Wilson twisted mass in the epsilon regime

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Introduction

- Why ϵ regime? Alternative and complementary to p regime

Wilson (twisted mass) fermions in the ϵ regime (Jansen,Nube,A.S.,Urbach,Wenger:2007)

$$S[\chi,\overline{\chi},U] = S_G[U] + S_F[\chi,\overline{\chi},U]$$

$$S_G[U] = \frac{\beta}{3}\sum_x \left\{ b_0 \sum_{\mu<\nu} \mathbb{R}\mathrm{e}\,\mathrm{Tr}\left[\mathbb{1} - P^{(1\times 1)}(x;\mu,\nu)\right] + b_1 \sum_{\mu\neq\nu} \mathbb{R}\mathrm{e}\,\mathrm{Tr}\left[\mathbb{1} - P^{(2\times 1)}(x;\mu,\nu)\right]\right\},$$

$$S_{\mathrm{F}}[\chi,\overline{\chi},U] = a^4 \sum_x \overline{\chi}(x)\left[D_{\mathrm{W}} + i\mu_{\mathrm{q}}\gamma_5\tau^3\right]\chi(x),$$

(Frezzotti,Grassi,Sint,Weisz:2000)

$$D_{\mathrm{W}} = \frac{1}{2}\{\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - a\nabla^*_\mu\nabla_\mu\} + m_0,$$

- Sample all topological sectors (Jansen,Nube,A.S.,Urbach,Wenger:2007)
- PHMC with exact reweighting
- Decrease quark mass without encountering instabilities and/or metastabilitites

Outline

- Well known facts
  - Wilson fermions phase diagram
  - ϵ expansion
- ϵ expansion of WχPT
  - Phase diagram
  - Cutoff effects
- Simulations with $N_{\rm f} = 2$ mass degenerate light Wtm quarks in the ϵ regime
  - Algorithm
  - Low energy constants
- Conclusions and outlooks

Phase diagram

(Aoki:1984;Sharpe,Singleton:1998)

(Münster;Scorzato;Sharpe,Wu:2003-2005)

Wilson twisted mass fermions $\longrightarrow$ metastabilities (Farchioni *et al.*:2004)

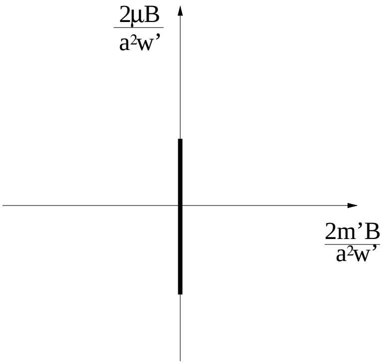
$$\mu_q > \frac{a^2|w'|}{B_0}$$

What happens in the ϵ regime?

Wilson fermions $\longrightarrow$ instabilities (Del Debbio,Giusti,Lüscher,Petronzio,Tantalo:2006)

$$m \geq \frac{n}{Z}\frac{a}{\sqrt{V}}$$

Reweighting can be useful (Hasenfratz,Hoffmann,Schaefer:2008)



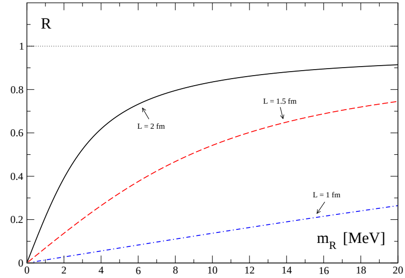
ϵ expansion in the continuum

- Integrate exactly over the costant zero modes, and treat the non-zero modes as standard perturbations
- Modify the power counting of the p regime, in a power counting where the pion mass M_π is small compared to the linear sizes of the box

$$\frac{1}{T} = \mathrm{O}(\epsilon), \quad \frac{1}{L} = \mathrm{O}(\epsilon), \quad M_\pi = \mathrm{O}(\epsilon^2).$$

