Infrared exponents and the strong coupling limit in lattice Landau gauge

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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Outline

- Introduction and motivation
  - What is the problem?
  - Why do we study SU(2) at β=0 ?
- Results
  - Gluon and ghost dressing function at β=0
  - Comparing results of standard and modified lattice Landau gauge
- Summary and Conclusion

The infrared behavior: a controversial issue

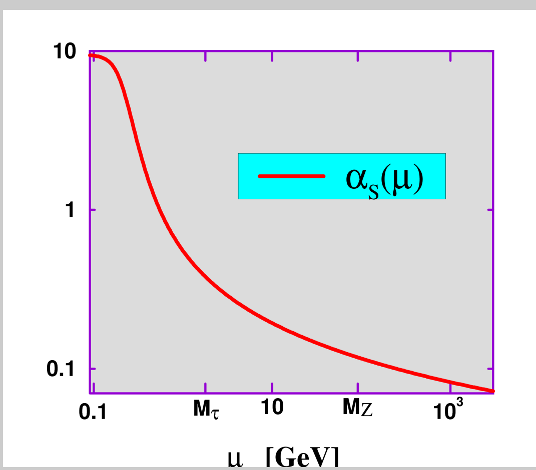
$\kappa_G \approx 0.596$

- Functional continuum methods favor $\kappa_D = 2\kappa_G$

$$Z(p^2,\mu^2) \propto (p^2/\mu^2)^{\kappa_D}$$
$$G(p^2,\mu^2) \propto (p^2/\mu^2)^{-\kappa_G}$$

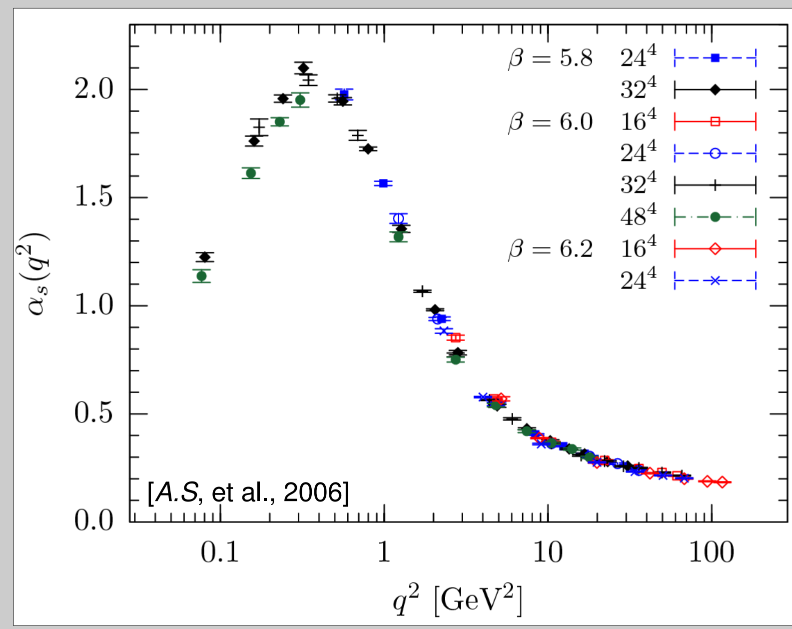
[von Smekal, Hauck, Alkofer, 1997]

- Infrared-finite coupling



- Lattice Landau gauge QCD
  - Neither has been found in standard lattice simulations

$\kappa_D \neq 2\kappa_G$



Why do we bother?

- Continuum methods in covariant gauges assume an unbroken BRST symmetry
- Then, with
  - an infrared diverging ghost dressing function and
  - a gluon mass gap

  confinement is realized by the quartet mechanism

  see [Kugo,Ojima, 1979]

- Lattice BRST has yet to be formulated
  - we are progressing towards it, see [von Smekal et al., arXiv:0710.2410]
- If ghost/gluon propagators turned out to be finite
  - Situation would be even worse than just having no power law
  - We would dump many achievements in covariant gauge theories

Motivation

- In those DSE studies ghost dominance is assumed
- What happens if, on the lattice, gluons are turned off?
  - Action is only given by F-P determinant and measure term
  - ghost dominance implemented by hand

$$\beta = 0$$

$$S = \cancel{S_g} + S_{\mathrm{meas}} + S_{\mathrm{FP}}$$

- We see the conformal infrared behavior in the strong coupling limit at large momenta
- deviations at small momenta can be described by a mass term,
  - depends on the definition of lattice gluon fields
  - might cause a non-trivial effect in simulations at finite coupling

Simulation

- Almost standard, besides we gauge-fixed “hot” configurations
- Used an overrelaxtion algo. that maximizes the gauge functional
- Calculated gluon and ghost propagators
- We also used another definition of lattice Landau gauge to compare with (see later)

