Spectrum of closed k-strings in $D=2+1$[*]

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Overview

1. **Introduction**
2. **Nambu-Goto Effective String Theory**
3. **Lattice Calculation**
4. **Results**
5. **Conclusions**
6. **In Progress:** $D = 3 + 1$

1. Introduction

◇ What effective string theory describes the confining flux-tube?

◇ If source transforms as $\psi(x) \longrightarrow z^k \psi(x)$, $z \in Z_N \Longrightarrow k$-string.

◇ k-strings

– AdS/CFT, MQCD. (Strassler, Armoni, Shiffman...)

– Hamiltonian approach. (Karabali, Nair, ...)

– **Lattice**.

◇ Recently:

– $k = 1$ string (fundamental string) is very well approximated by Nambu-Goto (+moderate corrections).

– k-string ground state: in the same universality class as N.G.

◇ The excitation spectrum still remains unexplored:

$\longrightarrow k = 2$ excitation spectrum.

2. Nambu-Goto Effective String Theory: Spectrum

The Spectrum of the Nambu-Goto (NG) String Model

- Spectrum given by:

$$E^2_{N_L,N_R,q,w} = (\sigma l w)^2 + 8\pi\sigma \left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 .$$

- Described by:
  1. The winding number w.
  2. The winding momentum $p_{||} = 2\pi q/l$ with $q = 0, \pm 1, \pm 2, \ldots$
  3. $N_L = \sum_{k>0} \sum_{n_L(k)>0} n_L(k)k$ and $N_R = \sum_{k'>0} \sum_{n_R(k')>0} n_R(k')k'$ connected through the relation: $N_R - N_L = qw$.

- String states can be characterised by irreducible representations of $\mathbf{SO(D-2)}$. In 2+1 dimensions this can be translated to Parity with eigenvalues:

$$P = (-1)^{\text{number of phonons}}.$$

2. Nambu-Goto effective string theory: Corrections

◇ Lüscher&Weisz(04) effective string action:

(Drummond '04, Dass and Matlock '06 for any D.)

$$E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{24}\right) - \frac{8\pi^2}{\sigma l^3}\left(n - \frac{1}{24}\right)^2 + \mathcal{O}\left(1/l^4\right).$$

◇ Equivalently:

$$E_n^2 = (\sigma l)^2 + 8\pi\sigma\left(n - \frac{1}{24}\right) + \mathcal{O}\left(1/l^3\right).$$

◇ Fitting Ansatz:

$$E_{\rm fit}^2 = E_{NG}^2(q=0) - \sigma\frac{C_p}{(l\sqrt{\sigma})^p} \qquad (p \geq 3).$$

3. Lattice Calculation

◇ A lot of effort has been invested in: 3D, 4D cases of Z_2, Z_4, $U(1)$, $SU(N \leq 8)$

(Caselle and collaborators, Gliozzi and collaborators, Kuti and collaborators, Lüscher&Weisz, Majumdar and collaborators, Teper and collaborators, Meyer)

◇ We study **closed flux tubes**.

◇ Our approach:

- Create a **large basis of operators** **(**$\sim 80 - 250$**)**.
- Calculate the correlation matrix $C_{ij,p,\pm}(t) = \langle \Phi^{\dagger}_{i,p,\pm}(t) \, \Phi_{j,p,\pm}(0) \rangle \rangle$.
- Use the **variational technique**.

◇ We define our theory on a 3D Euclidean lattice with $L \times L_{\perp} \times L_T$ sites.

◇ Monte Carlo simulations for $N = 4, 5$ and $\beta = 50.00$ and 80.00.

3. Lattice Calculation

◇ Operators: If U is a Polyakov loop then:

– For $k = 1$, $\phi \equiv \bigcirc \equiv \mathrm{Tr}\{U\}$

– For $k = 2$, $\phi_1 \equiv \circledcirc \equiv \mathrm{Tr}\{U^2\}$, $\phi_2 \equiv \bigcirc\!\!\!\bigcirc \equiv \mathrm{Tr}\{U\}\mathrm{Tr}\{U\}$

◇ Now, we are interested in excited states ($N_R \neq 0$, $N_L \neq 0$, $q \neq 0$)

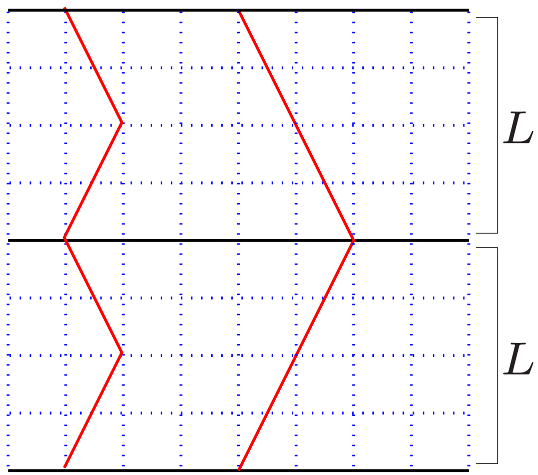
◇ Example: operators with tranverse deformations for $P = \pm$:

1st: $\phi = \mathrm{Tr}\ \{\quad\}^2 \pm \mathrm{Tr}\{\quad\}^2$

2nd: $\phi = \mathrm{Tr}\ \{\quad \cdot \quad\} \pm \mathrm{Tr}\{\quad \cdot \quad\}$

3rd Additionally ($w = 2$):

$\phi = \mathrm{Tr}\ \{\quad\} \pm \mathrm{Tr}\{\quad\}$



3. Lattice Calculation

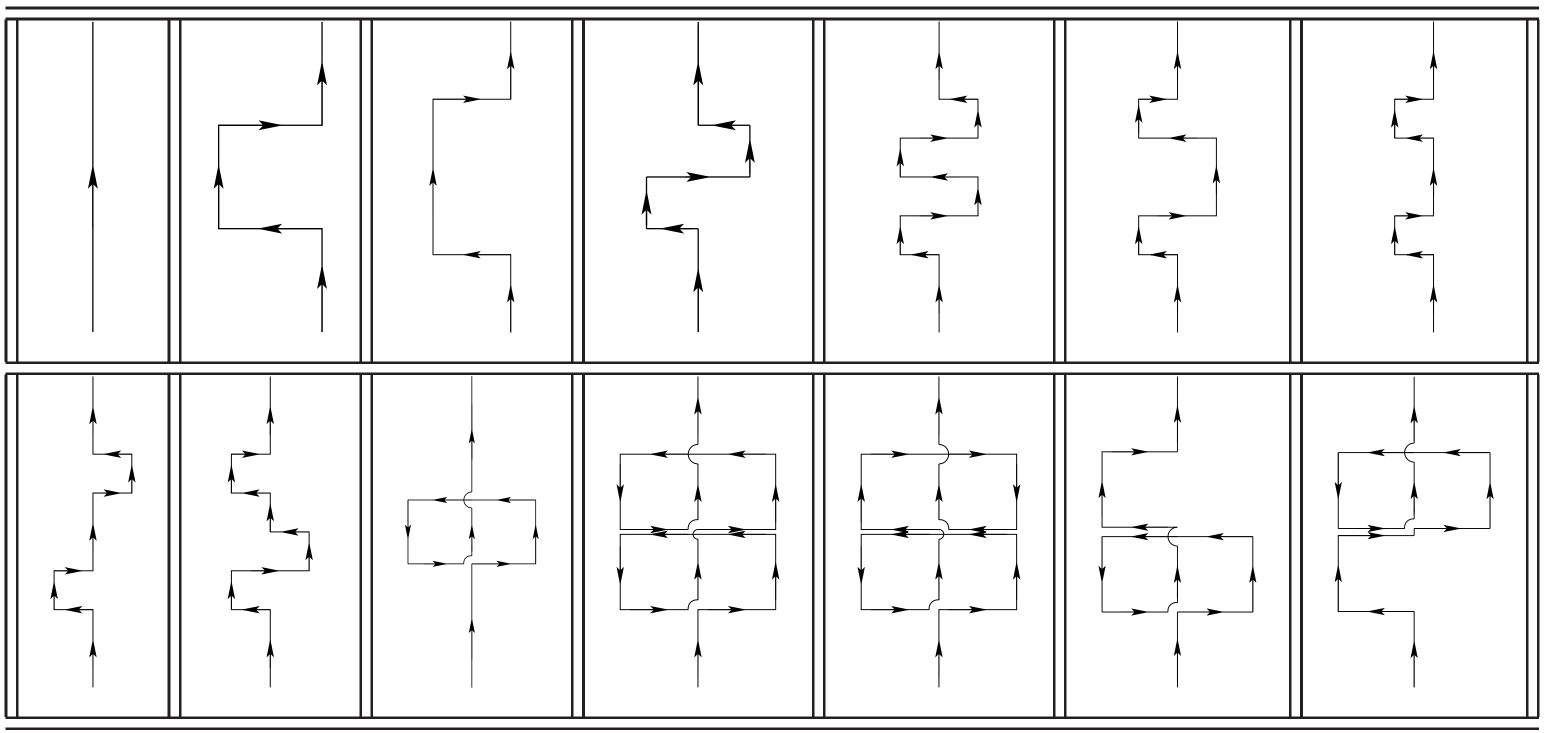
- Projection onto the Antisymmetric representation:

$$\phi = [\mathrm{Tr}\ \{\quad\}^2 - \mathrm{Tr}\ \{\quad \cdot \quad\}] \pm [\mathrm{Tr}\{\quad\}^2 - \mathrm{Tr}\{\quad \cdot \quad\}]$$