The order parameter, vanishes in the chiral limit at fixed finite volume (Gasser,Leutwyler:1987)

$$R = \frac{\langle \bar{q} q \rangle}{B_0 F^2}$$



ϵ expansion with Wilson fermions

(A.S.:in preparation)

- Continuum $\longrightarrow$ chiral symmetry restoration
- Include the effects of the non vanishing lattice spacing in the ϵ expansion
- Study the mass dependence of the chiral condensate

New suitable power counting in Wilson chiral perturbation theory (WχPT)

$$M = O(\epsilon^4), \quad \frac{1}{L} = O(\epsilon), \quad \frac{1}{T} = O(\epsilon) \quad a^2 = O(\epsilon^4)$$

Is this the appropriate power counting?

$$M \simeq 5MeV, \quad a \simeq 0.1fm, \quad L \simeq 1.5fm$$

$$F \simeq 90MeV, \quad B_0 \simeq 5.5GeV, \quad |w'| \simeq (570MeV)^4$$

$$\Rightarrow \quad MF^2B_0V \simeq 0.75, \quad a^2F^2|w'|V \simeq 0.75, \quad \frac{MB_0}{a^2|w'|} \simeq 1$$

- We are inside the Aoki region but with a finite volume
- The dependence of the “order parameter” on the “external field” is smooth

LO $\epsilon W\chi PT$

The partition function at leading order

$$\mathcal{Z} = \int \mathcal{D}[\Sigma_0] \mathrm{e}^{\frac{c_1 V}{2}\mathrm{Tr}\left[\Sigma_0+\Sigma_0^\dagger\right] - \frac{c_2 V}{4}\mathrm{Tr}\left[\Sigma_0+\Sigma_0^\dagger\right]^2 + \frac{c_3 V}{2}\mathrm{Tr}\left[i\tau^3\left(\Sigma_0^\dagger - \Sigma_0\right)\right]}$$

New scaling variable z_2

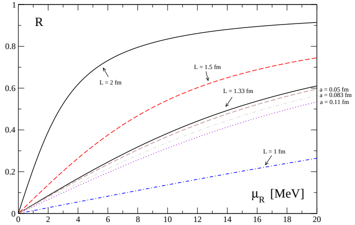
$$z_1 = c_1 V = B_0 F^2 m' V, \qquad z_2 = c_2 V = -\frac{F^2 w' V a^2}{4}, \qquad z_3 = c_3 V = B_0 F^2 \mu_{\mathrm{R}} V.$$

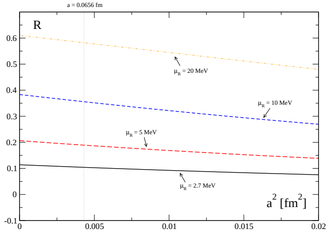
We can compute the chiral condensate

$$R = \frac{\langle \bar{q} q \rangle}{B_0 F^2}$$

$$R = \frac{1}{N_{\mathrm{f}}} \frac{\partial}{\partial z_3} \log \mathcal{Z}, \qquad z_1 = 0.$$

ϵ expansion with Wilson fermions





Smooth dependence on the quark mass.

No phase transition

Cutoff effects under control for the order parameter

ϵ expansion with Wilson fermions

- General power counting: valid also for Wilson fermions
- extension to NLO in progress
- No phase transition/minimal twisted mass in the ϵ regime
- It could be used to attack other problems like: interplay between a and V in the eigenvalues distribution
- Alternative way to extract the LEC $|w'|$ which parametrized the $O(a^2)$ effects
- Is this the correct power counting?