<u>Standard lattice LG</u>

$$F_U[g] = \frac{1}{4V}\sum_{x,\mu} \mathfrak{Re}\,\mathrm{Tr}\, U^g_{x\mu}$$

$$A_{x\mu} = \frac{1}{2iag_0}(U_{x\mu} - U^\dagger_{x\mu})$$

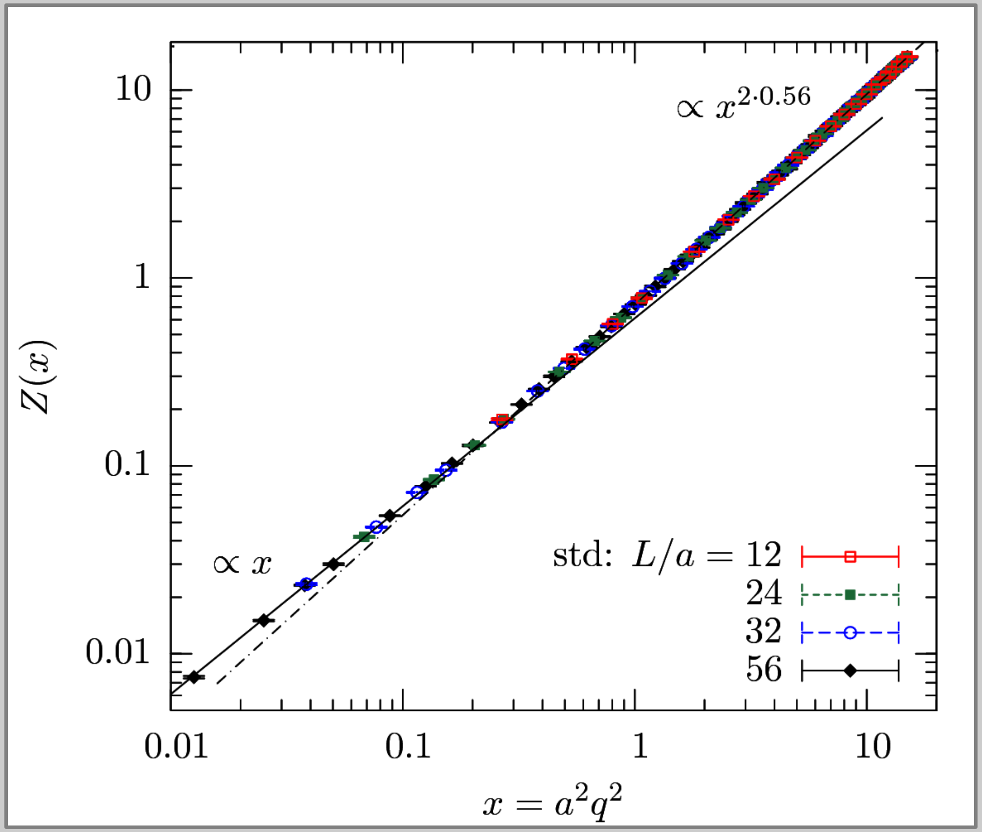
$$\partial_\mu A_{x,\mu} = 0$$

$$M^{ab}_{xy} = F''_U[g]\Big|_{U=U^g}$$

Gluon dressing function

- Two asymptotic regimes

$$x > 2.0 : cx^{2\kappa} \text{ with } \kappa \simeq 0.56$$
$$x < 0.1 : dx \qquad (x \equiv a^2 q^2)$$



- Minor finite-volume effects
- Gluon propagator

$$D^{ab}_{\mu\nu}(k) = \left\langle A^a_\mu(k) A^b_\mu(-k) \right\rangle_{U^g}$$

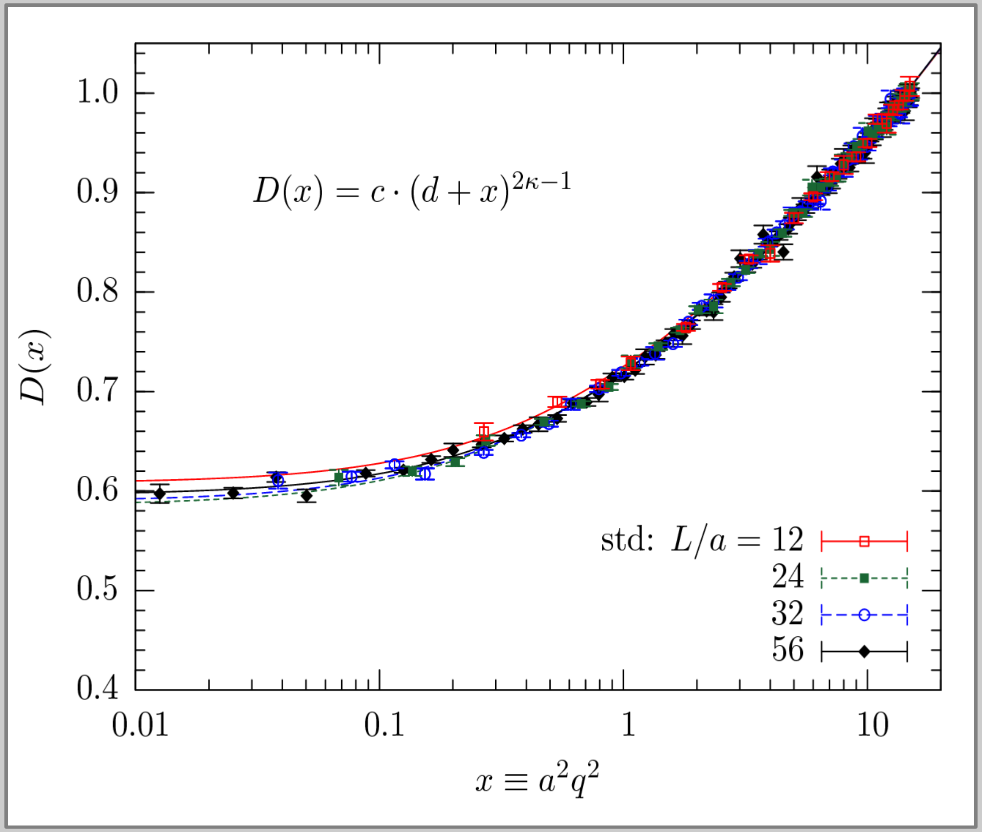
$$D^{ab}_{\mu\nu}(k) = \delta^{ab}\left(\delta^{\mu\nu} - \frac{q_\mu(k)q_\nu(k)}{q^2(k)}\right)\frac{Z(q^2(k))}{q^2(k)}$$

