K-meson vector decay constant and B-parameter from $N_f = 2$ tmQCD

Lattice 2008

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Generalities

- ETMC is performing state-of-the-art lattice QCD simulations with $N_f = 2$ dynamical flavours (sea quarks), with "lightish" masses ($300\ \text{MeV} \leq m_{PS} \leq 550\ \text{MeV}$).
- Several quantities are being analyzed for a couple of β's.
- With $N_f = 2$ sea quarks, strangeness enters the game in a partially quenched context.
- In this talk we will show **preliminary** results on the following quantities:
  - m_{K*}
  - f_{K*}
  - $[\, f_T/f_V \,]_{K*}$
  - B_K
- In parallel, other ETMC subgroups have been working on decay constants in the light and strange quark sector (see talks by C. McNeile and C. Tarantino).
- Collaborators: **P.Dimopoulos**, R.Frezzotti, V.Gimenez, V.Lubicz, **F.Mescia**, G.C. Rossi, S. Simula.

Theory

- ETMC simulations are performed with the tree-level Symanzik improved gauge action.
- The $N_f = 2$ sea quark flavours are regularized by the standard Wilson fermion action with a fully twisted mass term (standard tmQCD) .

$$\bar{\psi} = (\bar{u} \quad \bar{d})$$

$$\mathcal{L}_{tm} = \bar{\psi} \left[D_W + i\mu_q \tau^3 \gamma_5 \right] \psi$$

- This has the usual advantages:
  - Renormalization properties are, in many cases of interest (e.g. pseudoscaler decay constants, B_K...) much simpler than with standard Wilson quarks.

    Alpha Collab., R. Frezzotti, P.A. Grassi, S. Sint and P. Weisz, JHEP08 (2001) 058
  - Improvement is automatic with full twist (i.e. imaginary mass term only).

    R. Frezzotti, G.C. Rossi, JHEP08 (2004) 007

Theory

- ETMC simulations are performed with the tree-level Symanzik improved gauge action.
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$$\mathcal{L}_{tm} = \bar{\psi} \left[D_W + i\mu_q \tau^3 \gamma_5 \right] \psi$$

- **But** this is true for most, **not all,** quantities of interest .
- In particular, for WMEs of 4-fermion operators (e.g. B_K), it is not possible to have standard tmQCD formalism, with all flavours at full twist (i.e. automatic improvement), **and** multiplicative renormalization.

ALPHA P. Dimopoulos, J. Heitger, F. Palombi, C. Pena, S. Sint, A.V., NuclPhysB 749 (2006) 69

C. Pena, S. Sint, A.V., JHEP09 (2004)069

Theory

- ETMC simulations are performed with the tree-level Symanzik improved gauge action.
- The $N_f = 2$ sea quark flavours are regularized by the standard Wilson fermion action with a fully twisted mass term (standard tmQCD) .

$$\mathcal{L}_{OS} = \bar{\psi}_f \left[D_W + i\mu_f \gamma_5 \right] \psi_f \qquad\qquad f = u,\, d,\, s \cdots$$

- One way out is provided by the Osterwalder-Seiler (OS) variant of tmQCD.
- Valence quarks enter with a distinct action for each flavour, which is **fully twisted**.
    - quark fields are not organized in isospin doublets (i.e. no τ^3).
    - there is a separate mass term for each flavour, μ_f may be negative.

Theory

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- One way out is provided by the Osterwalder-Seiler (OS) variant of tmQCD.
- Valence quarks enter with a distinct action for each flavour, which is **fully twisted**.
  - Suitable combinations of μ_f signs for each flavour ensure automatic improvement **and** multiplicative renormalization for say, B_K.

R. Frezzotti, G.C. Rossi, JHEP10 (2004) 070

Theory

- ETMC simulations are performed with the tree-level Symanzik improved gauge action.
- The $N_f = 2$ sea quark flavours are regularized by the standard Wilson fermion action with a fully twisted mass term (standard tmQCD) .

$$\mathcal{L}_{OS} = \bar{\psi}_f \left[D_W + i\mu_f \gamma_5 \right] \psi_f \qquad\qquad f = u,\, d,\, s \cdots$$

- One way out is provided by the Osterwalder-Seiler (OS) variant of tmQCD.
- Valence quarks enter with a distinct action for each flavour, which is **fully twisted**.
- This is a compromise (unitarity issues arise when sea and valence flavours are treated differently) but in our partially quenched setup ($N_f = 2$ sea quark flavours and a valence strange quark) this is unavoidable for any regularization.

The Simulation

- The ETMC runs are performed at three gauge couplings β.
- The master run: 240 measurements at $\beta = 3.90$, corresponding to $a \approx 0.086(1)$ fm [i.e. $1/a \approx 2.3$ GeV] and volume $V = 24^3 \times 48$
  - 5 sea quark masses: $\mu = 0.0040, 0.0064, 0.0085\ 0.0100, 0.0150$

    (300 MeV $\leq m_{PS} \leq$ 550 MeV)

    ETMC, Ph. Boucauld et al., Phys. Lett. B650 (2007) 304
  - 7 valence quark masses; the extra ones are: $\mu = 0.0220, 0.0270$ ($\sim m_{strange}$)

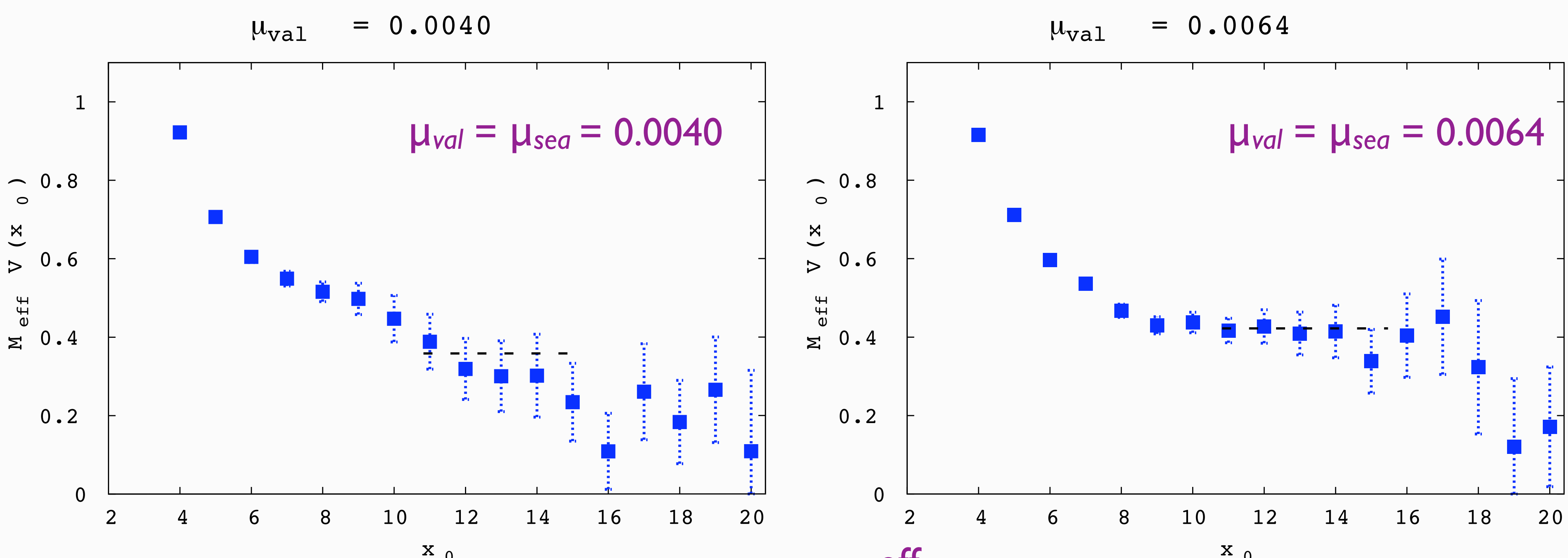
    ETMC, B. Blossier et al., JHEP 04 (2008) 020
  - use existing calibrations: $a\mu_d = a\mu(m_\pi) = 0.00079$ and $a\mu_s = a\mu(m_K) = 0.0217(10)$
- For B_K only, at $\beta = 3.90$, we did 200 measurements so far.
- For B_K only, we checked for finite volume effects at $V = 32^3 \times 64$ for $\mu = 0.0040$.
- For B_K only, we did a rough scaling test at $\beta = 4.05$, $\mu = 0.0030$, $V = 32^3 \times 64$ and 100 measurements.

K* meson mass and decay constant

- **P. Dimopoulos**, S. Simula, A.V.

Caveats

- We encountered low quality signals in two cases:
  - I: For all sea quark masses, when the valence quark masses are in the lightest range (say μ_{val} = 0.0040)



