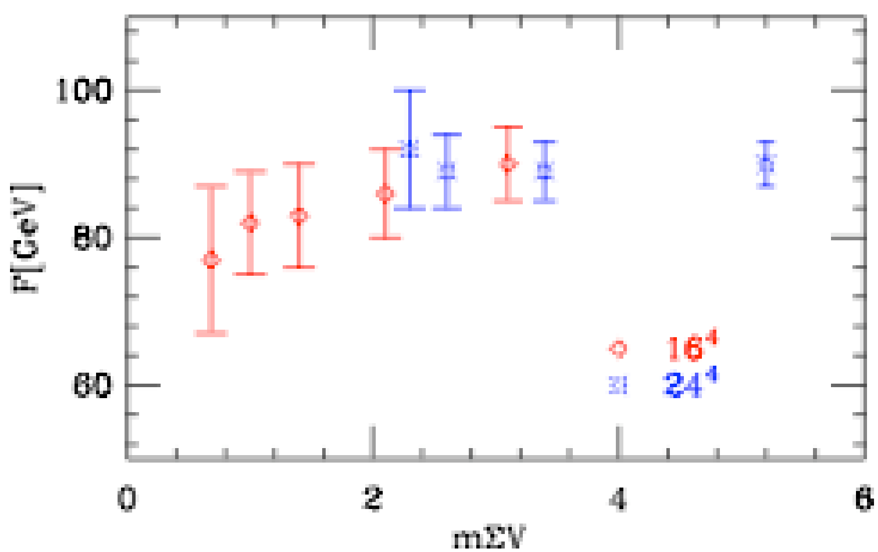
24^4

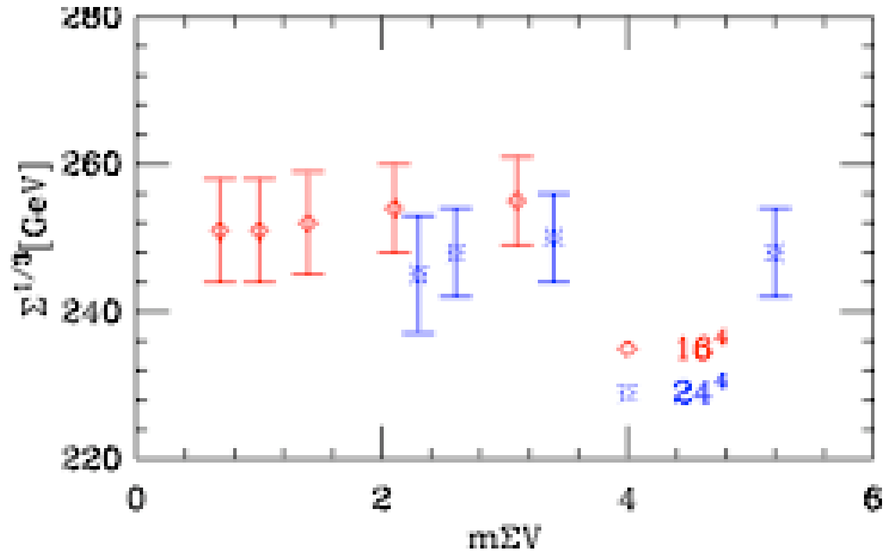
6MeV quarks

t : 8-16

Result for the low energy constants

($\overline{\text{MS}}$ at 2GeV)





(using $r_0 = 0.49\text{fm}$ and RI-MOM $Z_A = 0.99$, $Z_P = 0.9$)

Observable finite volume effects for F; Σ is stable.

Summary

- Reweighting in the quark mass is an effective method to reach small quark masses with Wilson fermions
  - Avoids long autocorrelation
  - Improves importance sampling
  - Stable algorithm
  - In most cases statistics is improved wrt partial quenched studies

- epsilon regime is within reach even on large volumes

  $F = 90(4)\ \mathrm{MeV},\ \Sigma^{1/3} = 248(6)\mathrm{MeV}$

- We find similar behavior in p-regime calculations as well