$$aq_\mu(k) = 2\sin(\pi k_\mu / L_\mu)$$

Gluon propagator

- Two asymptotic regimes

$$x > 2.0 : cx^{2\kappa-1} \text{ with } \kappa \simeq 0.56$$
$$x < 0.1 : d$$



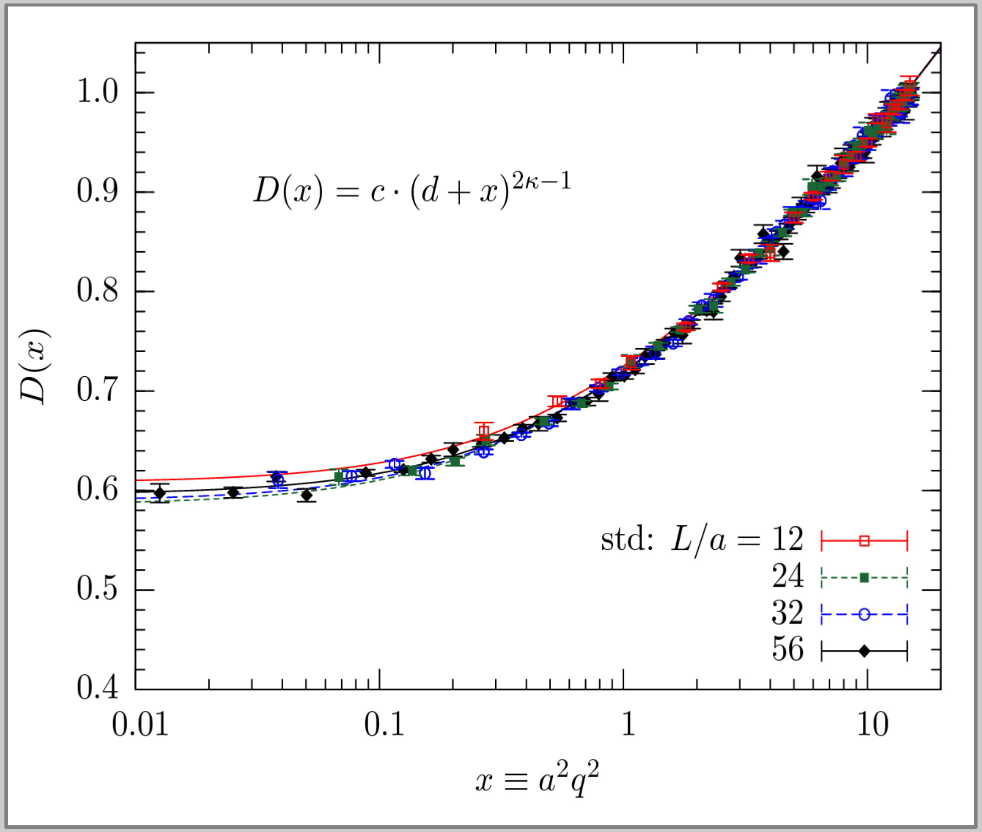
- Minor finite-volume effects
- Unphysical zero-momentum limit (see later)
  - Depends on how we define $A_\mu(x)$ on the lattice

$$D^{ab}_{\mu\nu}(k) = \left\langle A^a_\mu(k) A^b_\mu(-k) \right\rangle_{U^g}$$

Fitting the gluon exponent

- Apply different fit models
  - Intermediate momentum range difficult to accommodate
  - κ slightly varies with model

$$(1) \quad D(x) = cx^{2\kappa-1}$$
$$(2) \quad D(x) = d + cx^{2\kappa-1}$$
$$(3) \quad D(x) = c(d+x)^{2\kappa-1}$$



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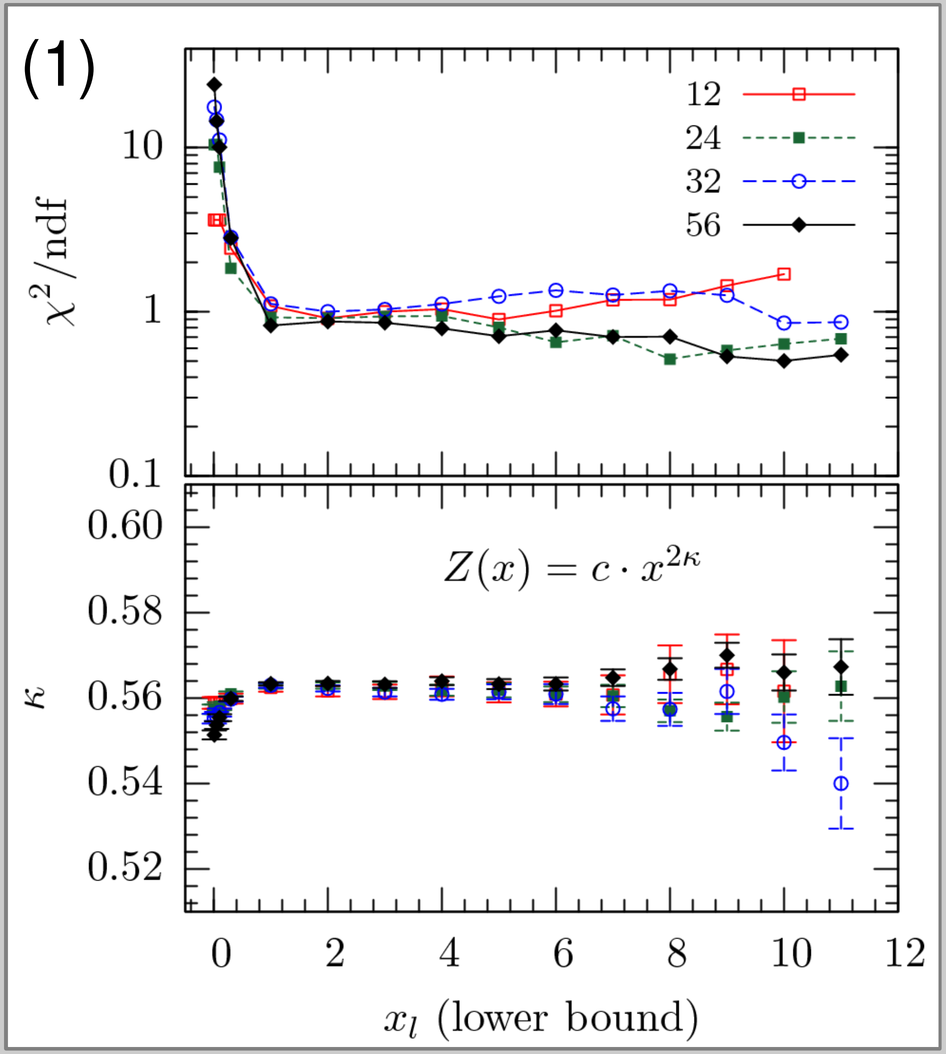