m_V^{eff}

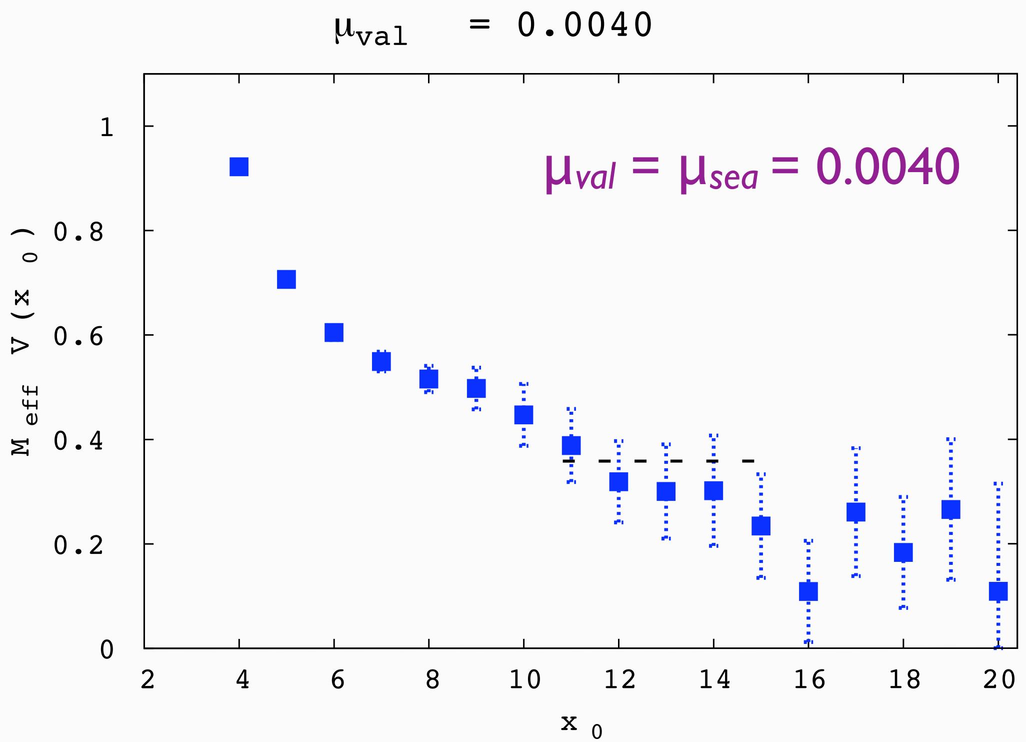
K* meson mass and decay constant

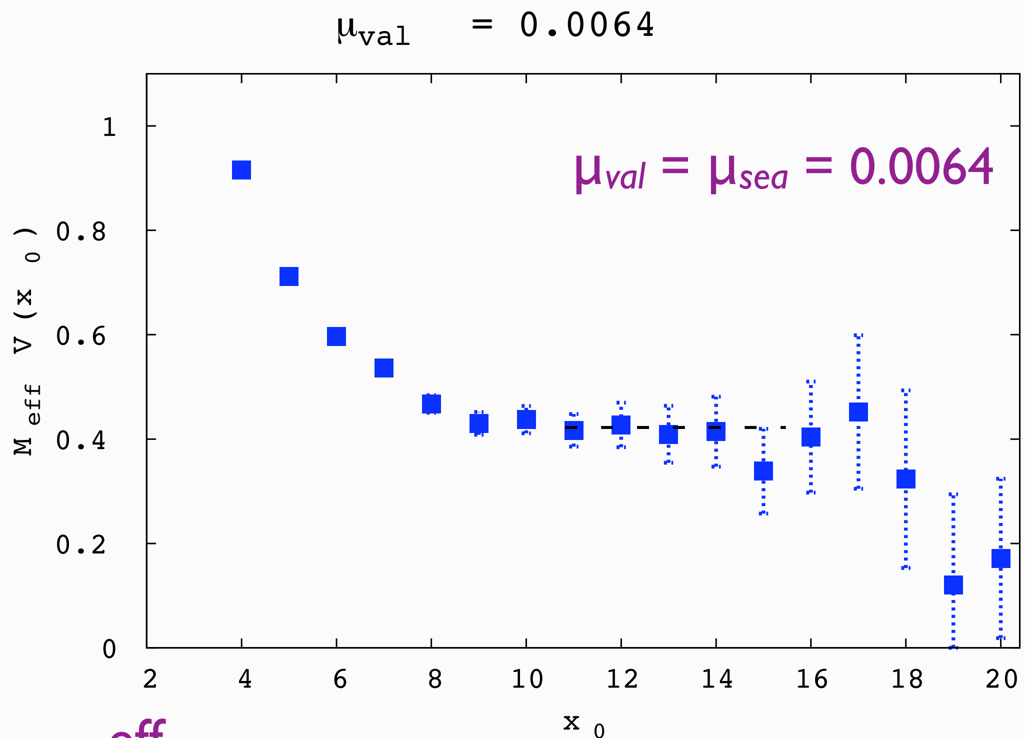
- **P. Dimopoulos**, S. Simula, A.V.

Caveats

- We encountered low quality signals in two cases:

- Nevertheless, since the signal-to-noise ratio is as expected ~ exp[- (m_V - m_{PS}) t]; ρ-meson mass and decay constant may be extracted (C. McNeile, this conference).





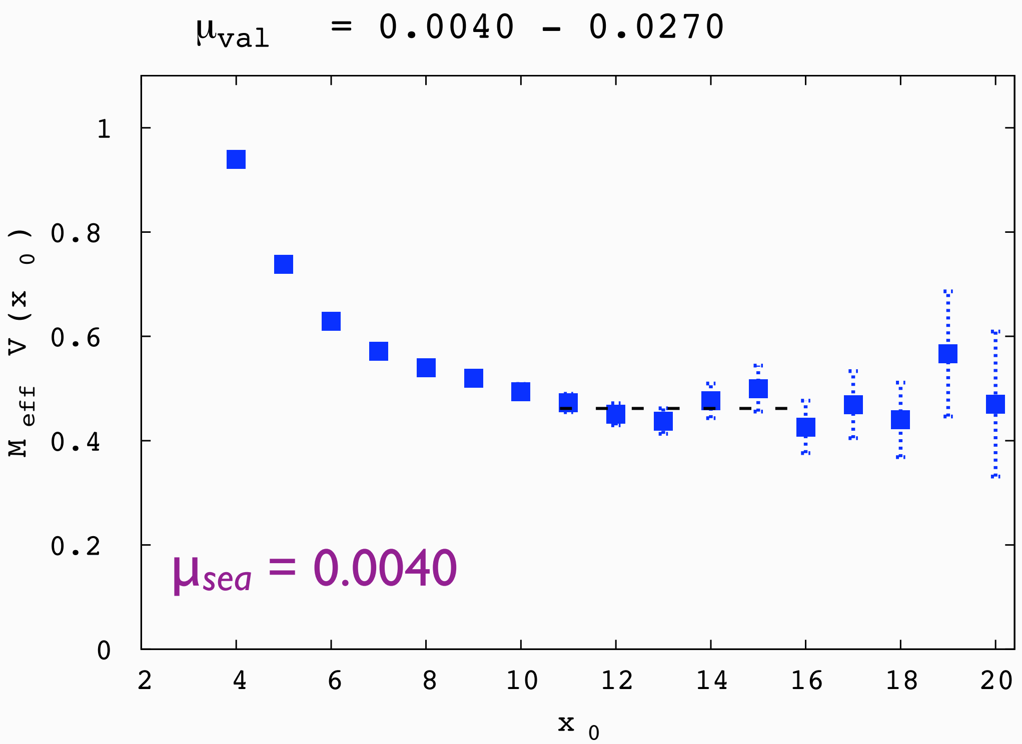
m_V^{eff}

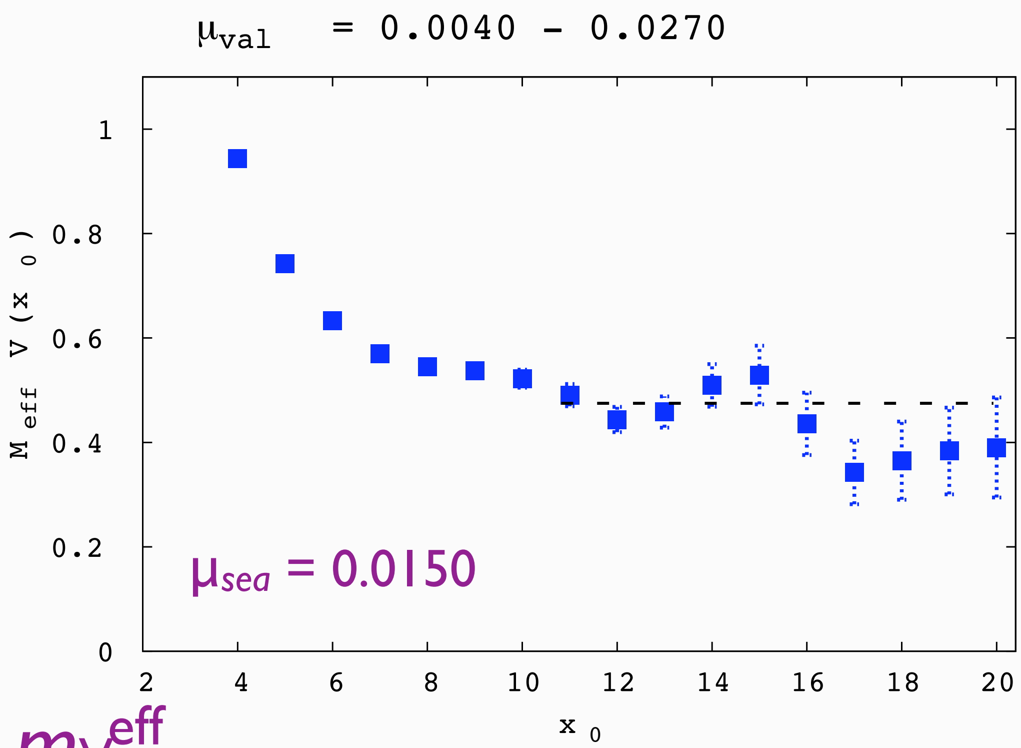
K* meson mass and decay constant

- **P. Dimopoulos**, S. Simula, A.V.

Caveats

- We encountered low quality signals in two cases:
  - II: For valence quarks lighter than the sea quarks ($\mu_{val} \leq \mu_{sea}$) (NB: unlike pseudoscalar case, where everything seems OK)