Algorithm

Simulate in the ϵ regime with Wtm using PHMC with exact reweighting (Jansen,Nube,A.S.,Urbach,Wenger:2007)

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] \mathrm{e}^{-S_G[U]} \det \left(QQ^\dagger [U] \right) \mathcal{O}[U] \quad Q = \gamma_5 \left[D_\mathrm{W} + i\mu_\mathrm{q}\gamma_5 \right],$$

Q single flavour operator

$$\det \left(QQ^\dagger [U] \right) = \frac{\det \left[QQ^\dagger P_{n,\tilde{\epsilon}} \left(QQ^\dagger \right) \right]}{\det \left[P_{n,\tilde{\epsilon}} \left(QQ^\dagger \right) \right]}, \quad P_{n,\tilde{\epsilon}} \left(QQ^\dagger \right) \simeq \left[QQ^\dagger \right]^{-1},$$

$$P_{n,\tilde{\epsilon}} \left(QQ^\dagger \right) \simeq \left[QQ^\dagger \right]^{-1} \qquad \{\lambda\} \in [\tilde{\epsilon}, 1]$$

Observables

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} W \rangle_P}{\langle W \rangle_P}, \quad W = \det \left[QQ^\dagger P_{n,\tilde{\epsilon}} \left(QQ^\dagger \right) \right] \simeq \prod_{\lambda_i < \tilde{\epsilon}} \left[\lambda_i P_{n,\tilde{\epsilon}} (\lambda_i) \right],$$

- Better sampling of configuration space
- With a twisted mass no instabilities issues
- In the ϵ regime no metastabilitites issues

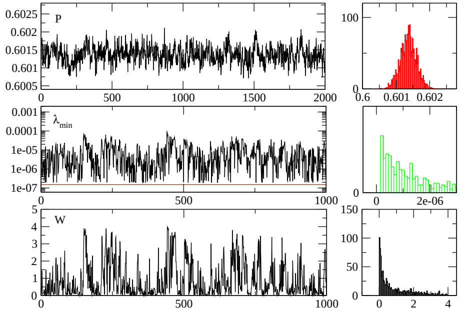
Simulation details

(Jansen,Michael,Nube,A.S.,Urbach,Wenger:in preparation)

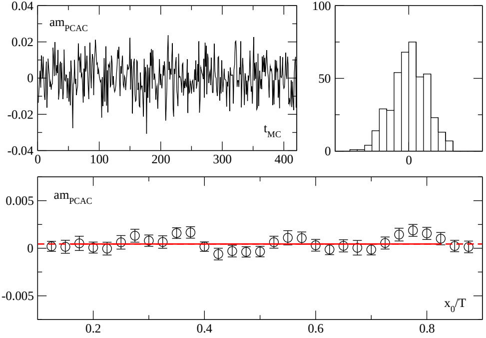
β	κ	L/a	T/a	$a\mu_q$
4.05	0.157010	20	40	0.00039

N_{traj}	N_{ana}	$\tau_{int}(P)$	$\tau_{int}(m_{PCAC})$
2500	421	~ 0.5	~ 0.5

r_0/a	$a[fm]$	$L[fm]$	am_{PCAC}
6.61(3)	0.0656(11)	1.31	0.00045(12)



PCAC mass



$$M_{\rm R} = \frac{1}{Z_{\rm P}} M \qquad M = \sqrt{(Z_{\rm A} m_{\rm PCAC})^2 + \mu_{\rm q}^2}$$

$$am_{\rm PCAC} = 0.00045(12)$$

$$\rightarrow \qquad aM_{\rm R} = 0.0012(2)$$

O(a) cutoff effects

(Frezzotti,Rossi:2003;Sint;A.S;Aoki,Bär:2005)

$$S_{\rm eff} = S_0 + aS_1 + \ldots \qquad S_0 = \int d^4x\overline{\chi}(x)\left[\gamma_\mu D_\mu + i\mu_{\rm R}\gamma_5\tau^3\right]\chi(x)$$

$$S_1 = \int d^4y\mathcal{L}_1(y) \qquad \mathcal{L}_1(y) = \sum_i c_i\mathcal{O}_i(y)$$