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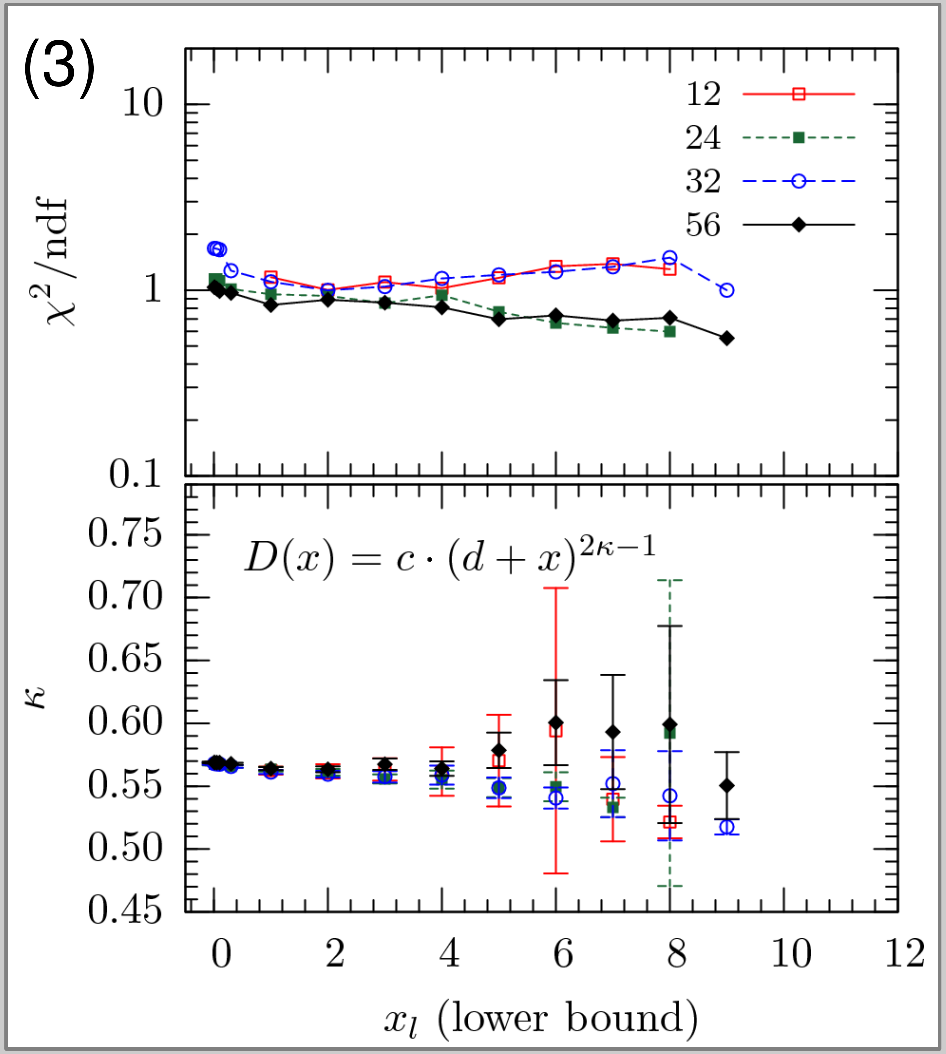

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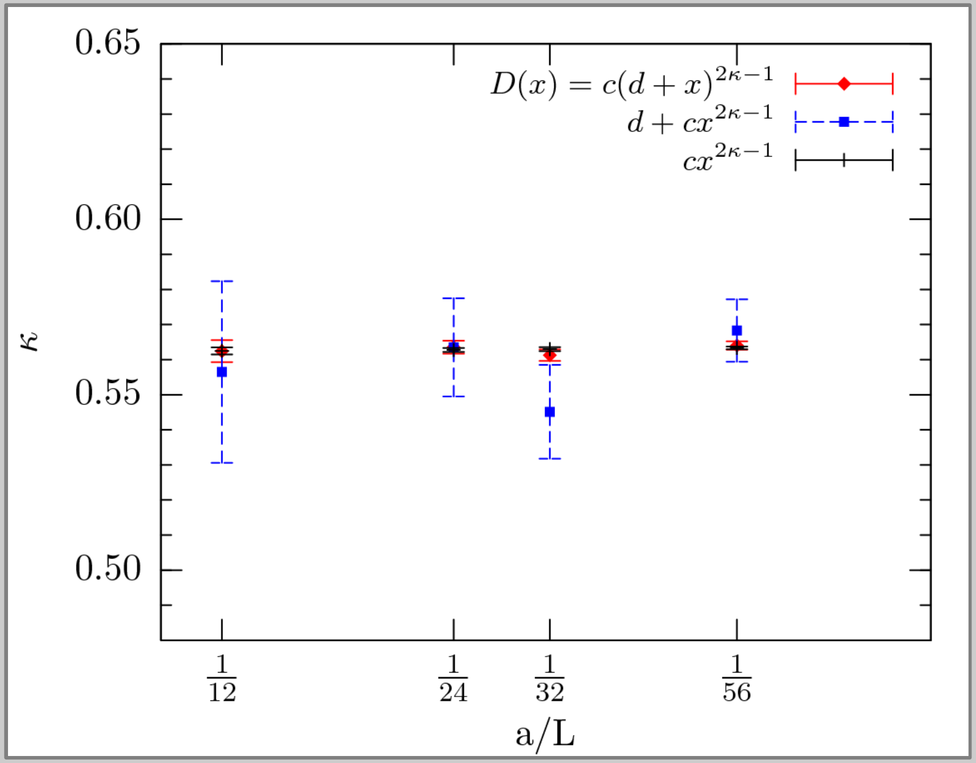
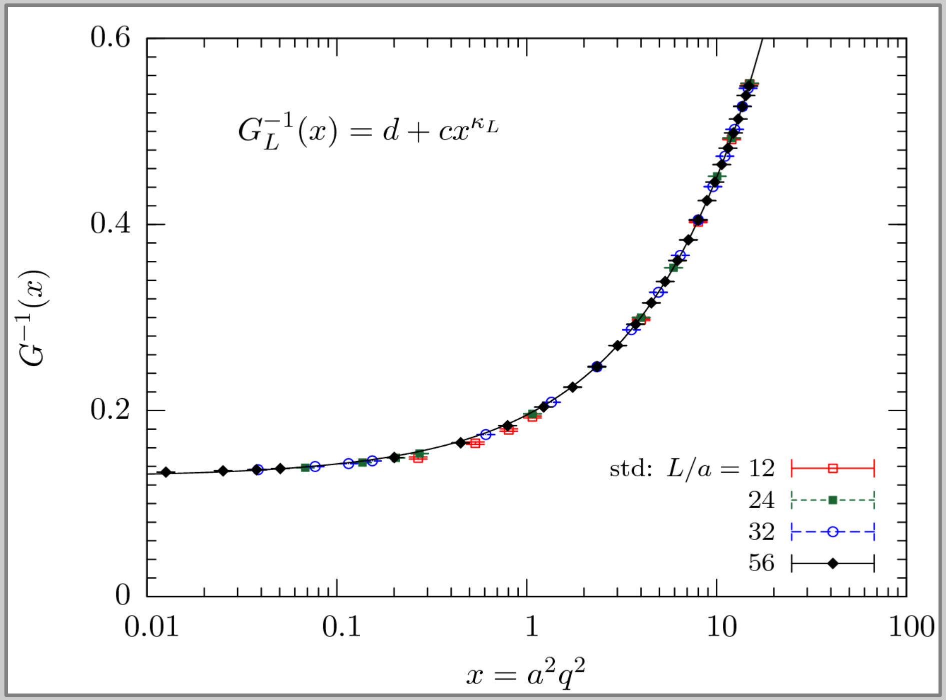