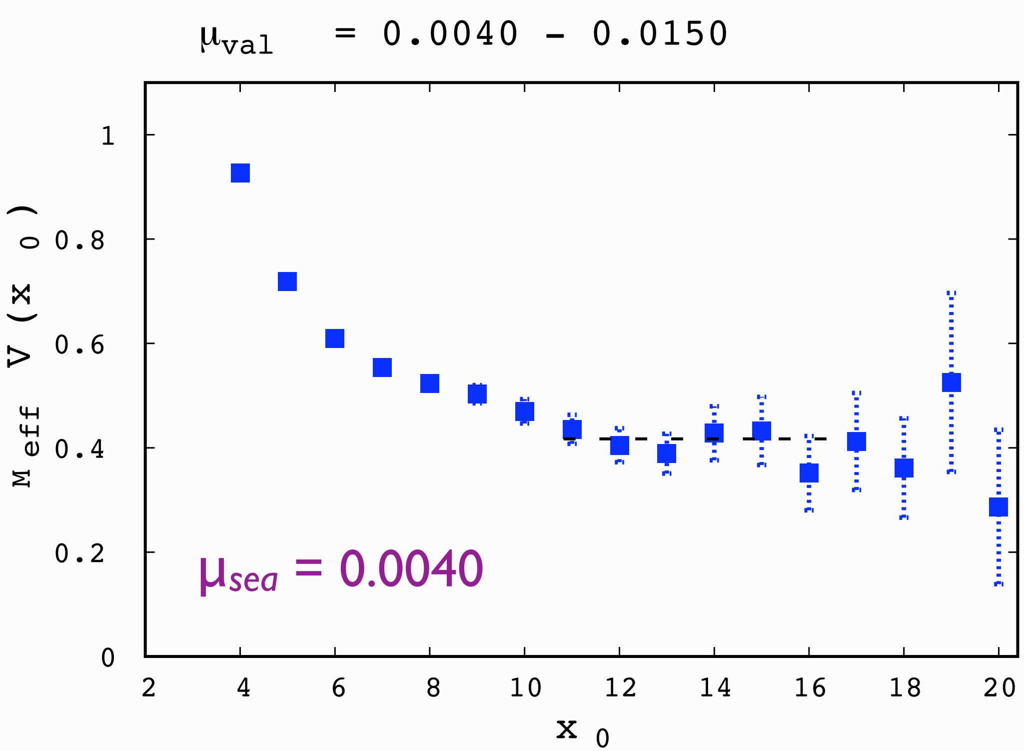


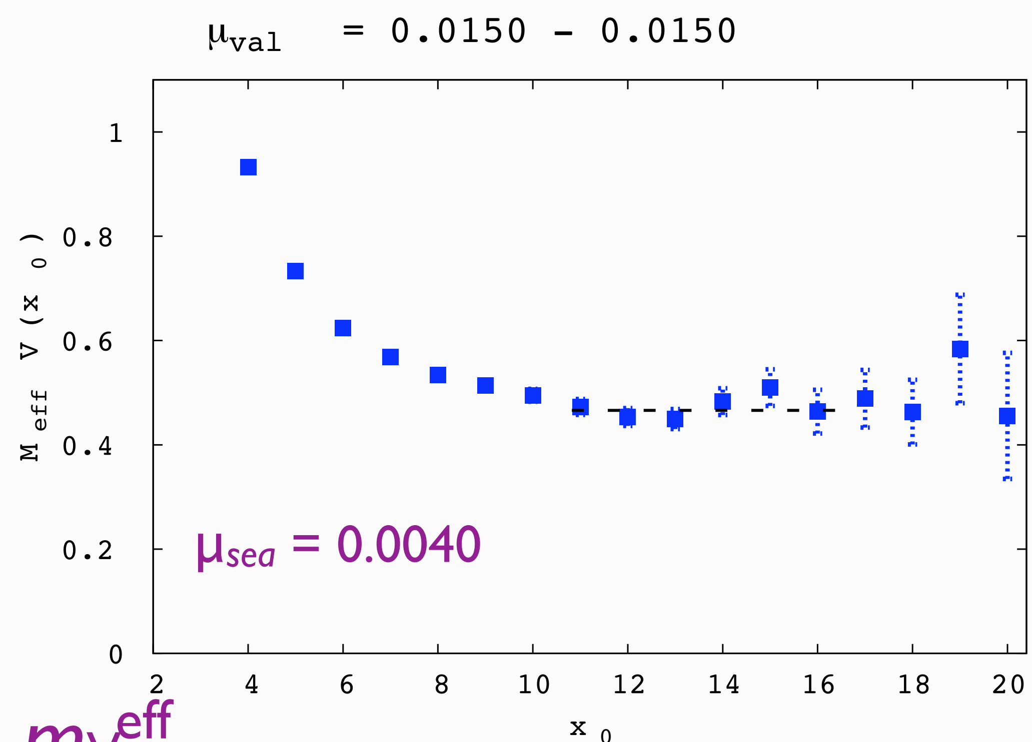


m_V^{eff}

K* meson mass and decay constant

- In all other cases the signal is satisfactory, so we analyze correlation functions consisting of:
  - one "light" valence quark ($\mu_l = \mu_{sea}$ = 0.0040, 0.0064, 0.0085 0.0100, 0.0150);
  - one "heavy" valence quark (μ_h = 0.0150, 0.0220. 0.0270).
  - Plateau: $11 \leq t \leq 16$





m_V^{eff}

K* meson mass and decay constant

- The vector meson mass and the observables of interest:

$$
\begin{aligned}
< 0 \,|\, V_k \,|\, V; \lambda > &= f_V \, m_V \, \epsilon_k^\lambda \\
< 0 \,|\, T_{0k} \,|\, V; \lambda > &= -i \, f_T \, m_V \, \epsilon_k^\lambda
\end{aligned}
$$

- are obtained from the correlation functions

$$
\begin{aligned}
C_{VV} &= \sum_{\vec{x},k} < V_k(x) \, V_k^\dagger(0) > \qquad k = 1,2,3 \\
C_{TT} &= \sum_{\vec{x},k} < T_{0k}(x) \, T_{0k}^\dagger(0) > \qquad k = 1,2,3
\end{aligned}
$$

- and the ratio $\dfrac{f_T}{f_V} \sim \left[\dfrac{C_{TT}(t)}{C_{VV}(t)} \right]^{1/2}$

- NB: valence quark propagators (also for B_K) are not computed from standard inversions of the Dirac operator (i.e. point-like sources), but from stochastic sources of the so-called extended one-end trick.

M. Foster, C. Michael, Phys.Rev.D59 (1999) 074503

C.McNeile, C.Michael, Phys.Rev.D73 (2006) 074506

K* meson mass and decay constant

- The vector meson mass and the observables of interest:

$$< 0\,|\,V_k\,|\,V;\lambda\ > \quad = \quad f_V\, m_V\, \epsilon_k^\lambda$$
$$< 0\,|\,T_{0k}\,|\,V;\lambda\ > \quad = \quad -i\, f_T\, m_V\, \epsilon_k^\lambda$$

- are obtained from the correlation functions

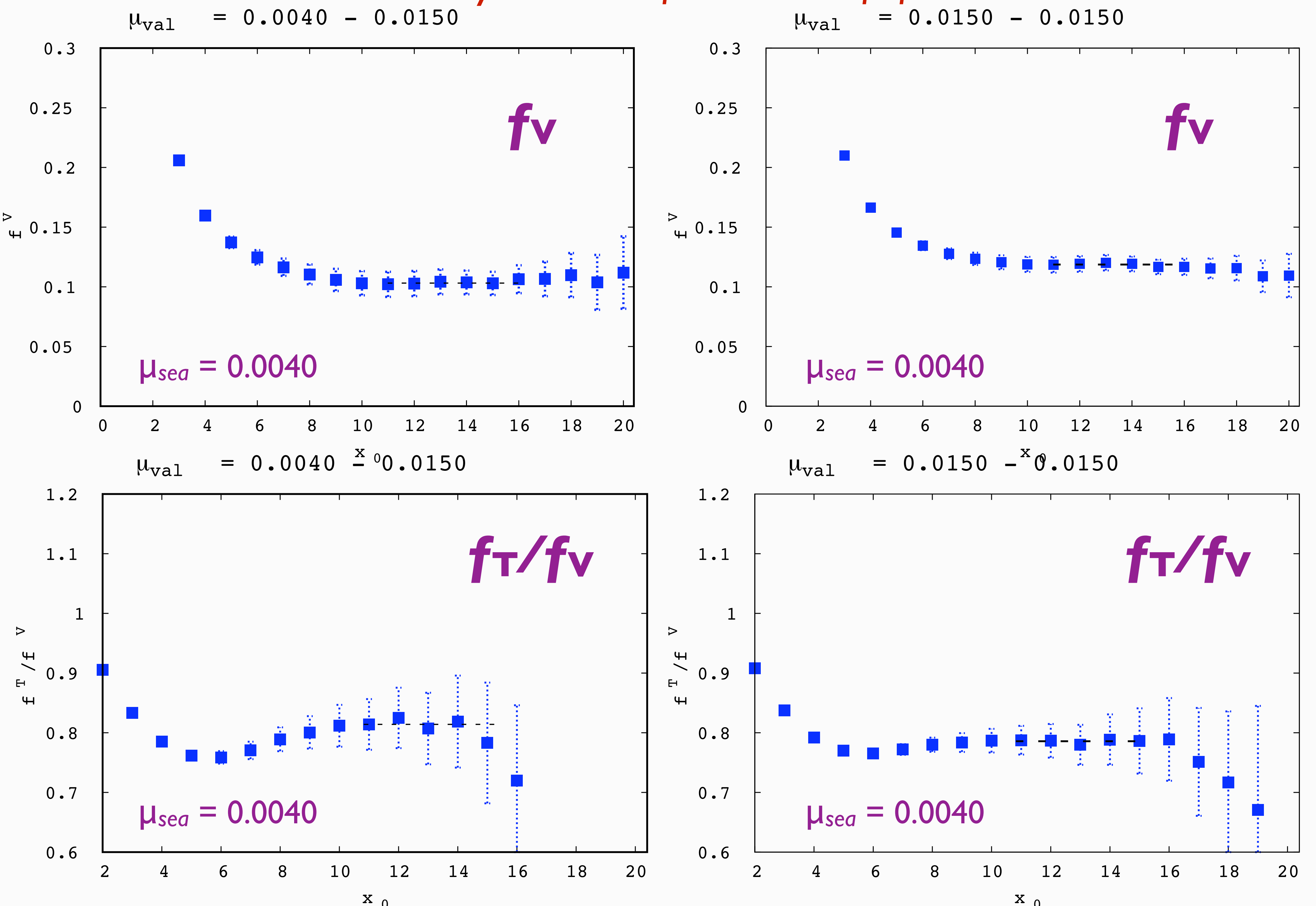
$$C_{VV} \quad = \quad \sum_{\vec{x},k} < V_k(x)\, V_k^\dagger(0) \ > \qquad k = 1,2,3$$
$$C_{TT} \quad = \quad \sum_{\vec{x},k} < T_{0k}(x)\, T_{0k}^\dagger(0) \ > \qquad k = 1,2,3$$

- and the ratio $\quad \dfrac{f_T}{f_V} \quad \sim \quad \left[\dfrac{C_{TT}(t)}{C_{VV}(t)} \right]^{1/2}$