$$\mathcal{O}_1 = i\bar{\chi}\sigma_{\mu\nu}F_{\mu\nu}\chi \qquad \mathcal{O}_5 = \mu_{\rm q}^2\bar{\chi}\chi$$

$$\langle\Phi\rangle = \langle\Phi\rangle_0 - a\int d^4y\langle\Phi\mathcal{L}_1(y)\rangle_0 + a\langle\Phi_1\rangle_0 + \ldots$$

$$\mathcal{R}_5^{1,2}: \begin{cases}\chi(x_0,\mathbf{x}) \longrightarrow i\gamma_5\tau^{1,2}\chi(x_0,\mathbf{x})\\ \overline{\chi}(x_0,\mathbf{x}) \longrightarrow \overline{\chi}(x_0,\mathbf{x})i\gamma_5\tau^{1,2}\end{cases} \qquad \mathcal{D}: \begin{cases}U(x;\mu) \longrightarrow U^\dagger(-x-a\hat{\mu};\mu),\\ \chi(x) \longrightarrow e^{3i\pi/2}\chi(-x)\\ \overline{\chi}(x) \longrightarrow \overline{\chi}(-x)e^{3i\pi/2}.\end{cases}$$

$\mathcal{R}_5^{1,2}$ is not spontaneously broken
Contact terms amount to a redifinition of Φ_1

- Symmetry restoration region (SRR) $\longrightarrow$ Automatic O(a) improvement in the chiral limit for Wilson fermions
- In the chiral limit of SRR the form of the Wilson term is actually irrelevant
- In the SRR only O(aM) expected $\Rightarrow$ very small O(a) even out full twist
- If the mass is of O(a^2) cutoff effects can become visible (observable dependent)

NLO ϵ expansion $N_f = 2$

(Hasenfratz,Leutwyler:1989;Hansen,Leutwyler:1990)

$$P^a(x) = \overline{\chi}(x) i\gamma_5 \frac{\tau^a}{2}\chi(x)$$

$$C_P(x_0) = \frac{1}{L^3}\int d^3x C_P(\mathbf{x}, x_0) \qquad \delta^{ab} C_P(\mathbf{x}, x_0) = \langle P^a(\mathbf{x}, x_0) P^a(\underline{0}, 0)\rangle$$

$$C_P(x_0) = a_P + \frac{T}{L^3} b_P \left[\frac{y^2}{2} - \frac{1}{24}\right] + \ldots \qquad y = \frac{x_0}{T} - \frac{1}{2}$$

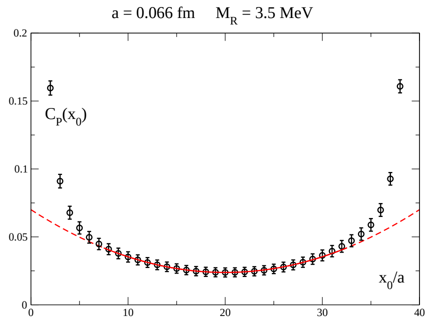
$$a_P = \frac{B_0^2 F^4 \rho^2}{8} G_1(u), \qquad b_P = F^2 B_0^2 \left[1 - \frac{1}{8} G_1(u)\right]$$

$$u = 2B_0 F^2 M V \rho, \qquad \rho = 1 + \frac{3}{2}\frac{\beta_1}{F^2\sqrt{V}}, \qquad G_1(u) = \frac{8}{u}\frac{Y'(u)}{Y(u)}, \qquad Y(u) = \frac{2I_1(u)}{u}$$

Fit formulæ

$$C_P(x_0) = A_0 + A_2 y^2 \qquad \Rightarrow \qquad a_P = A_0 + \frac{A_2}{12} \qquad b_P = A_2 \frac{2L^3}{T}$$