Fitting the ghost exponent

- Apply different fit models
  - Intermediate momentum range difficult to accommodate
  - κ varies with model, $\kappa \in [0.5, 0.7]$

$$(1) \quad G^{-1}(x) = cx^{\kappa}$$
$$(2) \quad G^{-1}(x) = d + cx^{\kappa}$$
$$(3) \quad G^{-1}(x) = c(d + x)^{\kappa}$$



  - More data at larger momentum might help

Fitting the ghost exponent

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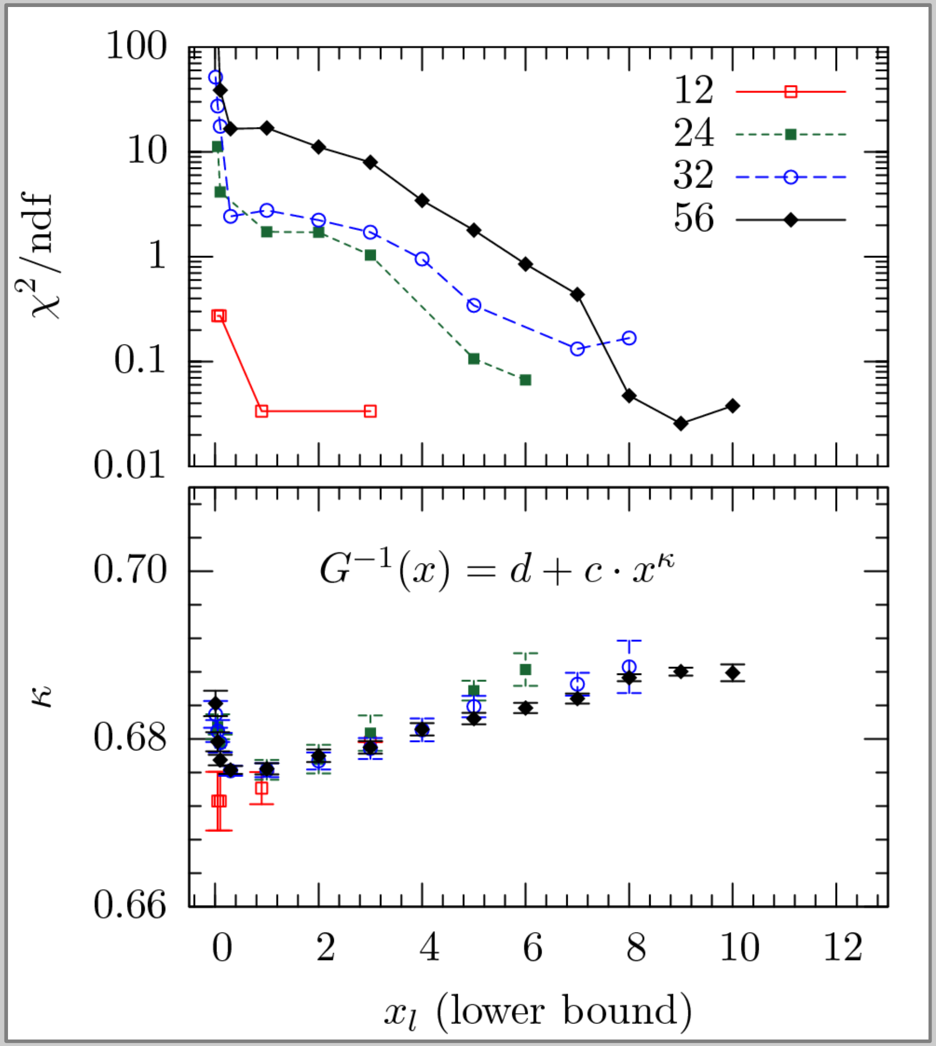
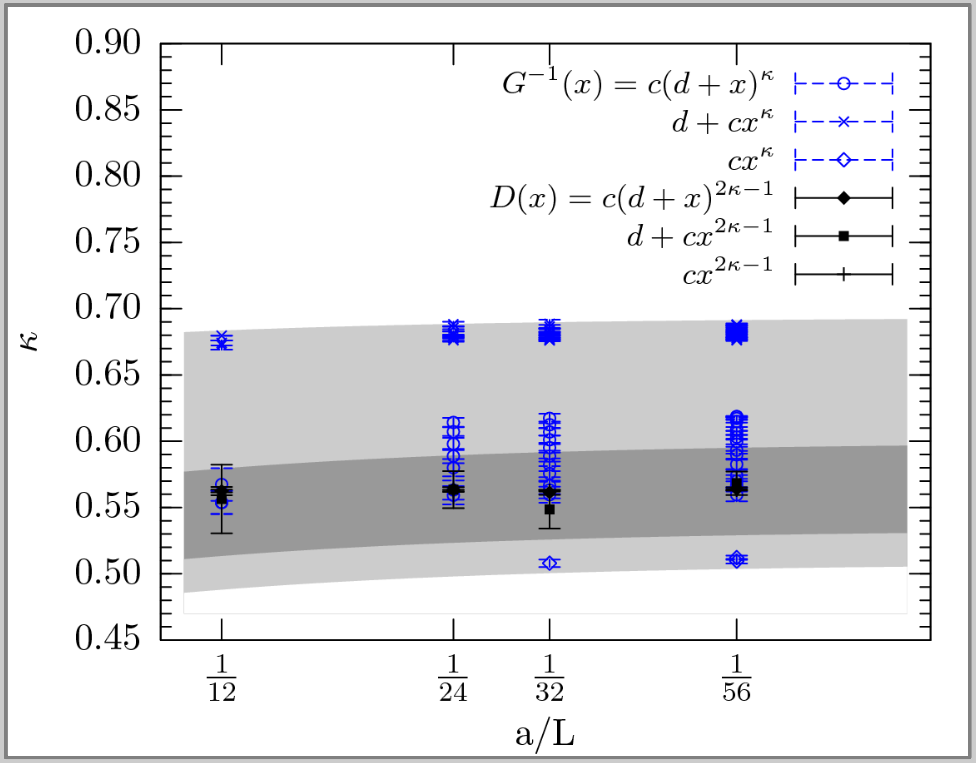