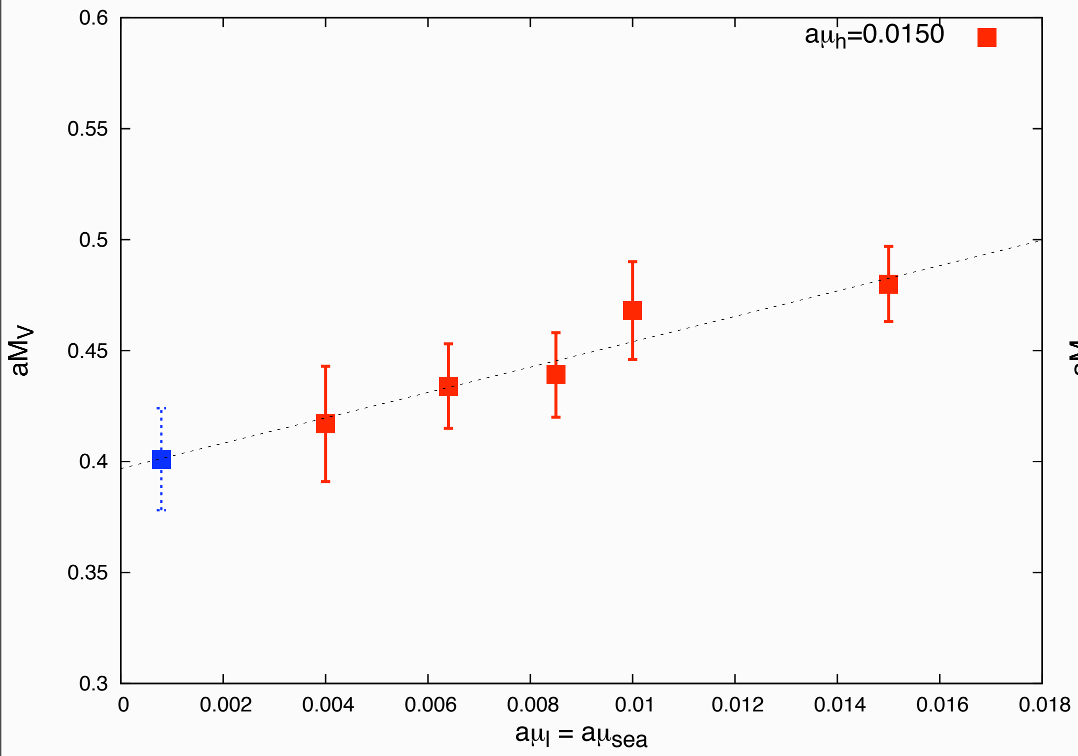
- NB: The required (re)normalization factors (Z_A, Z_T) are computed non-perturbatively in the RI/MOM scheme at a scale μ = 1/a ≈2.3 GeV
  - Z_A = 0.771 (4) $\qquad$ Z_T(1/a) = 0.769 (4)

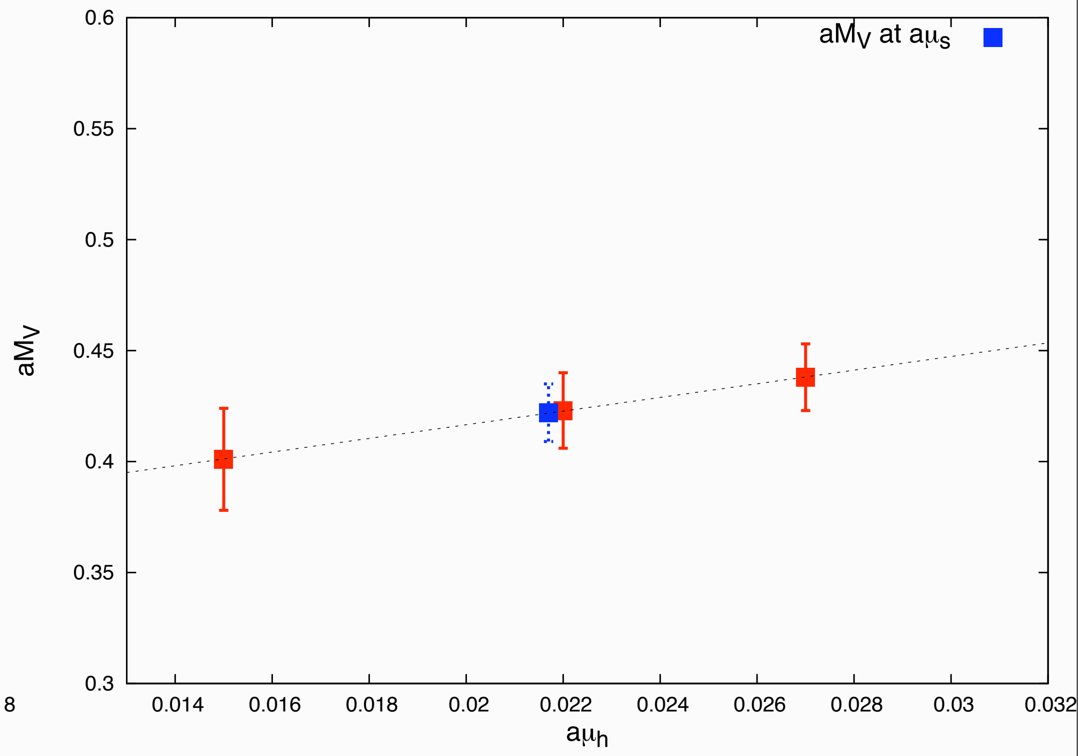
P. Dimopoulos et al., PoS LAT2007 (2007) 368

Decay constant f_V and ratio f_T/f_V



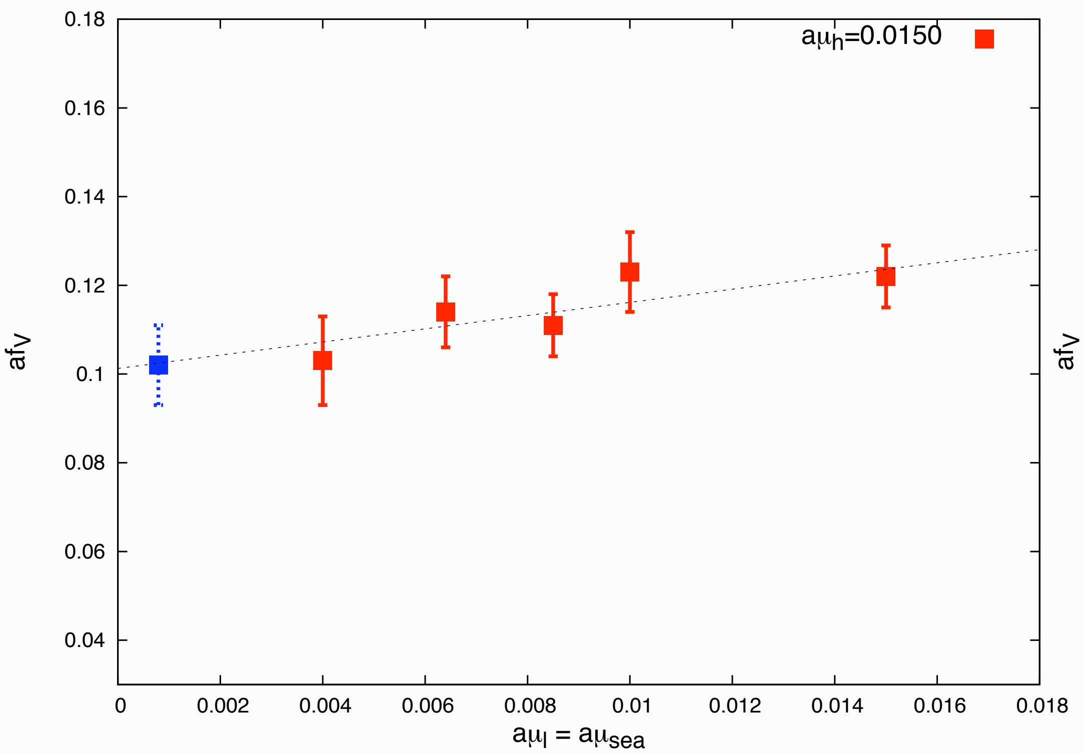
Mass extrapolations for m_V

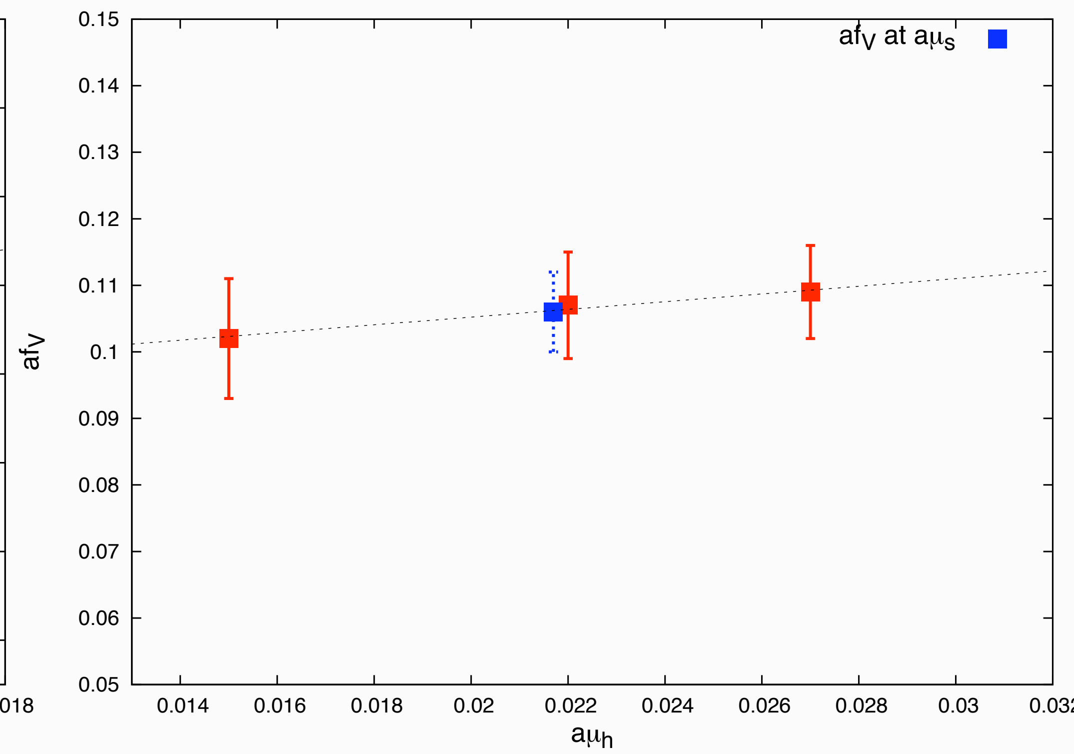




- At each fixed μ_h , we extrapolate linearly in $\mu_l \rightarrow \mu_d$
- We subsequently interpolate the μ_h results in $\mu_h \rightarrow \mu_s$