- Projection onto the Symmetric representation:

$$\phi = [\mathrm{Tr}\ \{\quad\}^2 + \mathrm{Tr}\ \{\quad \cdot \quad\}] \pm [\mathrm{Tr}\{\quad\}^2 + \mathrm{Tr}\{\quad \cdot \quad\}]$$

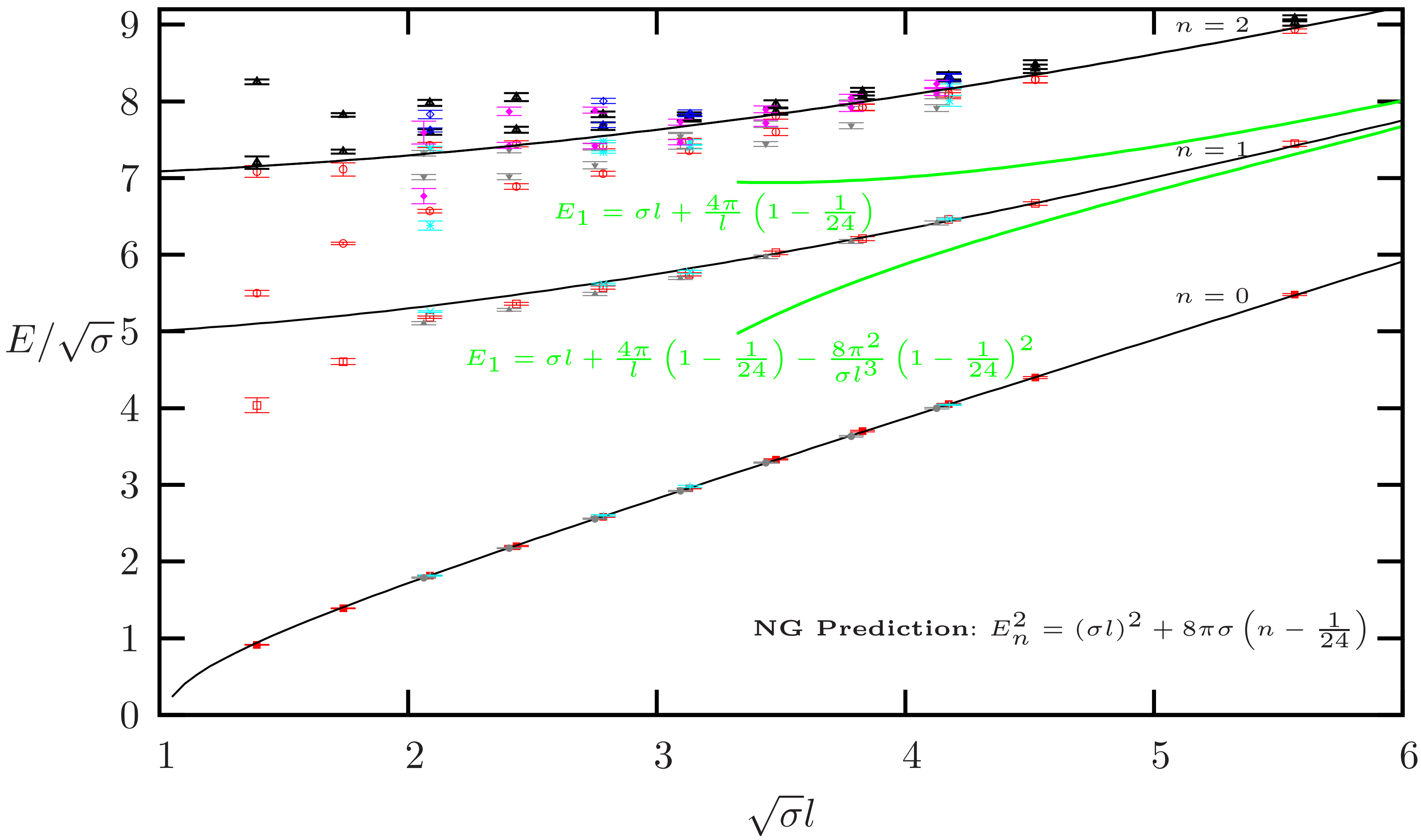
- Polyakov lines ($\times 5$ blocking-smearing levels):



4. Results: Fundamental representation

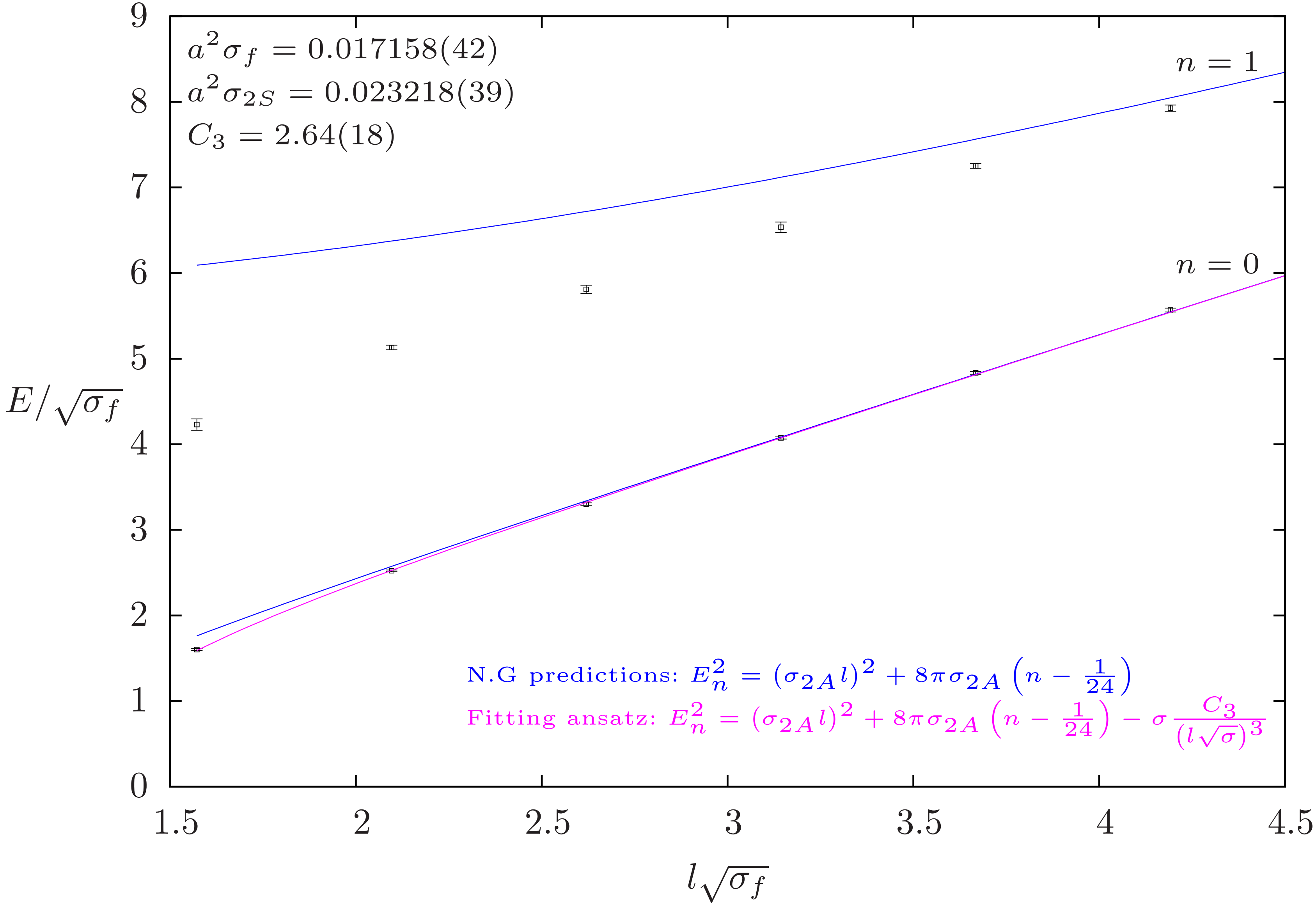
Groups: $SU(3)$ and $SU(6)$, $\underline{a} \simeq 0.04fm$ and $0.08fm$.