Comparing infrared exponents

- Use different models to fit data
  - Gluon: (black points) fitted κ values roughly agree
  - Ghost: (blue points) fit-model dependence dominates
  - General tendency: κ values grow with volume



- Within the given limits

$$\kappa_G \simeq 2\kappa_D$$

as expected from DSE studies

Standard vs. Modified lattice Landau gauge

[von Smekal et al., arXiv:0710.2410]

Standard lattice LG

$$F_U[g] \propto \sum_{x,\mu}\left(1 - \frac{1}{N_c}\mathfrak{Re}\,\mathrm{Tr}\,U^g_{x\mu}\right)$$

$$A_{x\mu} = \frac{1}{2iag_0}(U_{x\mu} - U^\dagger_{x\mu})$$

$$\partial_\mu A_{x,\mu} = 0$$

$$p_\mu(k)A_\mu(k) = 0$$

Modified lattice LG

$$\widetilde{F}_U[g] \propto -\sum_{x,\mu}\ln\left(\frac{1}{2} + \frac{1}{2N_c}\mathfrak{Re}\,\mathrm{Tr}\,U^g_{x\mu}\right)$$

$$\widetilde{A}_{x\mu} = \frac{1}{2iag_0}(\widetilde{U}_{x\mu} - \widetilde{U}^\dagger_{x\mu})$$

$$\partial_\mu \widetilde{A}_{x,\mu} = 0$$

$$p_\mu(k)\widetilde{A}_\mu(k) = 0$$

$$ap_\mu(k) = 2\sin(\pi k_\mu/L_\mu)$$

$$\widetilde{U}_{x\mu} := \frac{2N_cU_{x\mu}}{N_c + \mathfrak{Re}\,\mathrm{Tr}\,U_{x\mu}}$$

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Modified lattice LG

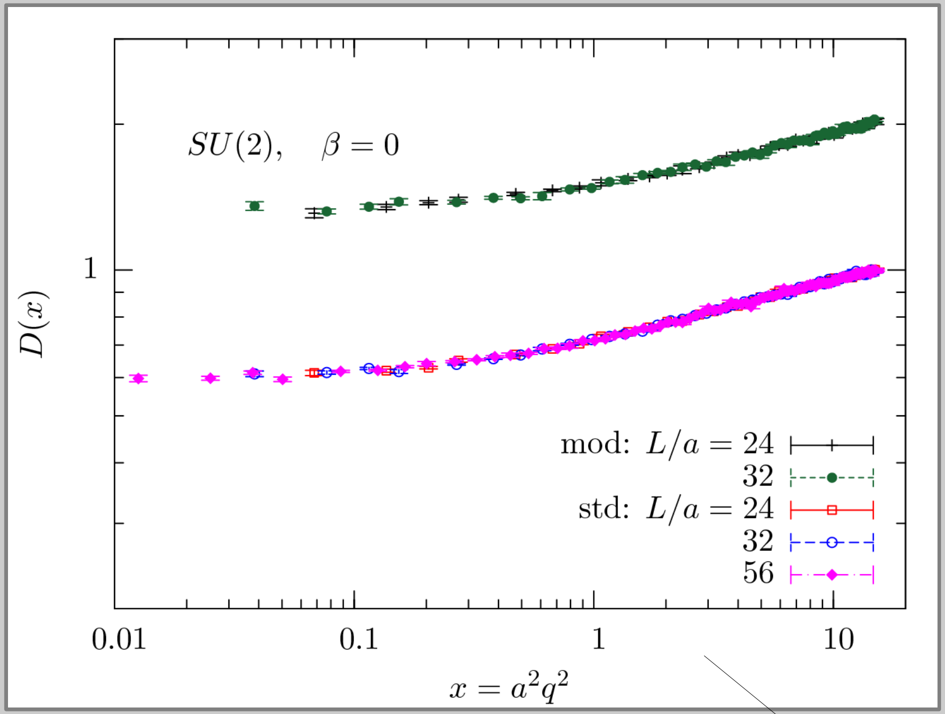
$$\widetilde{F}_U[g] \propto -\sum_{x,\mu}\ln\left(\frac{1}{2} + \frac{1}{2N_c}\mathfrak{Re}\,\mathrm{Tr}\,U^g_{x\mu}\right)$$

$$\widetilde{M}^{ab}_{xy}[\widetilde{U}] = M^{ab}_{xy}[\widetilde{U}] + \text{add. terms}$$

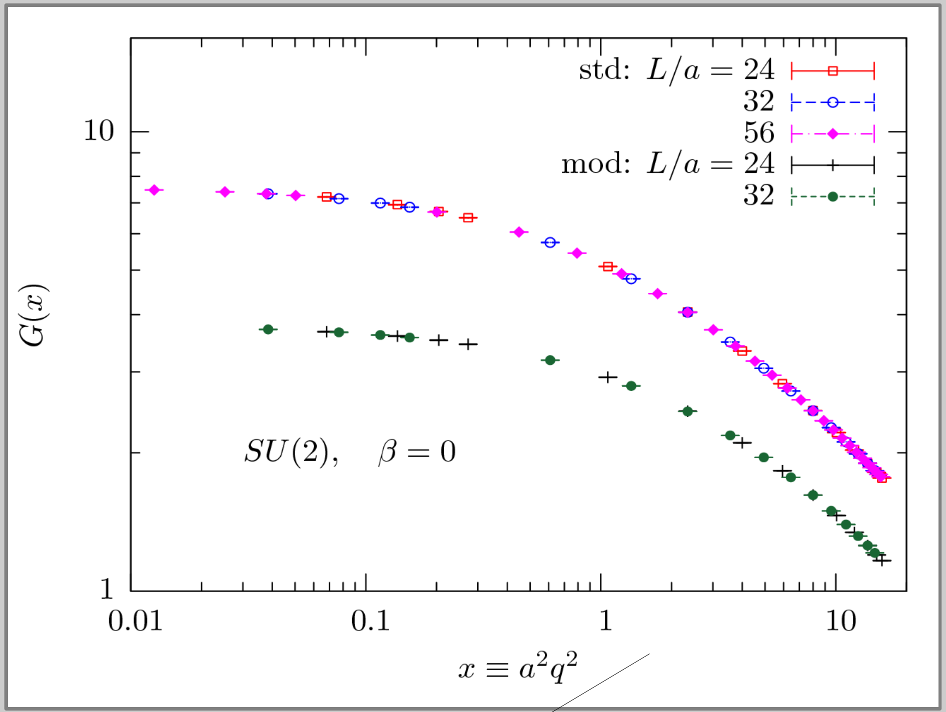
$$\widetilde{U}_{x\mu} := \frac{2N_cU_{x\mu}}{N_c + \mathfrak{Re}\,\mathrm{Tr}\,U_{x\mu}}$$