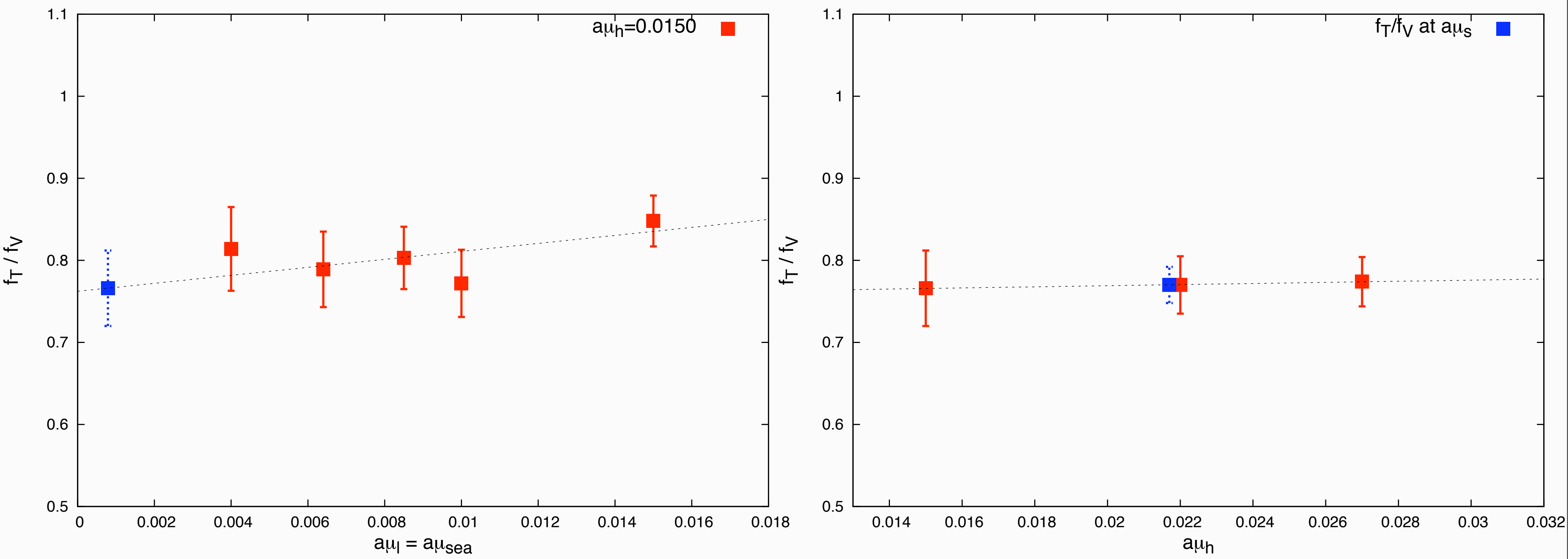
Mass extrapolations for f_V





- At each fixed μ_h , we extrapolate linearly in $\mu_l \to \mu_d$
- We subsequently interpolate the μ_h results in $\mu_h \to \mu_s$

Mass extrapolations for f_T/f_V

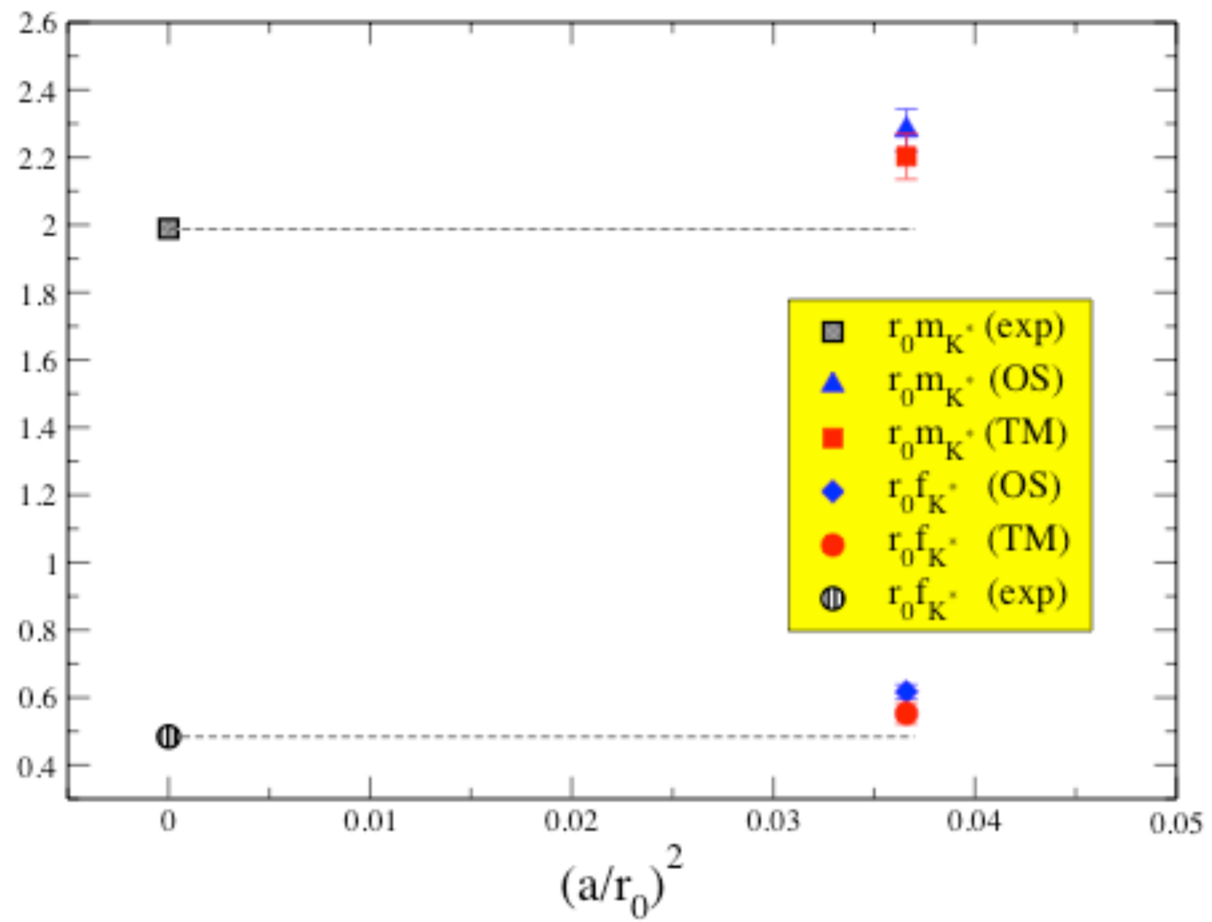


- At each fixed μ_h , we extrapolate linearly in $\mu_l \to \mu_d$
- We subsequently interpolate the μ_h results in $\mu_h \to \mu_s$

Resuts for m_V, f_V and f_T/f_V

$$aM_V^{K^*} = 0.437(08)(04)$$
$$af_V^{K^*} = 0.117(03)(01)$$
$$f_T/f_V|_{K^*} = 0.759(19)(03)$$

NB: analysis repeated with OS valence quarks



$r_0/a = 5.22$

Resuts for m_V, f_V and f_T/f_V

$$aM_V^{K^*} = 0.437(08)(04)$$
$$af_V^{K^*} = 0.117(03)(01)$$
$$f_T/f_V|_{K^*} = 0.759(19)(03)$$

Doing the RG running from 1/a = 2.3 GeV to 2 GeV we find:

$[\, f_T \,/\, f_V \,]_{K^*} = 0.764\ (19)(03)$

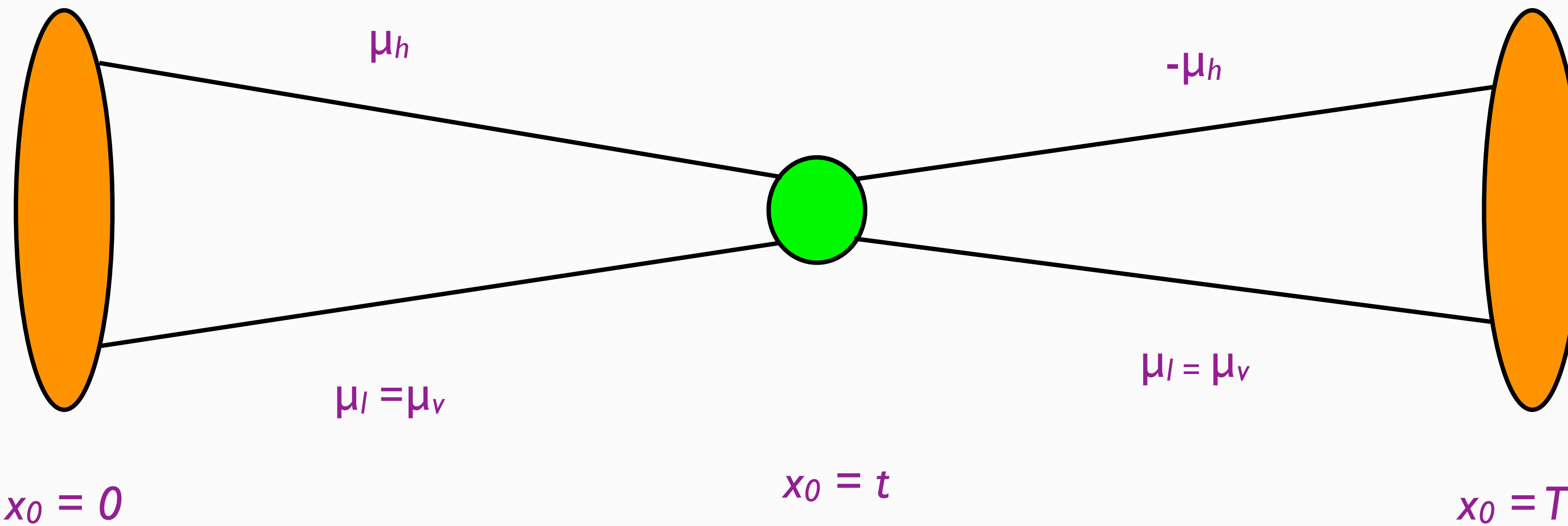
D. Becirevic, V. Lubicz, F. Mescia C. Tarantino, JHEP05 (2003) 007

NB: continuum quenched result $[\, f_T \,/\, f_V \,]_{K^*} = 0.74(2)$

B_K: a progress report

- Recall that we require both automatic improvement and multiplicative renormalization; thus the setup is that of OS valence quarks.