Quantum Numbers: $P = \pm$ and $q = 0$



4. Results: Antisymmetric Representation for $P = +$, $q = 0$

Group: $SU(4)$, $\underline{\beta} = 50.00$, $\underline{a} \simeq 0.06 fm$



4. Results: Symmetric Representation for $P = +$, $q = 0$

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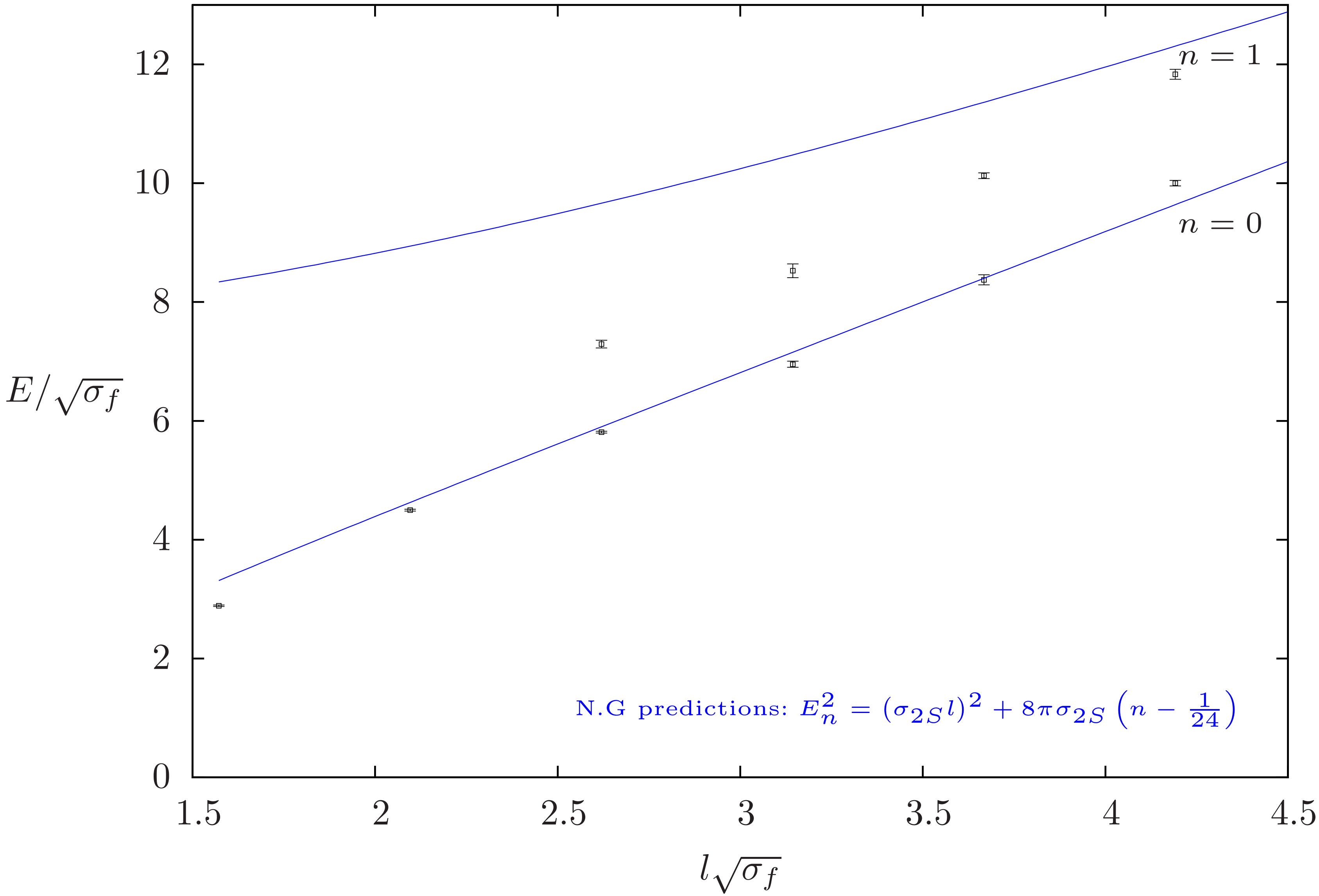
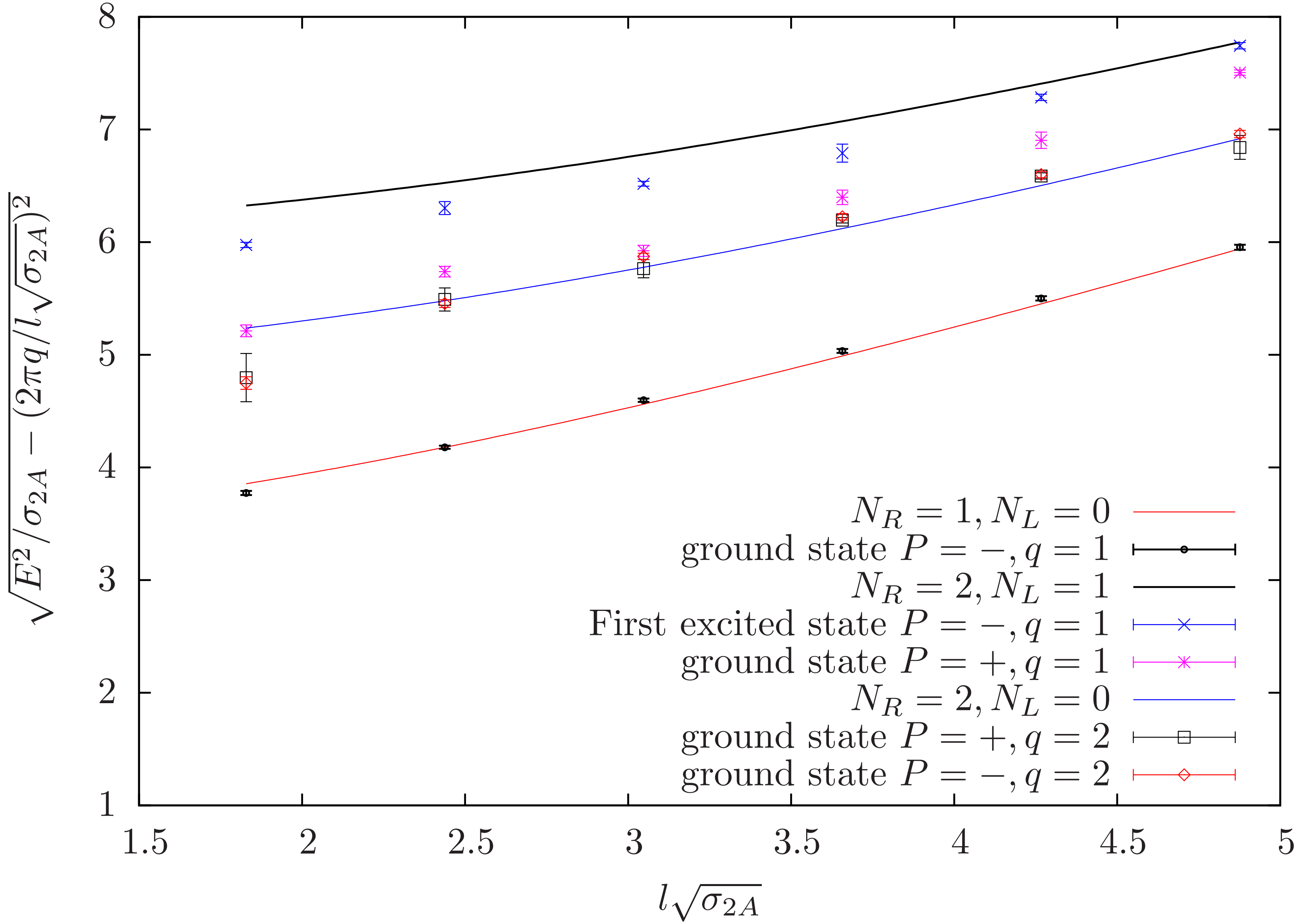