Pseudoscalar correlation function



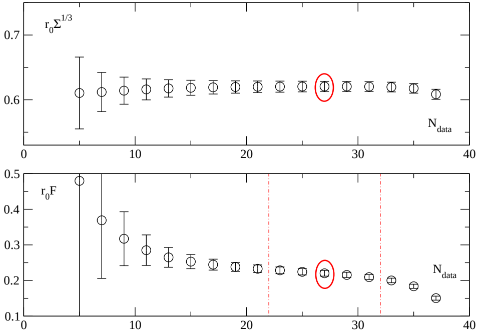
$$a^3 L^3 A_0 = (5.94(36)) \cdot 10^{-3}, \qquad a^3 L^3 A_2 = (4.81(30)) \cdot 10^{-2}$$

Random source locations (ETMC:2007)

Nested Jackknife/bootstrap errors

$Z_2 \times Z_2$ stochastic sources

Effective couplings plots



$$r_0\Sigma^{1/3} = 0.620(8), \qquad r_0F = 0.220(8)(10) \qquad [PRELIMINARY]$$

$$r_0\Sigma^{1/3} = 0.617(15), \qquad r_0F = 0.224(10)$$

(Hasenfratz,Hoffmann,Schaefer:2008)

Comparisons

Group	N_f	Σ(2GeV)
This work	2	$-(282 \pm 4\ \mathrm{MeV})^3$
ETMC (2007)	2	$-(272 \pm 4 \pm 7\ \mathrm{MeV})^3$
JLQCD (2007)	2	$-(251 \pm 7 \pm 11\ \mathrm{MeV})^3$
Lang et al (2007)	2	$-(276 \pm 11 \pm 16\ \mathrm{MeV})^3$
McNeile + MILC (2005)	2+1	$-(259 \pm 27\ \mathrm{MeV})^3$
McNeile + JLQCD (2005)	2	$-(209 \pm 8\ \mathrm{MeV})^3$

Group	N_f	F
This work	2	100(4)(5)MeV
ETMC(2007)	2	83(1)(3)MeV
QCDSF/UKQCD(2007)	2	79(5)MeV
JLQCD(2007)	2	78(3)(1)MeV

Conclusions and outlooks

Conclusions

- We are establishing the basic knoweledge to simulate with Wilson-like fermions in the ϵ regime
- Introduced a power counting to study the ϵ expansion with Wilson-like fermions
- LO Chiral condensate $\longrightarrow$ no phase transitions
- NLO and other observables ongoing
- Numerical simulations in the ϵ regime with Wtm
- PHMC with exact reweighting
- Sampling of all the topological sectors
- Extraction of LEC (Σ, F) without contamination from chiral logs

Outlooks

- Understand the usual systematic errors: discretization errors, quark mass and volume dependence
- Extend to more observables the current analysis
- Combine p and ϵ regime fits
- Alternatively compute LEC from spectral quantities

Comments on the power countings

$$\mathcal{L}_{W\chi}^{(2)} = \frac{F^2}{4}\left[\langle \partial_\mu \Sigma(x)^\dagger \partial_\mu \Sigma(x)\rangle + \langle \sigma(x)\Sigma(x)^\dagger + \sigma(x)^\dagger \Sigma(x)\rangle + \langle A(x)\Sigma(x)^\dagger + A(x)^\dagger \Sigma(x)\rangle\right].$$

$$\sigma' \equiv \sigma + A \qquad m_{\mathrm{R}} \longrightarrow m_{\mathrm{R}} + aW_0/B_0 \equiv m'.$$

$$(\Sigma)^{1/3} = 250MeV \qquad \Lambda = 200MeV \qquad m \sim a\Lambda^2 \simeq 20MeV \qquad V = (1.5fm)^4 \div (2.5fm)^4$$

$$\Rightarrow \qquad m\Sigma V = 1 \div 8$$

Is this region ϵ or p regime?

How do we define in which regime we are?

If we lower the quark mass $\longrightarrow$ certainly ϵ regime

$$m \sim a^2\Lambda^3 \simeq 2MeV$$

Different power counting has to be adopted and cutoff effects can become visible