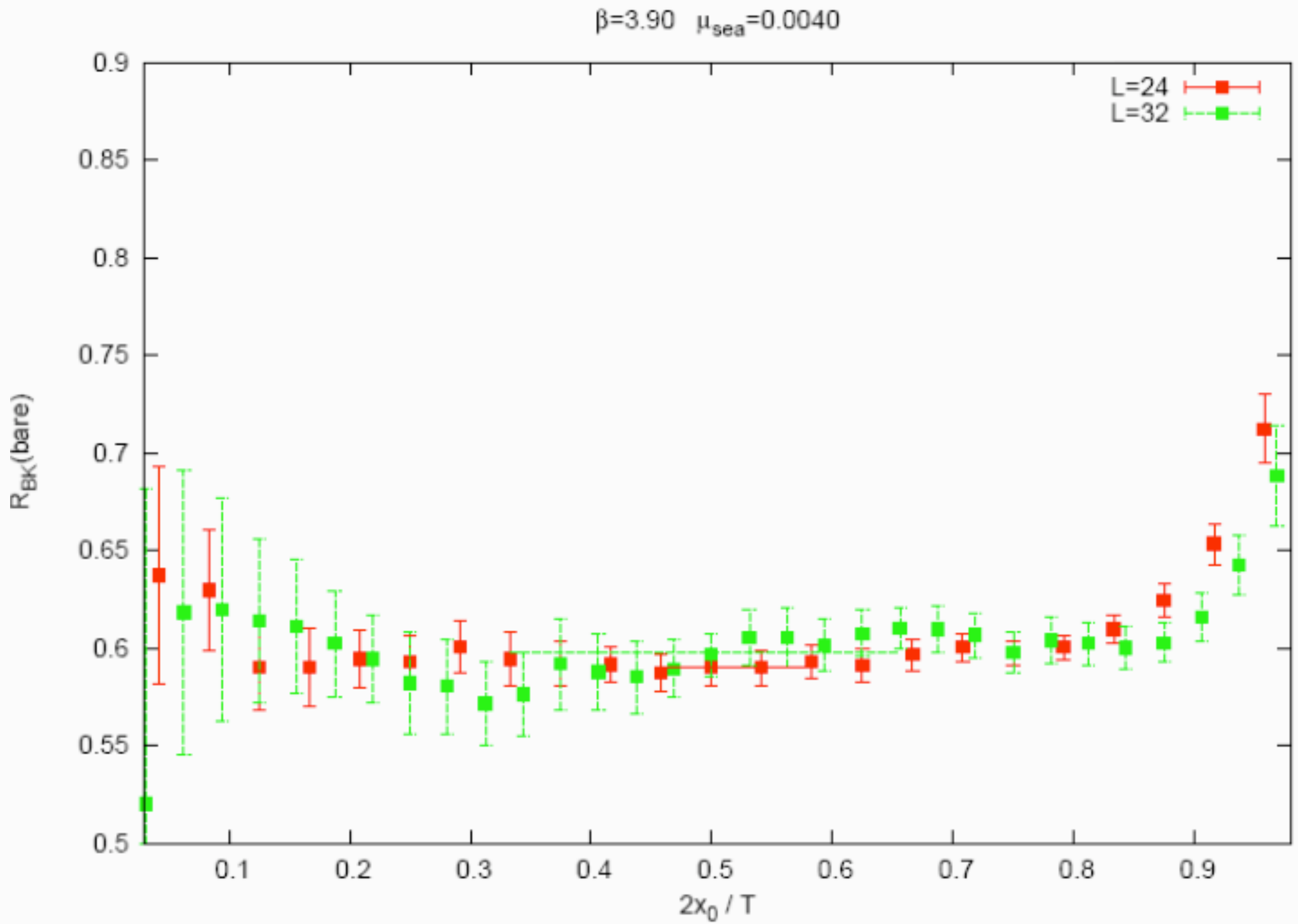
- We have two walls with noise sources at fixed times and a moving 4-fermion operator.



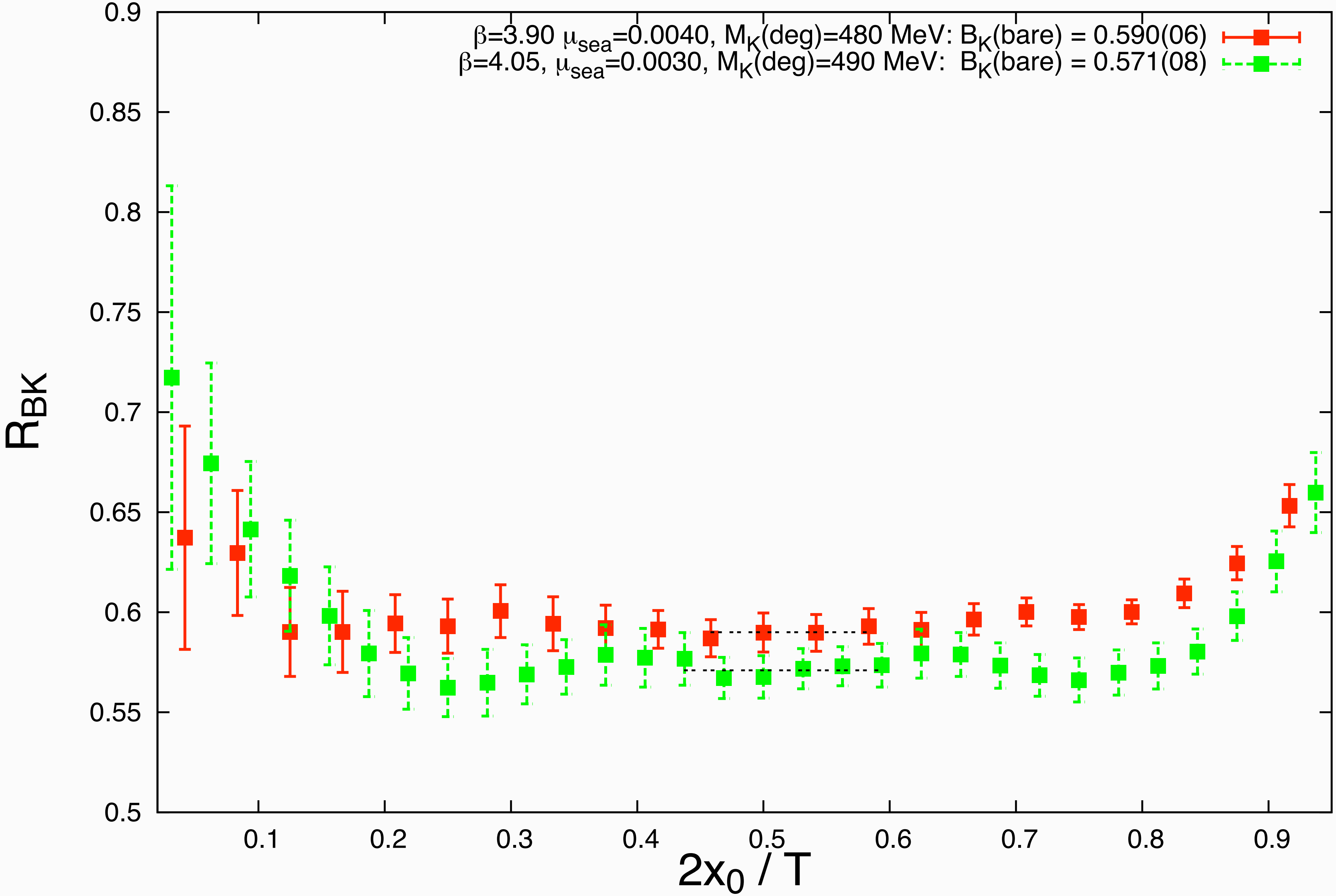
$$\bar{O}_{\mathrm{VA+AV}} = \lim_{a\to 0} Z_{\mathrm{VA+AV}}(g_0^2, a\mu)\, O_{\mathrm{VA+AV}}(a)$$

- **P. Dimopoulos**, R. Frezzotti, V. Gimenez, V. Lubicz, **F. Mescia**, G.C. Rossi, A.V.

B_K: finite volume effects



B_K: scaling effects (VERY ROUGH!!!)



B_K: chiral fits

- At fixed μ_h, we fit the light mass behaviour in $\mu_l = \mu_v$, using PQ-ChPT
- in general $\mu_v \neq \mu_{sea}$ gives rise to chiral logs

RBC/UKQCD C. Allton et al., 0804.0473 [hep-lat]

S.R.Sharpe and Y. Zhang, Phys. Rev. D53 (1996) 5125

$$B(\mu_h) \;=\; B_\chi(\mu_h)\left[1 + b_1(\mu_h)\,\frac{2B_0}{f^2}\,\mu_{sea} + b_2(\mu_h)\,\frac{2B_0}{f^2}\,\mu_v - \frac{2B_0}{32\pi^2 f^2}\mu_{sea}\,\ln\left(\frac{2B_0\mu_v}{\Lambda_\chi^2}\right)\right]$$

- This simplifies to a 2-parameter fit with a well defined chiral limit when $\mu_v = \mu_{sea}$.