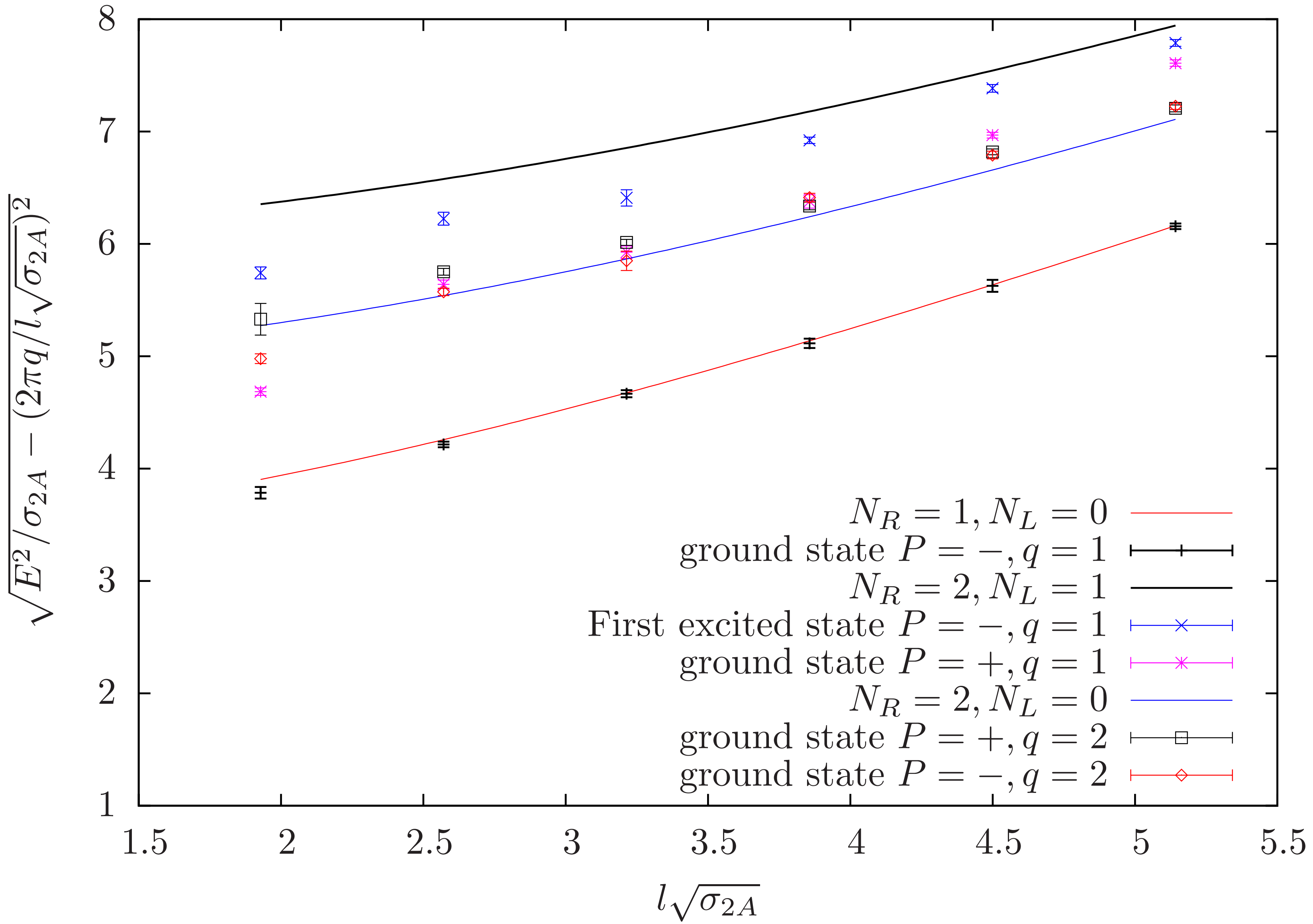
4. Results: Antisymmetric Representation, $q = 1, 2$

Projecting on the antisymmetric representation of $SU(4)$:



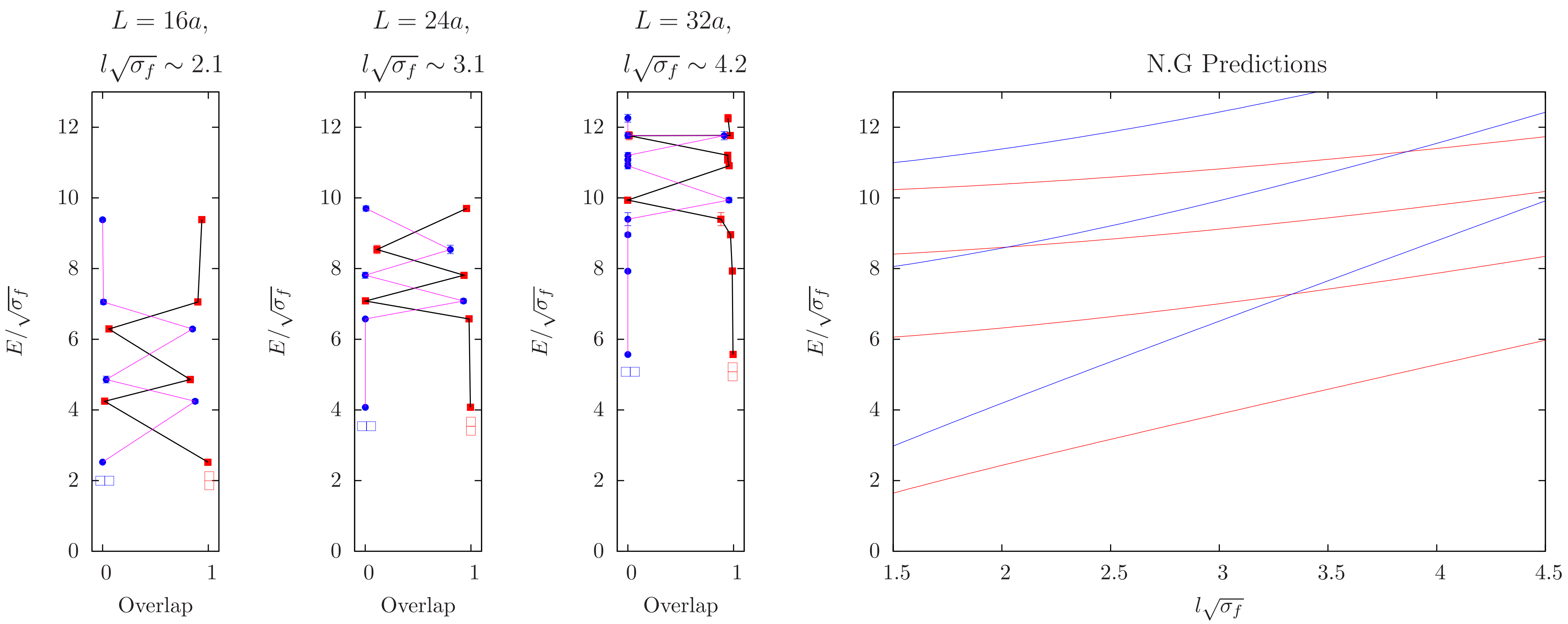
4. Results: Antisymmetric Representation, $q = 1, 2$

Projecting onto the antisymmetric representation of $SU(5)$:

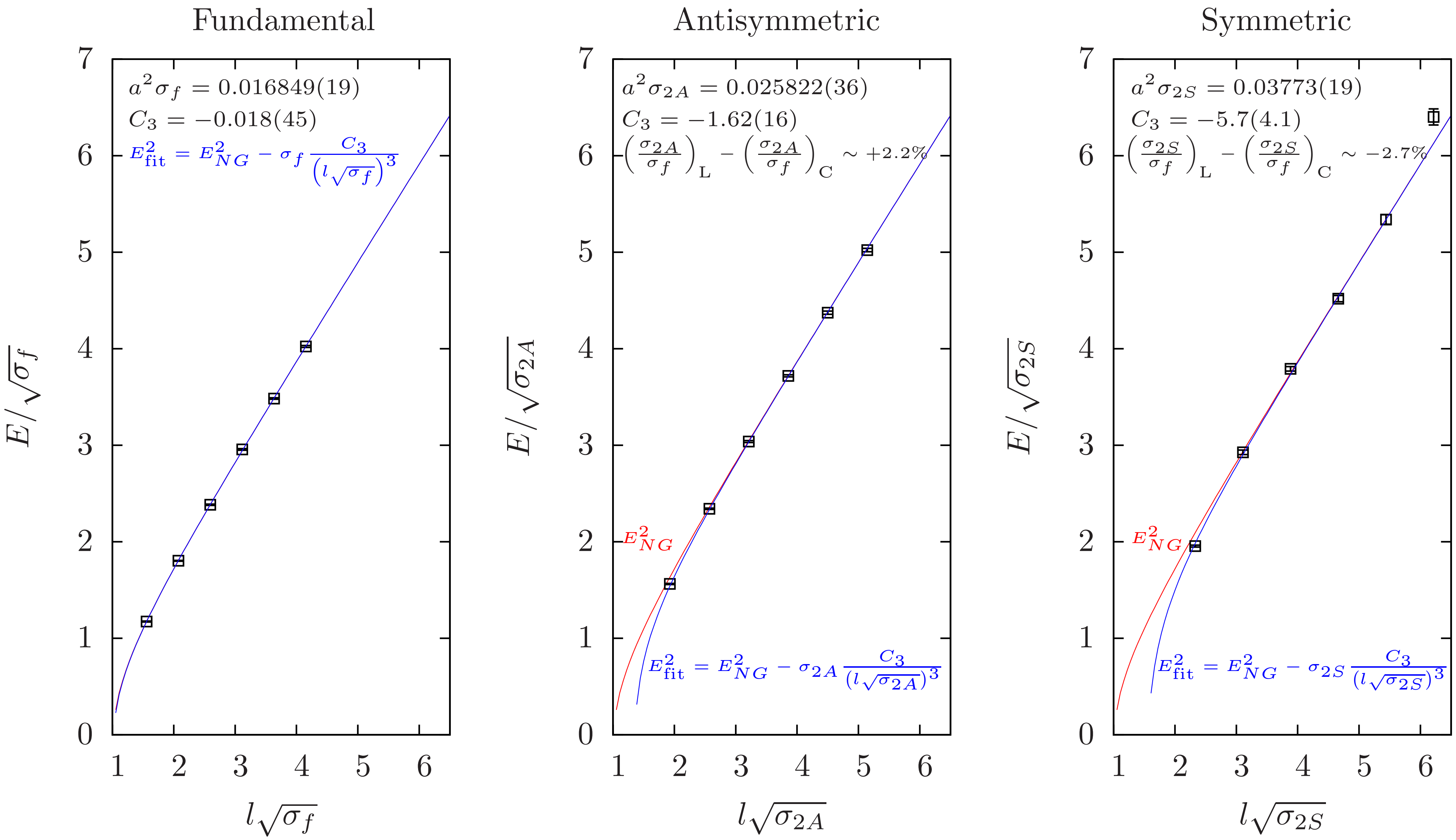


4. Results: $k = 2$ energy towers for $P = +$, $q = 0$, $SU(4)$

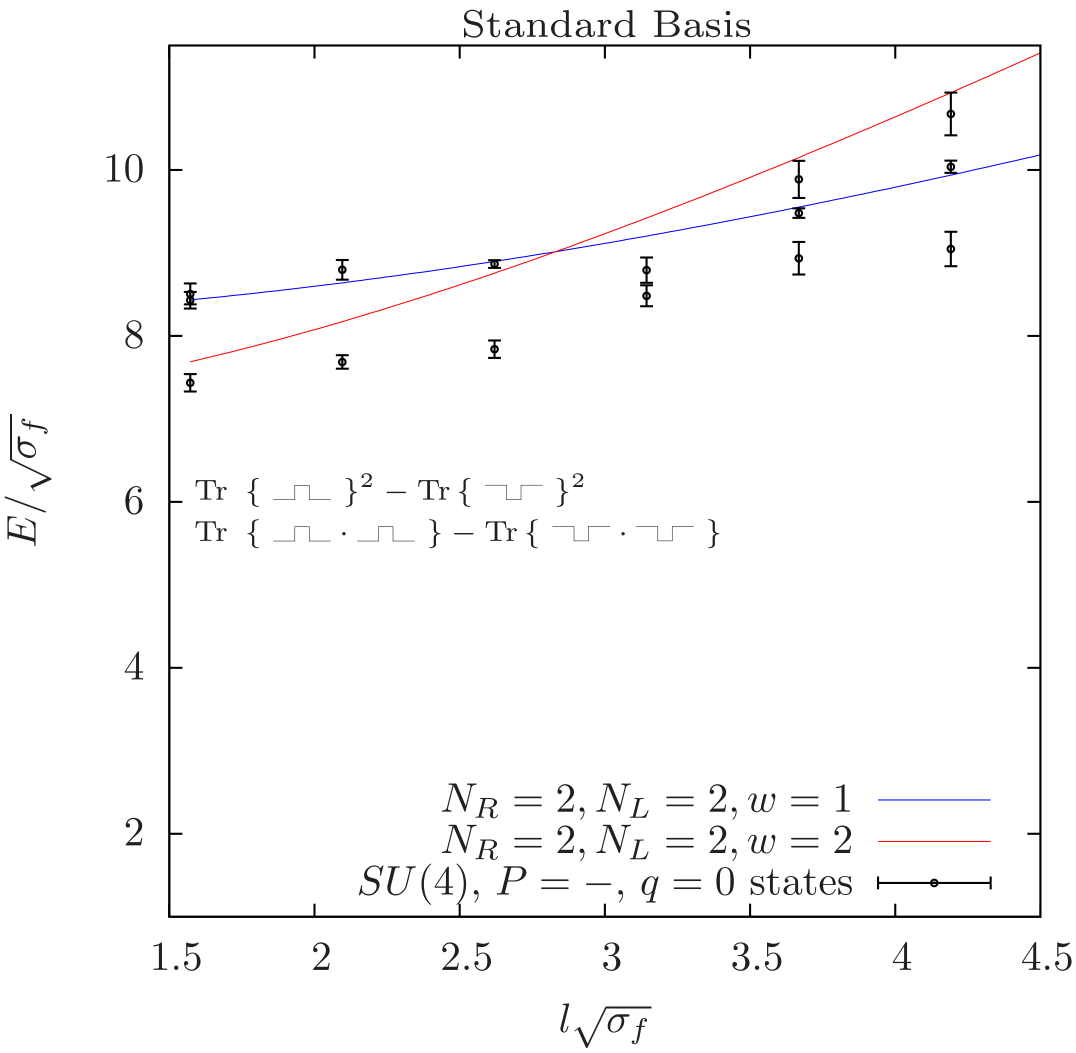
Energy Towers for $P = +$, $q = 0$, $SU(4)$