Comparing standard to modified LG

Gluon propagator



Ghost dressing function

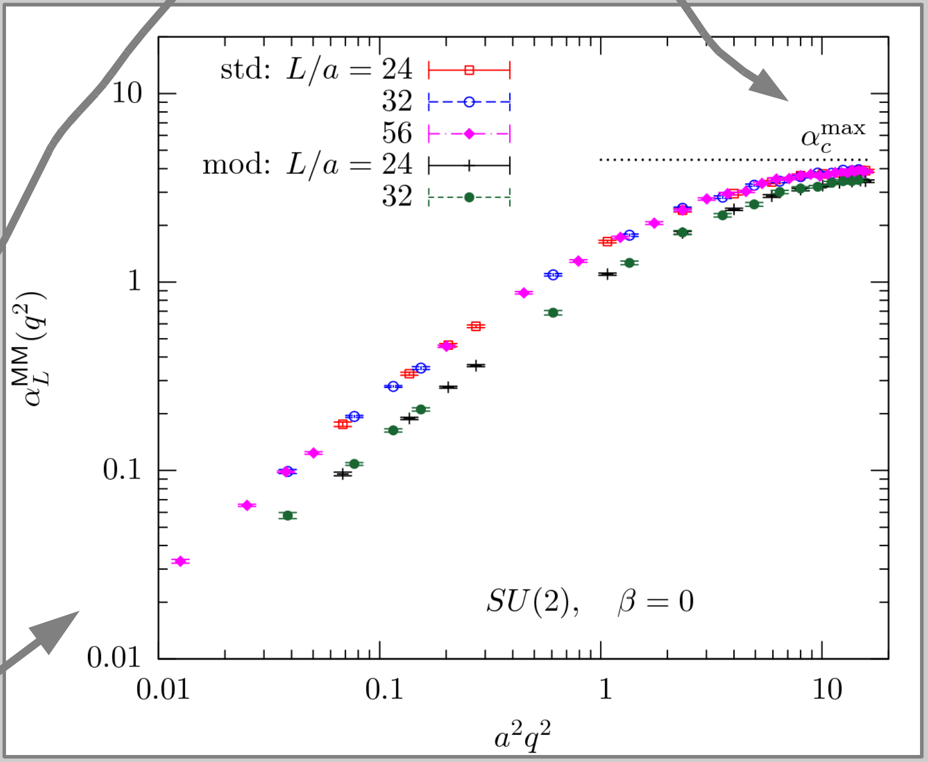


$$\alpha_L^{\mathrm{MM}}(p^2) = \frac{g_0^2}{4\pi} Z_L(a^2p^2) G_L^2(a^2p^2)$$

[von Smekal et al., 1997]

Critical coupling

- Compare MM coupling rather than individual propagators (no normalization necessary)
- In the large momentum limit we find the critical coupling as expected from DSE
- Findings independent of lattice implementation of LG



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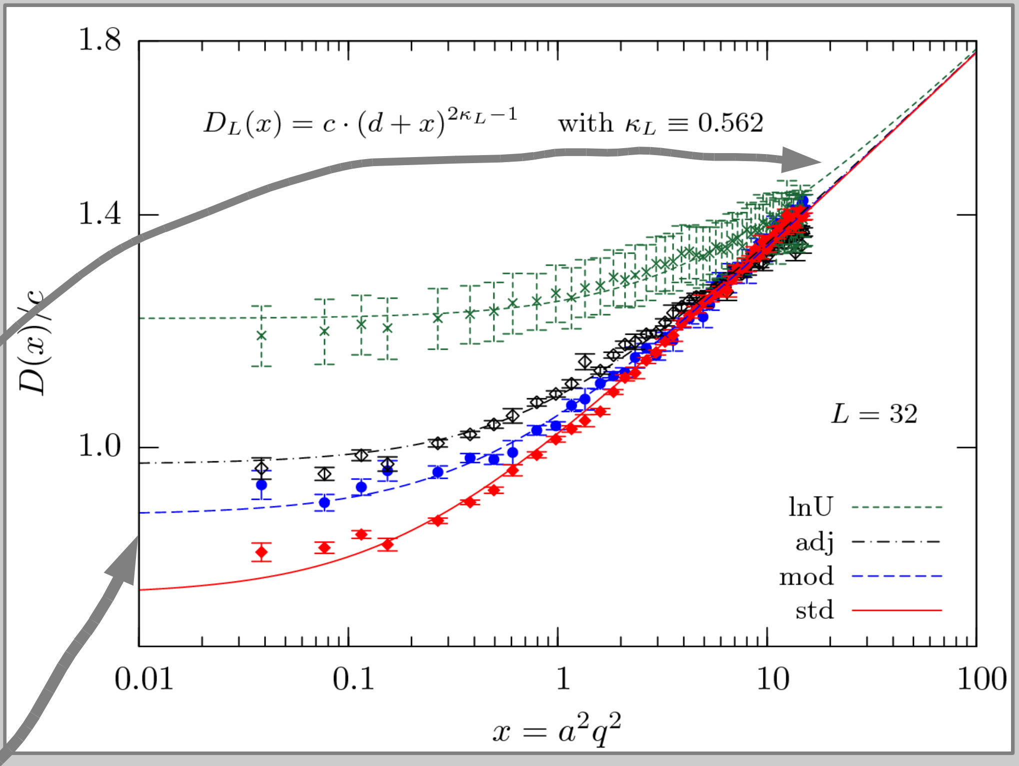
[von Smekal et al., 1997]

Different gluon field definitions at x=0

- We also checked
  - Log: $iag_0 A_{x,\mu} := \ln U_{x\mu}$
  - Adj: $ag_0 A^c_{x\mu} := 2u^0_{x,\mu} u^c_{x,\mu}$
- At large x, $D(x)$'s run into the same limit
  - κ fixed, works well for all
- At small x, $D(x)$'s differ
  - Not just a factor, but a additive constant



Depends on definition of lattice gauge potential

☞ unphysical!