B_K: chiral fits

- At fixed μ_h, we fit the light mass behaviour in $\mu_l = \mu_{val}$, using PQ-ChPT
- in general $\mu_{val} \neq \mu_{sea}$ gives rise to chiral logs

RBC/UKQCD C. Allton et al., 0804.0473 [hep-lat]

S.R.Sharpe and Y. Zhang, Phys. Rev. D53 (1996) 5125

$$B(\mu_h) = B_\chi(\mu_h)\left[1 + b_1(\mu_h)\frac{2B_0}{f^2}\mu_{sea} + b_2(\mu_h)\frac{2B_0}{f^2}\mu_v - \frac{2B_0}{32\pi^2 f^2}\mu_{sea}\ln\left(\frac{2B_0\mu_v}{\Lambda_\chi^2}\right)\right]$$

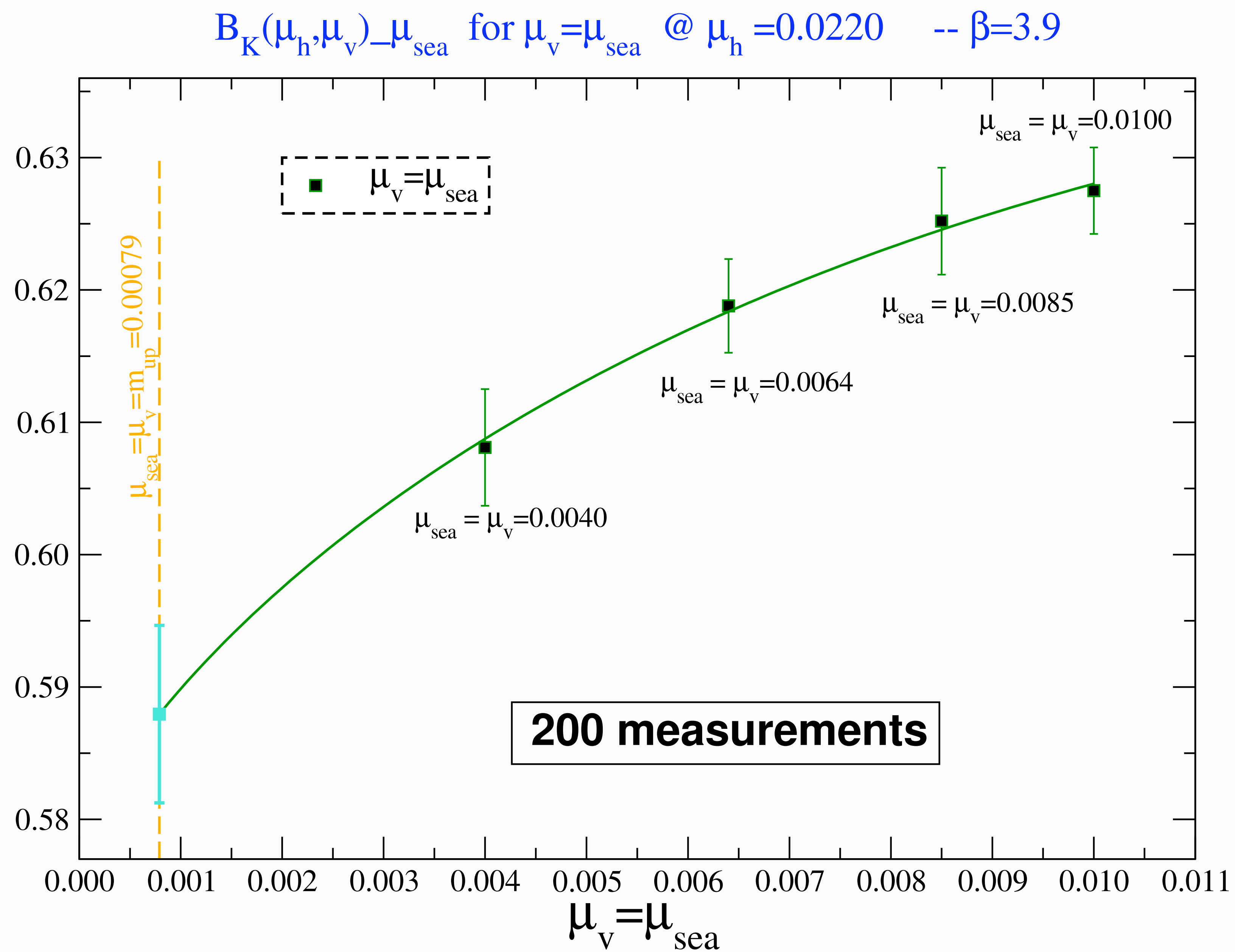
- This simplifies to a 2-parameter fit with a well defined chiral limit when $\mu_{val} = \mu_{sea}$.

Fixed by earlier ETMC chiral fit in the light sector

- Polynomial fitting alternatives are in the works!

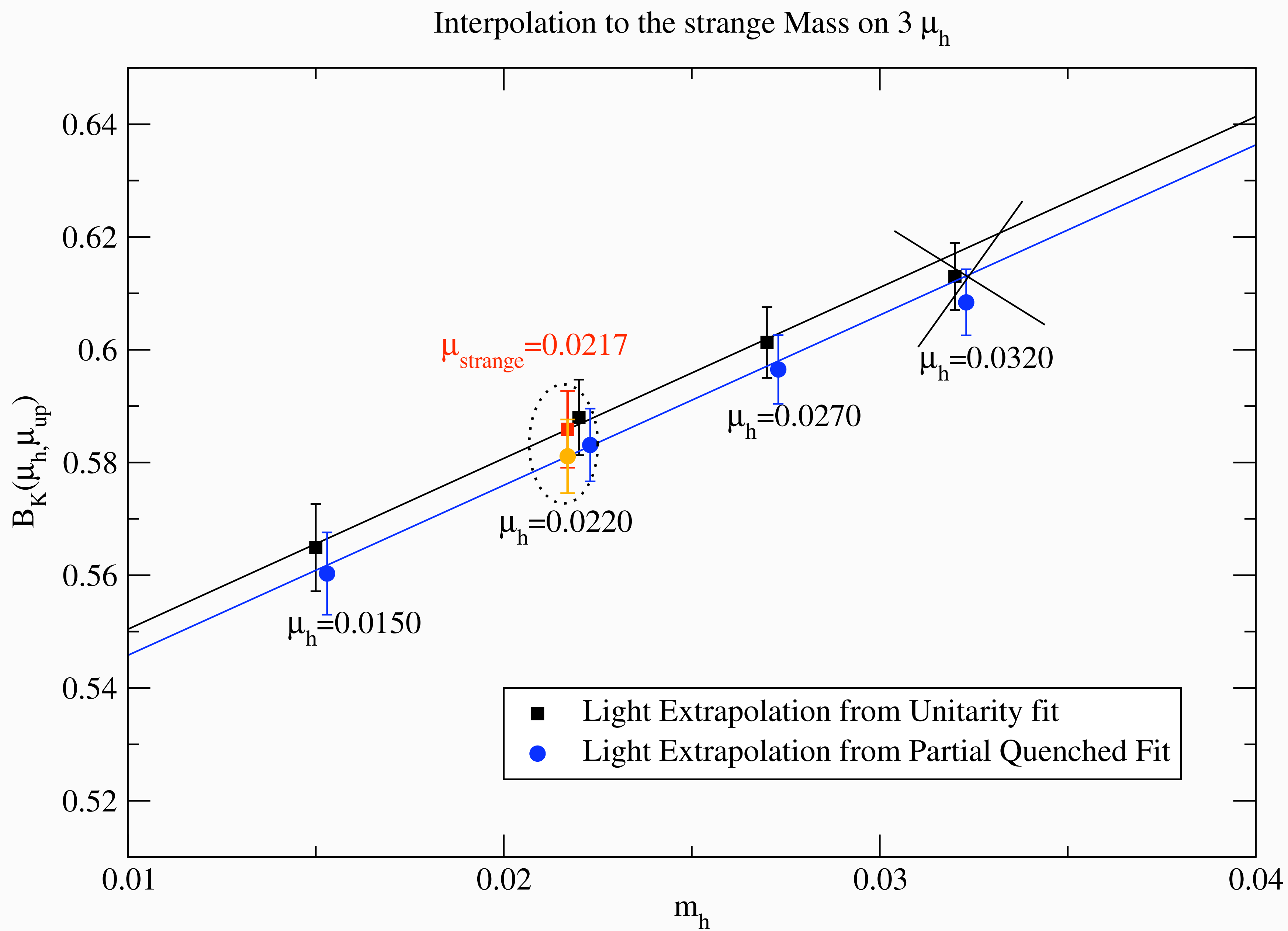
B_K: chiral fits

- At fixed μ_s, close to the physical strange mass we fit for $\mu_d = \mu_{sea}$