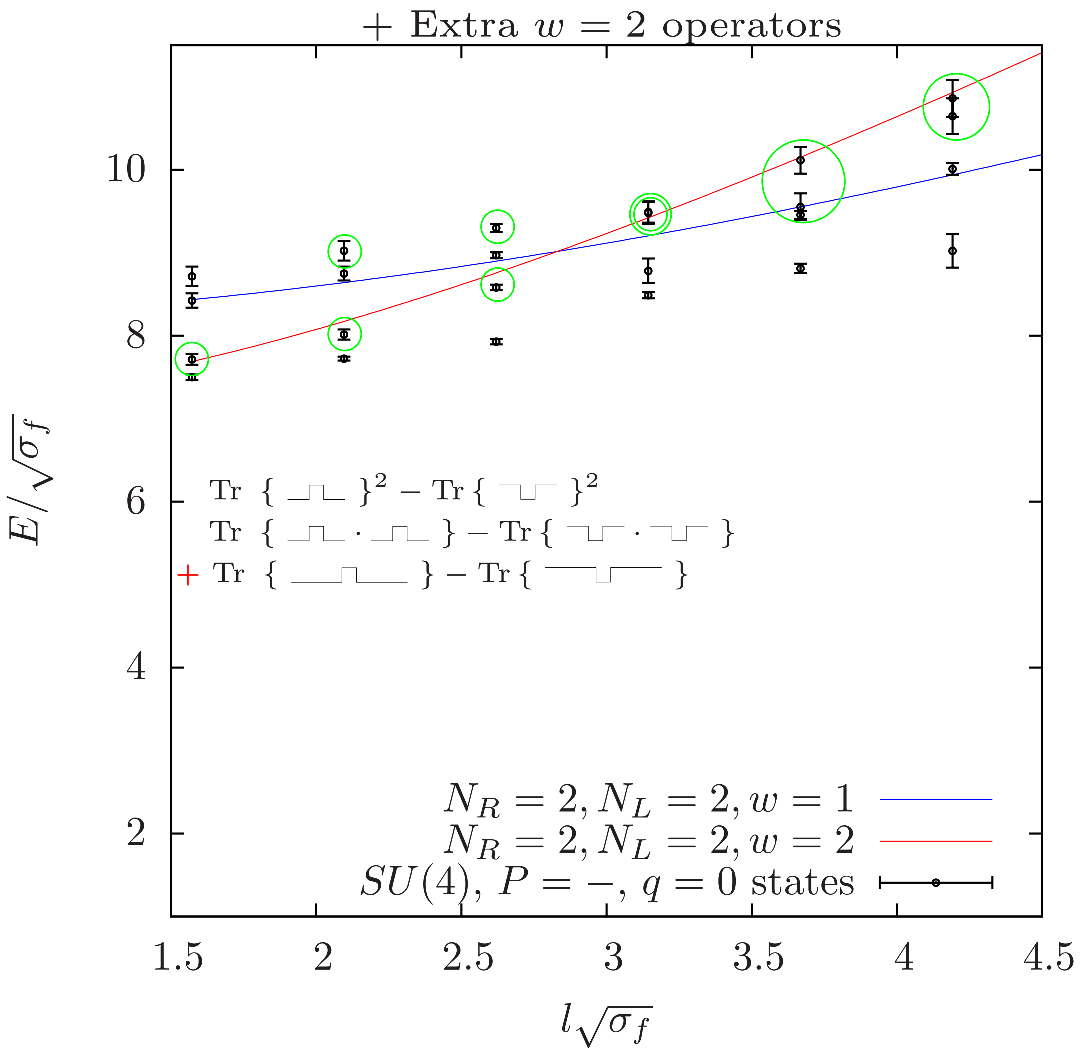


4. Results: Ground States for $P = +$, $q = 0$, $SU(5)$



4. Results: Extra states





1. $w = 2$ N.G states?

2. Massive states that cannot be accommodated in the N.G framework?

5. Conclusions

- Our $k = 2$ spectrum falls into the symmetric and antisymmetric representations

$\Longrightarrow$ k-strings know about the full gauge group and not just about its centre.

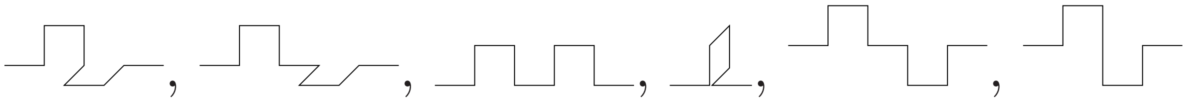
- $k = 2A$ spectrum is clearly well described by Nambu-Goto.

- Qualitative difference between the $k = 1$ and $k = 2$ ground states.

- $k = 2$ spectrum is rich.

6. In Progress: $D = 3 + 1$

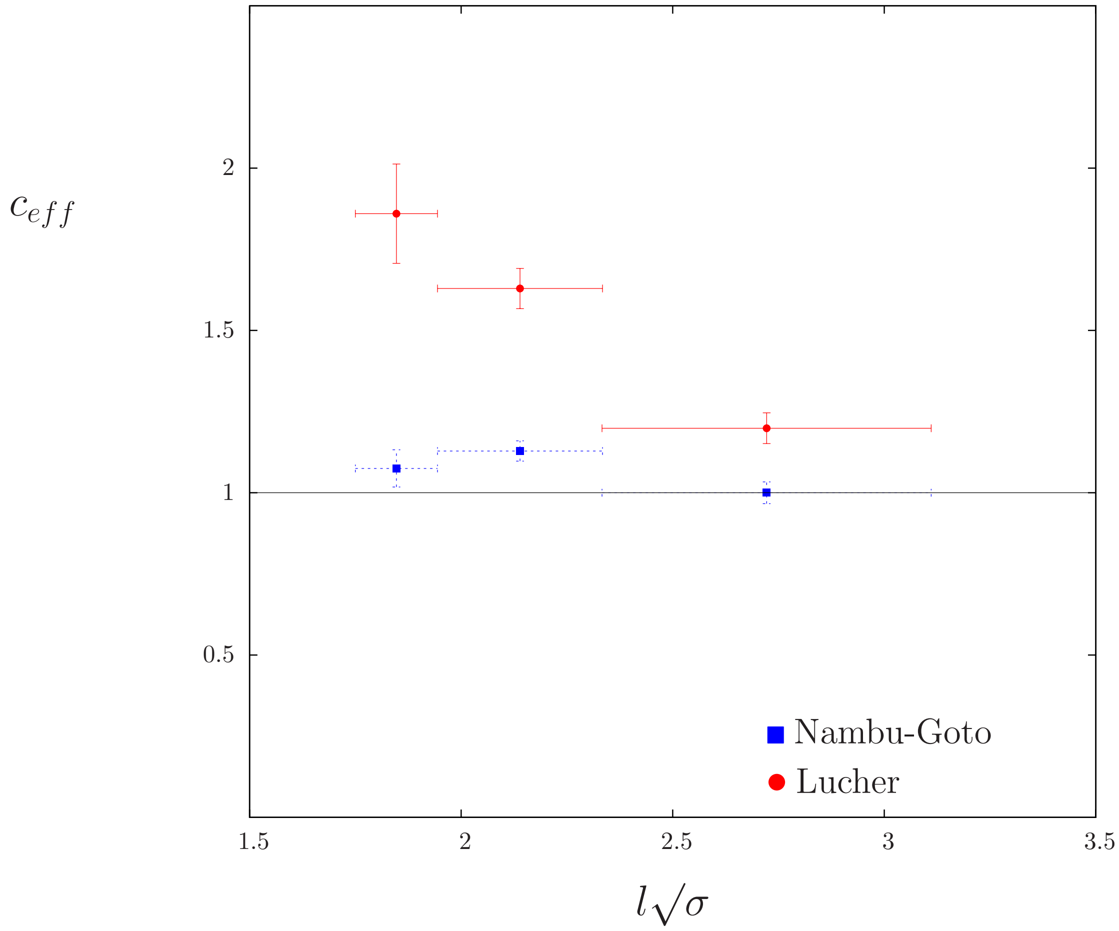
- Interested in the more complicated case of 3+1 dimensions. (calculations are under way).
  - Described by more irreducible representations.
  - Transverse deformations in two directions.

     , ...