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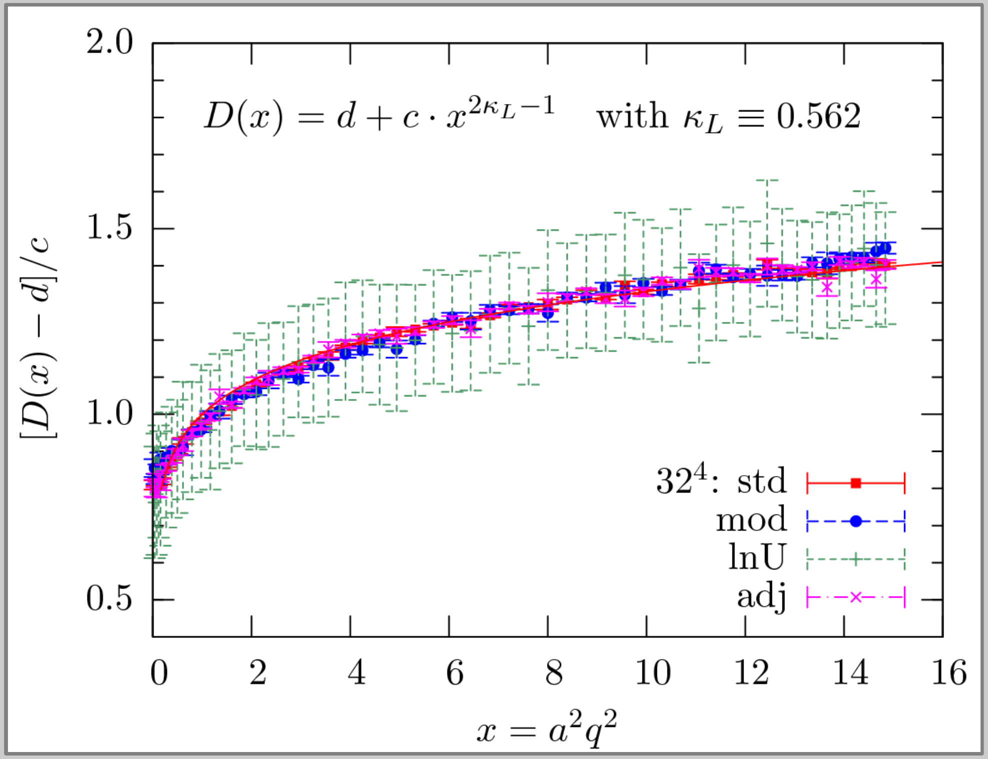


If we subtract individual constants large-x behavior is power-law like

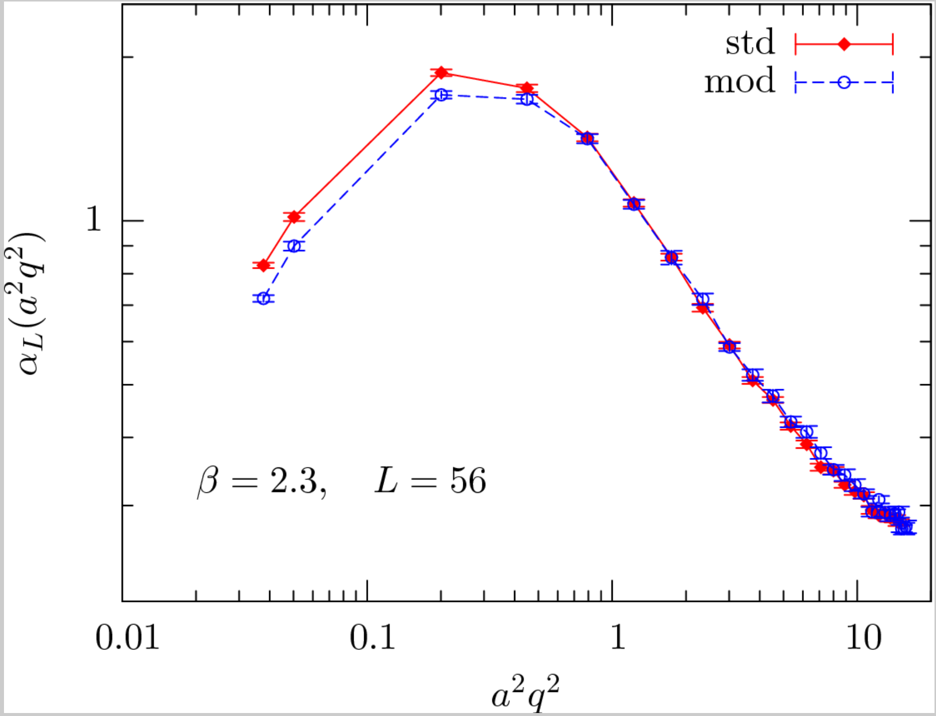
Summary and Conclusion

- Studied the gluon and ghost propagators of lattice Landau gauge at $\beta = 0$ in pure SU(2) lattice gauge theory
- Compared two implementations of lattice Landau gauge
  - within the available precision we find evidence (in both cases) of the conformal infrared behavior at larger a^2q^2

    $$\kappa_D \simeq 2\kappa_G$$
  - deviations at small a^2q^2 can be parametrized by a mass term that depends on the definition of lattice gluon and ghost fields
  - Mass term interferes with the power-law behavior larger momentum
- This might have a impact even at finite coupling and is the reason why we couldn't yet see the conformal infrared behavior

Checking at finite coupling

- Checked also at $\beta = 2.3, 2.5$
- "Discretization error" also affect data at small momentum
  - Could disappear in continuum limit though.
- To study infrared behavior integration measure needs to be properly incorporated
- Not yet done in standard implementations of lattice Landau gauge



We are working on it for the Modified lattice Landau gauge

[von Smekal et al., arXiv:0710.2410]