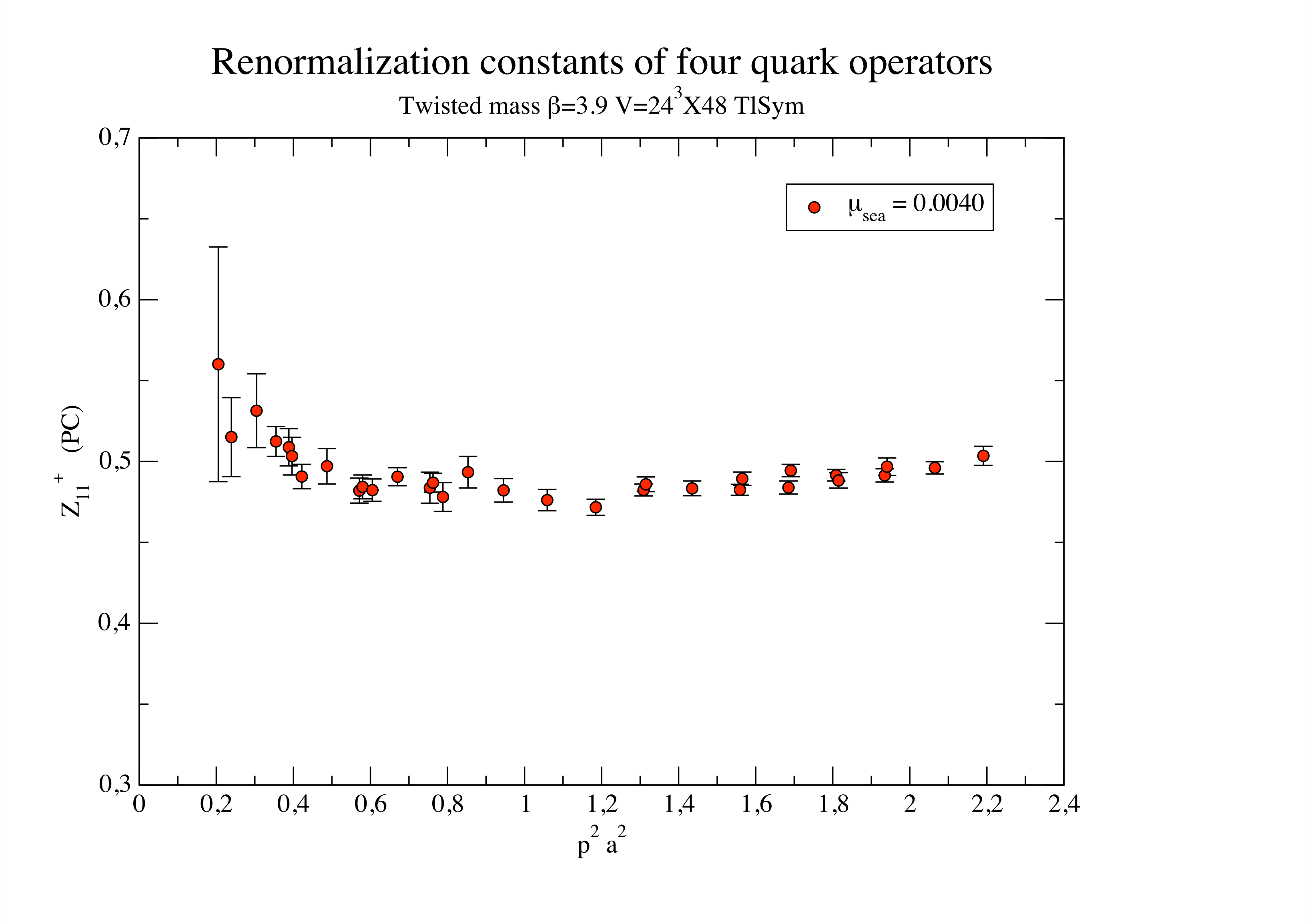
B_K: chiral fits

- Now interpolate the previous (physical μ_d result) in μ_s



B_K: renormalization

- RI/MOM scheme implemented



B_K: renormalization

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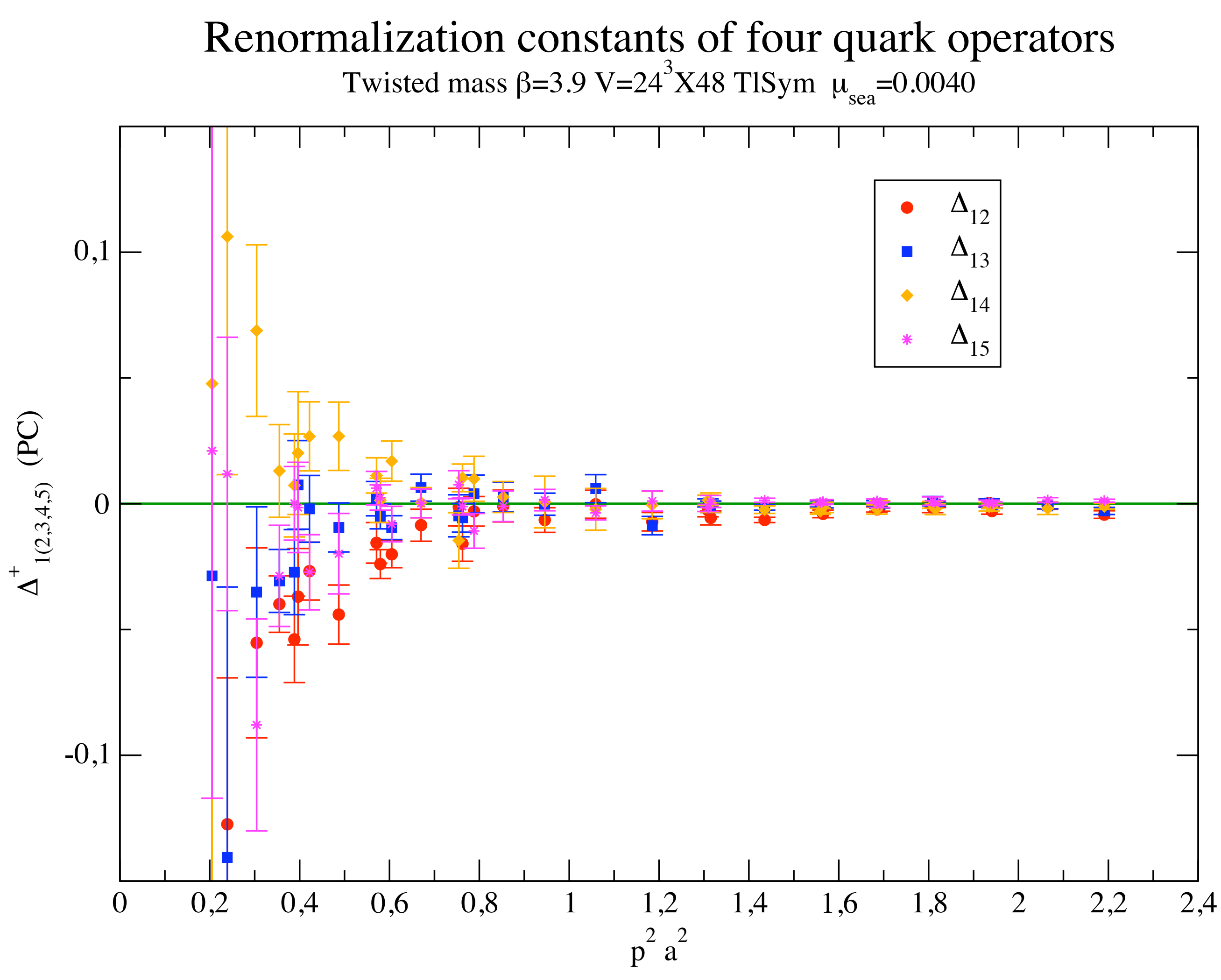
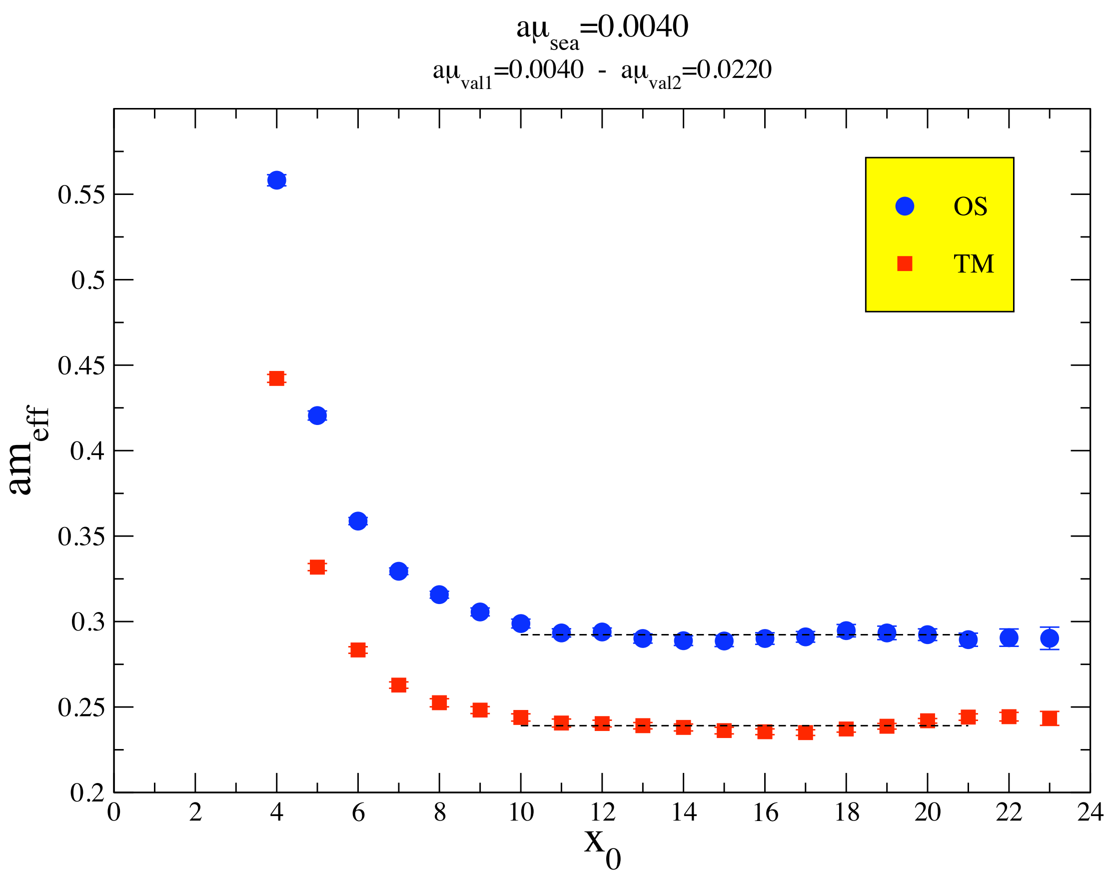


V. Gimenez, V. Lubicz

B_K: caveat

- At fixed β, the two Kaon states, obtained with different regularizations (i.e. standard tm and OS) are not degenerate, differing by $O(a^2)$ terms.
- The two different exponential decays cancel in the B_K ratio.
- We are left with a matrix element $< \bar{K}^0(m_K^{tm}) \,|\, O_{VA+AV} \,|\, K^0(m_K^{OS}) > \propto \; m_K^{tm}\, m_K^{OS}$



$am_{PS}^{OS} = 0.2923(16)$

$am_{PS}^{tm} = 0.2391(07)$

B_K: a our first **VERY ROUGH** estimate

Although a lot is still missing for giving a definitive result, we cannot resist from fooling around with our preliminary numbers:

$B_K(\beta=3.9) = 0.581(7)$ (bare)

$Z_{VA+AV}(\beta=3.9;\ 2\ \text{GeV};\ \text{RI/MOM}) = 0.454\ (18)$

$Z_V(\beta=3.9) = 0.771$

$Z_A(\beta=3.9) = 0.6104$

$B_K(2.0\ \text{GeV};\ \text{RI/MOM}) \approx 0.56(2)$ (renormalized)

$B_K(2.0\ \text{GeV};\ \text{RI/MOM}) \approx 0.77(3)$ (RGI)

NB: this is far from being our definitive, result!!!!!!

But how does it compare with the results of other groups?

B_K: a "ballpark plot"

- This is not a world data plot! It is a compilation of existing results, in order to confirm that our preliminary BK is in the right ballpark.