  - Quantum numbers of 3D-Parity, and angular momentum.
  - $k = 1$, $k = 2$

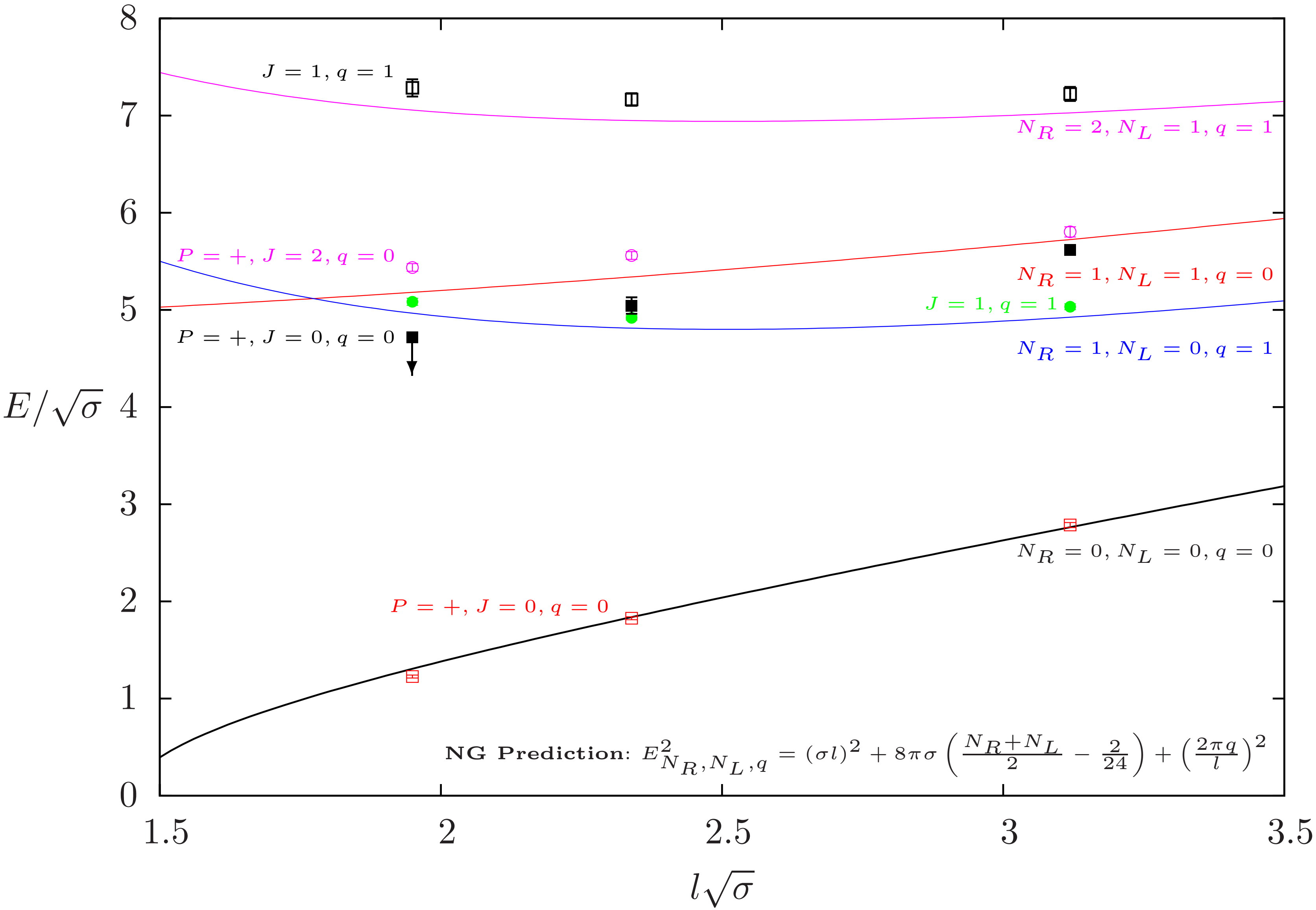
6. In Progress: $D = 3 + 1$

Effective charge for $SU(3)$, D=3+1, $\beta = 6.0625$.



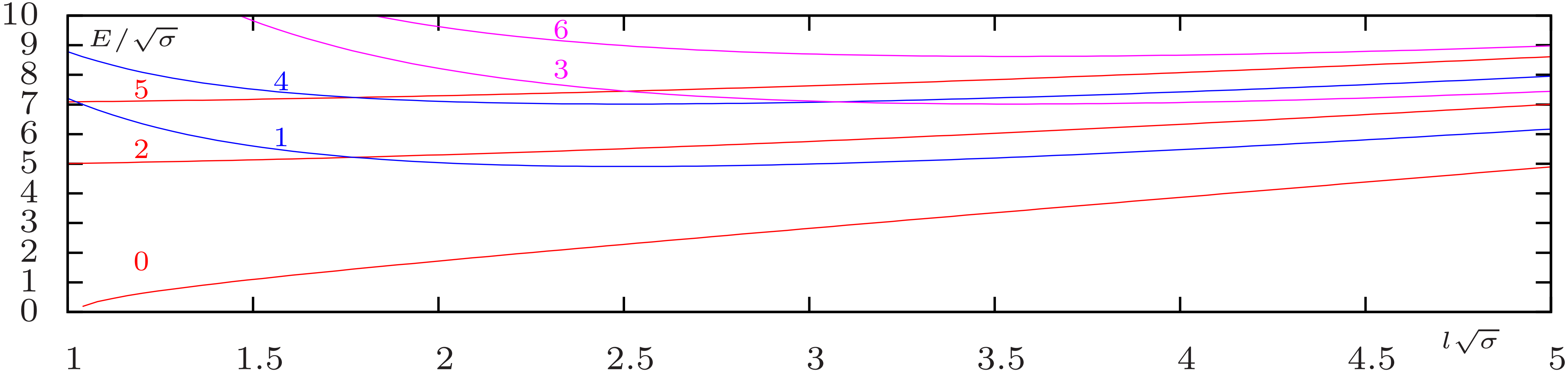
6. In Progress: $D = 3 + 1$

Preliminary results for the spectrum of $SU(3)$, $k = 1$ and $a \simeq 0.1fm$:



7. Appendix: Nambu-Goto States

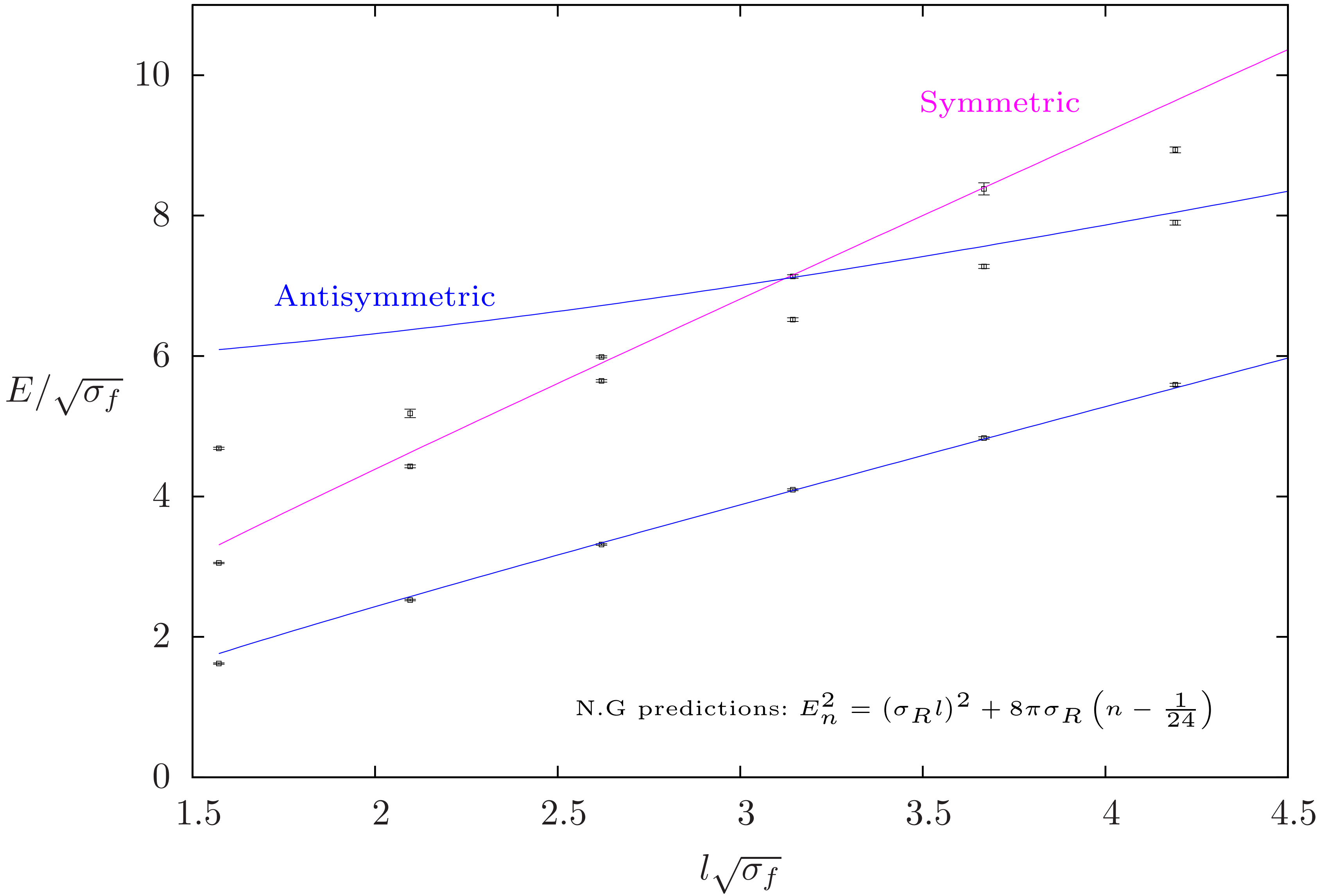
The seven lowest ($q = 0, 1, 2$) NG energy levels for the $w = 1$ closed string



level	N_R	N_L	q	$P=+$	$P=-$
0	0	0	0	$\|0\rangle$	
1	1	0	1		$\alpha_{-1}\|0\rangle$
2	1	1	0	$\alpha_{-1}\bar{\alpha}_{-1}\|0\rangle$	
3	2	0	2	$\alpha_{-1}\alpha_{-1}\|0\rangle$	$\alpha_{-2}\|0\rangle$
4	2	1	1	$\alpha_{-2}\bar{\alpha}_{-1}\|0\rangle$	$\alpha_{-1}\alpha_{-1}\bar{\alpha}_{-1}\|0\rangle$
5	2	2	0	$\alpha_{-2}\bar{\alpha}_{-2}\|0\rangle$, $\alpha_{-1}\alpha_{-1}\bar{\alpha}_{-1}\bar{\alpha}_{-1}\|0\rangle$	$\alpha_{-2}\bar{\alpha}_{-1}\bar{\alpha}_{-1}\|0\rangle$, $\alpha_{-1}\alpha_{-1}\bar{\alpha}_{-2}\|0\rangle$
6	3	1	2	$\alpha_{-3}\bar{\alpha}_{-1}\|0\rangle$, $\alpha_{-1}\alpha_{-1}\alpha_{-1}\bar{\alpha}_{-1}\|0\rangle$	$\alpha_{-2}\alpha_{-1}\bar{\alpha}_{-1}\|0\rangle$

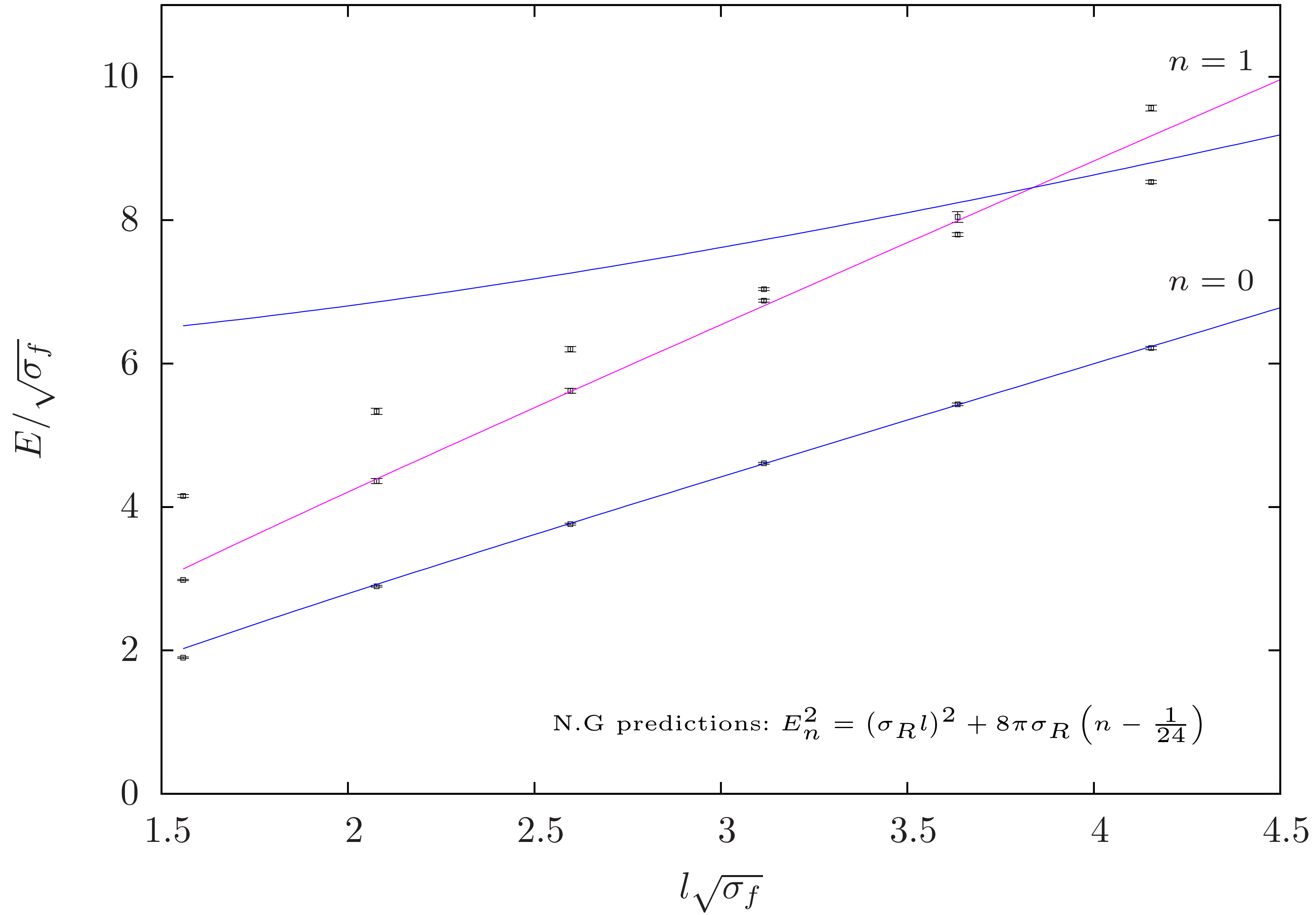
7. Appendix: $k = 2$ Spectrum for $P = +$, $q = 0$

Group: $SU(4)$, $\underline{\beta} = 50.00$, $\underline{a} \simeq 0.06 fm$



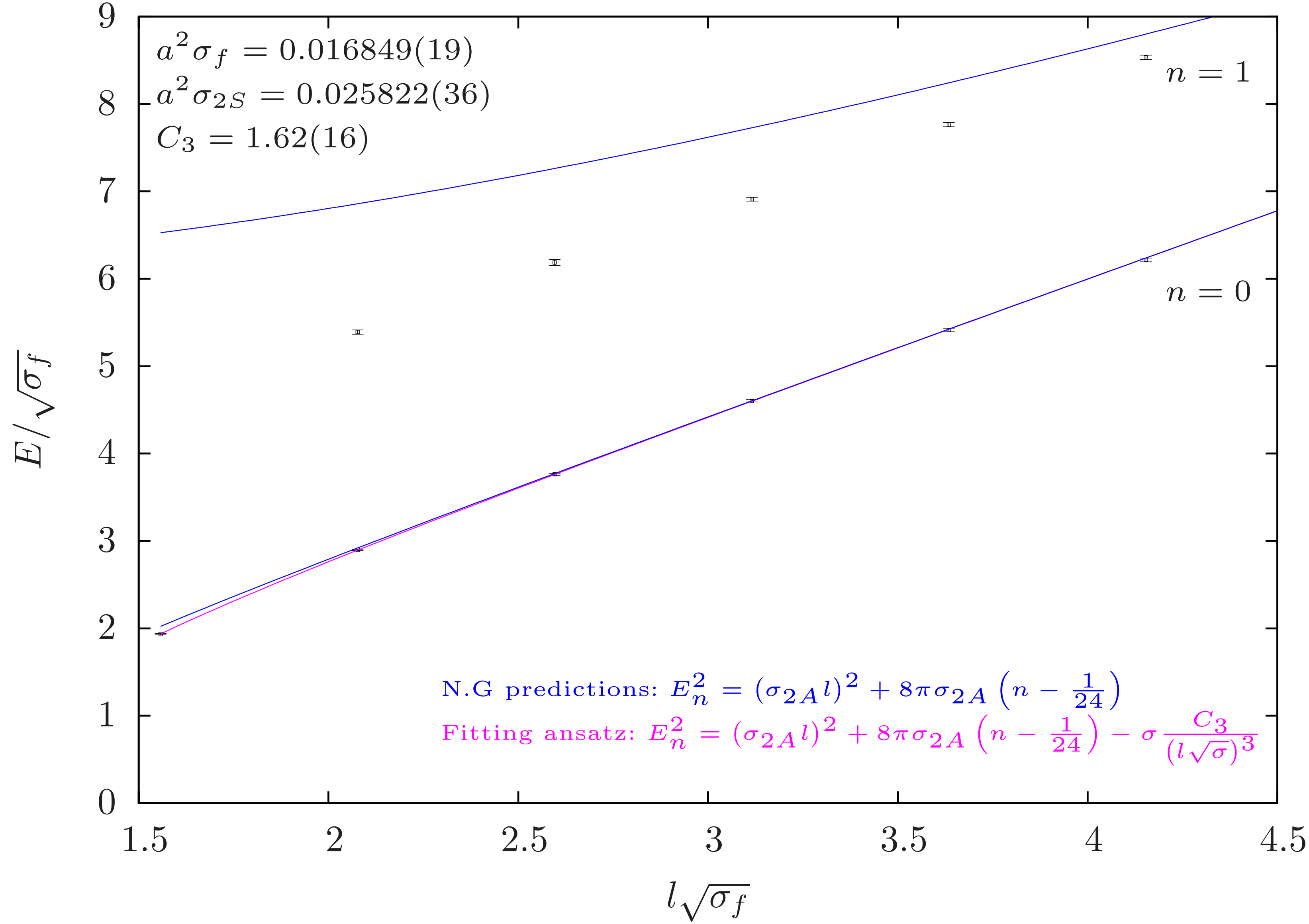
7. Appendix: $k = 2, P = +$, $q = 0$

Group: $SU(5)$, $\beta = 80.00$, $a \simeq 0.06 fm$



7. Appendix: Antisymmetric Representation for $P = +$, $q = 0$

Group: $SU(5)$, $\beta = 80.00$, $a \simeq 0.06 fm$



7. Appendix: Symmetric Representation for $P = +$, $q = 0$

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